*Proceedings of the Institute of Radio Engineers*
*Volume 24, Number 5* *May, 1936*

# A STUDY OF THE ELECTROMAGNETIC FIELD IN THE VICINITY OF A RADIATOR*

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***Summary***—*The complete equations for the electromagnetic field of an infinitesimal current element are given. The integration of these equations is considered for the case of a finite radiator having an empirical current distribution. Tables are included to facilitate computation and consideration is given to difference in phase of the current in various portions of the radiator.*

PROBLEMS in radio engineering involving the radiation and propagation of electromagnetic waves are frequently solved by the use of approximate expressions for the electromagnetic field which are, in general, not valid in a region less than one wave length

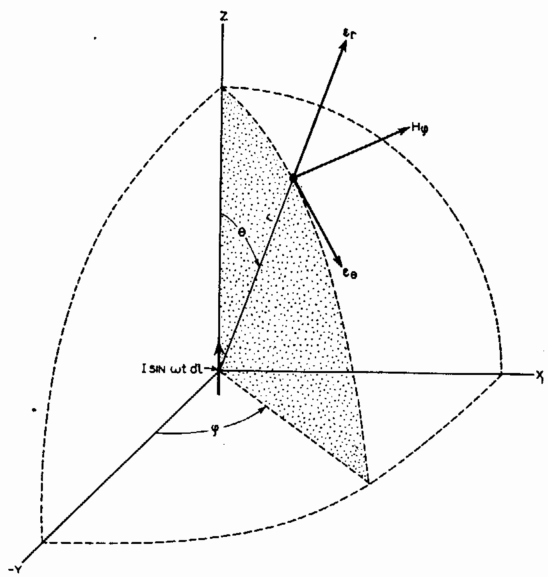


Fig. 1—Co-ordinate system.

from the radiator. When it becomes necessary to compute the electromagnetic field in the vicinity of the radiator, more generalized equations for the electromagnetic field must be employed.

* Decimal classification: R111.2. Original manuscript received by the Institute, June 26, 1935; revised manuscript received by the Institute, February 14, 1936.





These generalized equations may be derived by use of the Lorentz retarded vector and scalar potentials. Assuming a radiating element of infinitesimal length $dl$ carrying a sinusoidal current of the form $I \sin \omega t$ located at the center of the co-ordinate system shown in Fig. 1, the complete equations for the electromagnetic field at any point are[1]

$$\epsilon_r = 376.76 \frac{I}{r} \frac{dl}{\lambda} \cos \theta [A \cos \omega t + B \sin \omega t] \quad (1)$$

$$\epsilon_\theta = 188.38 \frac{I}{r} \frac{dl}{\lambda} \sin \theta [C \cos \omega t + D \sin \omega t] \quad (2)$$

$$H_\phi = \frac{I}{2r} \frac{dl}{\lambda} \sin \theta [F \cos \omega t + G \sin \omega t] \quad (3)$$

in which the factors, $A$, $B$, $C$, $D$, $F$, and $G$ are defined as follows:

$$\left.\begin{aligned} A &= -\beta \sin \alpha - \beta^2 \cos \alpha \\ B &= \beta \cos \alpha - \beta^2 \sin \alpha \\ C &= (1-\beta^2) \cos \alpha - \beta \sin \alpha \\ D &= (1-\beta^2) \sin \alpha + \beta \cos \alpha \\ F &= \cos \alpha - \beta \sin \alpha \\ G &= \sin \alpha + \beta \cos \alpha \end{aligned}\right\} \quad (4)$$

and,

$$\alpha = 1/\beta = 2\pi r/\lambda. \quad (5)$$

The above equations are expressed in the ohm-volt-centimeter-second system of units. In this system the electric field intensity is expressed in volts per centimeter and the magnetic field intensity in amperes per centimeter.

The computation of the field near a finite radiator is generally simplified by transforming (1), (2), and (3) to a cylindrical co-ordinate system. These equations may be still further generalized by assuming that the current has a phase angle; that is, that the current is of the form $I \sin (\omega t - \psi)$. When these assumptions are introduced the generalized radiation equations become

$$\epsilon_1 = \frac{Idl}{\lambda^2} \sin 2\theta [F_1 \cos \omega t + F_2 \sin \omega t] \quad (6)$$

[1] *See* Page and Adams, "Principles of Electricity," Chap. XVI; J. A. Fleming, "Principles of Electric Wave Telegraphy and Telephony," Chap. V; and G. W. Pierce, "Electric Oscillations and Electric Waves," Book II, Chap. VIII. See also the articles listed under reference No. 1 of Bibliography.

$$\epsilon_2 = \frac{Idl}{\lambda^2}[(F_3 + F_1 \cos 2\theta) \cos \omega t + (F_4 + F_2 \cos 2\theta) \sin \omega t] \quad (7)$$

$$H = \frac{Idl}{\lambda^2} \sin \theta [F_5 \cos \omega t + F_6 \sin \omega t]. \quad (8)$$

Here $\epsilon_1$ is the electric field component perpendicular to and directed away from the current element and $\epsilon_2$ is the electric field component parallel to and in the same direction as the current element. $H$ is the magnetic field component previously expressed by (3).

The functions $F_1$ through $F_6$ are defined as

$$\begin{aligned} F_1 &= 94.192 \frac{\lambda}{r}[\cos \delta - 3\beta(\sin \delta + \beta \cos \delta)] \\ F_2 &= 94.192 \frac{\lambda}{r}[\sin \delta + 3\beta(\cos \delta - \beta \sin \delta)] \\ F_3 &= 94.192 \frac{\lambda}{r}[- \cos \delta - \beta(\sin \delta + \beta \cos \delta)] \\ F_4 &= 94.192 \frac{\lambda}{r}[- \sin \delta + \beta(\cos \delta - \beta \sin \delta)] \\ F_5 &= \frac{\lambda}{2r}(\cos \delta - \beta \sin \delta) \\ F_6 &= \frac{\lambda}{2r}(\sin \delta + \beta \cos \delta) \end{aligned} \quad (9)$$

in which,

$$\delta = \alpha + \psi. \quad (10)$$

These six quantities are functions only of the distance from the radiator and the phase shift of the exciting current, and are tabulated in Tables I through VI for phase shifts up to 180 degrees. For greater phase shifts these tables may be extended by the relation

$$F_n(r, \psi + 180°) = - F_n(r, \psi). \quad (11)$$

The electromagnetic field of a radiator of finite dimensions is obtained by summing the effects of the infinitesimal elements of which the radiator may be assumed to be composed.[2] If the distribution of the current along the radiator can be expressed mathematically, the field components can be expressed in terms of definite integrals. These

[2] If the radiator is near the ground, reflection from the ground must also be taken into account. This is usually done by introducing an electric image. *See* Terman, "Radio Engineering," page 497. See also references Nos. 1 and 3 of bibliography for the case in which the ground is an imperfect conductor.



may be integrated by the usual methods or may be evaluated by the use of integral functions whose values have been tabulated.[3]

An alternative method is to integrate graphically. This method is specially advantageous when the current distribution does not follow a simple mathematical form or is given in the form of an empirical curve. The labor in a computation of this type may be shortened by use of Tables I through VI. The following example shows the application of this method.

*Example.* Given a half wave length dipole. The current distribution along this radiator is assumed to follow a sine wave having a value $I_0$ at the current loop (that is, $i = I_0 \sin 2\pi l/\lambda \sin \omega t$). Find the field components on the perpendicular bisector at a distance of one-half wave length from the dipole.

From the symmetry of the problem it is apparent that there is no $\epsilon_1$ component at this point. The increments of this component contributed by the upper half of the radiator exactly cancel the increments contributed by the lower half due to the reversal in the sign of $\sin 2\theta$ in (6).

The electric component parallel to the radiator, that is $\epsilon_2$, is equal to the sum of two integrals; viz,

$$\epsilon_2 = \frac{1}{\lambda^2}\int_0^{l=\lambda/2} I[F_3 + F_1 \cos 2\theta]dl \cos \omega t + \frac{1}{\lambda^2}\int_0^{l=\lambda/2} I[F_4 + F_2 \cos 2\theta]dl \sin \omega t. \quad (12)$$

To evaluate the first of these integrals take a number of points along the dipole (it will facilitate the later application of Simpson's rule if an odd number of points including the two extremities are taken) and at these points compute the value of $I$, $r$, $\theta$, $\cos 2\theta$. Read the values of $F_1$ and $F_3$ from Tables I and III. As the current is in phase throughout this radiator the values used are those in the $\psi = 0$ column. The values of these quantities are

| $l$ | I | r | $\theta$ | cos $2\theta$ | $F_1$ | $F_3$ |
|---|---|---|---|---|---|---|
| 0λ | 0 | 0.559λ | 116°34′ | −0.6000 | − 69.6 | 186 |
| 0.05λ | 0.309 $I_0$ | 0.537λ | 111°48′ | −0.7242 | − 82.7 | 197 |
| 0.10λ | 0.588 $I_0$ | 0.521λ | 106°42′ | −0.8349 | −106 | 203 |
| 0.15λ | 0.809 $I_0$ | 0.509λ | 101°19′ | −0.9230 | −121 | 206 |
| 0.20λ | 0.951 $I_0$ | 0.502λ | 95°43′ | −0.9802 | −128 | 207 |
| 0.25λ | $I_0$ | 0.500λ | 90°00′ | −1.0000 | −131 | 207 |
| 0.30λ | 0.951 $I_0$ | 0.502λ | 84°17′ | −0.9802 | −128 | 207 |
| 0.35λ | 0.809 $I_0$ | 0.509λ | 78°41′ | −0.9230 | −121 | 206 |
| 0.40λ | 0.588 $I_0$ | 9.521λ | 73°18′ | −0.8349 | −106 | 203 |
| 0.45λ | 0.309 $I_0$ | 0.537λ | 68°12′ | −0.7242 | − 82.7 | 197 |
| 0.50λ | 0 | 0.559λ | 63°26′ | −0.6000 | − 69.6 | 186 |

[3] The most important of these functions are the sine-integral [$Si(x)$] and cosine-integral [$Ci(x)$] functions. For an example of their use see S. Ballantine, "On radiation resistance of a simple vertical antenna at wavelength below the fundamental," Proc. I.R.E., vol. 12, pp. 823–832; December, (1924). For tables of these functions see references Nos. 8 through 10 of the bibliography.

Next compute the value of

$$I(F_3 + F_1 \cos 2\theta)$$

at each point on the dipole and plot the expressions as a function of the distance along the dipole as shown in Fig. 2. The value of the integral

$$\int_0^{l=\lambda/2} I[F_3 + F_1 \cos 2\theta]dl$$

is the area under this curve between the abscissas 0 and $\lambda/2$, or the shaded area in Fig. 2.

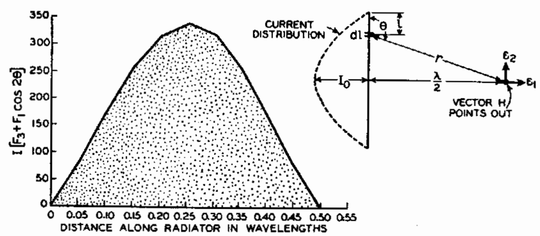


Fig. 2—Example of computation of the field by graphical integration.

This area is then evaluated by means of Simpson's rule, or any other suitable method. In this case the area, and hence the value of the integral, is 99.9 $I_0/\lambda$.

The second integral in (12) is evaluated in a like manner and the final result for $\epsilon_2$ obtained as

$$\epsilon_2(\text{volts per cm}) = 99.9 \frac{I_0}{\lambda} \cos \omega t + 38.9 \frac{I_0}{\lambda} \sin \omega t.$$

In a like manner the magnetic field components may be evaluated. They are

$$H(\text{amps per cm}) = -0.301 \frac{I_0}{\lambda} \cos \omega t - 0.1140 \frac{I_0}{\lambda} \sin \omega t.$$

## Acknowledgment

The material in this paper was originally contained in a thesis submitted to the Polytechnic Institute of Brooklyn and is published with the permission of the Institute. The writer wishes to acknowledge the assistance of Professor E. Weber of the Polytechnic Institute in preparing the mathematics of this paper and the helpful criticisms given him by Professor F. W. Grover of Union College.

TABLE I—$F_1$

| $r/\lambda$ \ $\psi$ | 0 | 18° | 36° | 54° | 72° | 90° | 108° | 126° | 144° | 162° | 180° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.05 | −58230 | −55380 | −47100 | −34220 | −17990 | 3.985 | 18000 | 34230 | 47110 | 55380 | 58230 |
| 0.10 | −7673 | −7292 | −6198 | −4497 | −2356 | 15.31 | 2385 | 4522 | 6216 | 7302 | 7673 |
| 0.15 | −2495 | −2362 | −1999 | −1440 | −739.6 | 32.80 | 802.0 | 1493 | 2037 | 2383 | 2495 |
| 0.20 | −1200 | −1124 | −938.4 | −660.5 | −318.0 | 55.59 | 423.8 | 750.5 | 1004 | 1159 | 1200 |
| 0.25 | −719.6 | −659.2 | −534.3 | −357.2 | −145.0 | 81.33 | 299.7 | 488.7 | 630.0 | 709.5 | 719.6 |
| 0.30 | −490.3 | −433.0 | −333.3 | −200.9 | −48.87 | 107.9 | 254.2 | 375.5 | 460.1 | 499.7 | 490.3 |
| 0.40 | −265.2 | −204.4 | −123.6 | −30.73 | 65.19 | 154.7 | 229.1 | 281.1 | 305.5 | 300.1 | 265.2 |
| 0.50 | −131.1 | −59.69 | −0.3408 | 68.46 | 130.6 | 179.9 | 211.6 | 222.6 | 211.8 | 180.1 | 131.1 |
| 0.60 | −26.77 | 28.27 | 80.54 | 124.9 | 157.1 | 173.9 | 173.6 | 156.4 | 123.8 | 79.18 | 26.77 |
| 0.70 | 52.38 | 91.78 | 121.9 | 141.1 | 145.9 | 136.5 | 113.7 | 79.76 | 38.03 | −7.295 | −52.38 |
| 0.80 | 98.90 | 117.8 | 125.2 | 120.4 | 103.8 | 76.96 | 42.64 | 4.136 | −34.77 | −70.27 | −98.90 |
| 0.90 | 109.4 | 107.4 | 94.84 | 73.04 | 44.09 | 10.83 | −23.50 | −55.52 | −82.11 | −100.7 | −109.4 |
| 1.0 | 87.03 | 68.88 | 43.98 | 14.77 | −15.88 | −44.97 | −69.67 | −87.54 | −96.85 | −96.67 | −87.03 |
| 1.2 | −6.727 | −31.23 | −52.68 | −68.97 | −78.51 | −80.36 | −74.35 | −61.06 | −41.79 | −18.44 | 6.727 |
| 1.4 | −65.81 | −68.60 | −64.67 | −54.42 | −38.83 | −19.45 | 1.838 | 22.95 | 41.81 | 56.58 | 65.81 |
| 1.6 | −35.89 | −19.36 | −0.9440 | 17.57 | 34.36 | 47.79 | 56.54 | 59.76 | 57.12 | 48.90 | 35.89 |
| 1.8 | 28.99 | 41.27 | 49.50 | 52.89 | 51.10 | 44.31 | 33.18 | 18.81 | 2.590 | −13.88 | −28.99 |
| 2.0 | 46.20 | 40.47 | 30.77 | 18.06 | 3.584 | −11.24 | −24.97 | −36.25 | −43.99 | −47.41 | −46.20 |
| 2.5 | −37.22 | −33.17 | −25.88 | −16.06 | −4.658 | 7.196 | 18.34 | 27.70 | 34.34 | 37.62 | 37.22 |
| 3.0 | 31.13 | 28.06 | 22.31 | 14.26 | 4.868 | −4.997 | −14.37 | −22.34 | −28.12 | −31.15 | −31.13 |

Note: The values of $\psi$ in Tables I through VI are the phase shift at intervals 0.05λ along a radiator.

TABLE II—$F_3$

| $r/\lambda$ \ $\psi$ | 0 | 18° | 36° | 54° | 72° | 90° | 108° | 126° | 144° | 162° | 180° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.05 | −3.985 | −18000 | −34230 | −47110 | −55380 | −58230 | −55380 | −47100 | −34220 | −17990 | 3.985 |
| 0.10 | −15.31 | −2385 | −4522 | −6216 | −7302 | −7673 | −7292 | −6198 | −4497 | −2356 | 15.31 |
| 0.15 | −32.80 | −802.0 | −1493 | −2037 | −2382 | −2495 | −2362 | −1999 | −1440 | −739.6 | 32.80 |
| 0.20 | −55.59 | −432.8 | −750.7 | −1004 | −1159 | −1200 | −1124 | −938.4 | −660.5 | −318.0 | 55.59 |
| 0.25 | −81.33 | −299.7 | −488.7 | −630.0 | −709.5 | −719.6 | −659.2 | −534.3 | −357.2 | −145.0 | 81.33 |
| 0.30 | −107.9 | −254.2 | −375.5 | −460.1 | −499.7 | −490.3 | −433.0 | −333.3 | −200.9 | −48.87 | 107.9 |
| 0.40 | −154.7 | −229.1 | −281.1 | −305.5 | −300.1 | −265.2 | −204.4 | −123.6 | −30.73 | 65.19 | 154.7 |
| 0.50 | −179.9 | −211.6 | −222.6 | −211.8 | −180.1 | −131.1 | −59.69 | −0.3408 | 68.46 | 130.6 | 179.9 |
| 0.60 | −173.9 | −173.6 | −156.4 | −123.8 | −79.18 | −26.77 | 28.27 | 80.54 | 124.9 | 157.1 | 173.9 |
| 0.70 | −136.5 | −113.7 | −79.76 | −38.03 | 7.295 | 52.38 | 91.78 | 121.9 | 141.1 | 145.9 | 136.5 |
| 0.80 | −76.96 | −42.64 | −4.136 | 34.77 | 70.27 | 98.90 | 117.8 | 125.2 | 120.4 | 103.8 | 76.96 |
| 0.90 | −10.83 | 23.50 | 55.52 | 82.11 | 100.7 | 109.4 | 107.4 | 94.84 | 73.04 | 44.09 | 10.83 |
| 1.0 | 44.97 | 69.67 | 87.54 | 96.85 | 96.67 | 87.03 | 68.88 | 43.98 | 14.77 | −15.88 | −44.97 |
| 1.2 | 80.36 | 74.35 | 61.06 | 41.79 | 18.44 | −6.727 | −31.23 | −52.68 | −68.97 | −78.51 | −80.36 |
| 1.4 | 19.45 | −1.838 | −22.95 | −41.81 | −56.58 | −65.81 | −68.60 | −64.67 | −54.42 | −38.83 | −19.45 |
| 1.6 | −47.49 | −56.54 | −59.76 | −57.12 | −48.90 | −35.89 | −19.36 | −0.9440 | 17.57 | 34.36 | 47.79 |
| 1.8 | −44.31 | −33.18 | −18.81 | −2.590 | 13.88 | 28.99 | 41.27 | 49.50 | 52.89 | 51.10 | 44.31 |
| 2.0 | 11.24 | 24.97 | 36.25 | 43.99 | 47.41 | 46.20 | 40.47 | 30.77 | 18.06 | 3.584 | −11.24 |
| 2.5 | −7.196 | −18.34 | −27.70 | −34.34 | −37.62 | −37.22 | −33.17 | −25.88 | −16.06 | −4.658 | 7.196 |
| 3.0 | 4.997 | 14.37 | 22.34 | 28.12 | 31.15 | 31.13 | 28.06 | 22.31 | 14.26 | 4.868 | −4.997 |

TABLE III—$F_3$

| $r/\lambda$ | $\psi$ = 0 | 18° | 36° | 54° | 72° | 90° | 108° | 126° | 144° | 162° | 180° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.05 | −21800 | −20490 | −17180 | −12180 | −5996 | 777.5 | 7475 | 13440 | 18090 | 20970 | 21800 |
| 0.10 | −3574 | −3169 | −2454 | −1499 | −397.4 | 743.3 | 1811 | 2659 | 3328 | 3628 | 3574 |
| 0.15 | −1324 | −1046 | −666.5 | −221.2 | 245.6 | 688.3 | 1064 | 1335 | 1475 | 1472 | 1324 |
| 0.20 | −594.1 | −374.8 | −118.9 | 148.9 | 402.0 | 615.7 | 769.2 | 847.4 | 842.6 | 755.3 | 594.1 |
| 0.25 | −239.9 | −64.50 | 117.2 | 287.4 | 429.4 | 529.5 | 577.7 | 569.3 | 505.3 | 391.7 | 239.9 |
| 0.30 | −34.08 | 40.22 | 227.6 | 331.2 | 402.3 | 434.1 | 423.4 | 371.2 | 282.7 | 166.6 | 34.08 |
| 0.40 | 165.6 | 230.5 | 272.8 | 288.4 | 275.7 | 236.1 | 173.4 | 93.69 | 4.821 | −84.52 | −165.6 |
| 0.50 | 207.5 | 215.8 | 203.1 | 170.5 | 121.1 | 59.96 | −7.083 | −73.43 | −132.6 | −179.1 | −207.5 |
| 0.60 | 160.4 | 101.7 | 91.53 | 41.64 | −12.32 | −65.08 | −111.5 | −146.9 | −168.0 | −172.7 | −160.4 |
| 0.70 | 72.90 | 30.59 | −14.63 | −58.43 | −96.51 | −125.1 | −141.5 | −144.0 | −132.5 | −107.9 | −72.90 |
| 0.80 | −15.55 | −52.99 | −85.26 | −109.2 | −122.4 | −123.6 | −112.8 | −90.90 | −60.10 | −23.42 | −15.55 |
| 0.90 | −76.44 | −96.93 | −107.9 | −108.4 | −98.20 | −78.41 | −50.95 | −18.51 | 15.75 | 48.47 | 76.44 |
| 1.0 | −96.58 | −96.48 | −86.94 | −68.90 | −44.10 | −14.99 | 15.59 | 44.64 | 69.32 | 87.22 | 96.58 |
| 1.2 | −34.58 | −10.41 | 14.78 | 38.53 | 58.50 | 72.75 | 79.87 | 79.18 | 70.74 | 55.37 | 34.58 |
| 1.4 | 50.64 | 62.45 | 68.15 | 67.18 | 59.63 | 46.25 | 28.33 | 7.648 | −13.79 | −33.87 | −50.64 |
| 1.6 | 51.54 | 39.68 | 23.94 | 5.856 | −12.80 | −30.21 | −44.66 | −54.73 | −59.45 | −58.35 | −51.54 |
| 1.8 | −11.90 | −27.26 | −39.95 | −48.73 | −52.74 | −51.86 | −45.39 | −34.74 | −20.70 | −4.627 | 11.90 |
| 2.0 | −47.39 | −46.23 | −40.55 | −30.89 | −18.21 | −3.748 | 11.08 | 24.83 | 36.14 | 43.92 | 47.39 |
| 2.5 | 37.83 | 36.72 | 32.01 | 24.18 | 13.97 | 2.399 | −9.409 | −20.30 | −29.19 | −35.24 | −37.83 |
| 3.0 | −31.49 | −30.46 | −26.43 | −19.85 | −11.31 | −1.666 | 8.145 | 17.16 | 24.49 | 29.43 | 31.49 |

TABLE IV—$F_4$

| $r/\lambda$ | $\psi$ = 0° | 18° | 36° | 54° | 72° | 90° | 108° | 126° | 144° | 162° | 180° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.05 | −777.5 | −7475 | −13440 | −18090 | −20970 | −21800 | −20490 | −17180 | −12180 | −5996 | 777.5 |
| 0.10 | −743.3 | −1811 | −2659 | −3328 | −3628 | −3574 | −3169 | −2454 | −1499 | −397.4 | 743.3 |
| 0.15 | −688.3 | −1064 | −1335 | −1475 | −1472 | −1324 | −1046 | −666.5 | −221.2 | 245.6 | 688.3 |
| 0.20 | −615.7 | −769.2 | −847.4 | −842.6 | −755.3 | −594.1 | −374.8 | −118.9 | 148.9 | 402.0 | 615.7 |
| 0.25 | −529.5 | −577.7 | −569.3 | −505.3 | −391.7 | −239.9 | −64.50 | 117.2 | 287.4 | 429.4 | 529.5 |
| 0.30 | −434.1 | −423.4 | −371.2 | −282.7 | −166.6 | −34.08 | 40.22 | 227.6 | 331.2 | 402.3 | 434.1 |
| 0.40 | −236.1 | −173.4 | −93.69 | −4.821 | 84.52 | 165.6 | 230.5 | 272.8 | 288.4 | 275.7 | 236.1 |
| 0.50 | −59.96 | 7.083 | 73.43 | 132.6 | 179.1 | 207.5 | 215.8 | 203.1 | 170.5 | 121.1 | 59.96 |
| 0.60 | 65.08 | 111.5 | 146.9 | 168.0 | 172.7 | 160.4 | 101.7 | 91.53 | 41.64 | −12.32 | −65.08 |
| 0.70 | 125.1 | 141.5 | 144.0 | 132.5 | 107.9 | 72.90 | 30.59 | −14.63 | −58.43 | −96.51 | −125.1 |
| 0.80 | 123.6 | 112.8 | 90.90 | 60.10 | 23.42 | −15.55 | −52.99 | −85.26 | −109.2 | −122.4 | −123.6 |
| 0.90 | 78.41 | 50.95 | 18.51 | −15.75 | −48.47 | −76.44 | −96.93 | −107.9 | −108.4 | −98.20 | −78.41 |
| 1.0 | 14.99 | −15.59 | −44.64 | −69.32 | −87.22 | −96.58 | −96.48 | −86.94 | −68.90 | −44.10 | −14.99 |
| 1.2 | −72.75 | −79.87 | −79.18 | −70.74 | −55.37 | −34.58 | −10.41 | 14.78 | 38.53 | 58.50 | 72.75 |
| 1.4 | −46.25 | −28.33 | −7.648 | 13.79 | 33.87 | 50.64 | 62.45 | 68.15 | 67.18 | 59.63 | 46.25 |
| 1.6 | 30.21 | 44.66 | 54.73 | 59.45 | 58.35 | 51.54 | 39.68 | 23.94 | 5.856 | −12.80 | −30.21 |
| 1.8 | 51.86 | 45.39 | 34.74 | 20.70 | 4.627 | −11.90 | −27.26 | −39.95 | −48.73 | −52.74 | −51.86 |
| 2.0 | 3.748 | −11.08 | −24.83 | −36.14 | −43.92 | −47.39 | −46.23 | −40.55 | −30.89 | −18.21 | −3.748 |
| 2.5 | −2.399 | 9.409 | 20.30 | 29.19 | 35.24 | 37.83 | 36.72 | 32.01 | 24.18 | 13.97 | 2.399 |
| 3.0 | 1.666 | −8.145 | −17.16 | −24.49 | −29.43 | −31.49 | −30.46 | −26.43 | −19.85 | −11.31 | −1.666 |

TABLE V—$F_5$

| $r/\lambda$ | $\psi$ = 0 | 18° | 36° | 54° | 72° | 90° | 108° | 126° | 144° | 162° | 180° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.05 | −0.3258 | −10.62 | −19.87 | −27.18 | −31.83 | −33.36 | −31.63 | −26.80 | −19.35 | −10.00 | 0.3258 |
| 0.10 | −0.6325 | −3.499 | −6.023 | −7.958 | −9.114 | −9.377 | −8.723 | −7.214 | −5.000 | −2.296 | 0.6325 |
| 0.15 | −0.9019 | −2.334 | −3.537 | −4.394 | −4.821 | −4.776 | −4.263 | −3.333 | −2.077 | −0.6179 | 0.9019 |
| 0.20 | −1.120 | −1.989 | −2.665 | −3.079 | −3.192 | −2.992 | −2.500 | −1.763 | −0.8532 | 0.1400 | 1.120 |
| 0.25 | −1.273 | −1.829 | −2.206 | −2.366 | −2.296 | −2.000 | −1.509 | −0.8696 | −0.1455 | 0.5929 | 1.273 |
| 0.30 | −1.356 | −1.695 | −1.868 | −1.858 | −1.667 | −1.312 | −0.8286 | −0.2643 | 0.3259 | 0.8842 | 1.356 |
| 0.40 | −1.304 | −1.343 | −1.250 | −1.035 | −0.7189 | −0.3324 | 0.08675 | 0.4974 | 0.8593 | 1.137 | 1.304 |
| 0.50 | −1.000 | −0.8527 | −0.6219 | −0.3303 | −0.006288 | 0.3183 | 0.6118 | 0.8453 | 0.9961 | 1.049 | 1.000 |
| 0.60 | −0.5443 | −0.3234 | −0.04728 | 0.2299 | 0.4865 | 0.6687 | 0.8363 | 0.8609 | 0.8667 | 0.7532 | 0.5443 |
| 0.70 | −0.06627 | 0.1624 | 0.3752 | 0.5512 | 0.6733 | 0.7312 | 0.7143 | 0.6291 | 0.4824 | 0.2885 | 0.06627 |
| 0.80 | 0.3114 | 0.4680 | 0.5787 | 0.6328 | 0.6250 | 0.5560 | 0.4325 | 0.2668 | 0.07488 | −0.1243 | −0.3114 |
| 0.90 | 0.5072 | 0.5587 | 0.5556 | 0.4901 | 0.3917 | 0.2471 | 0.07241 | −0.09824 | −0.2651 | −0.4060 | −0.5072 |
| 1.0 | 0.5000 | 0.4509 | 0.3577 | 0.2295 | 0.07882 | −0.07958 | −0.2302 | −0.3583 | −0.4513 | −0.5001 | −0.5000 |
| 1.2 | 0.07620 | −0.05526 | −0.1813 | −0.2896 | −0.3696 | −0.4134 | −0.4167 | −0.3792 | −0.3042 | −0.2002 | −0.07620 |
| 1.4 | −0.3128 | −0.3522 | −0.3571 | −0.3271 | −0.2651 | −0.1771 | −0.07175 | 0.04060 | 0.1490 | 0.2428 | 0.3128 |
| 1.6 | −0.2346 | −0.1585 | −0.06701 | 0.03109 | 0.1261 | 0.2088 | 0.2711 | 0.3068 | 0.3125 | 0.2876 | 0.2346 |
| 1.8 | 0.1092 | 0.1831 | 0.2392 | 0.2718 | 0.2778 | 0.2566 | 0.2103 | 0.1434 | 0.06248 | −0.02456 | −0.1092 |
| 2.0 | 0.2500 | 0.2316 | 0.1906 | 0.1309 | 0.05834 | −0.01989 | −0.09618 | −0.1630 | −0.2139 | −0.2441 | −0.2500 |
| 2.5 | −0.2000 | −0.1863 | −0.1543 | −0.1073 | −0.04969 | 0.01273 | 0.07391 | 0.1279 | 0.1693 | 0.1941 | 0.2000 |
| 3.0 | 0.1667 | 0.1558 | 0.1297 | 0.09081 | 0.04309 | −0.008842 | −0.05991 | −0.1032 | −0.1400 | −0.1612 | −0.1667 |

TABLE VI—$F_6$

| $r/\lambda$ | $\psi$ = 0 | 18° | 36° | 54° | 72° | 90° | 108° | 126° | 144° | 162° | 180° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.05 | 33.36 | 31.63 | 26.80 | 19.35 | 10.00 | −0.3258 | −10.62 | −19.87 | −27.18 | −31.83 | −33.36 |
| 0.10 | 9.377 | 8.723 | 7.214 | 5.000 | 2.296 | −0.6325 | −3.499 | −6.023 | −7.958 | −9.114 | −9.377 |
| 0.15 | 4.776 | 4.263 | 3.333 | 2.077 | 0.6179 | −0.9019 | −2.334 | −3.537 | −4.394 | −4.821 | −4.776 |
| 0.20 | 2.992 | 2.500 | 1.763 | 0.8532 | −0.1400 | −1.120 | −1.989 | −2.665 | −3.079 | −3.192 | −2.992 |
| 0.25 | 2.000 | 1.509 | 0.8696 | 0.1455 | −0.5929 | −1.273 | −1.829 | −2.206 | −2.366 | −2.296 | −2.000 |
| 0.30 | 1.312 | 0.8286 | 0.2643 | −0.3259 | −0.8842 | −1.356 | −1.695 | −1.868 | −1.858 | −1.667 | −1.312 |
| 0.40 | 0.3324 | −0.08675 | −0.4974 | −0.8593 | −1.137 | −1.304 | −1.343 | −1.250 | −1.035 | −0.7189 | −0.3324 |
| 0.50 | −0.3183 | −0.6118 | −0.8453 | −0.9961 | −1.049 | −1.000 | −0.8527 | −0.6219 | −0.3303 | −0.006288 | 0.3183 |
| 0.60 | −0.6687 | −0.8363 | −0.8609 | −0.8667 | −0.7532 | −0.5443 | −0.3234 | −0.04728 | 0.2299 | 0.4865 | 0.6687 |
| 0.70 | −0.7312 | −0.7143 | −0.6291 | −0.4824 | −0.2885 | −0.06627 | 0.1624 | 0.3752 | 0.5512 | 0.6733 | 0.7312 |
| 0.80 | −0.5560 | −0.4325 | −0.2668 | −0.07488 | 0.1243 | 0.3114 | 0.4680 | 0.5787 | 0.6328 | 0.6250 | 0.5560 |
| 0.90 | −0.2471 | −0.07241 | 0.09824 | 0.2651 | 0.4060 | 0.5072 | 0.5587 | 0.5556 | 0.4901 | 0.3917 | 0.2471 |
| 1.0 | 0.07958 | 0.2302 | 0.3583 | 0.4513 | 0.5001 | 0.5000 | 0.4509 | 0.3577 | 0.2295 | 0.07882 | −0.07958 |
| 1.2 | 0.4134 | 0.4167 | 0.3792 | 0.3042 | 0.2002 | 0.07620 | −0.05526 | −0.1813 | −0.2896 | −0.3696 | −0.4134 |
| 1.4 | 0.1771 | 0.07175 | −0.04060 | −0.1490 | −0.2428 | −0.3128 | −0.3522 | −0.3571 | −0.3271 | −0.2651 | −0.1771 |
| 1.6 | −0.2088 | −0.2711 | −0.3068 | −0.3125 | −0.2876 | −0.2346 | −0.1585 | −0.06701 | 0.03109 | 0.1261 | 0.2088 |
| 1.8 | −0.2566 | −0.2103 | −0.1434 | −0.06248 | 0.02456 | 0.1092 | 0.1831 | 0.2392 | 0.2718 | 0.2778 | 0.2566 |
| 2.0 | 0.01989 | 0.09618 | 0.1630 | 0.2139 | 0.2441 | 0.2500 | 0.2316 | 0.1906 | 0.1309 | 0.05834 | −0.01989 |
| 2.5 | −0.01273 | −0.07391 | −0.1279 | −0.1693 | −0.1941 | −0.2000 | −0.1863 | −0.1543 | −0.1073 | −0.04969 | 0.01273 |
| 3.0 | 0.008842 | 0.05991 | 0.1032 | 0.1400 | 0.1612 | 0.1667 | 0.1558 | 0.1297 | 0.09081 | 0.04309 | −0.008842 |

## Bibliography

In addition to references already cited in footnotes the following references may be of interest to the reader. This list does not include all available material and only lists articles on antennas which contain information on the field near the antenna or the generalized form of the radiation equations.

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# BOOK REVIEWS

**"Your Invention—How to Protect and Merchandise It,"** by Elmore B. Lyford, 210 Pages. Radio and Technical Publishing Company, 45 Astor Place, New York City, 1935. Price $1.50.

This book is written primarily to meet the needs of the experimenter and inventor who, when he considers applying for a patent in the United States, finds himself on unfamiliar ground.

The book is divided into five parts. Part 1 outlines the nature of a patentable invention and its relation to the prior art according to whether the invention is basic or is an improvement on other inventions. The procedure for applying for a patent is treated in some detail including the preliminary search; the preparation of the specification, drawings, and claims; the patent office action; and the probable cost of obtaining a simple patent. Interferences are discussed and the importance of recording the conception of an idea and showing diligence in following it up is emphasized. The question of patent agreements between employer and employees and the rights of each under varied conditions are mentioned. Advice is given concerning the selection of an attorney and the avoidance of certain classes of mail order attorneys.

Part 2 is concerned with the exploitation of the invention. It is shown that certain types of inventions are obviously most readily exploited by existing manufacturers or businesses in the particular field of the invention, while other inventions may be suitable for special exploitation. In the former event, advice is given for locating the most likely prospects and the questions of licenses, royalties, and promoter's fees are discussed with an indication as to how these matters are usually evaluated.

In case the invention is of such nature that it can be exploited on its own merits, suggestions are given as to how and where capital can be raised and advice is given as to the usual terms drawn up between the inventor and his financial backers.

This section also discusses briefly the question of foreign patents and shows a condensed summary of the patent laws of various foreign countries which includes the maximum term of the grant, whether the patents are taxed, and how soon the exclusive rights are lost if not used.

Information is also given concerning design patents.

Parts 3 and 4 discuss trade-marks and copyrights pointing out the similarities and differences as compared to patents. Information is given as to the procedure and cost of obtaining trade-marks and copyrights.

Part 5 contains a list of twenty-four suggested legal forms for petitions, oaths, assignments, etc. It also contains several pages of extracts from the patent laws of the United States.

The major sections are written in nontenchnical language but give a fairly comprehensive and accurate picture of the more important features of patent law.

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