and the Future," (page 355) in this same issue of *ELECTRICAL ENGINEERING*. We would appreciate anything you can do to put the facts straight.

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Forces Between Moving Charges

To the Editor:

Captain E. G. Cullwick in his interesting letter (*EE, May '47 pp 518–9*) voices the very reasonable expectation that the influence of one electric system upon another, will depend only on the relative velocity of the two systems, and not on any "absolute" velocity which may be assigned to either system. No doubt this is true, in a sense which requires careful definition.

However, it is not true that the *description* which an observer will give, of the influence of the one system upon the other, will depend only on the relative velocity of the two systems. It will depend also on the velocity of either system relative to the observer. This is because the observer describes his observation in terms of concepts whose definitions are relative to him.

Thus the observer defines the electric field at a point, as the force per unit charge on a charged body at *rest* relative to him at that point. He will also define the magnetic field, through the effects observed on a charged body, or on a probe coil, moved in some prescribed manner *relative* to him. Another observer, however, in motion relative to the first observer, in observing the electromagnetic field, will define the electric field at a point as the force per unit charge on a charged body at rest *relative* to him, and therefore in motion relative to the other observer. Similarly, with respect to the magnetic field. Thus, the electric and magnetic fields, in their very definition are relative to the observer, and the two observers, in our case, will not "see" the same electric and magnetic fields.

This relativity of the electric and magnetic fields with respect to the observer, arises not only from the relativity of the motion of the probes and probe coils used in the definition of the fields, but also from the relativity of the concept of force used in the definitions. Each observer will probably define force through some observation of the acceleration given to a particle of known mass. However, each observer, in measuring mass, will use a standard in a laboratory at rest *relative* to him, and in measuring acceleration, will use meter sticks and clocks at rest *relative* to him and in motion *relative* to the other observer. Hence, the concept of force which enters into the definition of the electric field, will also be relative to the observer.

Thus we find, as Captain Cullwick points out, that the electric and magnetic fields around a point charge, for an observer for whom the charge is at rest, are different from the electric and magnetic fields for an observer for whom the charge is moving.

One can hardly disagree with Captain Cullwick, that if absolute velocity is to have no meaning in nature, then the influence of one system upon another must depend only on their relative velocity. But this does not mean that the description of this influence as given by all observers will depend only upon this relative velocity of the two systems. We require only that these various descriptions shall be accounted for by taking account of the relativity of the concepts occurring in the various observers' descriptions, that is, that different probe charged bodies, probe coils, meter sticks, clocks, and so forth, are used by the different observers, in giving their descriptions of the influence of the one system on the other.

Captain Cullwick takes as his two systems, a coil, and a system of charges on a rigid body, in relative motion. For an observer for whom the system of charges is at rest, Captain Cullwick states that the integral of electric force along the wires of the coil is zero. For an observer for whom the coil is at rest, the integral of electric force along the wires of the coil is not zero. There is no contradiction in this. Each observer is able to account for the other's result by the different probe bodies, mass unit, meter sticks, and clocks which the other uses.

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To the Editor:

Reference is made to an article entitled "The Forces Between Moving Charges" by R. D. Sard, which appeared in the January 1947 issue of *ELECTRICAL ENGINEERING*, in which reference is made to an earlier article written by me (*EE, Oct '45, pp 351–6*). Professor Sard contends that my analysis proposes a drastic modification of electromagnetic theory, but it is worthy of note that he does not attempt to impeach the correctness of the results obtained by my methods. Indeed, he presents a lengthy demonstration to prove that for the case of charges moving parallel, line abreast, the textbook formulas yield the same results given by my formulas.

In writing the article to which Professor Sard refers I was under the impression that I was merely reviewing some of the basic concepts of electromagnetism in the light of prevailing thought, and offering a means for the practical engineer to bridge the gap between the two. Consequently, I have been somewhat surprised at the controversy which the article has aroused. While the most satisfactory reply would be to show the innumerable ways in which the analysis given in my article proves itself out, this would require an amount of demonstration and mathematical proof out of place in a communication of this character. Therefore I shall limit my remarks to matters related directly to the subject of Professor Sard's article, with the object of demonstrating the following:

1. There is not so much difference between my formulas and the textbook formulas as might appear from a casual review.
2. My formulas are closer to Maxwell's teachings than are the textbook formulas.
3. The differences between my formulas and Maxwell's teachings are in accord with precepts taught by the current textbooks.
4. It is widely recognized that the textbook formulas give incorrect or inconsistent results in many cases.

In the case of two charges moving line abreast, assume that the first charge is located at point $x=0$, $y=y$, $z=0$, with the other at the origin of co-ordinates. Professor Sard's equation 3*a* then reduces to a single component of force, which is in the y direction and equal to $F_{y2}=-e_1e_2(1-\beta_1\beta_2)/y^2\sqrt{1-\beta_1^2}$. Now, if the observer measures β_1 and β_2 equal in magnitude and in the same direction parallel to the x axis, this reduces to $F_{y2}=-e_1e_2\sqrt{1-\beta_x^2}/y^2$. But if the observer should attempt to transport his mental processes into the reference system of the second charge, which is also the reference system of the first charge since $\beta_2=\beta_1=\beta_x$, he knows from electrostatics that he would measure the force as $F_{y2}=-e_1e_2/y^2$ and is forced to introduce the concept of variable mass with respect to his own system, as Professor Sard has done in order to make his kinematic equations come out right.

However, there is available to the observer a simpler process which Professor Sard seems to have overlooked. He can use the transformation equations to relate E and H as measured in his own reference system to their values in the reference system of the charge on which the force is to be determined. In his own system the observer measures the velocities of the two charges as β_1 and β_2 and the electric and magnetic fields, according to Professor Sard's equation 1 for the conditions stated, as

$$E_x=0$$
$$E_y=-e_1/y^2\sqrt{1-\beta_1^2}$$
$$E_z=0$$
$$H_x=0$$
$$H_y=0$$
$$H_z=-\beta_1e_1/y^2\sqrt{1-\beta_1^2}$$

Transferring these field terms to the reference system of the second charge, according to the transformation equations as given by Page and Adams[1], yields:

$$E_{x2}=0$$
$$E_{y2}=-(1-\beta_1\beta_2)e_1/y^2\sqrt{1-\beta_1^2}\sqrt{1-\beta_2^2}$$
$$E_{z2}=0$$
$$H_{x2}=0$$
$$H_{y2}=0$$
$$H_{z2}=-(\beta_1-\beta_2)e_1/y^2\sqrt{1-\beta_1^2}\sqrt{1-\beta_2^2}$$

Putting these values in Professor Sard's equation 2 yields a single component of force, which is in the y direction and equal to:

$$F_{y2}=-\frac{(1-2\beta_1\beta_2+\beta_2^2)e_1e_2}{\sqrt{1-\beta_1^2}\sqrt{1-\beta_2^2}\,y^2} \qquad (1)$$

When $\beta_1=\beta_2=\beta_x$ this reduces to $F_{y2}=-e_1e_2/y^2$ just as though the second charge were at rest in the system of the first charge, which is my contention exactly.

It is to be noted that my article dealt with force effects as related to motion, which may be considered as the full electromagnetic effect. Professor Sard's analysis, following the usual textbook technique, combines the electrostatic and electromagnetic forces in the one formula. Any comparison with my formulas is to be made only with respect to terms in the combined formulas which contain β in any form. This is in accordance with the generally accepted concept that magnetism is a dynamic phenomenon, one of motion. To quote Maxwell,[2] "According to our hypothesis, we assume the kinetic energy to exist wherever there is magnetic force..." and "...this energy exists in the form of some kind of motion of the matter in every portion of space."

Since Professor Sard offers Maxwell as a background to his analysis, a few comments should be in order showing that my analysis follows Maxwell with very little change. In the first place, Maxwell based his development of electromagnetism on Ampère's work. In Chapter II of Section IV of his "Treatise" he gives an extensive discussion of Ampère's work, including Ampère's formulas for the interaction of currents. The basic formula for the force between current elements, as given by Ampère[3] and described by him as fundamental to his entire theory, is $F=ii'\,ds\,ds'(\sin\theta\sin\theta'\cos\omega+k\cos\theta\cos\theta')/r^2$ with a value of $-1/2$ for k. Recognizing the relation between $i\,ds$ and the quantity and velocity of the electricity making up a flow of current, this formula can be written: $F=QQ'(v\sin\theta\, v'\sin\theta'\cos\omega-1/2\, v\cos\theta v'\cos\theta')/r^2$, in which form it will be recognized as equivalent to equation 4 in my article except that I use the squares of relative velocity components instead of the products of individual velocity components. Maxwell claimed[4] that Ampère's formula required a factor of two for use in electromagnetic units, as Ampère used electrodynamic units. This undoubtedly is true, but does not apply to my formulas, as it is to be noted that in the days of both Ampère and Maxwell, as the latter admits,[5] it was not known whether a flow of current was more correctly treated in terms of the 1-fluid theory or the 2-fluid theory. Any formula designed for application to both positive and negative electricity combined, even though one kind is "bound" as we now consider the positive to be, should be expected to give twice the value given by formulas, such as mine, which treat the two kinds individually. When the results are summed up for the total of the two effects, as explained in my article and also used by Professor Sard, the two procedures become equivalent.

Utilization of the relative velocity as employed in my article, instead of the individual velocities of the current elements, is in accordance with prevailing concepts and may be considered a "modernization" of Ampère which is no more to be criticized than the Lorentz transformation, and, indeed, has been shown to be equivalent thereto in the case just investigated. Maxwell's "Treatise" abounds with references to relative motion. Modern textbooks introduce it through the concept that the field of a charge is carried along with the charge. For this I quote Page[6] and Richtmyer and Kennard,[7] the latter even amplifying to the extent of stating: "...relative to the charge the field seems to stand still." This is in conformity with the principle that the laws of nature are the same in any inertial system. Certainly, in accordance with this, the conditions when all charges have a common velocity must be the same as for static conditions. Professor Sard agrees with this. Then it follows that when the velocities are not the same the effects must depend upon the difference of the velocities. Of course I have not combined my velocities in a relativity manner. This does not seem to be necessary, but I see no objection to doing so if it gives the user more esthetic satisfaction.

The textbook formulas make a much more radical departure from Maxwell's teachings than any which I have discussed, in that they use Biot's law for the electromagnetic effects of current flow. Actually it is commonly believed that it is Ampère's law which is being used. Page[8] makes this misstatement, as do others, including Jeans.[9] In a recent article[10] in *ELECTRICAL ENGINEERING*, describing the method of teaching electricity and magnetism in the University of Kansas, the same historical error is made. This error has been pointed out by a number of sources.[11,12,13] While historical accuracy is not essential to scientific accuracy it frequently is helpful to a proper grasp of the significance of the ideas of past authorities. In this instance I believe that the introduction of Biot's formula, instead of Ampère's, as intended by Maxwell, is at the root of the present controversy. Maxwell seems to have discounted Biot's work rather completely. I have not noted in his "Treatise" a single direct reference to either Biot or Savart.

Use of Biot's formula leads to results which are inconsistent with the third law of motion (equality of action and reaction). This has been recognized widely, and has been the subject of a number of articles.[13,14,15] However, none of these references resolves the problem as completely and simply as does a straightforward application of my formulas. Robertson goes back to Ampère but does not employ the relative velocity concept. Howe attempts to reintroduce Maxwell's now outmoded displacement currents in free space. Both arrive at conclusions which should be recognized at once by any electrical engineer as contrary to the facts in generator and motor operation. Keller introduces the concept of the momentum of the field as expounded by J. J. Thomson,[16] but he uses formulas for E and H which, while purporting to be the same as the textbook formulas, actually differ from those in general use by a factor of $1-\beta^2$ although they appear at first glance to be equivalent but in slightly different form. This tends to cast doubt upon the correctness of his entire analysis.

It is worth noting that Maxwell repeatedly points out the usefulness of a close adherence to Ampère's analysis. For instance, following a comparison of possible formulas for the interaction of current elements, he concludes with this statement:[17] "Of these four different assumptions that of Ampère is undoubtedly the best, since it is the only one which makes the force on the two elements not only equal and opposite but in the straight line which joins them." This is a strikingly significant statement, and gives an unequivocal interpretation of the mental picture he had as he started upon the development of his field theory.

It even might be inferred that Maxwell felt impelled to utter a word of caution against breaking too far away from Ampère in the application of the field equations to individual charges. For after developing his field theory, we find he reverts to Ampère, repeating a thought previously mentioned in the course of his field development, in these terms:[18] "If we suppose our mathematical machinery to be so coarse that our line of integration cannot thread a molecular circuit, and that an immense number of magnetic molecules are contained in our element of volume, we shall still arrive at results similar to those of Part III, but if we suppose our machinery of a finer order, and capable of investigating all that goes on in the interior of the molecules, we must give up the old theory of magnetism, and adopt that of Ampère, which admits of no magnets except those which consist of electric currents."

Finally it seems worth while to point out that the writer's formulas, when put in vector form and combined with the Coulomb electrostatic force become:

$$F=e_1e_2\left[\frac{R}{r^3}+\frac{Rv^2}{c^2r^3}-\frac{3}{2}\cdot\frac{R(V\cdot R)^2}{c^2r^5}\right] \tag{2}$$

where

R = the vector distance between the charges
r = the scalar magnitude of R
V = the vector velocity difference of the charges
v = the scalar magnitude of V
c = the ratio of electromagnetic to electrostatic units
e_1, e_2 = the magnitudes of the charges
(All terms in centimeter-gram-second electrostatic units)

This formula follows from the work of Wilhelm Weber, published in 1846–48, and discussed at some length in the concluding chapter[19] of Maxwell's "Treatise." It is worthy of note that Maxwell admits the correctness of the results obtained by Weber's analysis, but rejects the method largely because it either did not require a medium or seemed to require instantaneous transmission in a medium. But it now appears that for unaccelerated charges (the case under discussion) the question of transmission does not enter, because if we accept, along with references quoted, the idea that "the field is carried along with the charge," then the effect of the velocity is uniformly present everywhere simultaneously. Of course if accelerations occur, a term for the acceleration effect must be added, and for large distances of separation the method of retarded potentials must be used. But this is only what should be expected, in accordance with the commonly accepted principle that radiation is produced only by accelerated charges.

This brings up a point which, while not specifically pertinent to the present discussion, is so fundamental and so generally misunderstood that it seems advisable to conclude with a few remarks concerning it. This is the fact that in electromagnetism there are two distinct processes: the one an effect due to the relative velocities of the charges, usually revealing itself as physical forces, as in the "motor" effect, but also frequently revealing itself as a voltage, as in the "generator" effect; and the other an effect due to the relative accelerations of the charges, usually revealing itself as an induced voltage, as in the "transformer" effect. . . . But the distinction between the effects of uniform velocities and changing velocities is not made clear in most textbooks. Even Maxwell does not draw a clear line of distinction in these matters. He discusses electromagnetism with respect to the work of Ampère and Faraday more or less correlatively, without seeming to realize that the former's discoveries were related mostly to the "motor" effect and the latter's mostly to the "transformer" and "generator" effects. It probably is due to the fact that two effects were discovered by the one individual and, especially, within a few days of each other that the distinctions as to causation have not become recognized generally. . . .

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Transmission Line Impedance Calculations

To the Editor:

In the usual method of calculating the attenuation function of a line, the series impedance per unit length, $Z/\underline{\theta_Z}$, and the shunt admittance per unit length, $Y/\underline{\theta_Y}$, first are determined. The propagation function per unit length, $\gamma/\underline{\theta_\gamma}$, is the square root of the product of these quantities. The real part of the propagation function is the attenuation function, α, and the imaginary part is the phase function, β.

For accuracy in the third significant figure in determining α, it frequently is found necessary to carry θ_Y and θ_Z to 1/100 of a degree, while in the determination of β, a degree change in θ_Y or θ_Z will not affect the value appreciably. This is because θ_γ is close to 90 degrees where the cosine is changing rapidly with the angle, and the sine is not.

As an example, consider the following case. A carrier cable, operated at 50,000 cycles per second, has the following constants per loop foot: $C=25.31$ micromicrofarads, $L=0.130$ microhenry, $R=0.00444$ ohm, and $G=0.0628$ micromho. Calculation gives

$$Y=(7.95\times10^{-6})/\underline{89.55^\circ}$$

$$Z=0.0409/\underline{83.78^\circ}$$

from which

$$\gamma=(0.571\times10^{-3})/\underline{86.67^\circ}$$

and

$$\alpha+j\beta=(0.0332+j0.569)\times10^{-3}$$

If one takes

$$Y=(7.95\times10^{-6})/\underline{90^\circ}$$

and

$$Z=0.0409/\underline{84^\circ}$$

γ becomes

$$\gamma=(0.571\times10^{-3})/\underline{87^\circ}$$

and

$$\alpha+j\beta=(0.0299+j0.569)\times10^{-3}$$

The error in α is about ten per cent, but the value of β is unchanged.

In the same way, the real part, R_k, of the characteristic impedance, $Z_k/\underline{\theta_k}$, usually may be calculated with small error without great accuracy in θ_Y and θ_Z, but the reactive component, X_k, may require more care. In the foregoing case

$$Z_k=71.8/\underline{-2.88^\circ}$$

and

$$R_k+jX_k=71.7-j3.61$$

Rounding of the angles to 90 and 84 degrees gives

$$R_k+jX_k=71.7-j3.76$$

causing an error of about four per cent in X_k.

In making these calculations with a slide rule, the determination of angles near 90 degrees to 1/100 of a degree is inconvenient. A pair of formulas which are convenient to use and which yield good accuracy are as follows:

$$\alpha=\frac{GZ_k^2+R}{2R_k} \tag{1}$$

and

$$X_k=\frac{GZ_k^2-R}{2\beta} \tag{2}$$

Since R_k, Z_k, and β may be determined accurately without great accuracy in θ_Y and θ_Z, these two formulas may be used to determine α and X_k accurately when θ_Y and θ_Z are only approximate values. In the foregoing example, if the approximate values $\theta_Y=90^\circ$ and $\theta_Z=84^\circ$ are used, $Z_k=71.8$, $R_k=71.7$, and

$$\alpha=\frac{(0.0628\times10^{-6}\times5150)+0.00444}{143.4}=0.0332\times10^{-3}$$

Similarly the value of X_k is found to be -3.62. These errors are well within the limits of slide rule accuracy.

It should be pointed out that these formulas for α and X_k reduce, at high frequency, to the well-known formulas for this case. For high frequencies, $Z_k=R_k$, and α becomes $(R/2Z_k)+(GZ_k/2)$; X_k becomes zero because β becomes large.

Proof of the formulas is as follows:

$$\gamma=\alpha+j\beta$$

$$=\sqrt{YZ}\;\Big/\underline{\frac{\theta_Z+\theta_Y}{2}}$$

$$=\sqrt{YZ}\cos\left(\frac{\theta_Z+\theta_Y}{2}\right)+j\sqrt{YZ}\sin\left(\frac{\theta_Z+\theta_Y}{2}\right) \tag{3}$$