curl of a velocity vector. Let $\mathbf{v}=\mathbf{e}_1v_1+\mathbf{e}_2v_2+\mathbf{e}_3v_3$ be any vector presented in the $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ coordinate frame. According to the Coriolis theorem the relation between the absolute rate $d\mathbf{v}/dt$ and the relative rate $\dot{\mathbf{v}}$ is

$$d\mathbf{v}/dt=\dot{\mathbf{v}}+\mathbf{\Omega}\times\mathbf{v}$$
$$=\mathbf{e}_1\dot{v}_1+\mathbf{e}_2\dot{v}_2+\mathbf{e}_3\dot{v}_3+\mathbf{\Omega}\times\mathbf{v}. \qquad (7)$$

To show an application of Eq. (6) consider a point moving in a spherical coordinate system where $h_r=1$, $h_\theta=r$, and $h_\varphi=r\sin\theta$. The use of Eq. (6) yields

$$\mathbf{\Omega}=\mathbf{e}_r\dot{\varphi}\cos\theta-\mathbf{e}_\theta\dot{\varphi}\sin\theta+\mathbf{e}_\varphi\dot{\theta}.$$

The purpose of this note is to call attention to a useful result of Ames and Murnaghan[1] which is difficult to find elsewhere in the literature. The only contribution of the present author has been to modernize the notation and derivation, and to express the result, Eq. (6), in a mnemonic form.

[1] J. S. Ames and F. D. Murnaghan, *Theoretical Mechanics* (Ginn and Company, Boston, Mass., 1929), pp. 88–98.
[2] L. J. Kijewski, Am. J. Phys. **33**, 816 (1965).

Batteries Connected in Parallel

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If the equation for the resultant of several arbitrary batteries connected in parallel has been previously published, it apparently is not common knowledge to most elementary physics instructors. Parallel cell connections are common enough to justify the inclusion of the following treatment in many elementary college physics courses.

The problem is to replace a number of generators (of arbitrary emf and internal resistance) connected in parallel with a single "equivalent" generator as indicated in Fig. 1.

The equation for r (the equivalent internal resistance) is the well-known

$$r^{-1}=\sum_{n=1}^{N}r_n^{-1}.$$

The equation for E (the equivalent emf) is

$$E=r\sum_{n=1}^{N}(E_n/r_n),$$

where r is as defined above.

To prove this relation we note that the voltage V across each parallel branch is the same (the load voltage), which implies $V=E_n-r_ni_n$ (for all n), where i_n is the current flowing through the nth branch. Dividing by r_n and summing, we get

$$V\sum r_n^{-1}=\sum(E_n/r_n)-I,$$

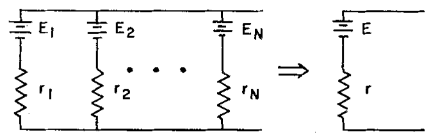

FIG. 1. Replacement of the parallel network by its "equivalent."

where $I\equiv\sum i_n$ is the total (load) current. Defining $\sum r_n^{-1}\equiv r^{-1}$ and multiplying by r, we get

$$V=r\sum(E_n/r_n)-Ir.$$

Since this holds for arbitrary I (arbitrary load resistance), we see that this is just the equation one would get for the terminal voltage of a single generator with an emf of

$$r\sum(E_n/r_n)$$

and internal resistance r. *Q.E.D.*

On the Abraham–Lorentz Electron*

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Recent papers[1,2] have brought up again the problem of the energy of a moving Abraham–Lorentz model of the electron. These papers attempt to show that the energy of the moving Abraham–Lorentz electron, $\tilde{U}$, is related to the energy of the stationary Abraham–Lorentz electron, $\mathcal{E}$, by, $\tilde{U}=\mathcal{E}/(1-\beta^2)^{1/2}$, which is the usual relativistic relation for a neutral particle of rest energy $\mathcal{E}$. That this relation does not hold and that special relativity is not consequently violated, has been shown in an earlier paper by the author.[3] The purpose of the present note is to show, by a simple example, the consequences of assuming the two different results.

It has been shown[3] by a direct calculation that the total energy of the moving Abraham–Lorentz electron is

$$U=\mathcal{E}'-\mathcal{E}_n, \qquad (1)$$

where

$$\mathcal{E}'=\frac{\mathcal{E}(1+\beta^2/3)}{(1-\beta^2)^{1/2}}, \qquad (2)$$

$$\mathcal{E}_n=\mathcal{E}\,[1-(1-\beta^2)^{1/2}]/3, \qquad (3)$$

and

$$\mathcal{E}=\int\frac{E^2}{8\pi}dv. \qquad (4)$$

The quantity, E, is the electric-field strength of the stationary Abraham–Lorentz electron and $\mathcal{E}$ is the total energy of the stationary Abraham–Lorentz electron. $\mathcal{E}'$ is the electromagnetic energy of the moving Abraham–Lorentz electron and $\mathcal{E}_n$ is the nonelectromagnetic energy of the moving Abraham–Lorentz electron.

The difference between U and $\tilde{U}$ can be brought out by the following thought experiment. Suppose an Abraham–Lorentz electron were allowed to fall from rest a distance x in a uniform gravitational field. According to the proponents of $\tilde{U}$, the velocity of the electron would be (ignoring radiation effects and assuming small velocity)

$$\tilde{v}=(2gx)^{1/2}. \tag{5}$$

According to the proponents of U, the velocity of the electron would be

$$v=(1.5gx)^{1/2}. \tag{6}$$

For the Abraham–Lorentz electron, the velocity v is about 15% smaller than the velocity $\tilde{v}$. Physically, the slower velocity given by Eq. (6) can be explained by recalling the effect of inductance on moving charges. Whenever a coherent motion of charges exists, the charges acquire an inertia in excess of their mass because of the magnetic interaction which arises between coherently moving charges.

The same effect occurs if, rather than an Abraham–Lorentz electron, a real charged sphere is used. Because of the mass of a real sphere, the difference between v and $\tilde{v}$ is very small. The proponents of $\tilde{U}$ obtain, as before, the velocity $\tilde{v}=(2gx)^{1/2}$ for the sphere, whereas the proponents of U obtain

$$v=(2gx)^{1/2}/\left[1+m/3(M+m)\right]^{1/2}, \tag{7}$$

where M is the rest mass of the charged sphere exclusive of the mass of the electromagnetic-field energy, and m is the mass of the electromagnetic-field energy. Since m is necessarily small as compared to M, the velocity can be written

$$v=(2gx)^{1/2}\left[1-(m/6M)\right]. \tag{8}$$

The largest value of m/M that can be realized in practice is about 10^{-12} ,so that it would be very difficult to measure the difference between v and $\tilde{v}$. The velocity v could be measured with a charged sphere in the earth's gravitational field and $\tilde{v}$ could be determined similarly with an uncharged mass.

The importance of considering the Abraham–Lorentz electron is that it brings out the fact that a *real* electron must have little or no electromagnetic mass associated with it. This is because a *real* electron is known to conform with the usual relativistic relation, $\mathcal{E}/(1-\beta^2)^{1/2}$, where $\mathcal{E}$ is the experimental rest energy of the electron. This relation can apply only to a mass that does not react on itself electromagnetically. The insistence that the energy of the *Abraham–Lorentz* electron should be governed by the relation, $\mathcal{E}/(1-\beta^2)^{1/2}$, pre-empts the important result that, as far as convected fields are concerned, a real electron cannot interact with itself.

* Work performed under the auspices of the U.S. Atomic Energy Commission.

[1] A. Gamba, Am. J. Phys. **35,** 83 (1967).

[2] J. W. Butler, Rept. Naval Res. Labs. Progr., Washington, D.C. (Jul., Aug., and Oct. 1967).

[3] J. W. Zink, Am. J. Phys. **34**, 211 (1966).

A Low-Temperature-Control System for Undergraduate Experiments

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The purpose of this note is to describe a simple and inexpensive temperature-control system which can be used in performing undergraduate laboratory experiments over the temperature range from −196°C to room temperature. The system was designed to meet two basic requirements. First, it was to be capable of changing the temperature of the experimental sample in a relatively short time and second, the temperature of the sample was to be maintained to within a few degrees of a desired temperature for periods of time of the order of ½h.

The temperature-control system which most closely approached the desired requirements is illustrated in Fig. 1. The temperature −196°C is maintained by keeping the Dewar in which the experiment is to be performed filled with liquid nitrogen throughout the experiment. For an experiment to be performed at higher temperatures, the liquid nitrogen in the Dewar is boiled away, and the temperature is then controlled by nitrogen vapor introduced through the vapor delivery tube.

The liquid-nitrogen vapor is produced by boiling the liquid nitrogen at a controlled rate in the 25-liter storage Dewar by means of an immersion heater connected to a variac. The temperature of the nitrogen vapor is controlled by an electric heating coil placed in the tube connecting the vapor delivery tube with the source of nitrogen vapor. The immersion heater used in this experiment was a Japanese heater made for heating water in a cup. The electric heating coil was about 10 ft. of No. 24 copper–nickel wire wound on a coil of ¼-in. diam. Both the immersion heater and the heating coil are connected to a variac as illustrated in Fig. 1. The Dewar which contained the experimental sample is about 25 cm high and 3 cm in diam. Butt joints in this system may be held and sealed with masking tape wrapped with wet asbestos paper. With cold vapor passing through the tube, the water in the paper freezes forming a solid joint.

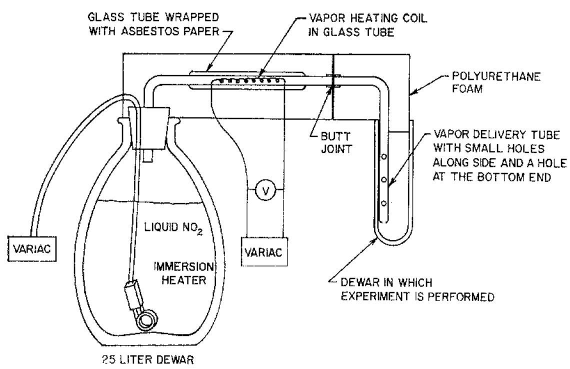

Fig. 1. Temperature control system.