On the Electrical Resistance of Moving Matter.

By Professor F. T. Trouton, F.R.S., and A. O. Rankine, B.Sc., University College, London.

(Received February 20,—Read March 5, 1908.)

The question of relative motion between the earth and the neighbouring ether has been under discussion for many years. It has, from time to time, been the subject of important investigations; but these have all resulted negatively. The experiment about to be described is not different from them in this respect, yielding, as it does, no definite information on the main point. It was suggested and commenced by one of us some years ago; but the serious difficulties which invariably attend measurements of such delicacy have delayed its completion till the present time. Indirectly, the aim was to measure the direction and magnitude of ether-drift; the actual method having been to attempt to demonstrate the existence of the Fitzgerald-Lorentz shrinkage which has been supposed to mask the effect in the direct experiments of Michelson and Morley, and of Trouton and Noble. It may be as well to say at once that if such a shrinkage be real, it is in this experiment also obscured by some other exactly compensating change or changes, no effect approaching that to be otherwise expected having been observed.

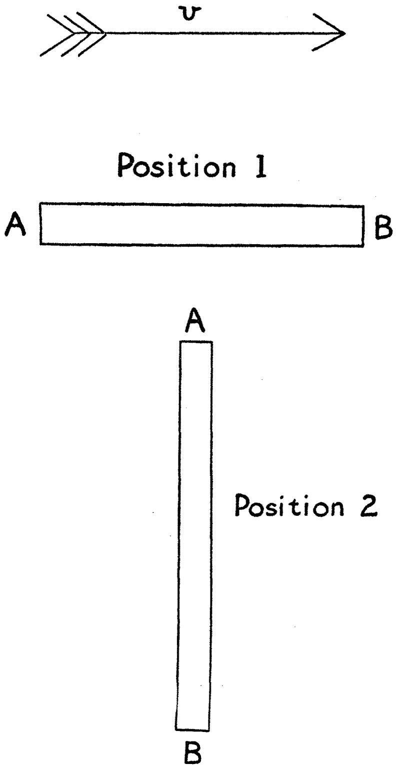


Fig. 1.

The principle of the measurement is a very simple one. Imagine a uniform wire AB (fig. 1) of length l and cross-sectional area a moving through the ether in a direction parallel to its length with a velocity v. Let ρ be its specific resistance and R its total electrical resistance. The relation

$$R = \rho \frac{l}{a} \qquad (1)$$

is then true. Differentiating logarithmically, we obtain

$$\frac{\delta R}{R}=\frac{\delta\rho}{\rho}+\frac{\delta l}{l}-\frac{\delta a}{a}. \tag{2}$$

Now suppose that the wire AB is turned through a right angle, so that its length is perpendicular to the velocity v. According to the Fitzgerald-Lorentz shrinkage hypothesis, the length of the wire will be thus increased by a small amount δl, such that $\frac{\delta l}{l}=\frac{1}{2}\left(\frac{v}{V}\right)^2$, where V is the velocity of light, and all powers of v/V higher than the second have been neglected, v being supposed very small compared with V. Writing β for v/V, we have

$$\frac{\delta l}{l}=\tfrac{1}{2}\beta^2.$$

This is not the only change in dimensions to be expected. In the first position of AB both dimensions of the cross section are perpendicular to the direction of motion, while in the second position one remains perpendicular, but the other becomes parallel. A decrease in this latter dimension of the relative amount $\frac{1}{2}\beta^2$ will cause a diminution of a in the same ratio. Therefore

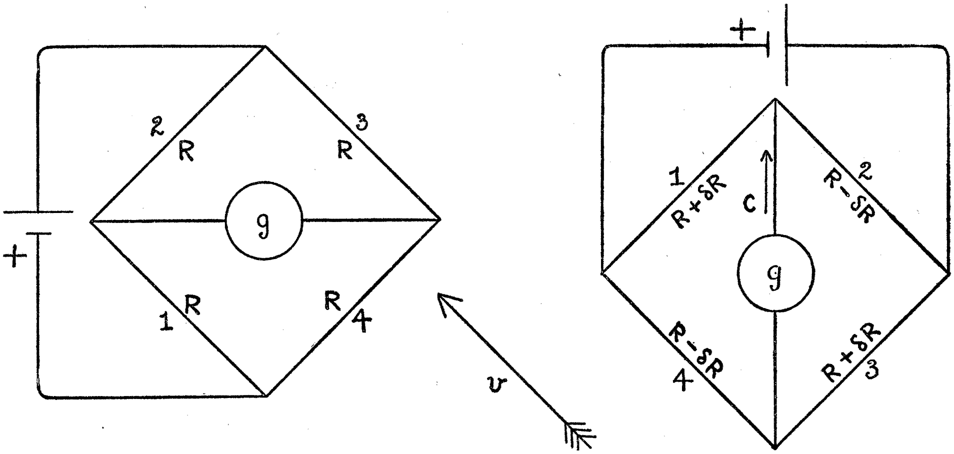
$$\frac{\delta a}{a}=-\tfrac{1}{2}\beta^2.$$

Substituting in (2), it follows that

$$\frac{\delta R}{R}=\frac{\delta\rho}{\rho}+\beta^2.$$

If it be supposed that the specific resistance of the material forming the wire is independent of the direction of motion, $\delta\rho/\rho=0$, and therefore $\delta R/R=\beta^2$. Hence, on the above assumptions, it is to be expected that the resistance of a wire with its length perpendicular to the ether-drift will be greater than when parallel in the ratio $(1+\beta^2):1$. On the other hand, should there be no change in total resistance, two alternatives present themselves. Either there is no alteration of length of the kind supposed, or the specific resistance changes in such a way as to compensate it. In the latter case the law of change would be that the specific resistance of a material to a current flowing parallel to the ether-drift is greater than that at right angles to this direction in the ratio $(1+\beta^2):1$. The present investigation was, however, based upon the assumption that the specific resistance was constant; and the object in view was to detect a variation of the resistance of a wire with direction.

The method used was the ordinary Wheatstone bridge method of comparing resistances, specially adapted, of course, to the particular requirements of this case. With certain modifications, to be described later, the arrangement was as follows:—Four coils of wire, each wound upon a flat rectangular frame, formed the four arms of a Wheatstone bridge. By suitably adjusting the position of any frame, the wire on it, with the exception of the small part used in turning the corners, could be made to take up any desired direction. The frames were arranged horizontally on a stand so that the wires forming opposite arms of the bridge were parallel, and those forming adjacent arms perpendicular to one another. The arrangement is shown diagrammatically in fig. 2. The lines marked 1, 2, 3, and 4, must be taken as representing the direction of the major part of the wire on the corresponding coil. If the resistances of 1, 2, 3, and 4 are equal, there will be no current through the galvanometer. Suppose that the coils 1 and 3 are parallel, and the coils 2 and 4 perpendicular to the ether-drift; also that balance is obtained, and the resistance of each coil is equal to R. If, now, the stand be rotated through a right angle, so that 1 and 3 become perpendicular, and 2 and 4 parallel to the drift, it is to be expected, on the previous assumptions, that 1 and 3 will each increase in resistance by an amount δR, and that 2 and 4 will each diminish by an equal amount. This would result in the balance being destroyed; and a current of magnitude $E\delta R/R(R+g)$ would flow through the galvanometer in the direction indicated by the arrow. Here E represents the E.M.F. of the battery, and g the galvanometer resistance; and the internal resistance of the battery has been neglected. Or, if for some reason it has been impossible to obtain perfect balance in the first instance, and a small current flows in the galvanometer all the time, the change to be expected upon rotation in the magnitude of this current is measured also by $E\delta R/R(R+g)$. In other words, and remembering that the expected value of



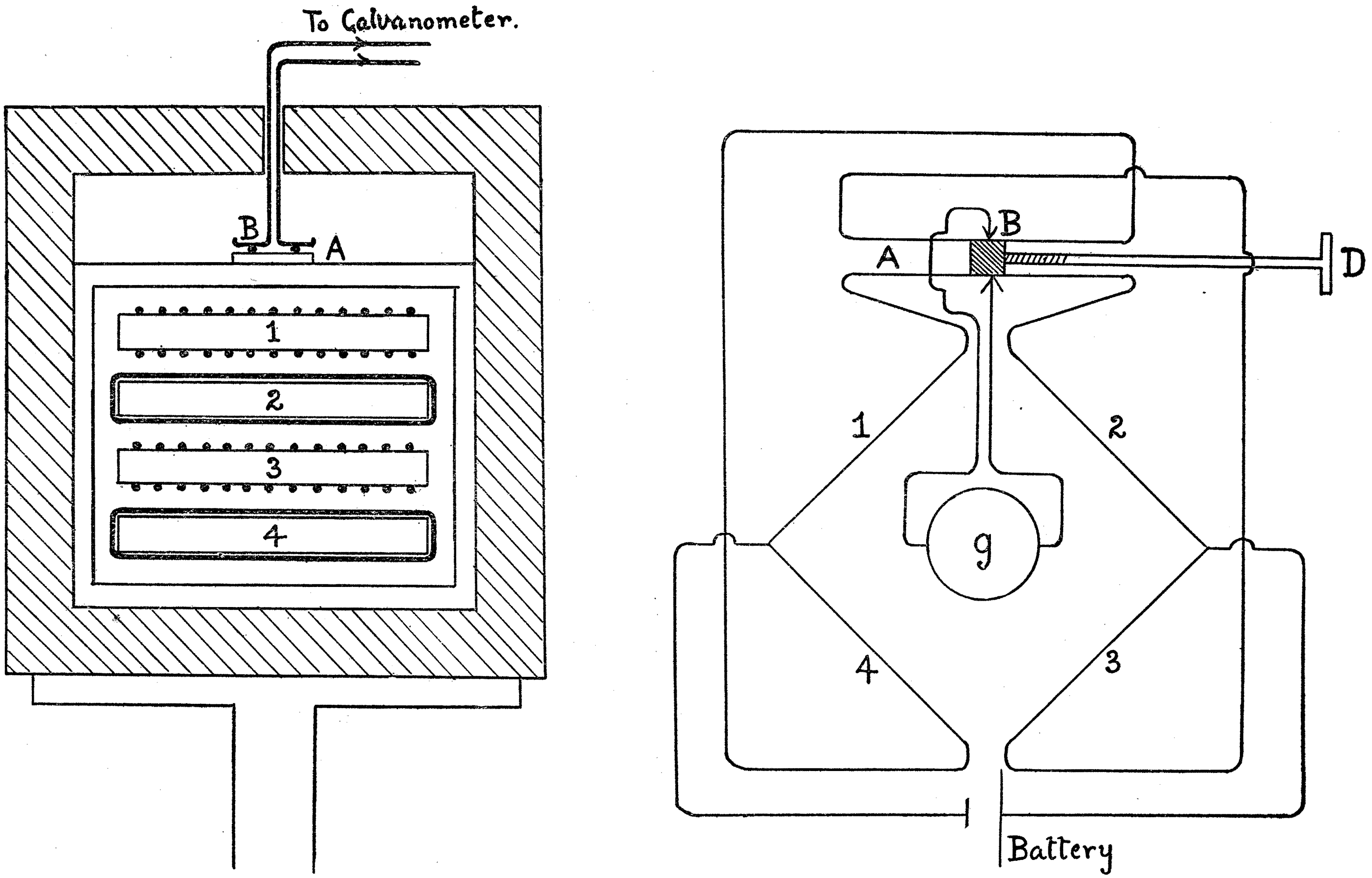
Position 1.

Position 2.

FIG. 2.

δR is $R\beta^2$, a variation of current of value $\frac{E}{R+g} \cdot \beta^2$ is to be looked for. Since this expression contains so small a factor as $\beta^2 = (u/V)^2$, it is obvious that the measurement will be a difficult one; and this was indeed found to be the case, many effects, usually negligible in resistance measurements, now being comparatively large. Their elimination—or rather, partial elimination, for they were to the last not entirely absent—was very tedious, and at times appeared almost hopeless; but it was at last effected in the manner about to be described.

The first difficulty was due to the presence of thermoelectric currents, which, at first, varied so rapidly as to make determinations impossible. It was very soon found that junctions, even between pieces of the same wire, which were originally in the resistance arms, and the key in the galvanometer arm produced disturbances of such a serious character that they had to be omitted. Again, it was at first intended to effect balance by the movements of a slider on a thick copper rod, the other end of the galvanometer arm being permanently attached to the junction between the opposite arms. It was also hoped to be able to re-balance after rotation by a further movement of this slider; this method, however, had to be abandoned, owing to the impossibility of moving the slider without producing further heating. Another objection to the use of this thick copper rod lay in the fact that it was the cause of a difference of temperature between the two ends of the galvanometer arm. Practically the same current flowed in this rod and in the thin wires of which the coils were made, and to which the other end of the galvanometer wire was attached. Owing, therefore, to the heating effect of the current itself, a permanent difference of temperature became established at the two terminals referred to. To effect the removal of these disturbances, the following means were adopted. The four bridge arms were made of two unbroken pieces of uniform wire soldered together at the points at which the current was led in from the battery. Here, of course, small variations in potential were ineffective, producing, in the case of perfect balance, no current through the galvanometer, and, even when a small current was flowing, causing changes of the second order only in it. There were no junctions at all in the wires whose resistances were being compared. The galvanometer was inserted by means of a slider (as indicated in fig. 3), which joined through the former the mid points of the two unbroken wires previously referred to. Contact was made by simple pressure between crossed wires. It was, of course, impossible to avoid using two junctions here; but, by arranging them very close together, and because they were now equally heated by the current, the thermoelectric effects were reduced practically to zero.



Vertical section through centre.

Diagram of arrangement of apparatus.

Fig. 3.

Another and more persistent disturbance arose from unequal heating producing changes of resistance in the coils. It soon became evident that it would be impossible to use uncovered wire; but even when the wire used (originally copper) was thickly covered with gutta-percha, the effect of heating was too large to make definite measurements. The coils were arranged one above the other on a stand, and were turned about a vertical axis inside an enclosure made of wood and felt. The temperature of the air varied from point to point of this enclosure; and, upon rotation of the stand, changes of resistance occurred, owing to the coils occupying different positions in it. Moreover, even when the coils were not rotated, the behaviour of the current in the galvanometer indicated a gradual increase of resistance in the upper coils relative to the lower ones. This was, doubtless, due to the warming of the air in the enclosure by the currents which were flowing; and, the warm air rising, the upper resistances increased more rapidly than those lower down. With the exception of this latter effect, the disturbances were removed by rotating the enclosure itself with the stand, thus carrying the temperature distribution of the air round, and by making the coils with manganin wire instead of copper, on account of the much smaller temperature coefficient of the former. The relative increase of resistance of the upper coils was thus made much more gradual, but it has been found impossible to entirely eliminate the effect, and it has been necessary, even in the final form of the apparatus, to take time readings of the current for the various positions of the stand. The variation of the current in the galvanometer, due to this cause, is now, however, sufficiently slow to make it quite easy to distinguish from it the immediate genuine effect which is looked for.

A further spurious effect was that due to alterations in resistance which were brought about by stresses introduced in rotating the stand. As it happened, the magnitude of this effect was just of the order of that expected; and this at one time led us to suspect a positive result. The apparatus at that time was not in its final form, and was not adapted for rotations other than a right angle; so it was impossible to make an absolutely conclusive test. The balancing bridge (shown in fig. 3) was not then rotated with the rest of the apparatus, and thus there arose a possibility of strain in the wires forming parts of the resistances which were being compared. This difficulty was finally surmounted by rotating the whole of the apparatus bodily, with the exceptions of the galvanometer and battery. This removed the strain to the wires leading to the two latter, *i.e.*, to places where small changes of resistance were unimportant.

Finally, it was necessary to remove an effect which can hardly be called a disturbance. As has been already pointed out, the use of a key was dispensed with in the galvanometer arm. The result of permanent contact was to produce an induction current in the galvanometer when the apparatus was turned round. This would not have mattered if it had not been necessary to take time readings on account of heating effects. Readings were eventually taken every quarter minute, and the galvanometer was not sufficiently damped to make this possible when the throws were large. In the whole region of space occupied by the stand, the magnetic field of the earth was reduced practically to zero by suitably disposing 16 permanent magnets in the neighbourhood. The temporary induction effect upon rotation died out then completely in about five or six seconds after that rotation.

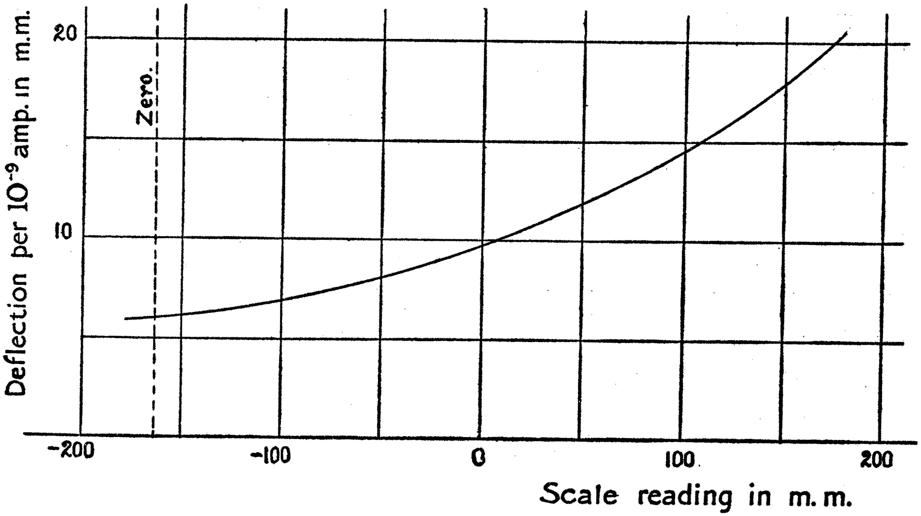
The final arrangement of the apparatus is shown diagrammatically in fig. 3. The four coils, 1, 2, 3, and 4, are arranged on a stand as before indicated, and above them (and also fixed to the stand) the balancing bridge A. This latter merely consists of about 5 or 6 cm. of bared wire drawn taut on a wooden stand. The wires are here parallel and about a centimetre apart, and the slider B, through which wires lead to the galvanometer, is movable along their length by means of a screw D. The slider B consists of an arrangement by which the two wires from the galvanometer are pressed down by springs, one on each wire of the balancing bridge, and balance is obtained by using the screw D. The whole of the apparatus, with the exception of the galvanometer and battery, is encased in a cubical double-walled enclosure, which is fixed to a horizontal turntable, the interspace between the two walls of the enclosure being filled with cork dust for purposes of thermal insulation. The screw-head D projects outside the enclosure, so that adjustments may be made without opening the latter, and the wires to the battery and galvanometer are led out through a small hole at the top on the vertical axis of rotation.

The battery used is a single-storage cell. The galvanometer is of the Du Bois type—a low-resistance suspended needle galvanometer, trebly shielded with soft iron. These shields are very effective in removing magnetic disturbances such as those caused by the neighbouring electric railway, and they are found to be very necessary in delicate work of this description. The behaviour of the needle is examined by using a Nernst lamp and scale at about $2\frac{1}{2}$ metres distant, and, in its most sensitive state, a scale deflection of about 4 cm. can be obtained with a current of 10^{-9} ampere.

In the actual experimental work the field about the needle was found to be variable, and the sensitiveness increased with the scale reading. This is shown in fig. 4, where the deflections produced by an additional 10^{-9} ampere are plotted against the scale reading. The expected deflection upon rotation, therefore, varied with the scale reading at that time, and use was made of this calibration in calculating the results.

With regard to the dimensions of the apparatus, the coils were made of gutta-percha-covered 24 manganin wire, each of them consisting of 16 turns round a 6-inch square flat frame. Almost exactly 1/7 part of the wire was not horizontal, *i.e.*, the parts used in turning corners, and in leading to the bridge. This fraction was, therefore, not expected to contribute to the calculated change of resistance. The resistance of each coil was 11·16 ohms and that of the galvanometer 10 ohms. The E.M.F. of the storage cell was slightly variable, but was taken as having an average value of 2·1 volts, and its internal resistance has been regarded as negligible.

FIG. 4.



Sensitiveness of galvanometer.

To calculate the expected variation in current upon rotation, therefore, we substitute the above values in the formula

$$\delta\mathrm{C} = \frac{\mathrm{E}}{\mathrm{R}+g}\beta^2.$$

This must, however, be reduced by 1/7 part, in order to allow for the non-contributing parts of the wire. We have

$$\delta\mathrm{C} = \frac{6 \times 2{\cdot}1}{7 \times 21{\cdot}16} \cdot \beta^2 \text{ ampere}$$

$$= \frac{6}{7} \cdot \frac{\beta^2}{10} \text{ ampere, approximately.}$$

If the earth's orbital motion only be taken into account, the value of β^2 is approximately 10^{-8}, and in this case the expected change in current would be

$$0{\cdot}86 \times 10^{-9} \text{ ampere.}$$

Such a current would produce, in the neighbourhood of the scale reading O (fig. 4), a deflection of 8·4 mm., but the particular deflection to be expected depends on the part of the scale at which the determination is made.

Moreover, if the sun's proper motion be allowed for, the value of β^2 is dependent upon the time of year, and a special calculation is required in each case. This has been done for the results given later in Tables II and IV. The values of β^2, the time of horizontal drift, and its azimuth when horizontal, were obtained from the values given in the paper describing the ether-drift experiments of Trouton and Noble.*

The method of taking observations was as follows:—A time was chosen when the calculated direction of the total drift was horizontal. By means of moving the slider B (fig. 3), an attempt was made to reduce the current in the galvanometer to zero. This, however, was very difficult, and not essential. Usually, the spot of light, whose position indicated the magnitude of the galvanometer current, was merely brought somewhere within the limits of the scale, and possibly there would be already a current of about 10^{-8} ampere. The spot of light would be found to slowly creep in the direction indicating a relative increase of resistance in the upper coils. Its velocity would, however, become much smaller after the current had been flowing for some time. (It may here be pointed out that, as a rule, the battery was, on this account, connected up with the bridge some hours before taking an observation.) The turntable was then rotated until one pair of coils became parallel to the drift, and a reading was taken at a particular instant. The turntable was then turned at once through a right angle, and a further reading taken after 15 seconds. Immediately the turntable was restored to its original position, another reading following after 15 seconds, and so on for about 20 reversals. Thus a set of 20 readings, at half-minute intervals, was obtained for each of the two positions of the stand.

Unfortunately, owing to mechanical shaking of the galvanometer (a disturbance which is never absent in London except in the early hours of the morning), it was impossible in the daytime to take readings nearer than 1 mm., although the optical definition was otherwise sufficiently good to admit of estimation to 1/10 mm.

The following set of observations is typical:—

---

* 'Phil. Trans.,' A, vol. 202, pp. 165—181.

Table I.

Date, December 16, 1907. Time, 4.40—4.50 P.M. Azimuth of horizontal drift, measured eastwards from meridian, $\frac{1}{2}\pi+42^\circ$.

Readings on scale (measuring the values of galvanometer current):—

Azimuth of 1 and 3 (parallel drift) -48°.	Azimuth of 1 and 3 (perpendicular drift) $+42^\circ$.	Azimuth of 1 and 3 (parallel drift) -48°.	Azimuth of 1 and 3 (perpendicular drift) $+42^\circ$.
mm.	mm.	mm.	mm.
−14		−39	
	−15		−41
−17		−42	
	−18		−43
−20		−45	
	−20		−45
−22		−47	
	−25		−47
−25		−49	
	−26		−51
−28		−52	
	−29		−53
−30		−53	
	−31		−54
−32		−54	
	−32		−56
−33		−56	
	−36		−57
−36		−57	
	−38		−58
−38		−60	
	−39		−63
		−65	

It will at once be apparent that there is no general tendency for the numbers in one column to be in excess of those in the other by the expected amount, viz., 10·9 mm. It is somewhat difficult, however, to determine the best method of interpreting them, for the purpose of discovering the limits of measurement of the apparatus. The form of the general time variation in current (so called to distinguish its immediate effects attributable to rotation) is unknown and not necessarily linear; hence, to take the difference of the means of the numbers in the two columns is only approximate. This latter was the method at first adopted, but although the results were satisfactory enough where the general current variation was practically linear, in cases where this condition did not exist discordant values of the difference were obtained, according to the number of observations utilised. Eventually the method about to be described was adopted as giving the most consistent results.

The following is an ideal set of readings, $a_1, a_2 \ldots a_n$ being those for one position of the coils, and $b_1, b_2 \ldots b_{n-1}$ those for the other position:—

Differences.	Readings.		Differences.
	a_1		
		b_1	$\frac{1}{2}(a_1+a_2)-b_1$
$a_2-\frac{1}{2}(b_1+b_2)$	a_2		
		b_2	$\frac{1}{2}(a_2+a_3)-b_2$
$a_3-\frac{1}{2}(b_2+b_3)$	a_3		
		b_3	
	⋮	⋮	⋮ ⋮
$a_{n-1}-\frac{1}{2}(b_{n-2}+b_{n-1})$...	a_{n-1}		
		b_{n-1}	$\frac{1}{2}(a_{n-1}+a_n)-b_{n-1}$
	a_n		

$$\frac{(a_2+a_3+\ldots+a_{n-1})-\frac{1}{2}(b_1+b_{n-1})-(b_2+b_3+\ldots+b_{n-2})}{n-2}$$

$$= \text{Mean of differences in first column.}$$

$$\frac{\frac{1}{2}(a_1+a_n)+(a_2+a_3+\ldots+a_{n-1})-(b_1+b_2+\ldots+b_{n-1})}{n-1}$$

$$= \text{Mean of differences in last column.}$$

The expression in the first column represents the difference between a particular value of *a* and the mean of the values of *b*, just before and just after, while that in the third column represents the difference between the mean of two successive values of *a* and the intermediate value of *b*. The means of the differences on the two sides respectively are given below, and they are to be expected to be equal to one another. This turns out to be very nearly true, and the final mean of these two numbers has been recorded in Tables II, etc., as measuring the observed excess of the "*a*" column over the "*b*" column.

For instance, take the numbers given in Table I. Here $n = 23$.

$$\text{Mean of differences in first column} = \frac{-835+39+799}{21} = \frac{3}{21} = +0{\cdot}14 \text{ mm.}$$

$$\text{Mean of differences in last column} = \frac{-39{\cdot}5-835+877}{22} = \frac{2{\cdot}5}{22} = +0{\cdot}11 \text{ mm.}$$

The final mean is therefore +0·13 mm., and this measured the change in current caused by the rotation of the coils in this particular case.

Now, by reference to the magnitude of the ether-drift on this particular day, and to the curve of sensitiveness of the galvanometer (fig. 4), it will be seen that the expected difference of scale reading is +10·9 mm. This certainly does not exist; and, in view of the fact that readings were made correct to 1 mm. only, there is reason for supposing that the observed difference is due to error in observation.

The following tables are records of the other observations taken:—

Table II.

These observations were made at the best times according to the calculations of Trouton and Noble, *i.e.*, when the resultant drift is horizontal.

Date.	Time.	Magnitude of drift.	1st azimuth of 1 and 3.	2nd azimuth of 1 and 3.	Calculated difference of reading.	Observed.
Dec., 1907.	P.M.	miles/sec.	°	°	mm.	mm.
11	4.50	22·1	−44	+46	+ 7·5	0·0
12	5.0	22·3	−44	+46	+11·5	+0·2
13	4.45	22·4	−45	+45	+ 7·7	+0·1
16	4.40	22·9	−48	+42	+10·9	+0·1
19	4.25	23·3	+40	−50	−11·3	+0·2
Jan., 1908.						
8	3.0	25·7	−63	+27	+11·4	−0·2

Table III.

In the following cases rotation was through 180°, so that no effect is to be expected.

Date.	Time.	1st azimuth of 1 and 3.	2nd azimuth of 1 and 3.	Observed difference.
Dec., 1907.	P.M.	°	°	mm.
13	4.45	−45	$(\pi/2)+45$	−0·2
16	12.5	+ 2	$\pi+2$	+0·3
16	4.25	−48	$(\pi/2)+42$	0·0
19	12.25	0	$-\pi$	−0·2
Jan., 1908.				
8	3.45	−63	$(\pi/2)+27$	−0·1

Table IV.

The following three observations are tests for the earth's orbital motion alone, no attention being paid to the effect of the sun's proper motion.

Date.	Time.	1st azimuth of 1 and 3.	2nd azimuth of 1 and 3.	Calculated difference.	Observed.
Dec., 1907.	P.M.	°	°	mm.	mm.
16	12.15	2	$(\pi/2)+2$	−5·5	+0·2
19	12.5	0	$(\pi/2)$	−6·0	+0·3
19	12.15	0	−90	−5·7	+0·3

It will be noticed that the observed difference is sometimes of the same sign as that calculated, and at other times of opposite sign. We are, therefore, inclined to attribute it, as before suggested, merely to error of observation; however, even supposing it to be a real effect, its maximum value is less than 2 per cent. of that looked for.

It may be objected that the above method of experimenting is not the correct one—that, to be quite conclusive, no assumption as to the direction of the ether-drift should be made. With a view to settling this point and, incidentally, making use of the increased accuracy of reading possible at night time, the following sets of observations were made throughout the early morning hours of January 18 last. The freedom from vibration made estimation to 1/10 mm. as easy as 1 mm. readings in the daytime. The observations were spread over the whole time from 12 midnight to 4.15 A.M., and were, in a sense, a search for ether-drift. The results are calculated in the way previously indicated, and tabulated in five sections, each section containing the results of exactly similar treatment as regards rotation. Thus, Section I contains the three cases in which the first azimuth of 1 and 3 was 0° and the second azimuth 90°.

Table V.

	Time.	1st azimuth of 1 and 3.	2nd azimuth of 1 and 3.	Observed difference of scale reading.
	A.M.	°	°	mm.
Section 1......	12.0	0	+90	+0·18
	2.0	0	+90	+0·41
	4.0	0	+90	+0·32
Section 2......	1.0	0	−90	−0·04
	3.0	0	−90	−0·04
Section 3......	12.15	−45	+45	+0·04
	2.15	−45	+45	+0·05
	4.15	−45	+45	−0·05
Section 4......	1.15	+45	−45	−0·04
	3.15	+45	−45	−0·31
Section 5......	12.35	−90	+90	+0·14

In interpreting the results above recorded, careful attention should be paid to the treatment of the coils in any particular case. Since the readings are now made to 1/10 mm., we think that a difference which affects the first place of decimals measures a real effect produced by rotation. Thus, in the first section, the differences are of this magnitude and of the same sign. That they are not, however, due to an effect of ether-drift is proved by the observations taken at intermediate times and recorded in Section 2. Here the coils were rotated in the opposite direction through 90° also, and no real effect was produced. It is obvious that, for measuring a genuine ether-drift effect, the direction of rotation through 90° is indifferent; and the fact that the observed differences of reading depend on the direction of rotation removes the possibility of attributing them, small as they are, to ether-drift. It should be noticed, too, that the observation recorded in Section 5 shows a difference of the same order, and that here also it must be due to a cause other than motion through the ether, because rotation is through 180°. In the other sections, with the exception of the second observation in Section 4, the differences are not large enough to justify any meaning being attached to them.

On the whole, therefore, this set of readings points to the conclusion that at no time during the night on which they were taken was there a change of resistance comparable with that looked for. We have, however, been unable up to the present to account for the small spurious effects observed. Several suggestions have presented themselves, but none appears to be valid. It was thought that possibly the twist on the galvanometer arm caused by the rotation might produce a sufficient change of resistance there to effect the small alteration in current. Calculations show, however, that a change of resistance of about 100 per cent. due to twisting copper wire through 90° would be necessary for this to be the case; so that the observed effects cannot be attributed to this cause. A second idea was that the relative change of resistance of the coils was brought about by the alteration of their distribution with respect to the magnetic field in which they stood. That magnetic field, as has been already pointed out, was very small, precautions having been taken to reduce it, as nearly as might be, to zero. This point was tested for by making the field purposely large, in the hope of magnifying the effect; but to no purpose. Finally, a small direct action of the rotating coils on the galvanometer was looked for when a much larger current than usual was passed through them. Here, again, there was no observable effect.

This question must therefore be left undecided. It does not really affect the main aim of these experiments. With regard to this we consider ourselves justified in making the following assertions:—

1. The total electrical resistance of a wire is not altered by an amount exceeding 5×10^{-10} of the whole amount by any change of its position relative to its motion through space.

2. On the assumption that the Fitzgerald-Lorentz shrinkage is a real effect, the specific resistance of a material is dependent upon the direction of flow of the current, being greater to a current flowing parallel to the velocity

of the material through space than to a current in a perpendicular direction. The magnitude of this change of specific resistance is shown by the experiments to be certainly within 2 per cent. of being sufficient to compensate the change of length.

Note.—In view of the very general acceptance of the Fitzgerald-Lorentz shrinkage theory, the negative results of these experiments will probably be attributed to a dependence of specific resistance on direction of current flow. In this connection it is worthy of note that certain independent considerations point to the same conclusion. The electronic theory of metallic conduction leads to the result* that the specific conductivity of a material is measured by the expression

$$\alpha \frac{e^2 n}{m} \cdot \frac{\lambda}{v},$$

where n is the number of electrons per unit volume, m the mass of an electron and e the charge upon it, v the mean velocity, λ the mean free path (*i.e.*, the mean distance traversed by an electron between successive collisions with atoms), and α a numerical constant. It is not here of importance whether this expression is absolutely correct or not, provided that it represents the facts dimensionally. The specific resistance is the reciprocal of the above quantity, and we therefore have

$$\rho = \frac{mv}{\alpha n e^2 \lambda}.$$

Of these quantities α and e are independent of the motion through space. The number of electrons per unit volume may also be supposed unaltered by changes of azimuth of the conductor, because the latter has the same volume in all azimuths. The changes of ρ, therefore, depend on the variations of the quantities m, v, and λ. Let us denote by the suffix l the values measured parallel to the drift, and by t the corresponding values in a direction at right angles. Hence,

$$\frac{\rho_l}{\rho_t} = \frac{m_l}{m_t} \cdot \frac{v_l}{v_t} \cdot \frac{\lambda_t}{\lambda_l}. \qquad (1)$$

On the shrinkage theory, we expect

$$\lambda_t/\lambda_l = 1 + \tfrac{1}{2}\beta^2,$$

and, following Lorentz,†

$$m_l/m_t = 1 + \beta^2.$$

---

* See J. J. Thomson, 'The Corpuscular Theory of Matter,' p. 53.

† 'Amsterdam Acad. Proc.,' 1903—04, p. 809. This value gives complete compensation, while Abraham's value, $m_l/m_t = 1 + \frac{4}{5}\beta^2$, does not. We have consequently taken it in our suggestion of the direction in which to look for the mechanism of compensation.

The only remaining ratio to be determined is v_l/v_t.

Now it is to be expected that the average kinetic energy of the electrons should be independent of the direction of motion; or, in other words, the total kinetic energy associated with any particular direction should be the same. On this assumption we obtain

$$m_l v_l^2 = m_t v_t^2, \quad \text{or} \quad \frac{m_l}{m_t} = \frac{v_t^2}{v_l^2};$$

hence

$$1+\beta^2 = \frac{v_t^2}{v_l^2}, \quad \text{or} \quad \frac{v_l}{v_t} = 1-\frac{\beta^2}{2}.$$

Returning to equation (1), it follows that

$$\frac{\rho_l}{\rho_t} = (1+\beta^2)\left(1-\frac{\beta^2}{2}\right)\left(1+\frac{\beta^2}{2}\right) = 1+\beta^2,$$

since β is a very small quantity.

That is to say, the specific resistance parallel to the ether-drift is greater than that at right angles in the ratio

$$1+\beta^2 : 1.$$

This corresponds exactly to the conclusions respecting specific resistance arrived at in the experiments above described.