by Mr. Pocklington. For given electrical force on the wire the vibrations become more persistent as the wire becomes thinner, the energy of the wave system increasing rapidly while the radiation is not much altered; but it was a mistake to express the result, which is sensibly though not rigorously true for thin wire rings, as if it were true for rings of large cross-section. The other corrections pointed out by Prof. Orr deserves my best thanks and afford gratifying evidence of the kindly interest he has taken in my work.]

LXXXIV. *The Magnetic Properties of Systems of Corpuscles describing Circular Orbits.* By J. J. Thomson, *M.A., F.R.S., Cavendish Professor of Experimental Physics, Cambridge**.

THE problems discussed in this paper are:—(1) The magnetic field due to a number of negatively electrified corpuscles situated at equal intervals round the circumference of a circle and rotating in one plane with uniform velocity round its centre; and (2) The effect of an external magnetic field on the motion and periods of vibration of such a system. These problems are met with when we attempt to develop the theory that the atoms of the chemical elements are built up of large numbers of negatively electrified corpuscles revolving around the centre of a sphere filled with uniform positive electrification.

If an electrified particle is moving uniformly with a velocity small compared with that of light, it produces a magnetic field whose components α, β, γ at a distance r from the particle are given by the equations

$$\alpha=e\left(v\frac{d}{dz}-w\frac{d}{dy}\right)\frac{1}{r},\quad \beta=e\left(w\frac{d}{dx}-u\frac{d}{dz}\right)\frac{1}{r},\quad \gamma=e\left(u\frac{d}{dy}-v\frac{d}{dx}\right)\frac{1}{r}, \tag{1}$$

where e is the charge on the particle and u, v, w the components of its velocity. These expressions will not give the magnetic force when the motion is variable; when, however, the velocity is periodic, proportional say to $\epsilon^{\iota pt}$, the equations for the components of the magnetic force can be derived at once from equations (1) by writing $\epsilon^{-\iota\frac{p}{V}r}/r$ for $1/r$ in those equations: V being the velocity of light through the medium. We can see this at once by noticing that the modified equations satisfy differential equations of the type

$$\frac{d^2\alpha}{dx^2}+\frac{d^2\alpha}{dy^2}+\frac{d^2\alpha}{dz^2}=\frac{1}{V^2}\frac{d^2\alpha}{dt^2};$$

* Communicated by the Author.

and for values of r small compared with the wave-length, $2\pi V/p$, of the vibrations, they give the same magnetic force as that which would be produced if the particle was moving uniformly with the velocity it possessed at the instant under consideration.

This derivation of the magnetic force when the velocity is variable from the solution obtained on the assumption that the velocity is uniform may be extended to cases more complicated than that of a single particle; if we have a collection of particles separated from each other by distances small compared with the wave-lengths of their vibrations, and if in the expression for the magnetic force due to these particles, calculated on the assumption that the velocity is uniform, there is the term

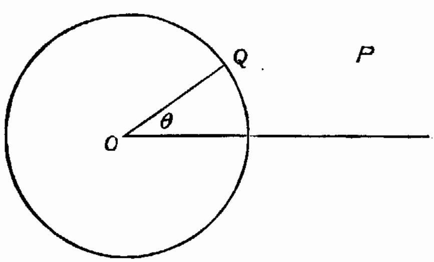
$$\phi(t)\frac{d^l}{dx^l}\frac{d^m}{dy^m}\frac{d^n}{dz^n}\frac{1}{r};$$

r being the distance from a point in the midst of the particles, then if $\phi(t)$ is proportional to $\epsilon^{\iota st}$, the solution, when we take into account the variability of the motion, will have for the corresponding term

$$\phi(t)\frac{d^l}{dx^l}\frac{d^m}{dy^m}\frac{d^n}{dz^n}\frac{\epsilon^{-\frac{\iota s}{V}r}}{r}.$$

2. We shall first find the magnetic field due to a particle with charge e describing with uniform velocity a circular orbit in the plane xy. Let O be the centre of the orbit, Q the position of the particle, P the point, not necessarily in the plane of xy, at which the magnetic force is to be determined; then if a is the radius of the orbit, ω the angular

Fig. 1.



velocity of rotation, θ the angle OQ makes with the axis of x,

$$u = -a\omega\sin\theta, \quad v = a\omega\cos\theta, \quad w = 0.$$

Hence γ the z component of the magnetic force at P, calculated on the assumption that the velocity of the particle is uniform, is given by the equation

$$\gamma = -ea\omega\left\{\sin\theta\frac{d}{dy} + \cos\theta\frac{d}{dx}\right\}\frac{1}{r'},$$

r' being written for PQ.

Writing r for OP we have

$$\frac{1}{r'} = \frac{1}{r} - a\left(\cos\theta\frac{d}{dx} + \sin\theta\frac{d}{dy}\right)\frac{1}{r} + \frac{a^2}{1.2}\left(\cos\theta\frac{d}{dx} + \sin\theta\frac{d}{dy}\right)^2\frac{1}{r} - \dots;$$

hence, writing ϑ for $\cos\theta\frac{d}{dx} + \sin\theta\frac{d}{dy}$, we have

$$\gamma = -ea\omega\left\{\vartheta\frac{1}{r} - a\vartheta^2\frac{1}{r} + \frac{a^2}{1.2}\vartheta^3\frac{1}{r} - \dots\right\}. \quad . \quad (2)$$

To pass to the solution in which the acceleration of the particle is taken into account we must express γ in terms of the time.

Now $\theta = \omega t$ when t the time is measured from the instant when the particle is on the axis of x; hence, substituting the exponential values for the cosine and sine, we have

$$\vartheta = \tfrac{1}{2}\left\{\epsilon^{\iota\omega t}\left(\frac{d}{dx} - \iota\frac{d}{dy}\right) + \epsilon^{-\iota\omega t}\left(\frac{d}{dx} + \iota\frac{d}{dy}\right)\right\}.$$

If we introduce two new variables ξ and η, defined by the equations

$$\xi = x + \iota y, \quad \eta = x - \iota y,$$

then
$$\frac{d}{dx} - \iota\frac{d}{dy} = 2\frac{d}{d\xi} \text{ and } \frac{d}{dx} + \iota\frac{d}{dy} = 2\frac{d}{d\eta};$$

thus
$$\vartheta = \left\{\epsilon^{\iota\omega t}\frac{d}{d\xi} + \epsilon^{-\iota\omega t}\frac{d}{d\eta}\right\}$$

and
$$\vartheta^n = \left\{\epsilon^{\iota n\omega t}\left(\frac{d}{d\xi}\right)^n + n\epsilon^{\iota(n-2)\omega t}\left(\frac{d}{d\xi}\right)^{n-1}\frac{d}{d\eta} + \frac{n.n-1}{1.2}\epsilon^{\iota(n-4)\omega t}\left(\frac{d}{d\xi}\right)^{n-2}\left(\frac{d}{d\eta}\right)^2 + \dots\right\}.$$

Hence to deduce from (2) the value of the magnetic force when we take into account the acceleration of the particle, we must write in that expression instead of $\vartheta^n\frac{1}{r}$

$$\epsilon^{\iota n\omega t}\left(\frac{d}{d\xi}\right)^n \frac{\epsilon^{\frac{-\iota n\omega r}{V}}}{r} + n\epsilon^{\iota(n-2)\omega t}\left(\frac{d}{d\xi}\right)^{n-1}\frac{d}{d\eta}\frac{\epsilon^{\frac{-\iota(n-2)\omega r}{V}}}{r}$$
$$+\frac{n\,.\,n-1}{2}\epsilon^{\iota(n-4)\omega t}\left(\frac{d}{d\xi}\right)^{n-2}\left(\frac{d}{d\eta}\right)^2\frac{\epsilon^{\frac{-\iota(n-4)\omega r}{V}}}{r}+\dots$$

when this substitution is made (2) will give the z component of the magnetic force.

The x component α is, on the assumption that the motion is steady, given by the equation

$$\alpha = ea\omega\cos\theta\frac{d}{dz}\frac{1}{r'}$$
$$=\tfrac{1}{2}ea\omega\left\{\epsilon^{\iota\theta}+\epsilon^{-\iota\theta}\right\}\frac{d}{dz}\left\{\frac{1}{r}-a\vartheta\frac{1}{r}+\frac{a^2\vartheta^2}{1\,.\,2}\frac{1}{r}-\dots\right\};$$

hence writing ωt for θ and proceeding as before, we find that when the acceleration of the particle is taken into account

$$\alpha=\tfrac{1}{2}ea\omega\frac{d}{dz}\left\{\frac{\epsilon^{\iota\left(\omega t-\frac{\omega r}{V}\right)}}{r}+\frac{\epsilon^{-\iota\left(\omega t-\frac{\omega r}{V}\right)}}{r}-a\left(\epsilon^{2\iota\omega t}\frac{d}{d\xi}\frac{\epsilon^{\frac{-2\iota\omega r}{V}}}{r}\right.\right.$$
$$\left.+\left(\frac{d}{d\xi}+\frac{d}{d\eta}\right)\frac{1}{r}+\epsilon^{-2\iota\omega t}\frac{d}{d\eta}\frac{\epsilon^{\frac{2\iota\omega r}{V}}}{r}\right)$$
$$+\frac{(-a)^n}{n\,!}\left(\epsilon^{(n+1)\iota\omega t}\left(\frac{d}{d\xi}\right)^n\frac{\epsilon^{-(n+1)\frac{\iota\omega r}{V}}}{r}\right.$$
$$+\epsilon^{(n-1)\iota\omega t}\left(\frac{d}{d\xi}\right)^{n-1}\left(\frac{d}{d\xi}+n\frac{d}{d\eta}\right)\frac{\epsilon^{-\frac{(n-1)\omega r}{V}}}{r}$$
$$\left.\left.+n\epsilon^{n-3\iota\omega t}\left(\frac{d}{d\xi}\right)^{n-2}\frac{d}{d\eta}\left(\frac{d}{d\xi}+\frac{n-1}{2}\frac{d}{d\eta}\right)\frac{\epsilon^{-\frac{(n-1)\iota\omega r}{V}}}{r}\right)\dots\right\}.$$

The value of $\iota\beta$ can be got in a similar way by multiplying $\frac{d}{dz}\frac{1}{r'}$ by $\frac{1}{2}(\epsilon^{\iota\theta}-\epsilon^{-\iota\theta})$.

3. These expressions consist of two parts: (1) periodic terms proportional to some power of $\epsilon^{\iota\omega t}$, and (2) terms independent of the time; in considering the magnetic force due to the particles we shall take these terms separately, retaining in each only the lowest power of a.

Let us begin with the single particle and take first the terms independent of the time. These terms are given by the equation

$$\gamma=ea^2\omega\left(\frac{d^2}{dx^2}+\frac{d^2}{dy^2}\right)\frac{1}{r}=-ea^2\omega\frac{d^2}{dz^2}\frac{1}{r};$$
$$\alpha=-ea^2\omega\frac{d}{dx}\frac{d}{dz}\frac{1}{r};$$
$$\beta=-ea^2\omega\frac{d}{dy}\frac{d}{dz}\frac{1}{r}.$$

i. e., the part of the magnetic force which is independent of the time is that due to a small magnet magnetized in the direction of the axis of z and whose moment is $ea^2\omega$.

For the periodic terms we have, retaining only the lowest powers of a,

$$\gamma=ea\omega\left\{\frac{x}{r^3}\cos\left(\omega t-\frac{\omega r}{V}\right)+\frac{y}{r^3}\sin\left(\omega t-\frac{\omega r}{V}\right)-\frac{x\omega}{Vr^2}\sin\left(\omega t-\frac{\omega r}{V}\right)\right.$$
$$\left.+\frac{y\omega}{Vr^2}\cos\left(\omega t-\frac{\omega r}{V}\right)\right\};$$
$$\alpha=ea\omega\left\{-\frac{z}{r^3}\cos\left(\omega t-\frac{\omega r}{V}\right)+\frac{z\omega}{Vr^2}\sin\left(\omega t-\frac{\omega r}{V}\right)\right\};$$
$$\beta=ea\omega\left\{-\frac{z}{r^3}\sin\left(\omega t-\frac{\omega r}{V}\right)-\frac{z\omega}{Vr^2}\cos\left(\omega t-\frac{\omega r}{V}\right)\right\}.$$

When r the distance from the centre of the orbit considerably exceeds V/ω, the terms involving the lowest powers of $1/r$ are the most important; confining our attention to these we have

$$\gamma=\frac{ea\omega^2}{Vr^2}\left\{y\cos\left(\omega t-\frac{\omega r}{V}\right)-x\sin\left(\omega t-\frac{\omega r}{V}\right)\right\};$$
$$\alpha=\frac{ea\omega^2}{Vr^2}z\sin\left(\omega t-\frac{\omega r}{V}\right);$$
$$\beta=-\frac{ea\omega^2}{Vr^2}z\cos\left(\omega t-\frac{\omega r}{V}\right).$$

Thus if O is the centre of the orbit, Oz the normal to its plane, then the magnetic force at the point P is equivalent to a magnetic force

$$-\frac{ea\omega^2}{Vr}\sin\left(\omega t-\frac{\omega r}{V}-\phi\right)$$

in the plane POz at right-angles to OP, together with a force, at right-angles to the plane POz, equal to

$$\frac{ea\omega^2}{\mathrm{V}r}\cos\theta\cos\left(\omega t-\frac{\omega r}{\mathrm{V}}-\phi\right).$$

Here θ is the angle OP makes with Oz, and ϕ the angle the plane POz makes with the plane of xz.

Using the language of the Electromagnetic Theory of Light, we may say that the rotating particle produces a wave of elliptically polarized light, the ratio of the axes of the ellipse being $\cos\theta$; thus along the normal to the orbit we have circular polarized light, while in the plane of the orbit the light is plane polarized.

Along with these magnetic forces we have in the plane POz at right-angles to OP an electric force equal to

$$\frac{ea\omega^2}{r}\cos\theta\cos\left(\omega t-\frac{\omega r}{\mathrm{V}}-\phi\right),$$

and at right-angles to the plane POz another electric force equal to

$$\frac{ea\omega^2}{r}\sin\left(\omega t-\frac{\omega r}{\mathrm{V}}-\phi\right);$$

applying Poynting's theorem we see that the rate at which energy is streaming through unit area at P is

$$\frac{1}{4\pi}\frac{e^2a^2\omega^4}{\mathrm{V}r^2}\left\{\cos^2\theta\cos^2\left(\omega t-\frac{\omega r}{\mathrm{V}}-\phi\right)+\sin^2\left(\omega t-\frac{\omega r}{\mathrm{V}}-\phi\right)\right\},$$

the mean value of this is

$$\frac{1}{8\pi}\frac{e^2a^2\omega^4}{\mathrm{V}r^2}(1+\cos^2\theta);$$

integrating this over the sphere through P we find that the mean rate at which the rotating corpuscle is emitting energy is

$$\frac{2}{3\mathrm{V}}e^2a^2\omega^4=\frac{2}{3}\frac{e^2}{\mathrm{V}}(\text{acceleration of the particle})^2.$$

4. Case of p particles separated by equal angular intervals rotating with uniform velocity ω round a circle.

Suppose that the particle we call (1) makes at the time t an angle ωt with the axis of x, the particle (2) will make an angle $\omega t+\frac{2\pi}{p}$, the particle (3) an angle $\omega t+2\,.\,\frac{2\pi}{p}$, and so on; hence if $\gamma_1, \gamma_2, \gamma_3, \ldots$ are the magnetic forces parallel to z

due to the particles (1) (2) (3) respectively, γ_1 will be given by the expression already found, γ_2 will be got from γ_1 by writing $\omega t+\frac{2\pi}{p}$ for ωt, γ_3 by writing $\omega t+2\cdot\frac{2\pi}{p}$ for ωt, and so on. The total magnetic force parallel to z due to the p particles will be

$$\gamma_1+\gamma_2+\gamma_3+\ldots\gamma_p.$$

The term independent of the time will be the same in $\gamma_1, \gamma_2, \ldots$, hence for the p particles it will be

$$-pea^2\omega\frac{d^2}{dz^2}\frac{1}{r}.$$

5. Let us now consider the periodic terms. The term corresponding to $\epsilon^{\iota n\omega t}$ in γ_1 will in γ_2 be $\epsilon^{\iota n\left(\omega t+\frac{2\pi}{p}\right)}$, in γ_3 $\epsilon^{\iota n\left(\omega t+2\,.\,\frac{2\pi}{p}\right)}$, and so on, hence the corresponding term in $\gamma_1+\gamma_2+\gamma_3+\ldots$ will be

$$\epsilon^{\iota n\omega t}+\epsilon^{\iota n\left(\omega t+\frac{2\pi}{p}\right)}+\epsilon^{\iota n\left(\omega t+2\,.\,\frac{2\pi}{p}\right)}+\epsilon^{\iota n\left(\omega t+3\,.\,\frac{2\pi}{p}\right)}+\ldots;$$

now this expression vanishes unless n is a multiple of p, when it equals $p\epsilon^{\iota n\omega t}$.

Hence the largest periodic term in the expression for the magnetic force due to the p corpuscles will be that corresponding to the term $\epsilon^{\iota p\omega t}$ in γ_1, and its magnitude will be p times this term; referring to the expression for γ_1 we see that the largest periodic term in the magnetic force due to the p particles will be

$$(-1)^p\frac{e\omega a^p}{1\,.\,2\,.\,3\,..\,p-1}\times p\left\{\epsilon^{\iota p\omega t}\left(\frac{d}{d\xi}\right)^p\frac{\epsilon^{-\frac{\iota p\omega r}{\mathrm{V}}}}{r}+\epsilon^{-\iota p\omega t}\left(\frac{d}{d\eta}\right)^p\frac{\epsilon^{\frac{\iota p\omega r}{\mathrm{V}}}}{r}\right\}$$

$$=(-1)^p\frac{e\omega a^p\,p}{1\,.\,2\,.\,3\,.\,(p-1)}\cdot\frac{1}{2^p}\left\{(x-\iota y)^p\left(\frac{d}{rdr}\right)^p\frac{\epsilon^{\iota\left(p\omega t-\frac{p\omega r}{\mathrm{V}}\right)}}{r}\right.$$

$$\left.+(x+\iota y)^p\left(\frac{d}{rdr}\right)^p\frac{\epsilon^{-\iota\left(p\omega t-\frac{p\omega r}{\mathrm{V}}\right)}}{r}\right\}.$$

Similarly the x and y components of the force due to the p particles are given by the equations

$$a=-(-1)^p\frac{1}{2}\frac{e\omega a^p p}{1.2.3..(p-1)}\frac{d}{dz}\left\{\epsilon^{\iota p\omega t}\left(\frac{d}{d\xi}\right)^{p-1}\frac{\epsilon^{-\frac{\iota p\omega r}{V}}}{r}\right.$$
$$\left.+\epsilon^{-\iota p\omega t}\left(\frac{d}{d\eta}\right)^{p-1}\frac{\epsilon^{\frac{\iota p\omega r}{V}}}{r}\right\}$$
$$=-(-1)^p\frac{e\omega a^p p}{1.2.3..p-1}\frac{1}{2^p}\frac{d}{dz}\left\{(x-\iota y)^{p-1}\left(\frac{d}{rdr}\right)^{p-1}\frac{\epsilon^{\iota(pt-\frac{\omega r}{V})}}{r}\right.$$
$$\left.+(x+\iota y)^{p-1}\left(\frac{d}{rdr}\right)^{p-1}\frac{\epsilon^{-\iota(pt-\frac{\omega r}{V})}}{r}\right\}$$

$$\iota\beta=(-1)^{p+1}\frac{e\omega a^p p}{1.2.3.p-1}\frac{1}{2^p}\frac{d}{dz}\left\{(x-\iota y)^{p-1}\left(\frac{d}{rdr}\right)^{p-1}\frac{\epsilon^{\iota(pt-\frac{\omega r}{V})}}{r}\right.$$
$$\left.-(x+\iota y)^{p-1}\left(\frac{d}{rdr}\right)^{p-1}\frac{\epsilon^{-\iota(pt-\frac{\omega r}{V})}}{r}\right\}.$$

These expressions give the intensity of the magnetic force at any distance from the rotating system; the terms in these expressions most important for the study of the physical properties of the system are those proportional to $1/r$, as at a distance from the system large compared with the wave-length of the vibration the other terms become insignificant. Confining ourselves to these terms we find after a little reduction that the magnetic force at a point P is equivalent to (1) a component M in the plane POz at right-angles to OP given by the equation

$$M=(-1)^p\frac{2e\omega p}{1.2.3\ldots(p-1)}\left(\frac{ap\omega}{2V}\right)^p$$
$$\frac{\sin^{p-1}\theta}{r}\cos p\left\{\left(\omega t-\frac{\omega r}{V}\right)-\phi-\frac{\pi}{2}\right\},$$

and to a component L at right-angles to this plane given by the equation

$$L=(-1)^p\frac{2e\omega p}{1.2.3\ldots(p-1)}\left(\frac{ap\omega}{2V}\right)^p$$
$$\frac{\sin^{p-1}\theta\cos\theta}{r}\sin p\left\{\left(\omega t-\frac{\omega r}{V}\right)-\phi-\frac{\pi}{2}\right\},$$

where, as before, θ is the angle between OP and Oz and ϕ the angle the plane POz makes with the plane of xz.

The components of the electric force are VL in the meridian plane and VM at right-angles to it.

We see that these expressions represent elliptically polarized light, the ratio of the axes being, as in the case when there is only one particle, $\cos\theta$; in this case, however, since L and M vanish when $\theta=0$, the intensity of the light vanishes along the normal to the plane of the orbit; thus a system of two or more particles rotating uniformly in a circular orbit does not give out any light along the axis of that orbit, a point which has to be kept in mind when considering the interpretation of the Zeeman effect. We see that as p increases the vibrations tend to become confined to the plane of the orbit.

6. The rate at which energy is streaming through unit area of the surface at P is by Poynting's theorem equal to

$$\frac{V}{4\pi}(L^2+M^2).$$

Substituting for L and M the values given above we find that the average rate at which energy is streaming through the surface of a sphere with its centre at O is

$$\frac{V}{4}\left(\frac{2e\omega p}{1.2.3\ldots p-1}\right)^2\left(\frac{ap\omega}{2V}\right)^{2p}\int_0^\pi\sin^{2p-2}\theta(1+\cos^2\theta)\sin\theta d\theta,$$
$$=\frac{V}{4}\left(\frac{2e\omega p}{1.2.3\ldots p-1}\right)^2\left(\frac{ap\omega}{2V}\right)^{2p}\left\{\frac{2^2.4\ldots(2p-2)}{1.3\ldots2p-1}-\frac{2.4\ldots2p}{1.3\ldots2p+1}\right\}$$
$$=\frac{V}{4}\left(\frac{2e\omega p}{1.2.3\ldots p-1}\right)^2\left(\frac{ap\omega}{2V}\right)^{2p}\frac{2.4\ldots2p-2.2p+2}{1.3\ldots2p+1}.$$

7. From this expression we see that when $a\omega$ the velocity of the particle is small compared with V the velocity of light, the rate at which energy radiates diminishes very rapidly as the number of particles increases. This is brought out by the following table, which gives the average radiation *per particle* for various groups of particles in terms of the radiation from a single particle moving with the same velocity in the same orbit; the table gives the results for two cases, in the first case the velocity of the particle is 1/10 that of light, in the second it is 1/100 of the same velocity. The radiation from a single particle is in each case taken as unity.

Number of Particles.	Radiation from each particle. $a\omega=V/10$.	$a\omega=V/10^2$.
1	1	1
2	$9{\cdot}6\times10^{-2}$	$9{\cdot}6\times10^{-4}$
3	$4{\cdot}6\times10^{-3}$	$4{\cdot}6\times10^{-7}$
4	$1{\cdot}7\times10^{-4}$	$1{\cdot}7\times10^{-10}$
5	$5{\cdot}6\times10^{-5}$	$5{\cdot}6\times10^{-13}$
6	$1{\cdot}6\times10^{-7}$	$1{\cdot}6\times10^{-17}$

From this we see that the radiation from each of a group of 6 particles moving with one tenth the velocity of light is considerably less than one five-millionth part of the radiation from a single particle describing the same orbit with the same velocity, while when the particles are moving with only 1/100 of the velocity of light the radiation falls to $1{\cdot}6\times10^{-17}$ of that from the single particle.

8. Action of an external magnetic field on a ring of rotating particles.

Since a ring of particles produces a magnetic field similar to that due to a current flowing round the orbit of the particles, it might seem at first sight as if a body whose atoms contained systems of such rotating particles ought to be strongly magnetic, the following investigation of the action of an external field on such a system shows that this is not the case.

Let us suppose that the external magnetic force H is uniform and parallel to the axis of X; and let the moving electrified particle be describing its orbit under a radial attractive force proportional to its distance from a fixed point. If m is the mass of the particle, e its electric charge, μ the force at unit distance, the equations of motion are

$$m\frac{d^2x}{dt^2}=-\mu x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$m\frac{d^2y}{dt^2}=-\mu y-\mathrm{H}e\frac{dz}{dt} \quad . \quad . \quad . \quad . \quad (2)$$

$$m\frac{d^2z}{dt^2}=-\mu z+\mathrm{H}e\frac{dy}{dt} \quad . \quad . \quad . \quad . \quad (3)$$

If η and ζ are the coordinates of the point referred to axes in the plane of yz rotating with the angular velocity $p=\frac{1}{2}\,\mathrm{H}e/m$ so that

$$y=\eta\cos pt-\zeta\sin pt, \qquad z=\eta\sin pt+\zeta\cos pt,$$

equations (2) and (3) become, if we neglect terms in H^2,

$$m\frac{d^2\eta}{dt^2}=-\mu\eta, \qquad m\frac{d^2\zeta}{dt^2}=-\mu\zeta.$$

The solutions of these equations are if $\omega^2=\mu/m$

$$x=\mathrm{A}\cos\omega t+\mathrm{B}\sin\omega t,$$
$$\eta=\mathrm{C}\cos\omega t+\mathrm{D}\sin\omega t,$$
$$\zeta=\mathrm{E}\cos\omega t+\mathrm{F}\sin\omega t.$$

Let the time be measured from the instant when the

magnetic force was applied. Let $y_0\, z_0,\, \dot{y}_0,\, \dot{z}_0$ be the values of y, z, $\frac{dy}{dt}$, $\frac{dz}{dt}$ at this time, then when $t=0$ we have

$$\eta=y_0, \qquad \zeta=z_0,$$
$$\frac{d\eta}{dt}=\dot{y}_0+pz_0,$$
$$\frac{d\zeta}{dt}=\dot{z}_0-py_0;$$

hence

$$\eta=\zeta_0\cos\omega t+\frac{1}{\omega}(\dot{y}_0+pz_0)\sin\omega t,$$
$$\zeta=z_0\cos\omega t+\frac{1}{\omega}(\dot{z}_0-py_0)\sin\omega t;$$

and similarly

$$x=x_0\cos\omega t+\frac{1}{\omega}\dot{x}_0\cos\omega t;$$

the motion parallel to x is not affected by the magnetic force.

The values of y and z are given by

$$y=\eta\cos pt-\zeta\sin pt, \quad z=\eta\sin pt+\zeta\cos pt.$$

The magnetic force α parallel to x calculated on the assumption that the motion is steady is given by the equation

$$\alpha=-e\left\{\frac{dz}{dt}\frac{d}{dy'}\frac{1}{r'}-\frac{dy}{dt}\frac{d}{dz'}\frac{1}{r'}\right\}$$

where x', y', z' are the coordinates of the point P at which the magnetic force is calculated.

Now,

$$\frac{1}{r'}=\frac{1}{r}-\left(x\frac{d}{dx'}+y\frac{d}{dy'}+z\frac{d}{dz'}\right)\frac{1}{r}+\ldots$$

Substituting this value of $1/r'$ in the expression for α, and expressing the coefficients of the differential coefficients of $1/r$ as periodic functions of the time, we can pass to the solution when we take the acceleration of the particles into account by the method explained at the beginning of this paper.

For the purpose of discussing the magnetic properties of the system under a steady magnetic field, the only terms which are of importance are the non-periodic ones. Substituting the values of x, y, z, $\frac{dy}{dt}$, $\frac{dz}{dt}$ in the expression for α, and picking out the non-periodic terms, we find, neglecting terms in p^2,

$$\alpha = e(y_0\dot{z}_0-\dot{y}_0 z_0)\left(\frac{d^2}{dy'^2}+\frac{d^2}{dz'^2}\right)\frac{1}{r}+\frac{ep}{4}\left\{\frac{\dot{y}_0^2+\dot{z}_0^2}{\omega^2}-(y_0^2+z_0^2)\right\}\left(\frac{d^2}{dy'^2}+\frac{d^2}{dz'^2}\right)\frac{1}{r},$$

or

$$\alpha = -e(y_0\dot{z}_0-\dot{y}_0 z_0)\frac{d^2}{dx'^2}\frac{1}{r}-\frac{\mathrm{H}e^2}{8m}\left\{\frac{\dot{y}_0^2+\dot{z}_0^2}{\omega^2}-(y_0^2+z_0^2)\right\}\frac{d^2}{dx'^2}\frac{1}{r}.$$

The first term represents the force due to the particle in its undisturbed orbit, and may for our present purpose be neglected. At the time $t=0$ the particle was describing a circular orbit with angular velocity ω, so that

$$\dot{y}_0^2+\dot{z}_0^2+\dot{x}_0^2=a^2\omega^2, \qquad x_0^2+y_0^2+z_0^2=a^2,$$

where a is the radius of the orbit: hence, as far as the term involving H is concerned,

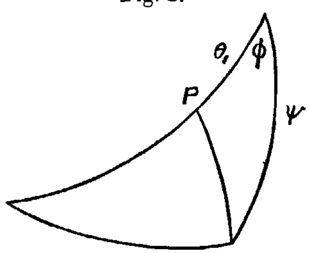
$$\alpha = \frac{\mathrm{H}e^2}{8m}\left(\frac{\dot{x}_0^2}{\omega^2}-x_0^2\right)\frac{d^2}{dx_0^2}\frac{1}{r}.$$

Thus the particle produces the same effect as a little magnet whose moment is parallel to the axis of x and equal to

$$\frac{\mathrm{H}e^2}{8m}\left(\frac{\dot{x}_0^2}{\omega^2}-x_0^2\right).$$

10. Suppose that the plane of the orbit makes an angle ϕ with the plane xz, the line of intersection of the orbit with xz making an angle ψ with the axis of x, then if θ_1 is the angle

Fig. 2.



the radius to the particle makes with the line of intersection at the time $t=0$, we have

$$x_0 = a(\cos\theta_1\cos\psi+\sin\theta_1\sin\psi\cos\phi),$$
$$\dot{x}_0 = a\omega(-\sin\theta_1\cos\psi+\cos\theta_1\sin\psi\cos\phi);$$

so that

$$\frac{\dot{x}_0^2}{\omega^2}-x_0^2 = a^2\{\cos 2\theta_1(\sin^2\psi\cos^2\phi-\cos^2\psi)-\sin 2\theta_1\sin 2\psi\cos\phi\}.$$

Thus each particle produces the same effect as a magnet magnetized parallel to the direction of H and having a moment equal to

$$\frac{\mathrm{H}e}{8m}a^2\{\cos 2\theta_1(\sin^2\psi\cos^2\phi-\cos^2\psi)-\sin 2\theta_1\sin 2\psi\cos\phi\}.$$

The coefficient of magnetization of any substance is the sum of these moments for the unit of volume when the external force H is unity. We see from the preceding expression for the magnetic moment due to a single orbit that if the particles and their orbits are uniformly distributed this sum must be zero. For consider orbits whose planes are all in any given direction: for such orbits ψ and ϕ will have constant values; the phase θ_1 at the time $t=0$ will, however, vary from orbit to orbit, and if these phases are equally distributed the mean values of $\cos 2\theta_1$ and $\sin 2\theta_1$ will be zero. Thus the coefficient of magnetization of the system will be zero; so that we cannot explain the magnetic or diamagnetic properties of bodies by the supposition that the atoms consist of charged particles describing closed periodic orbits under the action of a force proportional to the distance from a fixed point. I find that Professor Voigt has already come to this conclusion by a different method.

11. I find that the same absence of diamagnetic or magnetic properties will persist whatever be the law of force, provided the force is central and there is no dissipation of energy.

To prove this, let x, y, z be the coordinates of the particle; then α, the x-component of the magnetic force at the point (x', y', z'), is given by the equation

$$\alpha = -e\left\{\frac{dz}{dt}\frac{d}{dy'}\frac{1}{r'}-\frac{dy}{dt}\frac{d}{dz'}\frac{1}{r'}\right\},$$

where r' is the distance between the points (x, y, z) and (x', y', z'). If r is the distance of (x', y', z') from the centre of the orbit taken as the origin, then

$$\frac{1}{r'} = \frac{1}{r}-\left(x\frac{d}{dx'}+y\frac{d}{dy}+z\frac{d}{dz'}\right)\frac{1}{r}+\dots;$$

so that

$$\alpha = -e\left\{\frac{dz}{dt}\frac{d}{dy'}\frac{1}{r} - \frac{dy}{dt}\frac{d}{dz'}\frac{1}{r} - \frac{dz}{dt}\left(x\frac{d^2}{dx'\,dy'} + y\frac{d^2}{dy'^2} + z\frac{d^2}{dy'\,dz'}\right)\frac{1}{r} + \frac{dy}{dt}\left(x\frac{d^2}{dx'\,dz'} + y\frac{d^2}{dy'\,dz'} + z\frac{d^2}{dz'^2}\right)\frac{1}{r} + \ldots\right\}.$$

Now if the motion is periodic, the mean values of $\frac{dz}{dt}$, $\frac{dy}{dt}$, $z\frac{dz}{dt}$, and $y\frac{dy}{dt}$ are evidently zero.

Since

$$y\frac{dz}{dt} = \frac{1}{2}\left(y\frac{dz}{dt} - z\frac{dy}{dt}\right) + \frac{1}{2}\frac{d}{dt}(yz),$$

$$z\frac{dy}{dt} = -\frac{1}{2}\left(y\frac{dz}{dt} - z\frac{dy}{dt}\right) + \frac{1}{2}\frac{d}{dt}(yz),$$

the mean values of $y\frac{dz}{dt}$ and $-z\frac{dy}{dt}$ are one-half the mean value of $y\frac{dz}{dt} - z\frac{dy}{dt}$. We shall proceed to find these values and also to show that the mean values of $x\frac{dz}{dt}$ and $y\frac{dz}{dt}$ are zero.

The equations of motion are

$$m\frac{d^2x}{dt^2} = \mathrm{X}, \quad \ldots \ldots \quad (1)$$

$$m\frac{d^2y}{dt^2} = \mathrm{Y} - \mathrm{H}e\frac{dz}{dt}, \quad \ldots \ldots \quad (2)$$

$$m\frac{d^2z}{dt^2} = \mathrm{Z} + \mathrm{H}e\frac{dy}{dt}. \quad \ldots \ldots \quad (3)$$

The force whose components are X, Y, Z is supposed to be central; so that

$$\frac{\mathrm{X}}{x} = \frac{\mathrm{Y}}{y} = \frac{\mathrm{Z}}{z}.$$

From (1) and (2) we see

$$m\frac{d}{dt}\left(y\frac{dx}{dt} - x\frac{dy}{dt}\right) = \mathrm{H}e\,x\frac{dz}{dt};$$

so that the mean value of $x\frac{dz}{dt}$ vanishes. Similarly, we may show that the mean value of $y\frac{dz}{dt}$ vanishes.

From (2) and (3) we have

$$m\frac{d}{dt}\left(y\frac{dz}{dt} - z\frac{dy}{dt}\right) = \tfrac{1}{2}\mathrm{H}e\frac{d}{dt}(y^2+z^2)$$

or

$$y\frac{dz}{dt} - z\frac{dy}{dt} = \frac{1}{2}\frac{\mathrm{H}e}{m}\{(y^2+z^2) - (y_0{}^2+z_0{}^2)\} + y_0\frac{dz_0}{dt} - z_0\frac{dy_0}{dt},$$

where y_0, z_0, $\frac{dy_0}{dt}$, $\frac{dz_0}{dt}$ are the values of y, z, and their differential coefficients when $t=0$. Hence, if we retain in the expression for α only those terms whose mean values do not vanish, we have

$$\alpha = \left\{\tfrac{1}{2}e\left(y_0\frac{dz_0}{dt} - z_0\frac{dy_0}{dt}\right) + \frac{1}{4}\frac{\mathrm{H}e^2}{m}\left((\overline{y^2+z^2}) - (y_0{}^2+z_0{}^2)\right)\right\}\left(\frac{d^2}{dy'^2} + \frac{d^2}{dz'^2}\right)\frac{1}{r}.$$

Since

$$\left(\frac{d^2}{dy'^2} + \frac{d^2}{dz'^2}\right)\frac{1}{r} = -\frac{d^2}{dx'^2}\frac{1}{r},$$

we see that the orbit is equivalent to a little magnet whose moment is

$$\tfrac{1}{2}e\left(z_0\frac{dy_0}{dt} - y_0\frac{dz_0}{dt}\right) + \frac{1}{4}\frac{\mathrm{H}e^2}{m}\left(y_0{}^2+z_0{}^2 - (y^2+z^2)\right).$$

The part of the moment which depends upon the external magnetic force is

$$\frac{1}{4}\frac{\mathrm{H}e^2}{m}\{y_0{}^2+z_0{}^2-(y^2+z^2)\},$$

and the mean value of this is

$$\frac{1}{4}\frac{\mathrm{H}e^2}{m}\{y_0{}^2+z_0{}^2-(\overline{y}^2+\overline{z}^2)\},$$

where $\overline{y}^2$ and $\overline{z}^2$ are the mean values of y^2 and z^2 round the orbit.

If we consider a large number of particles describing orbits, the phases of the particles in their orbits being uniformly distributed, we can see that the total moment of their equivalent magnets will be zero; for consider a number of similar orbits which only differ from each other by the position of the particle when the magnetic force is first applied. For all these orbits $\overline{y}^2+\overline{z}^2$ is the same; so that if there are n orbits the sum of the second term inside the bracket will be $n(\bar{y}^2+\bar{z}^2)$; if the number of particles which at any time are situated between two neighbouring points P and Q on the orbit is proportional to the time taken by the particle to pass through PQ, then the sum of $y_0{}^2+z_0{}^2$ for the n orbits will be $n(\bar{y}^2+\bar{z}^2)$, and thus the two terms inside the bracket will balance each other and the resultant magnetic moment of the system will be zero. We thus see that we cannot explain the magnetic properties of bodies by means of charged particles describing without dissipation of energy closed orbits.

12. When there is dissipation of energy the collection of moving charged particles may, I think, possess magnetic properties. Let us represent the dissipation of energy by supposing that the motion of the particles is resisted by a force proportional to the velocity: the equations of motion are

$$m\frac{d^2y}{dt^2} = \mathrm{Y}-k\frac{dy}{dt}-\mathrm{H}e\frac{dz}{dt},$$

$$m\frac{d^2z}{dt^2} = \mathrm{Z}-k\frac{dz}{dt}+\mathrm{H}e\frac{dy}{dt},$$

where $k\frac{dy}{dt}$, $k\frac{dz}{dt}$ represent the forces due to the viscous resistance. Then, if the components Y and Z are those of a central force, we have

$$m\frac{d}{dt}\left(y\frac{dz}{dt}-z\frac{dy}{dt}\right)+k\left(y\frac{dz}{dt}-z\frac{dy}{dt}\right) = \tfrac{1}{2}\mathrm{H}e\frac{d}{dt}(y^2+z^2).$$

The solution of this equation is

$$y\frac{dz}{dt}-z\frac{dy}{dt} = \epsilon^{-\frac{k}{m}t}\left(y_0\frac{dz_0}{dt}-z_0\frac{dy_0}{dt}\right)+\frac{1}{2}\frac{\mathrm{H}e}{m}\epsilon^{-\frac{k}{m}t}\int_0^t\epsilon^{\frac{k}{m}t}\frac{d}{dt}(y^2+z^2)\,dt.$$

Thus, corresponding to the term $-\frac{1}{2}e\left(y\frac{dz}{dt}-z\frac{dy}{dt}\right)\frac{d^2}{dx^2}\frac{1}{r}$ in the expression for α, the magnetic force parallel to x, we have as the coefficient of $\frac{d^2}{dx^2}\frac{1}{r}$, and therefore as the moment of the equivalent magnet,

$$\tfrac{1}{2}e\left(z_0\frac{dy_0}{dt}-y_0\frac{dz_0}{dt}\right)\epsilon^{-\frac{k}{m}t}-\frac{1}{4}\frac{\mathrm{H}e^2}{m}\epsilon^{-\frac{k}{m}t}\int_0^t\epsilon^{\frac{k}{m}t}\frac{d}{dt}(y^2+z^2)\,dt.$$

The contribution of this particle to the coefficient of magnetization is the mean value of the coefficient of H in

the preceding expression, *i.e.* the mean value of

$$-\frac{1}{4}\frac{e^2}{m}\epsilon^{-\frac{k}{m}t}\int_0^t\epsilon^{\frac{k}{m}t}\frac{d}{dt}(y^2+z^2)\,dt. \quad . \quad . \quad . \quad (1)$$

If the motion of the particle is strongly retarded, the particle will tend to fall in to the centre of the orbit and $\frac{d}{dt}(y^2+z^2)$ therefore be negative; thus the mean value of the integral will be negative and the mean value of (1) positive. The contribution of this particle to the coefficient of magnetization will therefore be positive, and a collection of such particles will be paramagnetic.

13. Thus, to take a definite case, suppose the atoms of a substance, like the atoms of radio-active substances, were continually emitting corpuscles; the velocity of projection of the corpuscles under consideration being, however, insufficient to carry them clear of the atom, so that the corpuscles describe orbits round the centre of the atom: then, if the motion of the corpuscles were not accompanied by dissipation of energy, the corpuscles would not endow the body with either magnetic or diamagnetic properties; if, however, the energy of the corpuscles was dissipated during their motion outside the atom, so that they ultimately fell with but little energy into the atom, a system consisting of such atoms would be paramagnetic. If the energy of projection were derived from the internal energy of the atom, there would thus be a continual transference of energy from the atom to the surrounding systems; this would tend to raise the temperature of the system. I am not aware that any experiments have been made to find whether the temperature in the middle of a mass of a magnetic substance like iron whose surface is kept at a constant temperature differs from the temperature inside a mass of a non-magnetic substance like brass whose surface is kept at the same temperature. I hope, however, soon to be able to test this point.

14. The origin of the difference between the effects produced by charged particles describing free orbits, and those produced by constant electric currents describing circular circuits, as in Ampère's theory of magnetism, is that in the case of the particles describing their orbits we get, in addition to the effects due to the constant electric currents, effects of the same character as those due to the induction of currents in conductors from the variation of the magnetic field; these induced currents tend to make the body diamagnetic, while the Ampèrean currents tend to make it magnetic: in the case of

the particles describing free orbits these tendencies balance each other.

15. With a system of particles whose energy is being dissipated in the way described in § 13 we should get magnetic properties, these properties would be properties inherent in the atom, and would not explain the magnetic properties of iron for example, where the magnetization is so much affected by gross mechanical strain as to indicate that it depends largely if not entirely on the properties possessed by aggregations of large numbers of molecules. In the case of such aggregations, however, we may easily conceive that the orbits of charged bodies moving within them may not be free, but that in consequence of the forces exerted by the molecules in the aggregate, the orbit may be constrained to occupy an invariable position with respect to the aggregate —as if, to take a rough analogy, the orbit was a tube bored through the aggregate, so that the orbit and aggregate move like a rigid body, and in order to deflect the orbit it is necessary to deflect the aggregate. Under these conditions it is easy to see that the orbits would experience forces equivalent on the average to those acting on a continuous current flowing round the orbit; the aggregate and its orbit would under these forces act like a system of little magnets; and the body would exhibit magnetic properties quite analogous to those possessed by a system of Ampèrean circuits.

16. On the effect of a magnetic field on the frequency of the vibrations emitted by a system of n corpuscles rotating in a circular orbit.

The results obtained on pages (679) and (680) give the solution of this problem. We shall for the sake of brevity confine our attention (1) to the vibrations emitted along the axis of x, which is parallel to the magnetic force; for points along this axis $y=z=0$; and (2) to the vibrations emitted in a direction at right-angles to the magnetic force, say along the axis of y, so that $x=0$, $z=0$.

The components of the magnetic force are expressed by terms of the form

$$\frac{d^l}{dx^l}\frac{d^m}{dy^m}\frac{d^n}{dz^n}\frac{\epsilon^{\iota\left(pt-\frac{pr}{V}\right)}}{r} \quad . \quad . \quad . \quad . \quad (1)$$

When the distance from the vibrating system is a large number of wave-lengths, the most important term in the expression (1) is proportional to $1/r$, and this term can be got by writing

$$-\frac{\iota px}{Vr} \text{ for } \frac{d}{dx}, \quad -\frac{\iota py}{Vr} \text{ for } \frac{d}{dy}, \text{ and } -\frac{\iota pz}{Vr} \text{ for } \frac{d}{dx}$$

in that expression, so that as far as this term is concerned, (1) reduces to

$$\left(-\frac{\iota p}{V}\right)^{l+m+n}\left(\frac{x}{r}\right)^l\left(\frac{y}{r}\right)^m\left(\frac{z}{r}\right)^n\frac{\epsilon^{\iota\left(pt-\frac{pr}{V}\right)}}{r};$$

thus along the axis of x where y and z both vanish we need only consider terms for which $m=n=0$, and along the axis of y those for which $l=n=0$.

17. Let x, y, z denote the coordinates of one of the corpuscles, the y component of the magnetic force at the point (x', y', z') is given by the equation

$$\beta = \Sigma e\left(\frac{dz}{dt}\frac{d}{dx'}\frac{1}{r'} - \frac{dx}{dt}\frac{d}{dz'}\frac{1}{r'}\right) \quad . \quad . \quad . \quad (1)$$

where r' is the distance between (x, y, z) and (x', y', z'), and Σ denotes that the sum of the corresponding expressions for all the corpuscles is to be taken. If r is the distance of $x'\,y'\,z'$ from the origin of coordinates (the centre of the orbit) then

$$\frac{1}{r'} = \frac{1}{r} - \left(x\frac{d}{dx'} + y\frac{d}{dy'} + z\frac{d}{dz'}\right)\frac{1}{r} + \frac{1}{1\,.\,2}\left(x\frac{d}{dx'} + y\frac{d}{dy'} + z\frac{d}{dz'}\right)^2\frac{1}{r} - \ldots$$

Along the axis of x the only terms we need consider are those in which the differentiation is entirely with regard to x'; hence, confining ourselves to these terms, we see from § 5 that if there are n corpuscles regularly spaced round the orbit, the term in β with which we are concerned will be of the form

$$e\frac{dz}{dt}\cdot\frac{x^{n-1}}{1\,.\,2\,.\,3\,.\,n-1}\left(\frac{d}{dx'}\right)^{n-1}\frac{\epsilon^{\iota\left(pt-\frac{pr}{V}\right)}}{r} \quad . \quad . \quad . \quad (2)$$

Now x is of the form $A\cos\omega t + B\sin\omega t$,

while if $$\delta = \frac{1}{2}\frac{He}{m},$$

$$z = \zeta\cos\delta t + \eta\sin\delta t,$$

where $$\zeta = A'\cos\omega t + B'\sin\omega t,$$

$$\eta = A''\cos\omega t + B''\sin\omega t.$$

Substituting these values for x and z in $x^{n-1}\frac{dz}{dt}$, we see that (2) takes the form

$$\beta = \mathrm{A}\cos(n\omega t \pm \delta t)\frac{\epsilon^{-\iota\left(\frac{n\omega\pm\delta}{\mathrm{V}}\right)r}}{r};$$

thus, to use the language of spectrum analysis, instead of a single line of frequency $n\omega$ we have two lines whose frequencies are $n\omega+\delta$ and $n\omega-\delta$ respectively: it is easy to show that these lines are circularly polarized in opposite senses.

18. Let us now consider the effect at right-angles to the magnetic force, and first take the component of the magnetic force parallel to x; we have

$$\alpha = \Sigma e\left(\frac{dy}{dt}\frac{d}{dz'}\frac{1}{r'} - \frac{dz}{dt}\frac{d}{dy'}\frac{1}{r'}\right).$$

Along the axis of y the only terms we need consider are those in which the differentiation is entirely with regard to y'; picking out this term we have, when there are n corpuscles,

$$\alpha = e\frac{dz}{dt}y^{n-1}\frac{\epsilon^{\iota\left(pt-\frac{pr}{\mathrm{V}}\right)}}{r};$$

in this equation some constants have been omitted which do not affect the argument.

$$y = \eta\cos\delta t - \zeta\sin\delta t.$$

Substituting the values for z, η, and ζ given above, we find that α contains terms of the following types:—

$$\cos(n\omega t \pm n\delta t),\quad \cos(n\omega t \pm (n-2)\delta t),\quad \cos(n\omega t \pm (n-4)\delta t),$$

and so on, the last term being

$$\cos(n\omega t \pm 2\delta t) \text{ if } n \text{ is even,}$$
$$\cos(n\omega t \pm \delta t) \text{ if } n \text{ is odd.}$$

The equation for the magnetic force parallel to z is

$$\gamma = \Sigma e\left(\frac{dx}{dt}\frac{d}{dy'}\frac{1}{r'} - \frac{dy}{dt}\frac{d}{dz'}\frac{1}{r'}\right);$$

along the axis of y this reduces to terms proportional to

$$\frac{dx}{dt}\cdot y^{n-1}\frac{\epsilon^{\iota\left(pt-\frac{pr}{(\mathrm{V})}\right)}}{r}.$$

Substituting for x and y their values, we get in the expression for γ terms of the types

$$\cos(n\omega t \pm (n-1)\delta t),\qquad \cos(n\omega t \pm (n-3)\delta t),\ \ldots$$

the last term is $\cos n\omega t$ if n is odd, $\cos(n\omega t \pm \delta t)$ if n is even. Thus at right-angles to the direction of the magnetic force the effect is much more complicated than along that direction. At right-angles to the magnetic force the line whose frequency is $n\omega$ is split up into lines whose frequencies are $n\omega \pm n\delta$, $n\omega \pm (n-2)\delta$, $n\omega \pm (n-4)\delta$ . . : the last term being $n\omega \pm 2\delta$, or $n\omega \pm \delta$, as n is even or odd; all these lines are polarized in the plane parallel to the magnetic force: besides these we have lines whose frequencies are $n\omega \pm (n-1)\delta$, $n\omega \pm (n-3)\delta$, the last term being $n\omega$ or $n\omega \pm \delta$ as n is odd or even; all these lines are polarized in the plane at right-angles to the magnetic force. Thus while the original line is, looking in the direction of the magnetic force, only split into two whose frequencies are $n\omega \pm \delta$; in the direction at right-angles to the force it is split into $2n$ or $2n+1$ lines according as n is even or odd, while the maximum change of frequency is n times that for the radiation along the lines of force.

LXXXV. *The Influence of Stress and of Temperature on the Magnetic Change of Resistance in Iron, Nickel, and Nickel-Steel.* By W. E. Williams, *B.Sc.**

[Plate XXVII.]

THE influence of stress on the magnetic change of resistance in iron and nickel has been investigated by Tomlinson †; and in a paper previously communicated to the Philosophical Magazine ‡ the present writer described some experiments on the same subject. The results obtained seemed to indicate that a further investigation of this effect both in nickel and in iron would be desirable; and the experiments described below were undertaken with that object.

The present paper also contains an account of experiments on the effects of variations of temperature on the magnetic change of resistance. This effect has been investigated for the case of nickel by Dr. Knott §. As, however, his experiments were confined to fields below 60 C.G.S. units, while

* Communicated by Prof. E. Taylor Jones.
† Phil. Trans. Roy. Soc. 1883, Pt. I.
‡ Phil. Mag. Oct. 1902.
§ Trans. Roy. Soc. Edinburgh, xl. pt. iii. No. 23.