# A Tutorial on TEM Transmission Lines

by

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## Introduction

The concept of TEM transmission lines has been a distinct element of electronics engineering for well over 70 years [1, 2]. The term TEM (***T****ransverse* ***E****lectro****M****agnetic*, also known as ***T****ransverse* ***E****lectric and* ***M****agnetic*) refers to a condition in which both the electric and magnetic fields are parallel to a boundary plane [3] and there are no longitudial components of either field.

Other terms such as *transverse electric* (TE) and *transverse magnetic* (TM) refer to conditions in which the electric field or magnetic field, respectively, of a propagating wave is parallel to a boundary plane, in this case being the surface of the conductors of a transmission line, while at the same time the accompanying magnetic or electric fields, respectively, still have some longitudinal (or axial) components [4]. Both of these terms are normally associated with wave guides [3].

## Misconceptions

There are a number of misconceptions regarding the theory and practical applications of TEM transmission line, especially coaxial cable, which include:

> *"It is quite impossible to build a current balun of any ratio other than 1:1 using multiple transmission line transformers on a single core unless flux leakage between transmission lines is terrible.* [5]*"*

> *"It is physically impossible to build a transmission line current balun other than 1:1 on a single core when the windings have mutual coupling through the core.* [6]*"*

> *"I think the problem comes from the fact* (that) *circuit engineers look at physical appearance, and consider any two parallel conductors "transmission lines'...*[7]*"*

> *"Twisted pair, parallel lay, or coaxial...it only is excited with TEM mode (transmission line mode energy transfer) when fed with differential voltages at the "start" of the pair, and currents are equal and opposite* [8]*."*

> *"Coax does not always behave like a transmission line, …* [9]*"*

> *"…those who think any two parallel or concentric conductors when fed start-to-finish or end-to-end on one conductor forces a TEM mode are not viewing the system correctly*. [10]"

It is obvious from these remarks that there are a number of misconceptions regarding the general theory of TEM transmission line, which are contrary to established electromagnetics theory as well as engineering practice. These misconceptions are substantial obstacles in comprehending advanced applications of transmission lines, such as the design of transmission line transformers.

In order to avoid having these and other misconceptions regarding the theory and application of TEM transmission line become widespread, I'll provide here a brief tutorial on the subject, beginning with an overview of the fundaments of TEM theory as it applies to

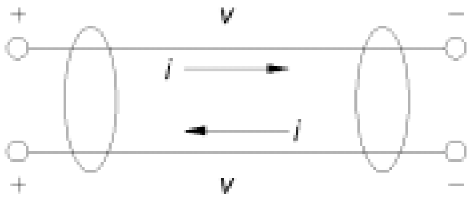


Figure 1 - Transmission Line in Transverse (TEM) Môde

TEM môde transmission lines, a generalisation of the parameters of TEM môde transmission lines, and then a detailed look at coaxial cable, which is a very familiar and readily understood form of TEM môde transmission line.

## Essential Concepts

Fig. 1 illustrates the essential concept of TEM môde transmission line in generalised form, where the two lines represent the two conductors of the transmission line, regardless of whether it is made of parallel wires, twisted wires, coaxial cable, or other means. Here, the currents are equal in magnitude and opposite in phase, which is consistent with Lenz's Law [3, 4, 11], while the voltages along the length of the two conductors are equal in both magnitude and phase. This may also be seen to say that the voltage across the two conductors is equal at all points along the transmission line.

These are the conditions that excite TEM môde, which again is equal and opposite currents in the two conductors, which results in no net magnetic field, and equal voltages either along or across the two conductors, which results on no net electric field. It is of no consequence how the external exciting voltage or current is applied, either across the terminals at one end or along the length of one side from one end to the other, and this point is underscored by the fact that discussions on the electromagnetics of TEM môde transmission line do so in terms of the currents in the conductors [12]. It only matters that the conditions of equal and opposite currents and equal voltages are met.

## Generalised TEM Transmission Lines

Let's examine the parameters that are common to all forms of TEM môde transmission line, and we'll later do a detailed study of coaxial cable as it is the easiest to comprehend since all of the electric and magnetic fields are contained between the conductors when in TEM môde. For all types of TEM môde transmission lines (coaxial, parallel wire, twisted wire, waveguide, etc.) the equations have the same basic form. If R, L, G, and C are the total series resistance, series inductance, shunt conductance, and shunt capacitance per unit length *z*, then the transmission line may be expressed as [3]:

$$\frac{\partial V}{\partial z} = -\left(R + j\omega L\right) I \qquad (1)$$

$$\frac{\partial I}{\partial z} = -\left(G + j\omega C\right) V \qquad (2)$$

which are derived from Maxwell's theorems and equations [1, 3, 4]. We'll dispense with the usual two or more chapters of differential calculus that normally bring us to this point.

From (1) and (2) we can derive the *characteristic impedance* $Z_o$ of the transmission line as related to the primary constants R, L, G, and C by [3]:

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad (3)$$

and the *complex propagation constant* $\gamma$ can be approximated as [3]:

$$\gamma = \alpha + j\beta = \sqrt{\left(R + j\omega L\right)\left(G + j\omega C\right)} \qquad (4)$$

where $\alpha$ is called the *attenuation constant* and $\beta$ is called the *phase constant*. The primary constants R, L, G, and C are directly related to the physical properties of the materials used in the transmission line and remain unaffected by the application of the transmission line.

For low-loss transmission line such as good quality coaxial cable [3]:

$$G \ll \omega C \tag{5}$$

$$R \ll \omega L \tag{6}$$

and the characteristic impedance $Z_o$ of (3) can be approximated as [3]:

$$Z_o \approx \sqrt{\frac{L}{C}} \tag{7}$$

and the propagation constant $\gamma$ of (4) can be approximated as [3]:

$$\gamma \approx j\omega\sqrt{L\,C} \tag{8}$$

The reciprocal of the square root of the product of L and C provides us with the *velocity of propagation* or *phase velocity* [3, 12]:

$$\upsilon_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L\,C}} = \frac{1}{\sqrt{\mu\,\varepsilon}} \tag{9}$$

where $\mu$ is the permeability of the transmission line medium in Farads/meter (F/m) and $\varepsilon$ is the permitivity of the transmission line medium in Henries/meter (H/m) [3].

**Coaxial Cable**

Let's now turn to the specific case of coaxial cable, using the illustration of Fig. 2 as a guide. Here, the inner conductor has a radius of $r_1$ and the outer conductor has an inner radius of $r_2$ and a thickness of t. The space between the inner and outer conductors is filled with an insulating dielectric material such as PTFE that has a *relative permitivity* (or *dielectric constant*) $\varepsilon_r$ and a *relative permeability* $\mu_r$. If the conductors are considered to be lossless as per our earlier appoximation, the unit shunt capacitance C is [12]:

$$C = \frac{2\pi\,\varepsilon}{ln\left(r_2/r_1\right)} = \frac{55.6\,\varepsilon_r}{ln\left(r_2/r_1\right)}\ \text{pF/m} \tag{10}$$

and the unit series inductance L is [12]:

$$\begin{aligned} L &= \frac{\mu}{2\pi} ln\left(r_2/r_1\right) = \\ &= 0.2\,\mu_r\, ln\left(r_2/r_1\right)\ \mu\text{H/m} \end{aligned} \tag{11}$$

The unit series resistance R is related to the conductivity of the conductors, and is frequency dependent by way of a phenomenon known as *skin effect*. We begin by first defining a quantity known as the *1/e depth of penetration* [4, 12, 13]:

$$\delta \approx \sqrt{\frac{2}{\omega\,\mu_v\,\mu_r\,\sigma}} \tag{12}$$

where $\sigma$ is the conductivity of the material. The current in a conductor will always concentrate on the surface that is nearest the wave that creates the current [12], and in the case of coaxial cable this is the electromagnetic field that exists between the inner surface of the outer conductor and the inner conductor. At high frequencies the skin effect causes the current to flow only on the outer surface of the inner conductor and the inner surface of the outer conductor [12], and this condition persists as long as the thickness t

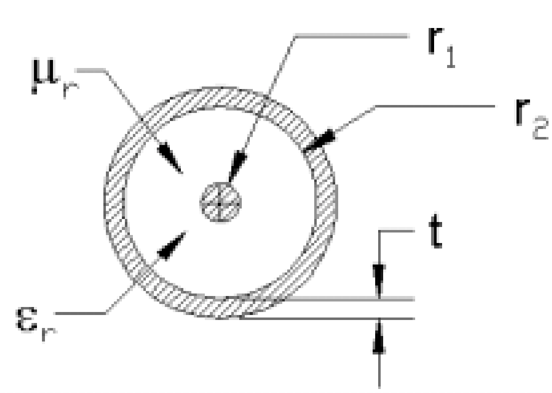


Figure 2 - Coaxial Transmission Line

of the outer conductor (see Fig. 2) is appreciably greater than the skin depth.

From (12) we can now define the surface resistance of the conducting material [3]:

$$R_s = \frac{1}{\sigma \delta}\ \Omega \tag{13}$$

The unit shunt conductance is related to the resistivity of the dielectric insulating material, due to a phenomenon known as *dielectric hysteresis*, which is analagous to the magnetic hysteresis in magnetic materials [4]. It is convenient to describe the total losses of the transmission line as the *equivalent conductivity* [4]:

$$\sigma' = \sigma + \omega \varepsilon'' \tag{14}$$

from which we can derive the *loss tangent* of the transmission line, which is [4]:

$$Tan\ \delta = \frac{\sigma'}{\omega \varepsilon'} \tag{15}$$

where ε' and ε" are often referred to as *dielectric dispersion formulas* [4, 14], which are very rigorous and very much beyond the scope of this discussion. It is sufficient for our purposes to use the loss tangent data provided by the manufacturer of the cable we are using and from that derive the surface resistance $R_s$.

Going back to (9), we can define a quantity that is well known to radio amateurs, which is the velocity factor of the cable:

$$VF = \frac{\upsilon_p}{c} = \frac{\sqrt{\mu_V\ \varepsilon_V}}{\sqrt{\mu\ \varepsilon}} = $$

$$= \frac{\sqrt{\mu_V\ \varepsilon_V}}{\sqrt{\mu_r\ \mu_V\ \varepsilon_r\ \varepsilon_V}} = \frac{1}{\sqrt{\mu_r\ \varepsilon_r}} \tag{16}$$

where c is the speed of light, $\mu_v$ is the permeability of free space ($4\pi$ x $10^7$ H/m). and $\varepsilon_v$ is the permitiviity of free space ($8.854\text{x}10^{-12}$ F/m). In general the relative permeability of most, if not all insulating materials is close to unity, so (10) can be comfortably approximated for coaxial cable as:

$$VF \approx \frac{1}{\sqrt{\varepsilon_r}} \tag{17}$$

Coaxial cable is a good practical example of TEM transmission line as the skin effect of the inner surface of the outer conductor causes the current of the outer conductor to be concentrated on the inside surface. The magnetic fields generated by the equal and opposite currents of the concentric inner conductor and the inner surface of the outer conduct cancel outside the outer conductor in both the near and far fields, leaving no net magnetic field outside of the outer conductor that would couple to nearby objects, such as the magnetic core of a transmission line transformer, which would cause additional losses beyond those of the cable itself [12, 15].

Similar equations may be developed for parallel wire transmission line [1, 3, 4] as well as twisted wire transmission line [16], the latter of which is an important element in the design of wideband transformers for RF applications [17].

## Advanced Applications

A thorough understanding of the basic theory of TEM transmission line is essential in comprehending advanced applications such as transmission line transformers (TLTs), which operate by transmitting energy by way of the TEM transmission line môde rather than on the more familiar coupling of magnetic flux as with a conventional transformer [18, 19].

## Closing Remarks

The theory of TEM transmission lines, especially coaxial cable, is a well- established element of RF design and has been part of the technology for over 70 years. The equations are both well established and easily understood, and have their foundations in basic electromagnetic theory such as Maxwell's theorems and equations and Lenz's Law. A basic background in the theory of TEM transmission line is an essential prerequisite in understanding advanced topics such as transmission line transformers, but not so difficult as to be beyond the understanding of those having entry level experience in the profession of RF circuit design.

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