A Direct Derivation of a Single-Antenna Reciprocity Relation for the Time Domain

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***Abstract*—In this paper, a single-antenna reciprocity relation is derived for the time domain. First, the antenna is considered on transmission; next, the same antenna is considered when it is receiving an incident plane wave. The two states, transmission and reception, are related by the application of a modified form of the reciprocity theorem for electromagnetic fields with general time dependence due to Cheo. The derivation of the reciprocity relation for the antenna makes use of simple geometric arguments to evaluate the spatial/temporal integrals that occur in the theorem. A few extensions of the reciprocity relation are also described.**

***Index Terms*—Antenna theory, reciprocity relation, time domain.**

I. Introduction

RECIPROCITY relations play an important role in several branches of physics. The earliest reciprocity relations for electromagnetism appear to be those of J. W. Strutt (Lord Rayleigh) presented in 1894 and H. A. Lorentz presented in 1896 [1], [2]. Rayleigh's theorem applies to electric circuits, whereas Lorentz's theorem applies to the electromagnetic field. Over the years, these two theorems have been elucidated and extended by a number of authors [3]–[11]. A version of these theorems specialized to the case of harmonic time dependence is now included in many textbooks on electromagnetic theory.

Surprisingly, the first detailed discussion of a reciprocity theorem for electromagnetic fields with arbitrary time dependence was given in a paper by Welch in 1960, some 64 years after the paper by Lorentz [12], [13].[1] Welch's paper was quickly followed by others presenting different formulations [14]–[16]. Reciprocity theorems for fields with arbitrary time dependence are discussed in only a few advanced textbooks on electromagnetism [17]–[19].

Since the earliest days of radio communication, the reciprocity theorems of Rayleigh and Lorentz have been applied to the antenna problem [3]–[7], [20]–[24]. Results obtained from these theorems for the special case of harmonic time dependence are now presented in most textbooks on antenna analysis, e.g., the fact that the far-zone field pattern for an antenna is the same on transmission and reception [25]–[27].

More recently, there has been considerable interest in antennas excited by signals with general time dependence, viz., pulses. This interest is motivated, in part, by a number of potential applications for ultra-wideband (UWB) technology. Different formulations have been offered for describing the performance of antennas in the time domain, some of which include a statement of reciprocity [28]–[32]. Often, the reciprocity relation is constructed by first considering a pair of antennas, then one of the antennas is eliminated to obtain a relationship between transmission and reception for the remaining antenna.

In this paper, we present a direct derivation of a single-antenna reciprocity relation for the time domain. First, the antenna is considered on transmission; next, the same antenna is considered when it is receiving an incident plane wave. The two states, transmission and reception, are related by the application of a modified form (presented in the Appendix) of the reciprocity theorem for electromagnetic fields with general time dependence due to Cheo [15]. The derivation of the single-antenna reciprocity relation presented in this paper makes use of simple geometric arguments to evaluate the spatial/temporal integrals that occur in the theorem.

In Section II of the paper, a physical model is constructed for the antenna that applies both for transmission and for reception. In Section III, the reciprocity theorem presented in the Appendix is applied to this model to obtain the reciprocity relation for the antenna, and in Section IV, a few extensions of the relation are given.

II. Model for the Antenna

In this section, we will present a fairly detailed electromagnetic model for the antenna that applies both for transmission and for reception. This model will later be used with the reciprocity theorem for Maxwell's equations, presented in the Appendix, to derive a reciprocity relation for the antenna. Many of the details of the model are included simply to provide physical significance in the derivation, and they will not appear in the final relation.

A. Antenna on Transmission

Fig. 1 shows the physical model for the antenna when it is transmitting. This is a schematic drawing giving the details of the feed region and only a very general representation for the antenna structure. We will assume that the antenna is constructed from simple materials and perfect electric conductors (PECs).

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[1] Actually, the first result in Lorentz's paper is for general time dependence; however, in the remainder of the paper the field is taken to be time-invariant or time-harmonic. Rayleigh's results are for the time-harmonic case.

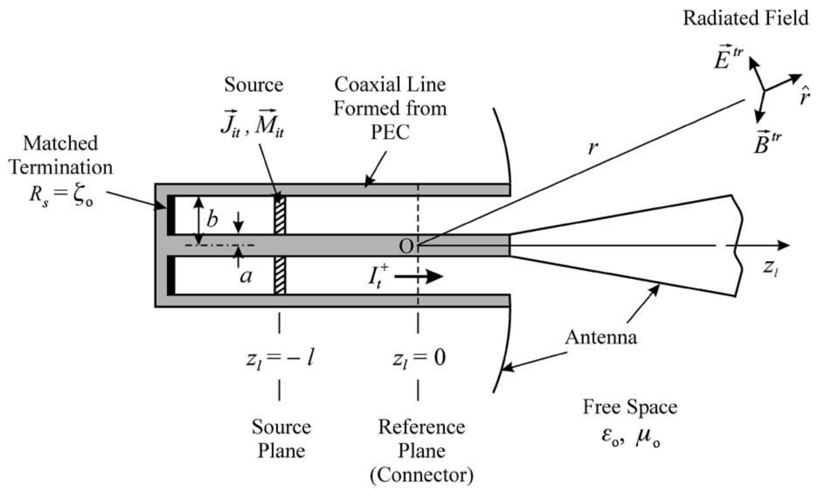

Fig. 1. Schematic drawing showing the model for the antenna on transmission.

The electromagnetic constitutive relations for the simple materials are

$$\vec{D}(\vec{r},t) = \varepsilon \vec{E}(\vec{r},t), \quad \vec{B}(\vec{r},t) = \mu \vec{H}(\vec{r},t),$$
$$\vec{J}_c(\vec{r},t) = \sigma \vec{E}(\vec{r},t) \tag{1}$$

in which ε, μ, and σ are the permittivity, permeability, and electrical conductivity, respectively, and the subscript c indicates a conduction current. The coaxial line attached to the antenna is formed from PECs and is filled with free space. It has an inner conductor of radius a and an outer conductor with inner radius b. At the reference plane $z_l = 0$, only the TEM mode is propagating within the coaxial line, and the fields of all other modes are negligible. The reference plane can be thought of as the junction or connector between the antenna and the feeding transmission line. At the reference plane, the current for the outward-propagating wave is $I_t^+(t)$. This is the current in the center conductor of the coaxial line, and it is taken to be positive when it is in the $+z_l$ direction. The source that produces this wave consists of discs of impressed electric and magnetic currents at the source plane $z_l = -l$. The volume densities for these impressed currents (subscript i) are

$$\vec{J}_{it}(\vec{r},t) = -\frac{I_t^+(t+l/c)}{2\pi\rho_l}\delta(z_l+l)\hat{\rho}_l,$$
$$\vec{M}_{it}(\vec{r},t) = -\frac{\zeta_o I_t^+(t+l/c)}{2\pi\rho_l}\delta(z_l+l)\hat{\phi}_l \tag{2}$$

in which (ρ_l, ϕ_l, z_l) are the cylindrical coordinates in the coaxial line, δ is the Dirac delta function, c is the speed of light in free space, and $\zeta_o = \sqrt{\mu_o/\varepsilon_o}$ is the wave impedance of free space. Note that these currents produce the outward traveling wave to the right of the source $(z_l > -l)$ and no field to the left of the source $(z_l < -l)$.[2] At the left-hand end of the coaxial line, there is a matched (reflectionless) termination formed by a disc of material with the surface resistance $R_s = \zeta_o$.

The radiated or far-zone field $(r \to \infty)$ of the antenna is[3]

$$\vec{E}^{tr}(\vec{r},t) = \frac{1}{r}\vec{e}^{tr}(\hat{r}, t-r/c)$$
$$\vec{B}^{tr}(\vec{r},t) = \frac{1}{r}\vec{b}^{tr}(\hat{r}, t-r/c)$$
$$= \frac{1}{cr}\hat{r}\times\vec{e}^{tr}(\hat{r}, t-r/c) \tag{3}$$

with $\vec{e}^{tr}$ and $\vec{b}^{tr}$ orthogonal to each other and orthogonal to $\hat{r}$. Notice, when lower case letters are used for the field, e.g., $\vec{e}^{tr}$, the direction in which the field is evaluated, $\hat{r}$, and the space-time interdependence are explicitly shown in the argument. We will assume that the source is on (is nonzero) for a finite length of time, so the radiated field has a maximum duration of t_t:

$$\vec{e}^{tr}(\hat{r}, t-r/c) = 0 \quad \begin{cases} t-r/c < 0 \\ t-r/c > t_t \end{cases}. \tag{4}$$

When an antenna is excited by a pulse, the antenna generally radiates a signal that is a different function of time for each direction in space. Here, t_t is the maximum duration when all directions are considered.

B. Antenna on Reception

Fig. 2 shows the physical model for the antenna when it is receiving. This is the same as the model used for transmission except the impressed currents are absent (the source is turned off). The incident plane wave is propagating in the direction $\hat{k}_i$ and has the electromagnetic field

$$\vec{E}^{i}(\vec{r},t) = \vec{e}^{i}(\hat{k}_i, t-\hat{k}_i\cdot\vec{r}/c)$$
$$\vec{B}^{i}(\vec{r},t) = \vec{b}^{i}(\hat{k}_i, t-\hat{k}_i\cdot\vec{r}/c)$$
$$= \frac{1}{c}\hat{k}_i\times\vec{e}^{i}(\hat{k}_i, t-\hat{k}_i\cdot\vec{r}/c) \tag{5}$$

[2]The derivation does not require both electric and magnetic currents for the source. An electric current density that is twice the value given in (2) will suffice. However, the field for $z_l < -l$ then will not be zero; this does not affect the derivation.

[3]The superscript r is used throughout to indicate the radiated or far-zone field.

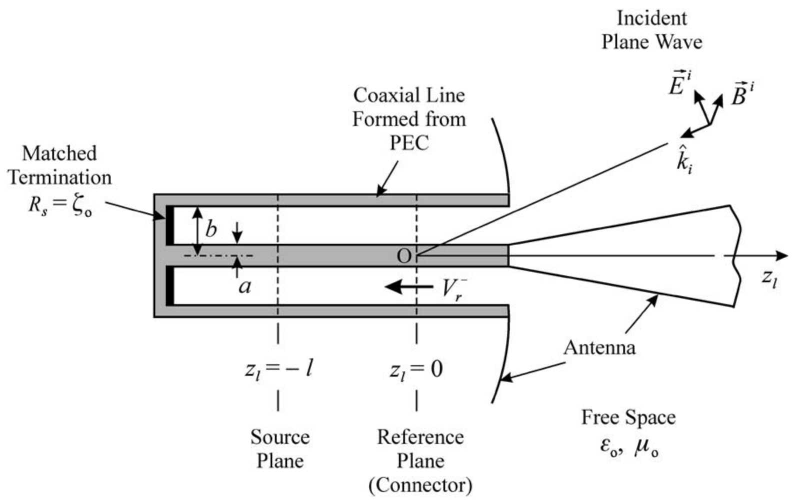

Fig. 2. Schematic drawing showing the model for the antenna on reception.

with $\vec{e}^{\,i}$ and $\vec{b}^{\,i}$ orthogonal to each other and orthogonal to $\hat{k}_i$. We will assume that the incident field is a pulse that exists (is nonzero) for a finite length of time t_i:

$$\vec{e}^{\,i}(\hat{k}_i, t - \hat{k}_i \cdot \vec{r}/c) = 0 \quad \begin{cases} t - \hat{k}_i \cdot \vec{r}/c < 0 \\ t - \hat{k}_i \cdot \vec{r}/c > t_i \end{cases}. \tag{6}$$

Notice that both the radiated field (3), (4) in the transmitting case and the incident field (5), (6) in the receiving case are referenced to the same point in space, $r = 0$ or O in Figs. 1 and 2, and at this point both fields are zero for $t < 0$.

For the receiving case, the total field in the space surrounding the antenna is the sum of the incident field and the scattered field

$$\begin{aligned} \vec{E}^r(\vec{r}, t) &= \vec{E}^i(\vec{r}, t) + \vec{E}^s(\vec{r}, t) \\ \vec{B}^r(\vec{r}, t) &= \vec{B}^i(\vec{r}, t) + \vec{B}^s(\vec{r}, t). \end{aligned} \tag{7}$$

In the far zone of the antenna $(r \to \infty)$, the scattered field has the form

$$\begin{aligned} \vec{E}^{sr}(\vec{r}, t) &= \frac{1}{r}\vec{e}^{\,sr}(\hat{r}, t - r/c) \\ \vec{B}^{sr}(\vec{r}, t) &= \frac{1}{r}\vec{b}^{\,sr}(\hat{r}, t - r/c) \\ &= \frac{1}{cr}\hat{r} \times \vec{e}^{\,sr}(\hat{r}, t - r/c) \end{aligned} \tag{8}$$

with $\vec{e}^{\,sr}$ and $\vec{b}^{\,sr}$ orthogonal to each other and orthogonal to $\hat{r}$.

The incident field (5) produces an inward-propagating wave in the coaxial line with the voltage $V_i^-(t)$ at the reference plane, $z = 0$. This is the voltage of the inner conductor of the coaxial line with respect to the outer conductor. The electromagnetic field for the TEM mode of this wave is

$$\begin{aligned} \vec{E}_r^-(\vec{r}, t) &= \frac{V_r^-(t + z_l/c)}{\rho_l \ln(b/a)}\hat{\rho}_l \\ \vec{B}_r^-(\vec{r}, t) &= -\frac{V_r^-(t + z_l/c)}{c\rho_l \ln(b/a)}\hat{\phi}_l. \end{aligned} \tag{9}$$

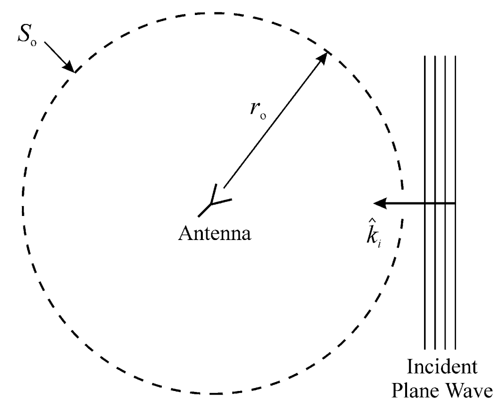

Fig. 3. Geometry used in applying the reciprocity theorem to the antenna.

III. Derivation of the Reciprocity Relation

Now we will apply the reciprocity theorem obtained in the Appendix to the geometry shown in Fig. 3. Here we have the antenna located at the center of a sphere of radius r_o. We will let $r_o \to \infty$ so that the surface of this sphere, S_o, is in the far zone of the antenna. The incident plane wave propagates in the direction $\hat{k}_i$, from right to left in the figure. The orientation of the antenna is arbitrary.

The reciprocity theorem (A11) with the transmitting antenna being case a and the receiving antenna being case b, becomes

$$\begin{aligned} -\int_{t=-\infty}^{\infty} \iiint_V &[\mu_o \vec{E}_r^-(\vec{r}, \tau - t) \cdot \vec{J}_{it}(\vec{r}, t) \\ &- \vec{B}_r^-(\vec{r}, \tau - t) \cdot \vec{M}_{it}(\vec{r}, t)]\, dV\, dt \\ = \int_{t=-\infty}^{\infty} \oiint_{S_o} &\Big\{ \Big[\vec{e}^{\,i}(\hat{k}_i, \tau - t - \hat{k}_i \cdot \vec{r}/c) \\ &+ \frac{1}{r}\vec{e}^{\,sr}(\hat{r}, \tau - t - r/c) \Big] \times \frac{1}{r}\vec{b}^{\,tr}(\hat{r}, t - r/c) \\ &- \frac{1}{r}\vec{e}^{\,tr}(\hat{r}, t - r/c) \times \Big[\vec{b}^{\,i}(\hat{k}_i, \tau - t - \hat{k}_i \cdot \vec{r}/c) \\ &+ \frac{1}{r}\vec{b}^{\,sr}(\hat{r}, \tau - t - r/c) \Big] \Big\} \cdot \hat{r}\, dS\, dt. \end{aligned} \tag{10}$$

After substituting (2) and (9), the volume integral on the left-hand side of (10) can be evaluated, see (11) at the bottom of the page. And, after using (3), (5), and (8), and considerable vector algebra, the right-hand side of (10) reduces to

$$\begin{aligned}\mathrm{RHS} = \frac{1}{c}\int_{t=-\infty}^{\infty}\oiint_{S_o}\{[\vec{e}^{\,i}(\hat{k}_i, \tau - t - \hat{k}_i\cdot\vec{r}/c)\\ \cdot\,\vec{e}^{\,tr}(\hat{r}, t - r/c)](1 - \hat{r}\cdot\hat{k}_i)\\ - c^2[(\hat{r}\times\hat{k}_i)\cdot\vec{b}^{tr}(\hat{r}, t - r/c)]\\ \cdot\,[(\hat{r}\times\hat{k}_i)\cdot\vec{b}^{i}(\hat{k}_i, \tau - t - \hat{k}_i\cdot\vec{r}/c)]\}r\,d\Omega\,dt. \quad (12)\end{aligned}$$

After substituting (11) and (12), (10) becomes

$$\begin{aligned}\int_{t=-\infty}^{\infty} V_r^-(\tau - t - l/c)I_t^+(t + l/c)\,dt\\ = \frac{1}{2\zeta_o}\int_{t=-\infty}^{\infty}\oiint_{S_o}\{[\vec{e}^{\,i}(\hat{k}_i, \tau - t - \hat{k}_i\cdot\vec{r}/c)\\ \cdot\,\vec{e}^{\,tr}(\hat{r}, t - r/c)](1 - \hat{r}\cdot\hat{k}_i)\\ - c^2[(\hat{r}\times\hat{k}_i)\cdot\vec{b}^{tr}(\hat{r}, t - r/c)]\\ \cdot\,[(\hat{r}\times\hat{k}_i)\cdot\vec{b}^{i}(\hat{k}_i, \tau - t - \hat{k}_i\cdot\vec{r}/c)]\}r\,d\Omega\,dt. \quad (13)\end{aligned}$$

Notice that this equation only involves the radiated field on transmission and the incident field on reception; the scattered field on reception does not appear. Now all that remains to complete the derivation is the evaluation of the surface integral on the right-hand side of (13).

In Fig. 4 we present snapshots for three instants in time ($t = 0, r_o/c, 3/2r_o/c$) showing the extent in space (gray areas) of the radiated field on transmission $\vec{e}^{\,tr}(\hat{r}, t - r/c)$, Fig. 4(a), and the incident field $\vec{e}^{\,i}(\hat{k}_i, t - \hat{k}_i\cdot\vec{r}/c)$, Fig. 4(b). The radiated field is a spherical wave that leaves the antenna at the time $t \approx 0$ and expands outward at the speed of light. The field of the incident plane wave travels from right to left (in the direction of $\hat{k}_i$) at the speed of light, and it is at the antenna when $t \approx 0$. In (13), we actually have the incident field with the argument $\tau - t - \hat{k}_i\cdot\vec{r}/c$, not the argument $t - \hat{k}_i\cdot\vec{r}/c$. This field, $\vec{e}^{\,i}(\hat{k}_i, \tau - t - \hat{k}_i\cdot\vec{r}/c)$, is shown in Fig. 4(c). It is a plane wave traveling from left to right: a time-reversed version of the incident plane wave (with a small shift $c\tau$). To visualize this situation, think of a movie made of the incident plane wave. In each frame of the movie, arrows are used to indicate the amplitude and direction of the field. When the movie is played, it shows the progression in space of the incident plane wave, Fig. 4(b). When the movie is played backward, it shows the progression in space of the time-reversed incident plane wave, Fig. 4(c).

Now, in the integrand on the right-hand side of (13), we have the scalar product $\vec{e}^{\,i}(\hat{k}_i, \tau - t - \hat{k}_i\cdot\vec{r}/c)\cdot\vec{e}^{\,tr}(\hat{r}, t - r/c)$, which is represented by the superposition of the corresponding pictures from Fig. 4(a) and (c). Notice, on the spherical surface S_o (the dashed circle), the gray areas in these pictures only overlap on the right-hand side of the surface, i.e., for $\hat{r} \approx -\hat{k}_i$, when $t = r_o/c$ [center pictures in Fig. 4(a) and (c)]. Thus, we can conclude that the contribution to the integral in (13) comes from a small area on the surface of the sphere S_o centered at $\hat{r} = -\hat{k}_i$ for times near $t = r_o/c$. We will now use this information to evaluate the surface integral.

We have assumed that τ is finite and that $r_o \to \infty$; therefore, $c\tau/r_o \ll 1$. This inequality was used in making the pictures in Fig. 4, and now it will be used in the evaluation of the surface integral on the right-hand side of (13). To help with the evaluation of the integral, we introduce the spherical coordinate system (r, θ, ϕ), shown in Fig. 5, with the polar axis in the direction $\hat{z} = -\hat{k}_i$. Equation (13) then becomes

$$\begin{aligned}\int_{t=-\infty}^{\infty} V_r^-(\tau - t - l/c)I_t^+(t + l/c)\,dt\\ = \frac{1}{2\zeta_o}\int_{t=-\infty}^{\infty}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\{[\vec{e}^{\,i}(\hat{k}_i, \tau - t + r_o\cos\theta/c)\\ \cdot\,\vec{e}^{\,tr}(\hat{r}, t - r_o/c)]\\ \cdot\,(1 + \cos\theta) - c^2\sin^2\theta[\hat{\phi}\cdot\vec{b}^{tr}(\hat{r}, t - r_o/c)]\\ \cdot\,[\hat{\phi}\cdot\vec{b}^{i}(\hat{k}_i, \tau - t + r_o\cos\theta/c)]\}r_o\sin\theta\,d\phi\,d\theta\,dt. \quad (14)\end{aligned}$$

In Fig. 5, we show the extent in space of the radiated field on transmission $\vec{e}^{\,tr}(\hat{r}, t - r/c)$ (dark gray) and the time-reversed incident field $\vec{e}^{\,i}(\hat{k}_i, \tau - t - \hat{k}_i\cdot\vec{r}/c)$ (light gray) at the instant $t = r_o/c$. From this figure, we see that the two fields overlap on the sphere S_o, and, therefore, the integrand of the right-hand side of (14) is nonzero, only for $0 \le \theta \le \theta_{\max}$. Here, the very small angle $\theta_{\max}$ is given by

$$\cos(\theta_{\max}) = 1 - c\tau/r_o. \quad (15)$$

After introducing the change of variable

$$t' = r_o(1 - \cos\theta)/c \quad (16)$$

$$\begin{aligned}\mathrm{LHS} = \int_{t=-\infty}^{\infty}\int_{z_l=-l-\Delta}^{-l+\Delta}\int_{\phi_l=0}^{2\pi}\int_{\rho_l=a}^{b}\left[\mu_o\frac{V_r^-(\tau - t + z_l/c)I_t^+(t + l/c)}{2\pi\rho_l^2\ln(b/a)}\delta(z_l + l)\right.\\ \left. + \frac{\zeta_o V_r^-(\tau - t + z_l/c)I_t^+(t + l/c)}{c2\pi\rho_l^2\ln(b/a)}\delta(z_l + l)\right]\rho_l\,d\rho_l\,d\phi_l\,dz_l\,dt = 2\mu_o\int_{t=-\infty}^{\infty} V_r^-(\tau - t - l/c)I_t^+(t + l/c)\,dt. \quad (11)\end{aligned}$$

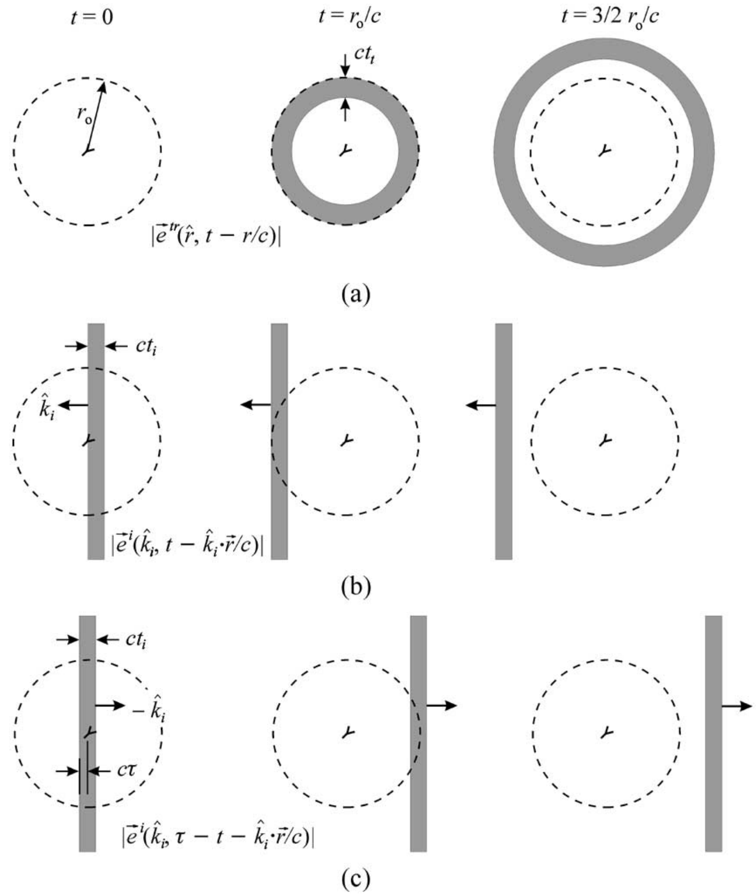

Fig. 4. Snapshots in time showing the extent of the electric field in space for (a) transmitting antenna, (b) incident plane wave, (c) incident plane wave time-reversed and shifted in time by τ.

and using (15), (14) becomes

$$\begin{aligned}&\int_{t=-\infty}^{\infty} V_r^-(\tau - t - l/c) I_t^+(t + l/c)\, dt\\ &= \frac{1}{2\mu_o}\int_{t=-\infty}^{\infty}\int_{t'=0}^{\tau}\int_{\phi=0}^{2\pi}\{[\vec{e}^{\,i}(\hat{k}_i, \tau - t + r_o/c - t')\\ &\cdot \vec{e}^{\,tr}(\hat{r}, t - r_o/c)]\\ &\cdot\left(2 - \frac{ct'}{r_o}\right) - c^2\left[\frac{2ct'}{r_o} - \left(\frac{ct'}{r_o}\right)^2\right][\hat{\phi}\cdot\vec{b}^{tr}(\hat{r}, t - r_o/c)]\\ &\cdot[\hat{\phi}\cdot\vec{b}^{i}(\hat{k}_i, \tau - t + r_o/c - t')]\}\, d\phi\, dt'\, dt. \qquad (17)\end{aligned}$$

Now, dropping terms of order $ct'/r_o \leq c\tau/r_o \ll 1$ or less, setting $\hat{r} = -\hat{k}_i$ in the argument of $\vec{e}^{\,tr}$, and evaluating the integral with respect to ϕ, (17) simplifies to become

$$\begin{aligned}&\int_{t=-\infty}^{\infty} V_r^-(\tau - t - l/c) I_t^+(t + l/c)\, dt\\ &= \frac{2\pi}{\mu_o}\int_{t=-\infty}^{\infty}\int_{t'=0}^{\tau}[\vec{e}^{\,i}(\hat{k}_i, \tau - t + r_o/c - t')\\ &\cdot \vec{e}^{\,tr}(-\hat{k}_i, t - r_o/c)]\, dt'\, dt. \qquad (18)\end{aligned}$$

After the successive changes of variable $t - r_o/c \to t$ and $\tau - t - t' \to t'$ to the right-hand side, (18) becomes

$$\begin{aligned}&\int_{t=-\infty}^{\infty} V_r^-(\tau - t - l/c) I_t^+(t + l/c)\, dt\\ &= \frac{2\pi}{\mu_o}\int_{t=-\infty}^{\infty}\left[\int_{t'=-t}^{\tau - t}\vec{e}^{\,i}(\hat{k}_i, t')\, dt'\right]\cdot \vec{e}^{\,tr}(-\hat{k}_i, t)\, dt. \qquad (19)\end{aligned}$$

The lower limit on the t' integration can be changed to $-\infty$ without affecting the outcome of the integration, because $\vec{e}^{\,tr} = 0, t < 0$, and $\vec{e}^{\,i} = 0, t' < 0$

$$\begin{aligned}&\int_{t=-\infty}^{\infty} V_r^-(\tau - t - l/c) I_t^+(t + l/c)\, dt\\ &= \frac{2\pi}{\mu_o}\int_{t=-\infty}^{\infty}\left[\int_{t'=-\infty}^{\tau - t}\vec{e}^{\,i}(\hat{k}_i, t') dt'\right]\cdot \vec{e}^{\,tr}(-\hat{k}_i, t)\, dt. \qquad (20)\end{aligned}$$

Equation (20) is essentially the single-antenna reciprocity relation we sought; however, it can be put in a more convenient form by removing some of the temporary notation we used in the derivation. First, we make the change of variable $t + l/c \to t$

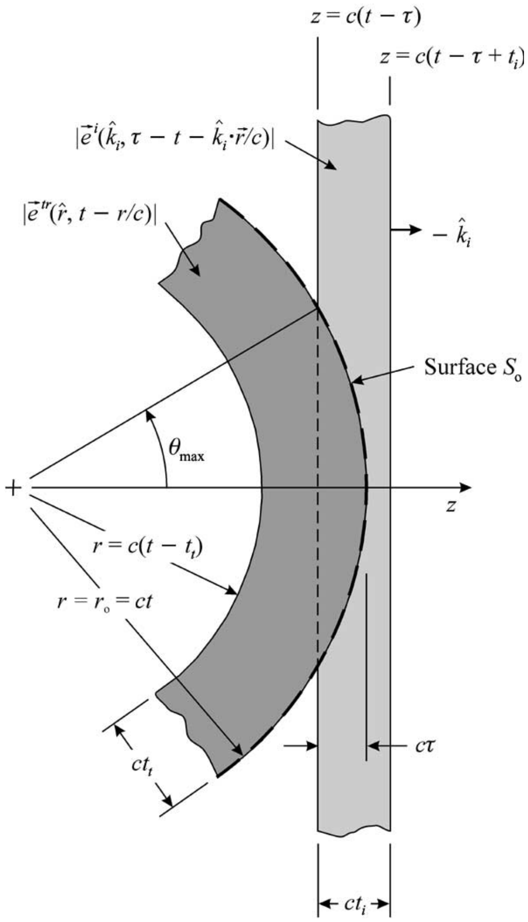

Fig. 5. Coordinates for evaluating the surface integral. Picture is for the time $t = r_{\mathrm{o}}/c$.

on the left-hand side to remove the length l. Another way to remove this length is to move the source plane to the reference plane in Fig. 1. On the right-hand side, we change r_{o} to r, and use (3) and (5) to rewrite the fields in their original form. Then, our final result for the *single-antenna reciprocity relation* is

$$\int_{t=-\infty}^{\infty} V_r^-(\tau - t) I_t^+(t)\, dt = \frac{2\pi}{\mu_{\mathrm{o}}} \int_{t=-\infty}^{\infty} \left[\int_{t'=-\infty}^{\tau - t} \vec{E}^i(0, t') dt' \right] \cdot r\vec{E}^{tr}(-r\hat{k}_i, t + r/c)\, dt \tag{21}$$

or

$$V_r^-(t) * I_t^+(t) = \frac{2\pi}{\mu_{\mathrm{o}}} \left[\int_{t'=-\infty}^{t} \vec{E}^i(0, t') dt' \right] \cdot * r\vec{E}^{tr}(-r\hat{k}_i, t + r/c) \tag{22}$$

in which $*$ indicates time convolution, and $\cdot*$ indicates the scalar product with time convolution. This relationship says that the time convolution of the voltage on reception with the current on transmission is equal to the scalar product with time convolution of the integral of the incident electric field, evaluated at the antenna ($\vec{r} = 0$), with the radiated electric field on transmission (field in limit $r \to \infty$), evaluated in the direction from which the incident field arrived ($-\hat{k}_i$), scaled by $2\pi r/\mu_{\mathrm{o}}$, and advanced in time by r/c.

An alternate form for the single-antenna reciprocity relation can be obtained by using the commutative property of convolution with the left-hand side of (21) and then differentiating with respect to τ:

$$\int_{t=-\infty}^{\infty} V_r^-(t) \left[\frac{dI_t^+(t')}{dt'} \right]_{t' = \tau - t} dt = \frac{2\pi}{\mu_{\mathrm{o}}} \int_{t=-\infty}^{\infty} \vec{E}^i(0, \tau - t) \cdot r\vec{E}^{tr}(-r\hat{k}_i, t + r/c)\, dt \tag{23}$$

or

$$V_r^-(t) * \frac{dI_t^+(t)}{dt} = \frac{2\pi}{\mu_{\mathrm{o}}} \vec{E}^i(0, t) \cdot * r\vec{E}^{tr}(-r\hat{k}_i, t + r/c). \tag{24}$$

In some applications, (23) may be easier to use than (21), because it does not involve an integration of the incident field with respect to time.

One result that we can immediately obtain from the reciprocity relation is the "derivative relation" for transmission and reception. In the transmitting case, let the current be the pulse $I_t^+(t) \propto p(t)$, and in the receiving case, let the jth component of the incident electric field vector be the same pulse, $E_j^i(0, t) \propto p(t)$. Then the jth component of the radiated electric field vector in the transmitting case is proportional to the time derivative of the voltage in the receiving case, $rE_j^{tr}(-r\hat{k}_i, t + r/c) \propto dV_r^-(t)/dt$. To show this, we use integration by parts with (21) to obtain

$$\begin{aligned}
\int_{t=-\infty}^{\infty} V_r^-(\tau - t) I_t^+(t)\, dt \quad &\propto \int_{t=-\infty}^{\infty} V_r^-(\tau - t) p(t)\, dt \\
&= \int_{t=-\infty}^{\infty} p(\tau - t) V_r^-(t)\, dt \\
&= \int_{t=-\infty}^{\infty} \left[\int_{t'=-\infty}^{\tau - t} p(t')\, dt' \right] \frac{dV_r^-(t)}{dt}\, dt \\
&\propto \int_{t=-\infty}^{\infty} \left[\int_{t'=-\infty}^{\tau - t} p(t') dt' \right] rE_j^{tr}(-r\hat{k}_i, t + r/c)\, dt
\end{aligned} \tag{25}$$

and, since $p(t)$ is arbitrary, it follows from the last line of this equation that $rE_j^{tr}(-r\hat{k}_i, t + r/c) \propto dV_r^-(t)/dt$.

In deriving the single-antenna reciprocity relation, we used a specific physical model for the antenna. Notice, many of the details of the model, such as the parameters for the source and the coaxial transmission line, do not appear in the final result, (21) or (23). This relation has general validity, it applies not only to an antenna fed by a coaxial transmission line propagating the TEM mode, but also to an antenna fed by any other lossless, wave-guiding structure propagating a single mode for which equivalent (symbolic) voltages and currents can be defined. Also, in the derivation, we assumed that the antenna was constructed from perfect conductors and simple materials with

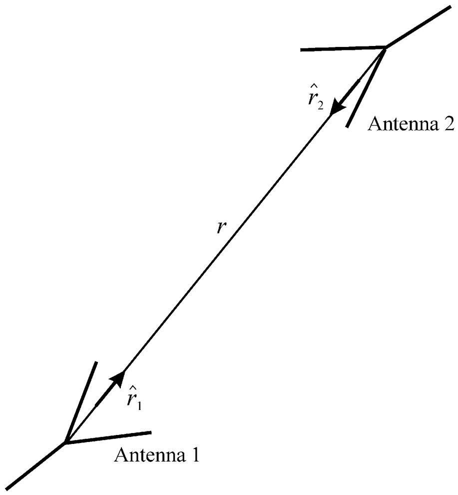

Fig. 6. Geometry for use with the two-antenna reciprocity relation.

the constitutive relations given by (1). As Cheo has shown, the reciprocity theorem on which the derivation is based applies for materials that have more general, linear constitutive relations, such as, [15]

$$\vec{D}(\vec{r},t) = \int_{t'=-\infty}^{\infty} \varepsilon(t')\vec{E}(\vec{r},t-t')\,dt'. \tag{26}$$

So we can infer that the reciprocity relation, (21) or (23), should also hold for antennas constructed from such materials.

IV. Extensions of the Reciprocity Relation

Starting from the basic single-antenna reciprocity relation, (21) or (23), it is possible to obtain other useful results for practical application. In this section, we will discuss a few of these results.

A. Two-Antenna Reciprocity Relation

Consider the two antennas (1 and 2) shown in Fig. 6. Each antenna is in the far zone of the other, so at one antenna the field of the other behaves locally as a plane wave. The separation between the antennas is r, and the unit vector pointing from antenna 1 (2) to antenna 2 (1) is $\hat{r}_1$ ($\hat{r}_2 = -\hat{r}_1$). Both antennas satisfy the requirements for the single-antenna reciprocity relation, e.g., the transmission line attached to the antenna is matched at the far end.

The field incident on antenna 1 when it is receiving is the field radiated by antenna 2 when it is transmitting

$$\vec{E}_1^i(0,t) = \vec{E}_2^{tr}(r\hat{r}_2,t) \tag{27}$$

and the field incident on antenna 2 when it is receiving is the field radiated by antenna 1 when it is transmitting,

$$\vec{E}_2^i(0,t) = \vec{E}_1^{tr}(r\hat{r}_1,t). \tag{28}$$

Now for antenna 1, the single-antenna reciprocity relation (22) with (27) gives

$$V_{r1}^-(t) * I_{t1}^+(t) = \frac{2\pi}{\mu_o}\left[\int_{t'=-\infty}^{t} \vec{E}_2^{tr}(r\hat{r}_2,t')dt'\right] \cdot * \, r\vec{E}_1^{tr}(r\hat{r}_1,t+r/c) \tag{29}$$

and for antenna 2, (22) with (28) gives

$$\begin{aligned} V_{r2}^-(t) * I_{t2}^+(t) &= \frac{2\pi}{\mu_o}\left[\int_{t'=-\infty}^{t} \vec{E}_1^{tr}(r\hat{r}_1,t')dt'\right] \cdot * \, r\vec{E}_2^{tr}(r\hat{r}_2,t+r/c) \\ &= \frac{2\pi}{\mu_o}\left[\int_{t'=-\infty}^{t+r/c} \vec{E}_1^{tr}(r\hat{r}_1,t')dt'\right] \cdot * \, r\vec{E}_2^{tr}(r\hat{r}_2,t) \\ &= \frac{2\pi}{\mu_o}\left[\int_{t'=-\infty}^{t} \vec{E}_2^{tr}(r\hat{r}_2,t')dt'\right] \cdot * \, r\vec{E}_1^{tr}(r\hat{r}_1,t+r/c) \end{aligned} \tag{30}$$

in which the last result was obtained using integration by parts. Notice that the right-hand sides of (29) and (30) are equal. Equating the left-hand sides of these equations gives the *two-antenna reciprocity relation*

$$V_{r1}^-(t) * I_{t1}^+(t) = V_{r2}^-(t) * I_{t2}^+(t). \tag{31}$$

The voltage at antenna 1 when it is receiving convolved with the current at antenna 1 when it is transmitting is equal to the voltage at antenna 2 when it is receiving convolved with the current at antenna 2 when it is transmitting.

B. Vector Effective Heights

For a linear, time-invariant system, the output $g(t)$ is related to the input $f(t)$ by a time convolution [33]

$$g(t) = \int_{t'=-\infty}^{\infty} h(t-t')f(t')dt' = h(t) * f(t) \tag{32}$$

or

$$g(t) = \int_{t'=-\infty}^{\infty} k(t-t')\frac{df(t')}{dt'}dt' = k(t) * \frac{df(t)}{dt} \tag{33}$$

when $f(t) \to 0, t \to \pm\infty$. Here, $h(t)$ or $k(t)$ is called the system function.

When the constitutive relations for all materials involved are linear and all objects are stationary, Maxwell's equations describe a linear, time-invariant system. So we can define system functions that characterize the antenna for transmission and reception. We can use (33) to relate the radiated electric field to the derivative of the current for the transmitting case

$$\begin{aligned} r\vec{E}^{tr}(\vec{r},t+r/c) &= \frac{\mu_o}{2\pi}\int_{t'=-\infty}^{\infty} \vec{h}_t(\hat{r},t-t')\frac{dI_t^+(t')}{dt'}\,dt' \\ &= \frac{\mu_o}{2\pi}\vec{h}_t(\hat{r},t) * \frac{dI_t^+(t)}{dt} \end{aligned} \tag{34}$$

and (32) to relate the voltage to the incident electric field for the receiving case

$$V_r^-(t) = \int_{t'=-\infty}^{\infty} \vec{h}_r(\hat{r} = -\hat{k}_i, t-t') \cdot \vec{E}^i(0,t')dt' = \vec{h}_r(\hat{r} = -\hat{k}_i, t) \cdot * \vec{E}^i(0,t). \quad (35)$$

Here, for historical reasons, we call the system function $\vec{h}_t(\hat{r}, t)$ the *vector effective height on transmission* and the system function $\vec{h}_r(\hat{r}, t)$ the *vector effective height on reception*. The components of both vector effective heights are orthogonal to the direction in space, $\hat{r}$. They are functions of the direction in space and time, and they have the units of m/s. The definitions for the vector effective heights are not unique or standard, and other authors have used different definitions from those presented here [28], [30], [32].

When (34) and (35) are substituted into (24), we obtain

$$\begin{aligned}\left[\vec{h}_r(\hat{r} = -\hat{k}_i, t) \cdot * \vec{E}^i(0,t)\right] * \frac{dI_t^+(t)}{dt} &= \vec{E}^i(0,t) \cdot * \left[\vec{h}_t(\hat{r} = -\hat{k}_i, t) * \frac{dI_t^+(t)}{dt}\right] \\ &= \left[\vec{h}_t(\hat{r} = -\hat{k}_i, t) \cdot * \vec{E}^i(0,t)\right] * \frac{dI_t^+(t)}{dt} \quad (36)\end{aligned}$$

in which we have used the associative and commutative properties of convolution on the right-hand side of the equation. On comparing the first and last entries in (36), and noting that $\vec{E}^i(0,t)$ and $I_t^+(t)$ are arbitrary functions of time, we see that

$$\vec{h}_t(\hat{r}, t) = \vec{h}_r(\hat{r}, t) \quad (37)$$

i.e., the vector effective heights (the system functions) are the same for transmission and reception.[4] Instead of using (22) or (24), (37) together with (34) and (35) can be used to express reciprocity for the antenna.

The vector effective heights are convenient for determining the response of a two-antenna link, such as the one shown in Fig. 6. For example, the voltage at antenna 2 due to the current at antenna 1 is simply obtained by using (28) and (34) with (35)

$$\begin{aligned}V_{r2}^-(t) &= \frac{\mu_o}{2\pi r}\vec{h}_2(-\hat{r}_1, t) \cdot * \left[\vec{h}_1(\hat{r}_1, t) * \frac{dI_{t1}^+(t)}{dt}\right]_{t-r/c} \\ &= \frac{\mu_o}{2\pi r}\left\{\vec{h}_2(-\hat{r}_1, t) \cdot * \left[\vec{h}_1(\hat{r}_1, t) * \frac{dI_{t1}^+(t)}{dt}\right]\right\}_{t-r/c}. \quad (38)\end{aligned}$$

C. Frequency Domain

Any of the results we have obtained for the time domain can be converted to results for the frequency domain by using the Fourier transformation: $f(t) \leftrightarrow f(\omega)$. Specifically, for a time convolution we have

$$A(t) * B(t) \leftrightarrow A(\omega)B(\omega)$$

where

$$A(t) \leftrightarrow A(\omega), B(t) \leftrightarrow B(\omega). \quad (39)$$

As an example, we will consider the two-antenna reciprocity relation given by (31). After Fourier transformation, it becomes

$$V_{r1}^-(\omega)I_{t1}^+(\omega) = V_{r2}^-(\omega)I_{t2}^+(\omega)$$

where

$$V_{r1}^-(t) \leftrightarrow V_{r1}^-(\omega), \text{ etc.} \quad (40)$$

This expression is in terms of partial voltages and currents, e.g., I_t^+, not the total voltages and currents, e.g., $I_t = I_t^+ + I_t^-$, that are customarily used in frequency-domain analysis. We will now rewrite (40) in terms of the total current on transmission I_t and the total open-circuit voltage on reception V_{oc}.

For the two cases we have considered so far (transmission and reception), the total voltages and currents at the plane $z_l = 0$ in the transmission line are

$$V_t(\omega) = V_t^+(\omega) + V_t^-(\omega) = [1 + \Gamma_V(\omega)]V_t^+(\omega) \quad (41)$$

$$I_t(\omega) = I_t^+(\omega) + I_t^-(\omega) = [1 - \Gamma_V(\omega)]I_t^+(\omega) = \frac{1}{Z_o}[1 - \Gamma_V(\omega)]V_t^+(\omega) \quad (42)$$

$$V_r(\omega) = V_r^-(\omega) \quad (43)$$

$$I_r(\omega) = I_r^-(\omega) = -\frac{1}{Z_o}V_r^-(\omega) \quad (44)$$

in which Z_o is the characteristic impedance of the transmission line, and $\Gamma_V(\omega)$ is the voltage reflection coefficient for the antenna. Solving (42) for $I_t^+(\omega)$, we obtain

$$I_t^+(\omega) = I_t(\omega)/[1 - \Gamma_V(\omega)]. \quad (45)$$

The case where the antenna is receiving with the transmission line open-circuited at the plane $z_l = 0$ can be obtained as a superposition of the transmitting and receiving cases already discussed, with the currents for these two cases adjusted to make their sum zero[5]

$$I_{\text{oc}}(\omega) = I_t(\omega) + I_r(\omega) = 0. \quad (46)$$

After substituting (42) and (44), (46) gives

$$V_t^+(\omega) = V_r^-(\omega)/[1 - \Gamma_V(\omega)]. \quad (47)$$

So the open-circuit voltage is

$$\begin{aligned}V_{\text{oc}}(\omega) &= V_t(\omega) + V_r(\omega) = [1 + \Gamma(\omega)]V_t^+(\omega) + V_r^-(\omega) \\ &= 2V_r^-(\omega)/[1 - \Gamma(\omega)] \quad (48)\end{aligned}$$

[4]This result is in keeping with earlier work for the frequency domain, in which the vector effective heights are defined to be equal on transmission and reception [23].

[5]The two electromagnetics problems, the open-circuit case and the superposition of the transmitting and receiving cases, are equivalent for the exterior volume, i.e., the volume bounded on the interior by the surface of the coaxial line and the disc at $z_l = 0$ and on the exterior by a spherical surface of infinite radius. For both problems, the sources within the volume are the same (the incident field), and the tangential components of either the electric field or the magnetic field are the same on the interior surface: On the surface of the PEC transmission line $\vec{E}_{\tan} = 0$, and on the disc $\vec{B}_{\tan} = 0 (I_{\text{oc}} = 0)$.

in which we have used (41), (43), and (47). Solving (48) for $V_r^-(\omega)$, we have

$$V_r^-(\omega) = \frac{1}{2}[1 - \Gamma_V(\omega)]V_{\rm oc}(\omega). \tag{49}$$

Finally, using (45) and (49) with (40), we obtain

$$V_{\rm oc1}(\omega)I_{t1}(\omega) = V_{\rm oc2}(\omega)I_{t2}(\omega) \tag{50}$$

which is the two-antenna reciprocity relation for the frequency domain presented in most textbooks on antenna analysis, e.g., [25]–[27].

APPENDIX
RECIPROCITY THEOREM FOR ELECTROMAGNETIC FIELDS WITH GENERAL TIME DEPENDENCE

In this Appendix, we present a brief derivation for a modified version of a reciprocity theorem due to Cheo [15]. Fig. A1 is a schematic drawing of the geometry to be used in the derivation of the theorem. The surface S separates space into two regions: the interior volume V that contains simple materials and perfect conductors and the exterior volume that contains free space. The electromagnetic constitutive relations for the simple materials are given in (1). There are impressed currents (subscript i) of the electric and magnetic type. The volume densities for these currents are $\vec{J}_{ia}$ and $\vec{M}_{ia}$ within V and $\vec{J}_{ib}$ and $\vec{M}_{ib}$ outside of V. We will assume that these sources are on (are nonzero) for a finite duration in time, so that within V all electromagnetic quantities are zero for $t \to -\infty$ and $t \to \infty$.

In each material region, Maxwell's equations for the field $\vec{E}_a, \vec{B}_a$ produced by the sources $\vec{J}_{ia}, \vec{M}_{ia}$ are

$$\nabla \times \vec{E}_a(\vec{r},t) = -\frac{\partial \vec{B}_a(\vec{r},t)}{\partial t} - \vec{M}_{ia}(\vec{r},t) \tag{A1}$$

$$\nabla \times \vec{B}_a(\vec{r},t) = \mu\left[\sigma\vec{E}_a(\vec{r},t) + \vec{J}_{ia}(\vec{r},t) + \varepsilon\frac{\partial \vec{E}_a(\vec{r},t)}{\partial t}\right] \tag{A2}$$

and similarly for the field $\vec{E}_b, \vec{B}_b$ produced by the sources $\vec{J}_{ib}, \vec{M}_{ib}$

$$\nabla \times \vec{E}_b(\vec{r},t) = -\frac{\partial \vec{B}_b(\vec{r},t)}{\partial t} - \vec{M}_{ib}(\vec{r},t) \tag{A3}$$

$$\nabla \times \vec{B}_b(\vec{r},t) = \mu\left[\sigma\vec{E}_b(\vec{r},t) + \vec{J}_{ib}(\vec{r},t) + \varepsilon\frac{\partial \vec{E}_b(\vec{r},t)}{\partial t}\right]. \tag{A4}$$

Now we introduce the change of variable $t \to \tau - t$ with τ finite into (A3) and (A4) to obtain

$$\nabla \times \vec{E}_b(\vec{r},\tau - t) = \frac{\partial \vec{B}_b(\vec{r},\tau - t)}{\partial t} - \vec{M}_{ib}(\vec{r},\tau - t) \tag{A5}$$

$$\nabla \times \vec{B}_b(\vec{r},\tau - t) = \mu\left[\sigma\vec{E}_b(\vec{r},\tau - t) + \vec{J}_{ib}(\vec{r},\tau - t) - \varepsilon\frac{\partial \vec{E}_b(\vec{r},\tau - t)}{\partial t}\right]. \tag{A6}$$

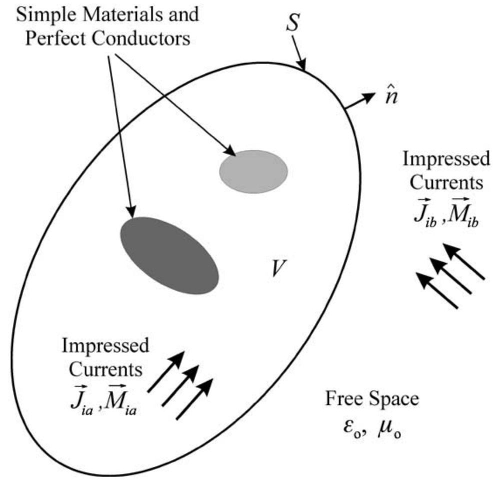

Fig. A1. Schematic drawing showing the geometry used in deriving the reciprocity theorem.

Next, we make use of the vector relation [34]

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B}) \tag{A7}$$

to write

$$\begin{aligned}\nabla \cdot [\vec{E}_b(\vec{r},\tau - t) \times \vec{B}_a(\vec{r},t) - \vec{E}_a(\vec{r},t) \times \vec{B}_b(\vec{r},\tau - t)] \\ = \vec{B}_a(\vec{r},t) \cdot [\nabla \times \vec{E}_b(\vec{r},\tau - t)] \\ - \vec{E}_b(\vec{r},\tau - t) \cdot [\nabla \times \vec{B}_a(\vec{r},t)] \\ - \{\vec{B}_b(\vec{r},\tau - t) \cdot [\nabla \times \vec{E}_a(\vec{r},t)] - \vec{E}_a(\vec{r},t) \\ \cdot [\nabla \times \vec{B}_b(\vec{r},\tau - t)]\}\end{aligned} \tag{A8}$$

which, after substituting (A1), (A2) and (A5), (A6), becomes

$$\begin{aligned}\nabla \cdot [\vec{E}_b(\vec{r},\tau - t) \times \vec{B}_a(\vec{r},t) - \vec{E}_a(\vec{r},t) \times \vec{B}_b(\vec{r},\tau - t)] \\ = \frac{\partial}{\partial t}[\vec{B}_b(\vec{r},\tau - t) \cdot \vec{B}_a(\vec{r},t)] \\ - \mu\varepsilon\frac{\partial}{\partial t}[\vec{E}_b(\vec{r},\tau - t) \cdot \vec{E}_a(\vec{r},t)] \\ - \mu[\vec{E}_b(\vec{r},\tau - t) \cdot \vec{J}_{ia}(\vec{r},t) - \vec{E}_a(\vec{r},t) \cdot \vec{J}_{ib}(\vec{r},\tau - t)] \\ - [\vec{B}_a(\vec{r},t) \cdot \vec{M}_{ib}(\vec{r},\tau - t) - \vec{B}_b(\vec{r},\tau - t) \cdot \vec{M}_{ia}(\vec{r},t)].\end{aligned} \tag{A9}$$

Now we integrate this expression over the volume V and over all time to obtain

$$\begin{aligned}\int_{t=-\infty}^{\infty} \iiint_V \nabla \cdot [\vec{E}_b(\vec{r},\tau - t) \times \vec{B}_a(\vec{r},t) \\ - \vec{E}_a(\vec{r},t) \times \vec{B}_b(\vec{r},\tau - t)]\, dV\, dt \\ = \iiint_V [\vec{B}_b(\vec{r},\tau - t) \cdot \vec{B}_a(\vec{r},t) \\ - \mu\varepsilon\vec{E}_b(\vec{r},\tau - t) \cdot \vec{E}_a(\vec{r},t)]_{t=-\infty}^{\infty}\, dV \\ - \int_{t=-\infty}^{\infty} \iiint_V [\mu\vec{E}_b(\vec{r},\tau - t) \cdot \vec{J}_{ia}(\vec{r},t) \\ - \vec{B}_b(\vec{r},\tau - t) \cdot \vec{M}_{ia}(\vec{r},t)]\, dV\, dt.\end{aligned} \tag{A10}$$

After applying the divergence theorem [34] to the volume integral on the left-hand side of this equation, and recognizing that the first term on the right-hand side is zero because of the conditions already assumed at $t \to \pm\infty$, we obtain our final result for the reciprocity theorem

$$\int_{t=-\infty}^{\infty} \oiint_{S} [\vec{E}_b(\vec{r},\tau - t) \times \vec{B}_a(\vec{r},t) - \vec{E}_a(\vec{r},t) \times \vec{B}_b(\vec{r},\tau - t)] \cdot d\vec{S}\, dt = -\int_{t=-\infty}^{\infty} \iiint_{V} [\mu \vec{E}_b(\vec{r},\tau - t) \cdot \vec{J}_{ia}(\vec{r},t) - \vec{B}_b(\vec{r},\tau - t) \cdot \vec{M}_{ia}(\vec{r},t)]\, dV\, dt. \quad \text{(A11)}$$

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