On the Interpretation for Radiation from Simple Current Distributions

Glenn S. Smith
School of Electrical and Computer Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0250
Tel: +1 (404) 894-2922
Fax: +1 (404) 894-4641
E-mail: glenn.smith@ece.gatech.edu

Keywords: Electromagnetic radiation, antenna theory, current

[Editor's note: Because of a software problem, the version of this article that appeared in the previous issue of the *Magazine* contained numerous instances in which a key symbol was omitted. The following is a corrected version, which should be used and referenced instead of the previous version. WRS]

1. Introduction

It is indisputable that Maxwell's equations provide a unique electromagnetic field for any reasonable distribution of charge/current. However, the interpretation of the results from such calculations may not be unique; different physical models may be used to explain the same results.

In this *Magazine* and at the recent [1997] IEEE Antennas and Propagation Symposium, Ed Miller has raised some interesting questions about the interpretation for the radiation from simple, filamentary current distributions [1, 2]. The length of the filament is h, and the current is time-harmonic with frequency ω (free-space wave number $k_0 = \omega/c = 2\pi/\lambda_0$). He shows that for an electrically long filament (limit $k_0 h \to \infty$), the total time-averaged power radiated by a *sinusoidal or standing-wave distribution* is $\langle \mathcal{P}_{rad} \rangle \propto \ln(k_0 h)$, whereas for a *uniform distribution* it is $\langle \mathcal{P}_{rad} \rangle \propto k_0 h$. For an electrically short filament (limit $k_0 h \to 0$), both distributions have $\langle \mathcal{P}_{rad} \rangle \propto (k_0 h)^2$. He asks interesting questions about the physical interpretation of these results, which we will summarize with the single question: Where does the radiation originate for the two distributions?

The use of harmonic time dependence complicates the answer to this question. The current at each point on the filament has been oscillating for an infinite time; this complicates the causal relationship between elements of current on the filament and the radiated field at a particular point. In this note, we will examine the radiation from similar filamentary distributions (traveling-wave[1] and uniform) for pulse excitation. For pulse excitation, it is easier to establish the causal relationship between the elements of current and the radiated field. We will also show, for these simple distributions, that the total energy radiated, U_{rad}, for pulse excitation depends on the length of the filament, h, in the same way as does $\langle \mathcal{P}_{rad} \rangle$ for the time-harmonic case.

[1]A detailed calculation shows that $\langle \mathcal{P}_{rad} \rangle$ for a filament with a traveling-wave distribution has the same asymptotic behavior as for the sinusoidal or standing-wave distribution.

It is important to understand the problem we are addressing: the calculation from Maxwell's equations of the radiated field of a well-defined current distribution. We are not solving the boundary-value problem associated with the linear antenna: the determination of the electromagnetic field of a perfectly-conducting, thin wire excited by a specified source. This is so even though, in some cases, the current distributions we are using are reasonable approximations to the current on the linear antenna.

2. Pulse-excited filaments

The filament is aligned with the z axis, as in Figure 1. For the *traveling-wave distribution*, the current and radiated electric field are [3]

$$\mathcal{I}(z,t) = \mathcal{I}_s(t - z/c)\left[U(z) - U(z-h)\right], \qquad (1)$$

and

$$\vec{\mathcal{E}}^r(\vec{r},t) = \frac{\mu_0 c \sin\theta}{4\pi r(1-\cos\theta)}\left\{\mathcal{I}_s(t - r/c) - \mathcal{I}_s\left[t - r/c - (h/c)(1-\cos\theta)\right]\right\}\hat{\theta}. \qquad (2)$$

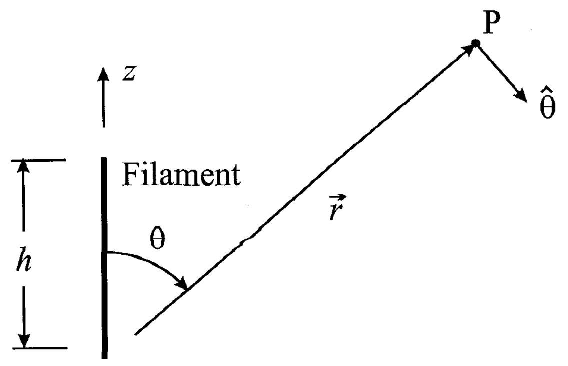


Figure 1. The geometry for a filament of current.

Here, the function $\mathcal{I}_s(t)$ is the pulse excitation, and U is the Heaviside unit step function. For the *uniform distribution*, the current and radiated electric field are

$$\mathcal{I}(z,t)=\mathcal{I}_s(t)\left[U(z)-U(z-h)\right], \tag{3}$$

and

$$\bar{\mathcal{E}}^r(\bar{r},t)=\frac{\mu_0 c\tan\theta}{4\pi r}\left\{\mathcal{I}_s\left[t-r/c+(h/c)\cos\theta\right]-\mathcal{I}_s(t-r/c)\right\}\hat{\theta}. \tag{4}$$

The total energy radiated by the filament is

$$U_{rad}=\frac{2\pi}{\zeta_0}\int_{t=-\infty}^{\infty}\int_{\theta=0}^{\pi}\left|\bar{\mathcal{E}}^r(t,r,\theta)\right|^2 r^2\sin\theta\, d\theta\, dt, \tag{5}$$

where ζ_0 is the wave impedance for free space.

Now we will assume that the excitation is the Gaussian pulse shown in Figure 2a, with the characteristic time τ:

$$\mathcal{I}_s(t)=I_0 e^{-(t/\tau)^2}. \tag{6}$$

After using Equation (6) with Equations (2), (4), and (5), and performing some tedious integrations, the total energy radiated by the traveling-wave distribution becomes

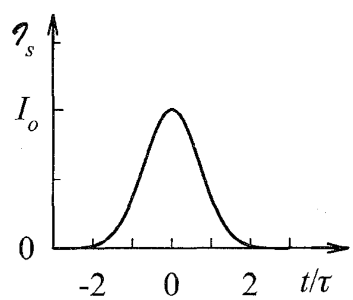


Figure 2a. A Gaussian pulse of current.

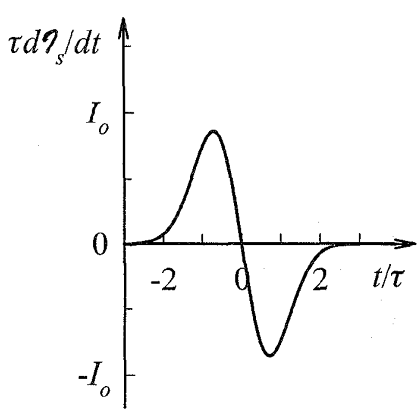


Figure 2b. The derivative of the Gaussian pulse of current.

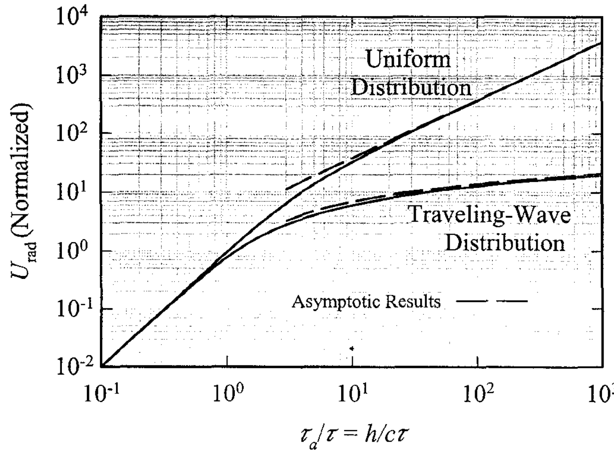


Figure 3. The total energy radiated by a filament excited by a Gaussian pulse of characteristic time τ, $\tau_a = h/c$.

$$U_{rad}=\frac{\zeta_0\tau I_0^2}{4\sqrt{2\pi}}\Big[\gamma-2+\ln\left(2\tau_a^2/\tau^2\right)+\sqrt{\frac{\pi}{2}}\left(\frac{\tau}{\tau_a}\right)\mathrm{erf}\left(\sqrt{2}\,\tau_a/\tau\right)+\mathrm{E}_1\left(2\tau_a^2/\tau^2\right)\Big], \tag{7}$$

and for the uniform distribution,

$$U_{rad}=\frac{\zeta_0\tau I_0^2}{2\sqrt{2\pi}}\Big[\exp\left(-\tau_a^2/2\tau^2\right)-2+\sqrt{\frac{\pi}{2}}\left(\frac{\tau}{\tau_a}+\frac{\tau_a}{\tau}\right)\mathrm{erf}\left(\tau_a/\sqrt{2}\tau\right)\Big]. \tag{8}$$

Here, $\tau_a = h/c$ is the time for light to travel the length of the filament, $\gamma = 0.57721...$ is Euler's constant, erf is the error function, and E_1 is an exponential integral [4].

In Figure 3, the total energy radiated is plotted in normalized form,

$$U_{rad}(\text{Normalized})=U_{rad}\Big/\left(\frac{\zeta_0\tau I_0^2}{6\sqrt{2\pi}}\right), \tag{9}$$

versus the parameter $\tau_a/\tau = h/c\tau$, which is the characteristic time for a filament of length h, divided by the characteristic time for the pulse. For both distributions, traveling-wave and uniform, the energy is seen to increase monotonically with increasing τ_a/τ. The dashed lines show the asymptotic behavior in the limit $\tau_a/\tau\to\infty$:

$$U_{rad}(\text{Normalized})\sim 3\ln(\tau_a/\tau),\quad \text{traveling-wave}, \tag{10}$$

$$U_{rad}(\text{Normalized})\sim 3\sqrt{\frac{\pi}{2}}(\tau_a/\tau),\ \text{uniform}. \tag{11}$$

In the limit $\tau/\tau_a\to 0$, $U_{rad}(\text{Normalized})\sim(\tau_a/\tau)^2$ for both distributions. These behaviors are seen to be the same as those for the time-average power radiated, $\langle\mathcal{P}_{rad}\rangle$, with time-harmonic excitation, once we recognize that τ plays the same role as $1/\omega$, so that $\tau_a/\tau = h/c\tau$ is like $k_0 h = \omega h/c$.

3. Radiation from the ends of the filament

Figures 4 and 5 show the radiated electric fields, Equations (2) and (4), for the two distributions. The excitation is the Gaussian pulse, Equation (6), with the characteristic time $\tau/\tau_a = 0.076$. These figures require some explanation. Consider a large spherical surface centered on the lower end of the filament. Observers are located at the angles $\theta = 0°, 22.5°, 45°, 67.5°,...$ around the sphere, and they record the radiated electric field, $\vec{\mathcal{E}}_\theta^r$, as a function of the normalized time, t/τ_a. The individual plots in these figures are graphs of their results.

Each dashed line in these figures connects times of arrival associated with a spherical wavefront centered at an end of the filament: W_1 is for a wavefront centered on the lower end of the filament, while W_2 is for a wavefront centered on the upper end of the filament. Notice that the amplitude of the field on a wavefront and the separation in time of the two wavefronts change with the angle of observation, θ.

For the traveling-wave distribution, the two wavefronts increase in amplitude and begin to overlap as θ approaches 0°. The region of overlap is determined by the ratio τ_a/τ; when $\tau_a/\tau >> 1$, overlap starts at $\theta \approx 2\sqrt{2\tau/\tau_a}$ and extends over the solid angle $\Omega \approx 8\pi(\tau/\tau_a)$. An examination of Equation (2) when θ is near 0° shows that the electric field is proportional to the temporal derivative of the exciting current:

$$\vec{\mathcal{E}}^r(\vec{r},t) \sim \frac{\mu_0 h}{4\pi r}\frac{d\mathcal{I}_s(t-r/c)}{dt}\theta\hat{\theta}. \quad (12)$$

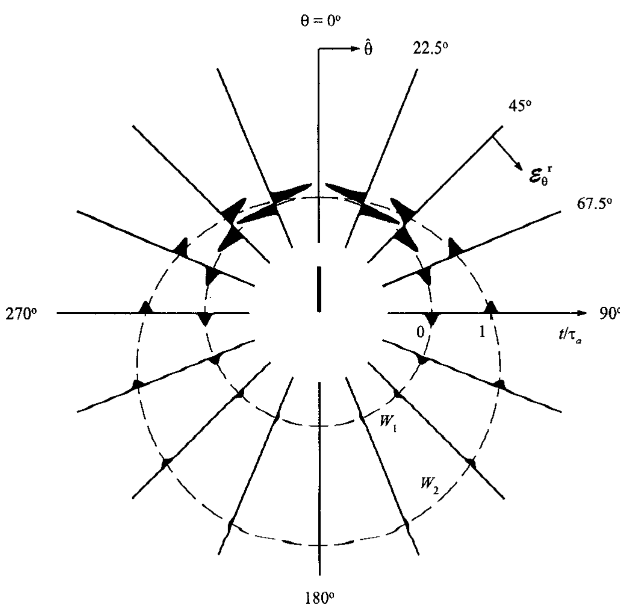


Figure 4. The radiated electric field of a filament with a traveling-wave distribution of current. The excitation is a Gaussian pulse with $\tau/\tau_a = 0.076$.

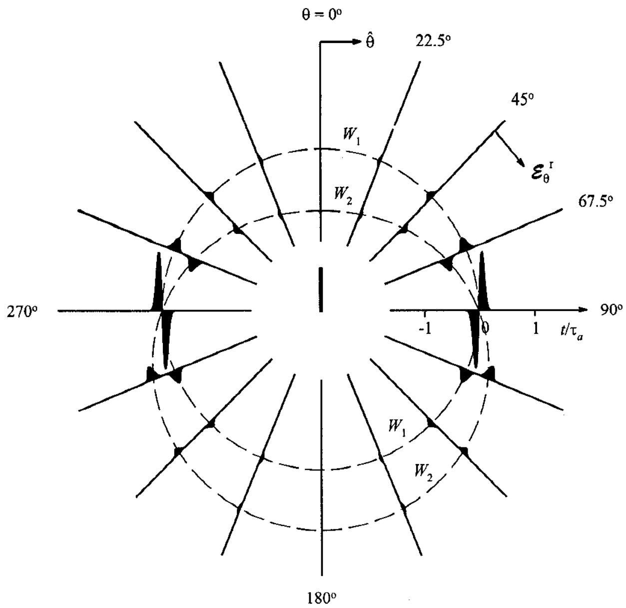


Figure 5. The radiated electric field of a filament with a uniform distribution of current. The excitation is a Gaussian pulse with $\tau/\tau_a = 0.076$.

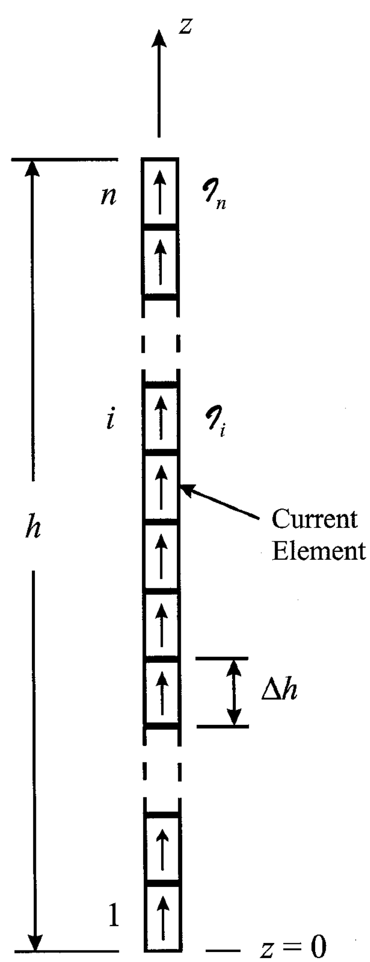


Figure 6. The current filament divided into n elements.

This behavior can be seen in Figure 4. At $\theta = 0°$, the field is zero, due to the factor of θ in Equation (12). However, at $\theta = 22.5°$, the field clearly resembles the derivative of the Gaussian pulse, which is shown in Figure 2b.

For the uniform distribution, the two wavefronts increase in amplitude and begin to overlap as θ approaches 90°. When $\tau_a/\tau >> 1$, overlap starts at $\pi/2 - \theta \approx 4\tau/\tau_a$, and extends over the solid angle $\Omega \approx 16\pi(\tau/\tau_a)$. An examination of Equation (4) when θ is near 90° shows that the electric field is proportional to the temporal derivative of the exciting current:

$$\vec{\mathcal{E}}^r(\vec{r},t) \sim \frac{\mu_0 h}{4\pi r}\frac{d\mathcal{I}_s(t-r/c)}{dt}\hat{\theta}\ . \tag{13}$$

This behavior can be clearly seen in Figure 5, where the field at $\theta = 90°$ is the derivative of the Gaussian pulse.

The dependence of the total energy radiated, U_{rad}, on the length of the filament (τ_a/τ), shown in Figure 3, is caused by the behavior of the field in the region where the wavefronts overlap. If there were no overlap, U_{rad} would simply be the sum of the energies associated with the two wavefronts, and it would be independent of the length of the filament. For the traveling-wave distribution, overlap occurs near where the field is zero ($\theta = 0°$), whereas for the uniform distribution, overlap occurs near where the field is maximum ($\theta = 90°$). This difference causes the energy radiated by the uniform distribution to be greater than that radiated by the traveling-wave distribution.

The radiation from the pulse-excited filament is analogous to the radiation from a moving point charge [3]. For example, a charge accelerated at $z = 0$, allowed to drift at constant velocity over the length h, and decelerated at $z = h$ would produce pulses of radiation only at the end points, a situation similar to that for the filament with the traveling-wave distribution.

4. Radiation from the entire filament

The radiation from these filaments can be described in an alternate way. Consider the schematic drawing in Figure 6. The filament is divided into a large number, n, of elements, each of length $\Delta h = h/n$, and the current, $\mathcal{I}_i$, in the ith element is assumed to be uniform over the length of the element. The radiation from the filament can be viewed as the superposition of the radiation from the n elements. The radiated electric field for the traveling-wave distribution is then

$$\vec{\mathcal{E}}^r(\vec{r},t) = \frac{\mu_0 c\sin\theta}{4\pi r}\left\{\sum_{i=1}^{n}\left(\frac{\Delta h}{c}\right)\frac{d\mathcal{I}_s\left[t - r/c - (i-1/2)(\Delta h/c)(1-\cos\theta)\right]}{dt}\right\}\hat{\theta}, \tag{14}$$

and for the uniform distribution,

$$\vec{\mathcal{E}}^r(\vec{r},t) = \frac{\mu_0 c\sin\theta}{4\pi r}\left\{\sum_{i=1}^{n}\left(\frac{\Delta h}{c}\right)\frac{d\mathcal{I}_s\left[t - r/c + (i-1/2)(\Delta h/c)\cos\theta\right]}{dt}\right\}\hat{\theta}. \tag{15}$$

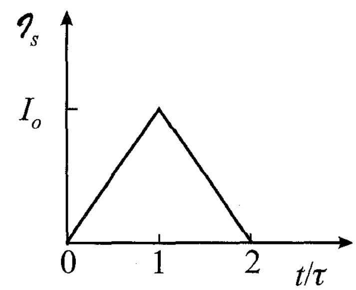


Figure 7a. A triangular pulse of current.

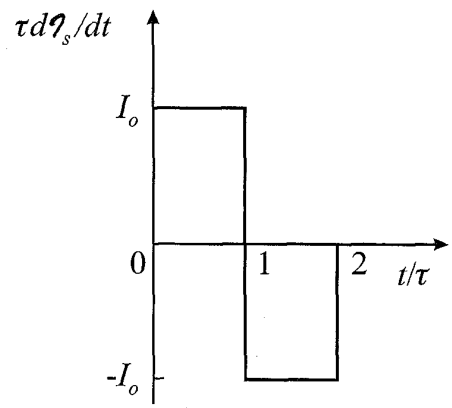


Figure 7b. The derivative of the triangular pulse of current.

Notice that the radiation is due to the time derivative of the current in each of the elements. In these expressions, the factor $\sin\theta$ determines the directional characteristics of an individual current element. It is the "element factor" used with conventional, time-harmonic arrays. The sum within the braces determines the directional characteristic for the group of elements, and it is analogous to the "array factor" used with conventional, time-harmonic arrays.

For the purpose of illustration, we will assume that the excitation is a triangular pulse with the characteristic time τ:

$$\begin{aligned}\mathcal{I}_s(t) &= I_0\left(1 - \left|t/\tau - 1\right|\right), \quad \left|t/\tau - 1\right| < 1\\ &= 0 \qquad\qquad\qquad\quad, \quad \left|t/\tau - 1\right| \geq 1.\end{aligned} \tag{16}$$

This function and its temporal derivative are shown in Figure 7. We use this pulse instead of the Gaussian pulse, Equation (6), because the discontinuities in the derivative of this pulse clearly delineate the contributions of the individual current elements to the sums in Equations (14) and (15).

Figure 8 shows the sums (array factors) from Equation (14) and (15) for the angles $\theta = 0°$ and 90° when $n = 32$, and $\tau/\tau_a = 0.125$. Notice that the results are similar for the two distributions. The contributions from the elements add destructively (the sum is nearly zero) everywhere except at the times corresponding to radiation from the ends of the filament (at $\tau/\tau_a = 0.0$ and 1.0 for the traveling-wave distribution, and at $\tau/\tau_a = -1.0$ and 0.0 for the uniform distribution). The process that causes this cancellation is shown schematically in Figure 9 for the traveling-wave distribution at $\theta = 90°$. The radiation from each element is the deriva-

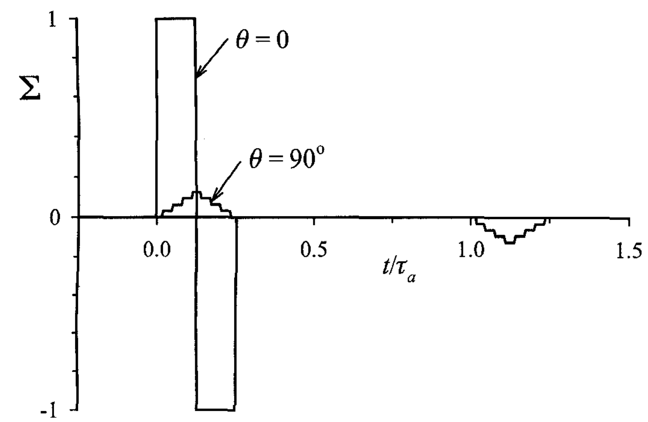


Figure 8a. The sum (array factor) from the expression for the radiated electric field, for the traveling-wave distribution of current. The excitation is a triangular function with $\tau/\tau_a = 0.125$.

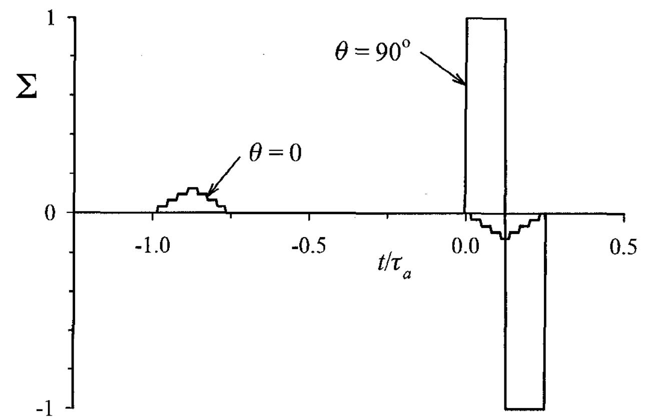


Figure 8b. The sum (array factor) from the expression for the radiated electric field, for the uniform distribution of current. The excitation is a triangular function with $\tau/\tau_a = 0.125$.

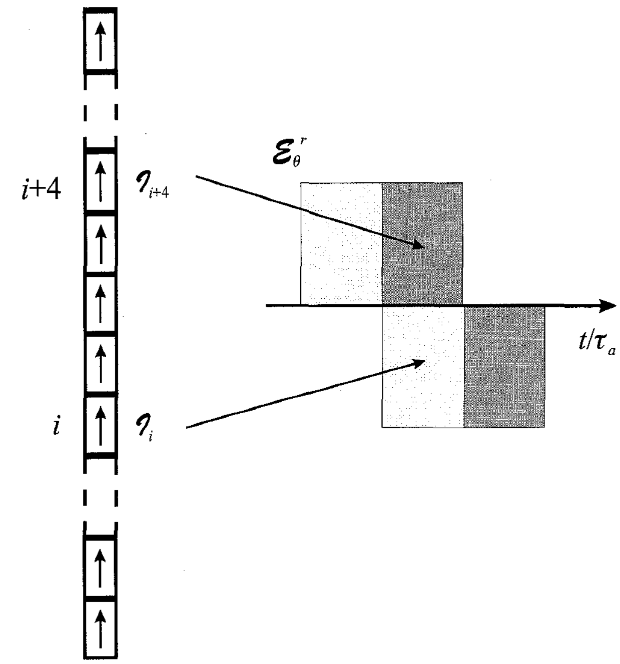


Figure 9. An illustration showing the partial cancellation of radiation from two current elements. The result is for a traveling-wave distribution of current at $\theta = 90°$, and $n = 32$.

tive of the triangular pulse shown in Figure 7b: a positive rectangular pulse followed by a negative rectangular pulse. The positive pulse from one element overlaps the negative pulse from another to produce the cancellation. For this example, the positive pulse from element $i + c\tau/\Delta h = i + n\tau/\tau_a = i + 4$ overlaps the negative pulse from element *i*. This cancellation occurs everywhere except at the times corresponding to radiation from the ends of the filament, near $t/\tau_a = 0.0$ and 1.0 in Figure 8a. At these points, radiation from the elements adds to produce a staircase approximation to the triangular pulse.

For the traveling-wave distribution, the sum (array factor) is largest at $\theta = 0°$, precisely where the element factor ($\sin\theta$) is zero. For the uniform distribution, the sum (array factor) is largest at $\theta = 90°$, precisely where the element factor is maximum. This difference accounts for the greater energy radiated from the uniform distribution.

5. Conclusions

We have considered the radiation from two simple filamentary current distributions: traveling-wave and uniform. For an excitation that is a Gaussian pulse of characteristic time τ, the total energy radiated by the distributions, U_{rad}, was shown to behave as $\ln(\tau_a/\tau)$ for the traveling-wave distribution, and as τ_a/τ for the uniform distribution, where $\tau_a = h/c$ is the time for light to travel the length of the filament.

An examination of numerical results shows that two physical interpretations can be used for the radiation. Radiation can be considered to arise at the two ends of the filament in the form of spherical wavefronts centered at the ends. The overlap of these wavefronts, which changes with the ratio τ_a/τ, is the cause of the observed dependencies for U_{rad}. An alternate explanation is that radiation occurs along the entire length of the filament. Destructive interference of the radiation from different points on the filament then causes the radiation to be insignificant except at the times corresponding to radiation from the ends of the filament.

6. Acknowledgment

This research was supported in part by the Joint Services Electronics Program under contract DAAH-04-96-1-0161.

7. References

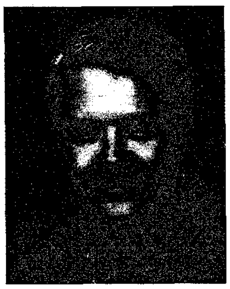
1. E. K. Miller, "PC's for AP and Other EM Reflections," *IEEE Antennas and Propagation Magazine*, **38**, June 1996, pp. 90-95.

2. E. K. Miller, "An Exploration of Radiation Physics in Electromagnetics," IEEE International Symposium on Antennas and Propagation *Digest*, Volume 3, Montreal Canada, July 1997, pp. 2048-2051.

3. G. S. Smith, *An Introduction to Classical Electromagnetic Radiation*, Cambridge University Press, New York, 1997.

4. M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, US Government Printing Office, Washington, DC, 1964.

Introducing Feature Article Author



Glenn S. Smith received the BSEE degree from Tufts University, Medford, MA, in 1967, and the SM and PhD degrees in applied physics from Harvard University, Cambridge, MA, in 1968 and 1972, respectively.

From 1972 to 1975, he served as a Postdoctoral Research Fellow at Harvard University, and also as a part-time Research Associate and Instructor at Northeastern University, Boston, MA. In 1975, he joined the faculty of the Georgia Institute of Technology, Atlanta, GA, where he is currently Regents' Professor of Electrical and Computer Engineering.

He is the author of the book, *An Introduction to Classical Electromagnetic Radiation* (Cambridge, 1997), and co-author of the book, *Antennas in Matter: Fundamentals, Theory and Applications* (MIT Press, 1981). He also authored the chapter "Loop Antennas" in the book, *Antenna Engineering Handbook* (McGraw-Hill, 1993). He is a member of Tau Beta Pi, Eta Kappa Nu, and Sigma Xi, a Fellow of the IEEE, and a member of USNC/URSI Commissions A and B. His technical interests include basic electromagnetic theory and measurements, antennas and wave propagation in materials, and the radiation and reception of pulses by antennas.

Editor's Comments *Continued from page 11*

Part of the answer is that the situation is a bit more straightforward outside North America. The country code for the US is "1." The North American Numbering Plan (NANP) consists of ten digits: a three-digit area code, a three-digit prefix (or, more properly, local-exchange code), and a four-digit number. However, within the US and Canada, if you want to dial a long-distance number (and part of the difficulty is that what constitutes "long distance" has become *extremely* complex), you often dial a 1 in front of the NANP digits. Indeed, many businesses within the US list their numbers as "1" followed by the NANP digits.

But it isn't that simple: you may also have to dial a one-, two-, five-, seven-, or ten-digit access code to select a long-distance carrier, and the "long-distance 1" may or may not be used. It's actually more complicated than all of this, but I think you get the idea. [Part of the reason for all of the complication is that we now have more-or-less totally open competition for local, intra- and inter-Local Access and Transport Area ("LATA"), and long-distance services. There are literally hundreds of local and long-distance carriers in the US. Of course, it's going to get more complicated. With an average of between three and four telephone lines per large-city household, we're rapidly outgrowing the NANP!] Of course, if you are within the same area code (and, sometimes, within the same LATA - or sometimes not!), then you often omit the "1" and the area code, and simply use the last seven NANP digits.

The answer to his question is that given the above confusing situation, I had felt it was less confusing to simply list the NANP digits, and leave the reader to add what was appropriate for the location from which he or she was dialing. However, it is true that this is not consistent transnationally. Thus, with this issue, I'm going to try to standardize on having numbers listed with the country code (preceded by a "+") first, followed by the numbering-plan digits appropriate for the country–including doing this for those in North America. The change isn't going to happen all at once. The numbers for *Magazine* Staff members, for example, may take two or three issues to get changed. In the interim, if you have an opinion on this issue, let me know. It may well be that maintaining the inconsistent status quo would be the least confusing!

Please, send *yourself* an e-mail! I get a lot of e-mail. Fortunately, almost all of it is useful. However, over the last few months, I have noticed several disturbing trends in a lot of the e-mail I receive (in what follows, when I refer to a "text" message, I mean a message that uses the reduced-character-set seven-bit ASCII text characters that can be transmitted over the Internet without requiring encoding; see my From the Screen of Stone column in the February, 1997, issue of the *Magazine* for a discussion of e-mail formats and "attachments"):

1. Messages that consist of a text version of the message, with an HTML version appended, usually as a binary "attachment;"

2. Messages that could be sent in "plain" text format, but are instead sent as HTML, as if they were designed to be viewed with a Web browser;

3. Messages that consist of a text version for the body of the message, and an "attached" binary or HTML file, containing the name and contact information of the sender;

4. Messages that are text messages (as opposed to formatted, word processor documents), but have been sent by "attaching" a (seven-bit ASCII) file as a binary "attachment."

Why do these create problems? HTML is designed to be viewed by a Web browser. Many e-mail programs do not display it properly, if at all. Often, what you get is a text message with all of the HTML commands appearing as so much "garbage" in the text. Unless you are trying to send someone a Web page via e-mail, sending an e-mail message that includes HTML is really counter-productive. Sending a message that involves an attachment requires that the recipient's e-mail program be able to decode that form of attachment. Worse (as discussed in my above-referenced column), opening an attachment without first scanning it for viruses is a good way to get a virus (and virus-scanning software isn't perfect).

Continued on page 49