On the Lower Bound of the Radiation Q for Electrically Small Antennas

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***Abstract*—The fundamental question of the lower bound on the radiation Q of an electrically small antenna is of practical importance because of its relationship to the antenna bandwidth. Previous works predict a lower bound on the radiation Q that is usually too low and, hence, a bandwidth that can be optimistically large. This paper addresses why this is so and offers a new prediction for a realizable lower bound on the radiation Q. This new prediction is based on the far-field pattern, in both the visible and invisible spatial regions, in contrast to previous works based upon a near-field modal approach. Results for a linear dipole, bow-tie, and end-loaded dipole are presented to illustrate the validity of the lower bound presented herein.**

Radiation Q can be related to bandwidth provided the Q is adequately large. Implicit is the presence of a matching network as a part of the antenna system. Both the losses in the antenna and the losses in the matching network have an effect on the system bandwidth, the system efficiency and the system Q, of which the radiation Q is a part. These various relationships are also discussed.

***Index Terms*—Antenna theory, antennas, apertures, dipole antennas, electrically small antennas, electromagnetic radiation, linear antennas, Q factor, wire antennas.**

I. Introduction

THE radiation Q of electrically small antennas is a quantity of fundamental interest. On the other hand, a quantity of practical interest is the bandwidth of electrically small antennas. When the radiation Q, denoted herein by $\mathrm{Q_A}$, is adequately large (i.e., $\mathrm{Q_A} > 10$), the fractional bandwidth is approximately equal to the inverse of $\mathrm{Q_A}$. It is important to know the maximum possible bandwidth and hence the lower bound on $\mathrm{Q_A}$ in a number of practical situations. For example, in order to achieve a specified bandwidth for an antenna for a wireless device, a knowledge of the lower bound on the radiation Q would set a limit on how small the antenna could be. More emphatically, if bandwidth and size were specifications given to a designer, a knowledge of the lower bound on the radiation Q could determine if the two specifications were compatible and could be satisfied simultaneously.

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A fundamental definition for the radiation Q, or $\mathrm{Q_A}$, of an antenna is

$$\mathrm{Q_A} = \begin{cases} \frac{2\omega W_e}{P_r} & W_e > W_m \\ \frac{2\omega W_m}{P_r} & W_m > W_e \end{cases} \tag{1}$$

where $\mathrm{W_e}$ is the time average, nonpropagating stored electric energy, $\mathrm{W_m}$ is the time-average, nonpropagating stored magnetic energy, ω is the radian frequency, and $\mathrm{P_r}$ is the radiated power.

An approximate representation of $\mathrm{Q_A}$, when the antenna input impedance can be adequately represented by a lumped circuit (i.e., an electrically small antenna), is

$$\mathrm{Q_A} = \frac{X_A}{R_A} \quad \text{provided} \quad \mathrm{Q_A} > 10 \tag{2}$$

where $\mathrm{X_A}$ is the antenna input reactance and $\mathrm{R_A}$ is the antenna input resistance. For $5 < \mathrm{Q_A} < 10$, the relationship in (2) becomes questionable and for $\mathrm{Q_A} < 5$ it is not likely to be valid.

An antenna is generally considered electrically small if it can fit inside a radiansphere. The term radiansphere was originated by Wheeler [1], [2] in his early works and denotes an imaginary sphere of radius equal to $\lambda/2\pi$ (a diameter of about 1/3 wavelength, λ). In 1948 Chu [3] published a classic and widely quoted paper in which he derived an expression for the minimum radiation Q by expressing the fields around a small hypothetical antenna in series of spherical modes, which give the total energy surrounding the hypothetical antenna. In an effort to separate the nonpropagating energy from the total energy so as to know both the propagating and nonpropagating energy separately, Chu used an ingenious ladder network approximate representation of the modes to indirectly calculate the radiation Q. Chu's approximate result was subsequently quoted by others in related works over the following years [4]–[9].

In 1996 McLean [10] improved on the work of Chu by deriving an exact result for the radiation Q using the fields for the $\mathrm{TM_{01}}$ mode directly. McLean's result is nearly the same as Chu's for very small antennas and predicts a slightly higher lower bound on $\mathrm{Q_A}$ for antennas approaching a length of about 1/3 wavelength. McLean's result is

$$\mathrm{Q_A} = \left[\frac{1}{\beta^3 a^3} + \frac{1}{\beta a}\right] \tag{3}$$

where *a* is the radius of the smallest sphere that encloses a hypothetical radiator and β is the wavenumber.

The lower bounds provided by Chu and McLean have been found to be elusive to achieve in practice (i.e., the $\mathrm{Q_A}$ is always higher and the bandwidth less than expected). The primary purpose of this paper is to address why this is so.

One argument [7] often given for practical designs having a higher $\mathrm{Q_A}$ than the lower bounds given in [3] and [10], is that the entire spherical volume is not utilized by the antenna. In an attempt to overcome this factor, Foltz and McLean [11] expanded the fields around an antenna using prolate spheroidal functions (instead of spherical functions) and found that $\mathrm{Q_A}$ was dependent on the ratio of the major and minor axis of the prolate bounding surface. This served to raised the expected $\mathrm{Q_A}$ for antennas, such as a small dipole. A small dipole, of course, does not consume much of a spherical volume and is better represented by a prolate spheroidal volume. However, it is not apparent in their work what ratio of the major to minor axis of the prolate bounding surface should be used for a specific antenna configuration.

In this paper, a much different approach is taken to establish the lower bound on the radiation Q for electrically small antennas. Instead of expanding the near-fields around an antenna, the far-field pattern is used. The approach here relies on the concept of superdirectivity. It is well-known that electrically small antennas are superdirective, since they have more directivity than their small size warrants. Radiation Q can be related to superdirectivity. Results using this different approach give a higher lower bound on the radiation Q and suggest that the lower bound on $\mathrm{Q_A}$ for an electrically small antenna can be determined from how superdirective the antenna pattern is.

II. Superdirective Approach

Superdirectivity is produced by an interference process whereby a portion of the antenna pattern is scanned into the invisible region. This causes energy to be stored in the near-field resulting in a large radiation Q. A superdirective ratio $\mathrm{R_{SD}}$ has been defined as the ratio of an integration of the far-field pattern function over all space (i.e., visible + invisible) to an integration over visible space as follows [12]–[15]:

$$R_{\mathrm{SD}} = \frac{\int_{-\infty}^{\infty} |f(u)|^2\, du}{\int_{\frac{-\pi L}{\lambda}}^{\frac{\pi L}{\lambda}} |f(u)|^2\, du} \tag{4}$$

where $f(u)$ is a normalized far-field pattern function in u-space and $u = (\beta \mathrm{L}/2)\cos\theta$ when main beam radiation is broadside to the source (i.e., linear source of length L with constant phase) and the source is along the z axis. The superdirective ratio is not new [15] and is recognized as a measure of the realizability of an antenna. Rhodes in [16] comments that (4) above "is something like $\mathrm{Q_A}$" and "is somewhat similar to $\mathrm{Q_A}+1$."

In [12]–[14] $\mathrm{R_{SD}}$ in (4) is equal to $1+\mathrm{Q_A}$ *if*

$$\mathrm{Q_A} = \frac{\int_{-\infty}^{\frac{-\pi L}{\lambda}} |f(u)|^2\, du + \int_{\frac{\pi L}{\lambda}}^{\infty} |f(u)|^2\, du}{\int_{\frac{-\pi L}{\lambda}}^{\frac{\pi L}{\lambda}} |f(u)|^2\, du}. \tag{5}$$

However, we have found in our investigation that it is not adequate to use just the far-field pattern function $f(\mathrm{u})$ in (5) as is written in [12]–[14]. The expression in (5) should be consistent with the fundamental definition in (1) and, therefore, should include the element pattern $g(u)$ where

$$\mathrm{E}_n(\mathrm{u}) = \mathrm{g(u)f(u)} \tag{6}$$

and $E_n(u)$ is the normalized field pattern. Rhodes [16] had no need to consider the element pattern, g(u), in his synthesis work since g(u) "is not a controllable part of the radiation pattern."

Therefore, the expression used to calculate $\mathrm{Q_A}$ here is

$$\mathrm{Q_A} = \frac{\int_{-\infty}^{\frac{-\pi L}{\lambda}} |E_n(u)|^2\, du + \int_{\frac{\pi L}{\lambda}}^{\infty} |E_n(u)|^2\, du}{\int_{\frac{-\pi L}{\lambda}}^{\frac{\pi L}{\lambda}} |E_n(u)|^2\, du}. \tag{7}$$

We will use (7) to determine the lower bound on $\mathrm{Q_A}$ for an electrically small dipole with a sinusoidal current distribution.

For a dipole of arbitrary length along the z axis with a sinusoidal current distribution, it is well-known that the far-zone electric field is [14]

$$E_\theta = j\eta \frac{e^{-j\beta r}}{2\pi r} I_m \frac{\cos\left[\left(\frac{\beta L}{2}\right)\cos\theta\right] - \cos\left(\frac{\beta L}{2}\right)}{\sin\theta}. \tag{8}$$

For $\mathrm{L} < \lambda$, the maximum value of (8) occurs at $\theta = \pi/2$, so

$$E_{\theta_{\max}} = j\eta \frac{e^{-j\beta r}}{2\pi r} I_m \frac{1-\cos\left(\frac{\beta L}{2}\right)}{1}. \tag{9}$$

Thus, the normalized field pattern, $E_n(\theta)$, can be written as

$$\begin{aligned} E_n(\theta) &= \frac{E_\theta(\theta)}{E_{\theta_{\max}}} \\ &= \sin\theta \frac{\cos\left[\left(\frac{\beta L}{2}\right)\cos\theta\right] - \cos\left(\frac{\beta L}{2}\right)}{\left[1-\cos\left(\frac{\beta L}{2}\right)\right]\sin^2\theta} \\ &= g(\theta) f(\theta). \end{aligned} \tag{10}$$

The normalization is for convenience. Since $\mathrm{u} = ((\beta L)/2)\cos\theta$, it follows that $\sin\theta$ is given by

$$\begin{aligned} \sin\theta &= \sqrt{1-\cos^2\theta} \\ &= \sqrt{1-\left(\frac{2}{\beta L}\right)^2 \left(\frac{\beta L}{2}\cos\theta\right)^2} \\ &= \sqrt{1-\left(\frac{2u}{\beta L}\right)^2} \\ &= g(u) \end{aligned} \tag{11}$$

and $E_n(u)$ in turn is given by

$$E_n(u) = \frac{\cos u - \cos\left(\frac{\beta L}{2}\right)}{\left[1-\cos\left(\frac{\beta L}{2}\right)\right]\sqrt{1-\left(\frac{2u}{\beta L}\right)}^2}. \tag{12}$$

Thus, (12) is used in (7) to compute $\mathrm{Q_A}$ of an electrically small dipole with a sinusoidal current distribution. Note that (12) vanishes at infinity. The integrations in (7) could be carried out in θ-space. Our preference is to work in u space.

III. Results

Fig. 1 shows a curve based on (7) and (12), denoted "far-field Q," for a dipole that generates the far-field pattern given by (8). That (middle) curve is above the lower curve from (3) due to McLean but below the third (top) curve based on (2) for the ratio of $\mathrm{X_A}/\mathrm{R_A}$. The (top) curve based on (2) was calculated using the Method of Moments for a dipole of radius 0.005λ.

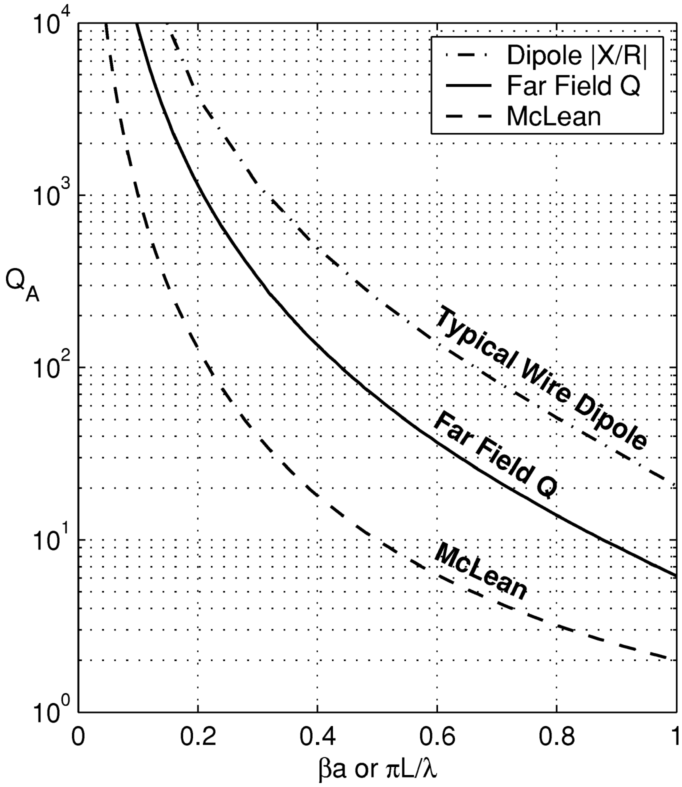

Fig. 1. Radiation Q versus βa or πL/λ where β is the wavenumber, a is the radius (L is the diameter) of the smallest sphere that can enclose the radiator. Bottom curve is the *lower bound* for an ideal or Hertzian dipole, which has a uniform current distribution, using (3). Middle curve is the *lower bound* for a dipole with a sinusoidal current distribution using (7). Top curve is a typical radiation Q for a thin wire dipole using the ratio of input reactance to radiation resistance.

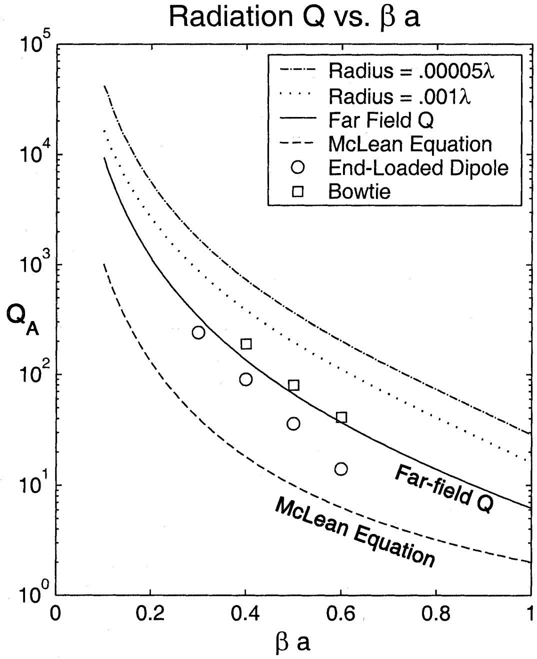

Fig. 2. Radiation Q versus βa. Top two curves are for thin linear dipoles using X/R. Bottom curve is the lower bound for an ideal or Hertzian dipole, which has a uniform current distribution. Solid line curve is the lower bound for a sinusoidal current dipole. Bow-tie (Fig. 3) and end-loaded dipole (Fig. 4) data points are determined using X/R.

For several reasons we believe that the middle curve in Fig. 1, derived from the far-field pattern, more accurately represents the lower bound on Q_A for an electrically small linear dipole with a sinusoidal current distribution.

First, the curve from McLean's result is derived using the lowest order TM mode and that mode *only*. The TM_{01} mode, as pointed out by McLean, generates fields that are equivalent to the fields of an ideal (or Hertzian) dipole with a $\sin\theta$ pattern. Such fields are associated with a uniform current that obviously does not go to zero at the dipole ends. The fields from such a radiator are not the same as those from a small dipole with a sinusoidal (almost triangular) current distribution. Even the far-field *patterns* are not exactly the same. That is, the ideal dipole has a $\sin\theta$ pattern independent of its length whereas the pattern in (10) only approaches $\sin\theta$ as the dipole becomes very small. Thus, McLean's curve can be considered to be the lower bound for an ideal or Hertzian dipole.

Second, the pattern for a small dipole with a sinusoidal current distribution is slightly narrower than the pattern of an ideal dipole, and is, therefore, more superdirective (higher radiation Q). Thus, the lower bound for a sinusoidal distribution should be higher than the lower bound for a uniform distribution.

Third, practical designs for antennas with a sinusoidal current distribution fall above our "far-field Q" curve. In view of (7), this suggests that there are two components to the current, one of which accounts for the far-field pattern and a certain minimum amount of stored energy in the near-field, and another component of the current which only contributes to near-field stored energy.

Fourth, the top curve in Fig. 1 using the ratio of antenna input reactance to antenna input resistance for a thin linear dipole, while associated with an identical far-field pattern as in (10), shows a higher value of Q_A. This suggests that there are other geometries of the same maximum dimension that are capable of producing the pattern in (10) while yielding a lower Q_A. To illustrate that this is so, consider Fig. 2, which shows that dipoles of thicker radii have a lower Q_A, a fact which is well-known. The dipole radiation Q curves were obtained using (2) and the Method of Moments.

Also in Fig. 2 are data points for a "bow-tie" dipole of a size that fits into the spherical volume of radius "a" (see Fig. 3), has a current that goes to zero at its ends, and has the same far-field pattern as in (10). The radiation Q of the bow-tie has the lowest Q_A of those with the pattern of (10), but is still above the "far-field" Q_A curve in Fig. 2 obtained using (7). On the other hand, the end-loaded dipole of Fig. 4 has a somewhat uniform current on its radiating portion (i.e., approximates a Hertzian dipole), and its radiation Q using (2) lies between the McLean curve and our "far-field" curve, as expected. The current distribution on the electrically small end-loaded dipole is that of a triangle on a pedestal or uniform value. As shown in Fig. 5, as the dipole becomes electrically longer, the *slope* of the triangle portion of the triangle on a pedestal becomes less (i.e., the distribution becomes more uniform). Thus, the data points in Fig. 2 for the end-loaded dipole move closer to the McLean curve for an ideal or Hertzian dipole as the dipole becomes longer.

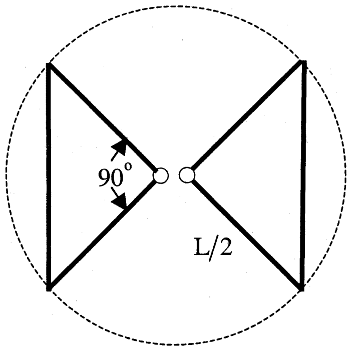

Fig. 3. Bow-tie antenna inside a sphere of diameter L.

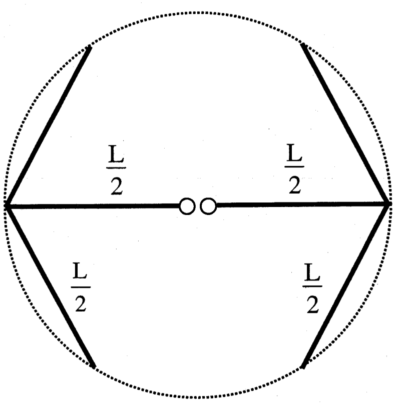

Fig. 4. End-loaded dipole inside a sphere of diameter L.

The sinusoidal current distribution on a dipole becomes nearly triangular when the dipole becomes small. The normalized field pattern for a triangular distribution is [14]

$$E_n(u) = g(u)\left[\frac{\sin\left(\frac{u}{2}\right)}{\frac{u}{2}}\right]^2. \tag{13}$$

Using (11) and (13) in (7) gives a curve identical to that in Figs. 1 and 2 for the sinusoidal distribution (i.e., far-field Q), except very slightly at the extreme right $(\beta \mathrm{a} \sim 1)$ where the dipole is marginally electrically small and the current distribution is beginning to depart somewhat from a triangular distribution. Note that (13) goes to zero at $\pm\infty$ and is, therefore, integrable at $\pm\infty$.

The various results of this section leads to the following two hypotheses.

Hypothesis #1: A realizable lower bound on $\mathrm{Q_A}$ for a linearly polarized electrically small antenna can be determined from its far-field pattern according to (7).

To use this hypothesis, the field pattern squared should be integrable at $\pm$ infinity. Further, the results will be somewhat sensitive to the accuracy with which the field pattern is known, a fact that is consistent with the superdirective nature of electrically small antennas.

Hypothesis #2: The radiation Q of a practical antenna may be viewed as having two parts: the first part is associated with that component of the current distribution which generates the far-field pattern (in both the visible and invisible regions); the second part is associated with all other components of the current distribution which contribute to stored energy but not to the far-field pattern. The first part determines the lower bound on the radiation Q.

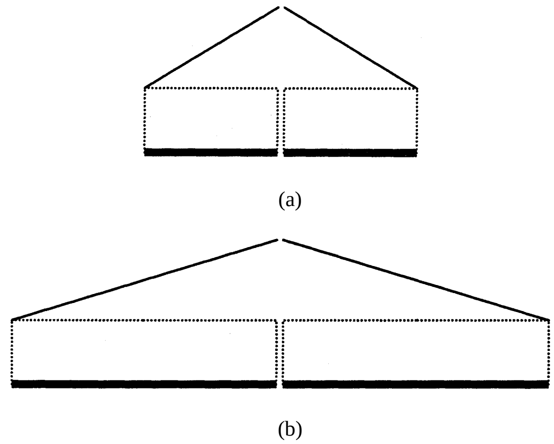

Fig. 5. Triangle on a pedestal current distributions on two dipoles. The dipole in (a) of half the length of the dipole in (b), but its triangular portion has twice the *slope* of dipole (b). Both dipoles have the same tenninal current value.

For example, the far-field pattern is obtained from an integration of the current distribution. Since integration is a smoothing process, various relatively minor discontinuities in the current may not affect the far-field pattern but can contribute to near-field stored energy (e.g., feed-point design).

IV. Circularly Polarized Antennas

It is written in [4] that the lowest possible radiation Q is obtained for an antenna that excites both of the lowest order modes, TM_{01} and TE_{01}, and that this radiation Q can be half that for either mode alone. This is correct and clearly the resulting field can be circularly polarized when the modes have the proper relative complex strengths.

The TE_{01} mode represents a fictitious ideal magnetic dipole which in turn represents a small loop of electric current. So, if circular polarization is generated with an ideal electric dipole and a small electric loop, the Q of that two element system will be one-half that of either radiator alone. (Note that the fields of the two radiators are uncoupled.) On the other hand, if circular polarization is generated by two crossed electric dipoles with a $\pi/2$ phase difference, the radiation Q of such a system will differ from the dipole-loop combination and differ from the radiation Q of either element alone. Grimes and Grimes [18], using a time-dependent Poynting theorem approach, have recently showed that for the crossed Hertizian electric dipole case, the radiation Q approaches 1/3 that of a single dipole as the dipoles become electrically small. This factor of 1/3 cannot be obtained using the frequency domain approaches employed in [3]–[6], [10], [11] because the near-fields of the two dipoles are coupled and there is a phase difference between them.

It is interesting to consider the normal mode helix, a popular antenna for hand held wireless transceivers. Each turn of the normal mode helix can be modeled by an ideal electric dipole and a small electric loop, and radiates elliptical polarization [14]. Thus, the normal mode helix excites both the TM_{01} and TE_{01} modes, but not of the proper relative amplitudes to generate circular polarization. It follows that the normal mode helix should have a lower $\mathrm{Q_A}$ than a linear dipole of the same max-

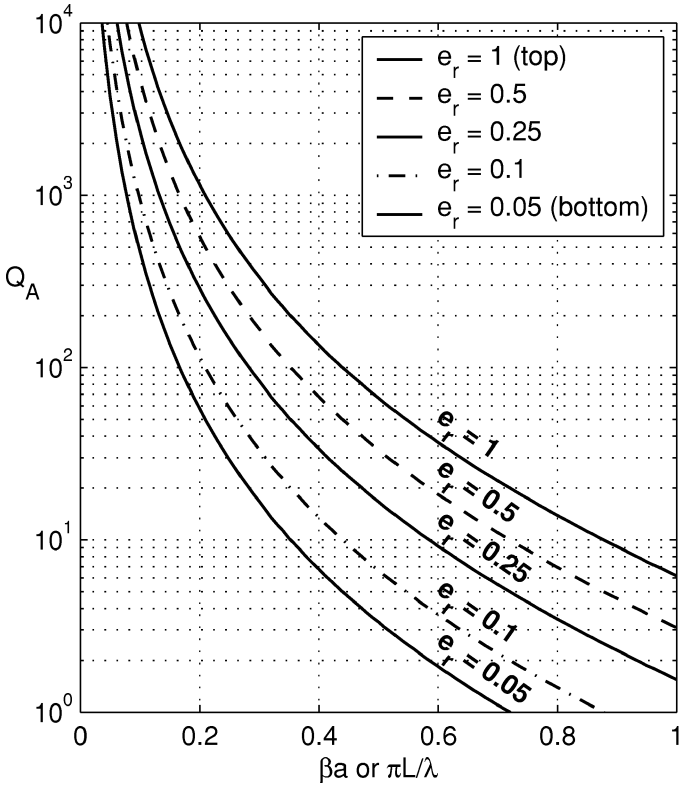

Fig. 6. Lower bounds on the radiation Q of a sinusoidal current dipole for various values of radiation efficiency versus $\beta \mathrm{a}$ or $\pi \mathrm{L}/\lambda$.

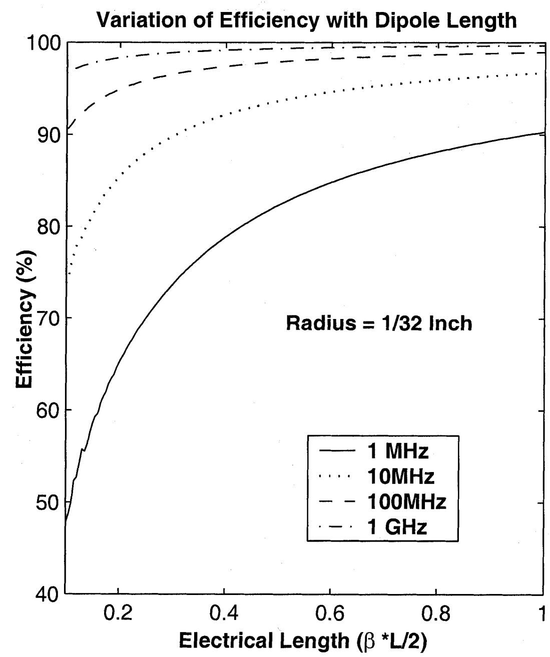

Fig. 7. Radiation efficiency versus $\beta \mathrm{a}$ or $\pi \mathrm{L}/\lambda$ for a linear dipole with sinusoidal current distribution at four frequencies. Dipole wire radius is 1/32 in. Material is aluminum.

imum dimension. In fact, it is known that the normal mode helix has a wider bandwidth than a linear dipole of the same length, and thus has a lower $\mathrm{Q_A}$.

V. System Q

Implicit in the relationships in (1) and (2) is the assumption that the antenna will be resonated with a lossless matching circuit so as to exhibit a purely real input impedance at a resonant frequency. In this situation $\mathrm{Q_A}$ is often referred to as the unloaded Q. However, if the matching circuit is not lossless, then the matching circuit has a finite Q, denoted $\mathrm{Q_m}$, and the Q of the system, $\mathrm{Q_s}$, often called the loaded Q, are related by [19]

$$\frac{1}{\mathrm{Q_s}} = \left[\frac{1}{\mathrm{Q_A}} + \frac{1}{\mathrm{Q_m}}\right]. \tag{14}$$

Clearly, if the matching circuit is lossless, $\mathrm{Q_m}$ is infinite, and the loaded and unloaded Q's are the same. If the matching circuit is a conjugate matching circuit, then $\mathrm{Q_A} = \mathrm{Q_m}$ and $\mathrm{Q_s} = \mathrm{Q_A}/2$ according to the definition in (1), since the time-averaged stored energy of the system remains unchanged while the dissipation of the system is doubled.

VI. System Efficiency

Electrically small antennas are known to be inefficient radiators due to the relative magnitudes of the radiation and ohmic loss resistances. The antenna radiation efficiency, $\mathrm{e_r}$, is given by the relationship [14]

$$e_r = \frac{R_r}{R_r + R_{\mathrm{ohmic}}} \tag{15}$$

where $\mathrm{R_r}$ is the radiation resistance and $\mathrm{R_{ohmic}}$ is the ohmic loss resistance.

The radiation Q of a lossy antenna is that of the lossless antenna reduced by the fractional value of the radiation efficiency according to (15). Fig. 6 shows the unloaded $\mathrm{Q_A}$ for several values of the radiation efficiency. Just as in a lumped circuit, the addition of loss decreases Q.

Fig. 7 shows the radiation efficiency of a small dipole at four frequencies when the radius is fixed at 1/32 in. The curves in Fig. 7 suggest that many practical electrically small dipole antennas at VHF and above will likely have radiation efficiencies greater than 50%.

If a matching circuit is present with an efficiency of $\mathrm{e_m}$, defined by the ratio of power out to power in, the efficiency of the system $\mathrm{e_s}$ is [8], [17]

$$e_s = e_r e_m = \frac{e_r}{1 + \frac{\mathrm{Q_A}}{\mathrm{Q_m}}}. \tag{16}$$

The above relationship is valid when the matching network contains no elements which store energy in the same form (i.e., electric or magnetic) as the antenna [17].

When the matching circuit is lossless, (16) reduces to $\mathrm{e_s} = \mathrm{e_r}$. In [17] it is shown that many electrically small antennas can be efficiently matched with a simple reactive L section. Matching is also discussed by Wheeler in [20] and [21].

VII. Summary and Conclusion

A different approach for predicting the realizable radiation Q of an electrically small antenna is presented. The method utilizes the superdirective property of the far-field radiation pattern and predicts a higher lower bound on the radiation Q for a dipole type element than previous works. This higher lower bound appears to be closer to achievable values of $\mathrm{Q_A}$

associated with dipole elements, wherein the current goes to zero at its end points, such as the linear dipole and bow-tie. This higher lower bound also appears to be in agreement with recently published results for the radiation Q of fractal dipole-type elements [22].

In the previous literature a relationship is published between the superdirectivity ratio, $\mathrm{R_{SD}}$, and the radiation Q, both of which use only the pattern factor $f(u)$. While one can define $\mathrm{R_{SD}}$ in any reasonable way, the expression for radiation Q must be consistent with the fundamental definition in (1). Therefore, a correct expression for $\mathrm{Q_A}$ is given by (7) and not by (5). In fact, the omission of the element factor, g(u), as in (5), produces meaningless results for the radiation Q of a electrically small radiator. The expression in (5) may be approximately true for large broadside sources, but it is not true in general.

Results for previous works [3]–[6], [10] are based upon the near-fields surrounding the antenna without regard for the art used to produce those fields. When only the lowest order TM or TE mode is used, those fields are the same as those of an ideal or Hertzian dipole. Thus, these previous results could be expected to predict a lower bound for an antenna reasonably well-modeled by an ideal dipole, such as the end-loaded dipole in Fig. 4, but not for a linear electric dipole-type element (wherein the current goes to zero at the ends).

When only the lowest order TM or TE mode is used as in [3], [10], a fundamental limit for the lower bound on the radiation Q is obtained. This fundamental limit is a fundamental limit in much the same sense that the maximum gain from an aperture occurs when the aperture is uniformly filled everywhere with fields of the same phase. In both cases the source distribution is of uniform amplitude and constant phase. In both cases, the source distribution is difficult to achieve in practice.

Finally, this paper, particularly the two hypotheses and Fig. 2, serve to explain when and why an electrically small antenna of the electric type can approach either the McLean (Chu) lower bound or the new (far-field derived) lower bound in this paper.

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References

[1] H. A. Wheeler, "Fundamental limitations of small antennas," *Proc. IRE*, vol. 35, pp. 1479–1484, Dec. 1947.

[2] ——, "Small antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-23, pp. 462–469, July 1975.

[3] L. J. Chu, "Physical limitations on omni-directional antennas," *J. Appl. Phys.*, vol. 19, pp. 1163–1175, Dec. 1948.

[4] R. F. Harrington, "Effect of antenna size on gain, bandwidth and efficiency," *J. Res. Natl. Bur. Stand.*, vol. 64-D, pp. 1–12, Jan./Feb. 1960.

[5] R. E. Collin and S. Rothschild, "Evaluation of antenna Q," *IEEE Trans. Antennas Propagat.*, vol. AP-12, pp. 23–27, Jan 1964.

[6] R. L. Fante, "Quality factor of general ideal antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-17, pp. 151–155, Mar. 1969.

[7] R. C. Hansen, "Fundamental limitations in antennas," *Proc. IEEE*, vol. 69, pp. 170–182, Feb. 1981.

[8] K. Fujimoto *et al.*, *Small Antennas*. New York: Wiley, 1987.

[9] Y. T. Lo and S. W. Lee, *Antenna Handbook: Theory, Applications and Design*. New York: Wiley, 1998.

[10] J. S. McLean, "A re-examination of the fundamental limits on the radiation Q of electrically small antennas," *IEEE Trans. Antennas Propagat.*, vol. 44, pp. 672–675, May 1996.

[11] H. D. Foltz and J. S. McLean, "Limits on the radiation Q of electrically small antennas restricted to oblong bounding regions," in *Proc. IEEE Antennas and Propagation Int. Symp.*, vol. 4, July 11–16, 1999, pp. 2702–2705.

[12] R. C. Hansen, *Microwave Scanning Antennas*. Los Altos, CA: Peninsula, 1985.

[13] C. H. Walter, *Traveling Wave Antennas*. Los Altos, CA: Peninsula, 1990.

[14] W. L. Stutzman and G. A. Thiele, *Antenna Theory & Design*, 2nd ed. New York: Wiley, 1998.

[15] T. L. Taylor, "Design of line-source antennas for narrow beamwidth and low sidelobes," *IEEE Trans. Antennas Propagat.*, vol. AP-3, pp. 16–28, 1955.

[16] D. R. Rhodes, *Synthesis of Planar Antenna Sources*. Oxford, U.K.: Clarendon, 1974.

[17] G. S. Smith, "Efficiency of electrically small antennas combined with matching networks," *IEEE Trans. Antennas Propagat.*, vol. AP-25, pp. 369–373, May 1977.

[18] D. M. Grimes and C. A. Grimes, "Radiation Q of dipole-generated fields," *Radio Sci.*, vol. 34, no. 2, pp. 281–296, Mar.–Apr. 1999.

[19] P. A. Rizzi, *Microwave Engineering, Passive Circuits*. Englewood Cliffs, NJ: Prentice-Hall, 1988.

[20] H. A. Wheeler, "The wide-band matching area for a small antenna," *IEEE Trans. Antennas Propagat.*, vol. AP-31, pp. 364–367, Mar. 1983.

[21] ——, "Small antennas," in *Antenna Engineering Handbook*, 3rd ed. New York: McGraw Hill, 1993, ch. 6.

[22] J. P. Gianvittorio and Y. Rahmat-Samii, "Fractal antennas: a novel antenna miniaturization technique," *IEEE Trans. Antennas Propagat. Mag*, vol. 44, pp. 20–36, Feb. 2002.

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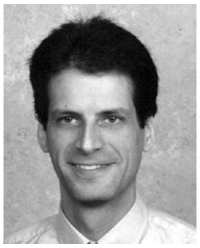

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