ROMAN SURVEYING

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1.-AIM

Roman engineering works are distinguished in particular by their use of very careful measurements. Main roads follow long straight lines with gentle gradients wherever the terrain allows. Cities and fields are strictly squared off, sometimes over extremely large areas. However, it is probable that it is the aqueducts, those lengthy conduits that brought water from natural springs and other water sources to the cities for human consumption, that reach the high point of geometric perfection, simply because it was essential to their functioning.

In analyzing their planning and construction processes, we have been struck with doubts as to the way in which Roman engineers could have surveyed and constructed each piece with precision on site. These works were at times so complex to carry out, even with today's techniques, as to require the greatest skill and greatest care in planning to ensure they functioned properly.

The time spent in these studies has been encouraged to a large extent by the little that is known and by how unconvincing are pronouncements on Roman surveying, mainly in modern writings. This work explains how our thoughts on the subject have developed.

At the same time as we describe some of the instruments that we have reconstructed, their functioning and the results of our experiences in using them, we shall summarize briefly some of the techniques that, being well within the capabilities of Roman technicians, could have been used successfully in their engineering works.

2.-ANCIENT TECHNOLOGY

Roman engineers gained most of their knowledge to resolve problems of measurement and calculation from Greece.

And not only Roman engineers, but also those of Renaissance Europe used ancient texts to advance the meager science inherited directly from the Middle Ages. Juan de Herrera was the first to be in charge of setting up the first scientific academies in the Spain of Philip II[1]. It is noteworthy how Juan de Herrera specifies the books and texts to be read in order to meet his students' objectives. For example, he instructs those who wish to become arithmeticians to know "the nine books of Euclid, some other theoretical arithmetic, such as Jordanus Nemorarius (1225-1260) or Boëthius (c. A.D. 480-524), and the practical aspects to be taken out of the works of Frate Luca Pacioli (1445-1517) or Niccolo Cartaglia (c. A.D.1506-1558): those who intend to become geometricians and surveyors "must know the first books of Euclid, the doctrine of triangles of Regiomontanus (Johann Müller, 1436-1476), the last five books of Euclid, including book X, *Sphaerica* of Theodosius (c.1st century B.C.), *Conics* by Apollonius of Perga (c. 3rd century B.C.), and the works of Archimedes (c.287-212 B.C.) on the sphere and the cylinder". At the same time, those who intended to dedicate themselves to mechanics as well as astrologers, gnomonists, perspectivers, musicians, architects, fortification engineers, surveyors, gunnery experts, needed to know Euclid's *Geometry* as well as other listed specialist works.

Particularly interesting is the case of cosmographers and pilots, for whom it was considered essential to know the *Sphere* and the *Theories of the planets* and to understand in depth Ptolemy's *Geography*, together with the use of maritime charts, the astrolabe, the *ballista* and the compass.

To these ends, Pedro Ambrosio de Ondériz (¿?-1596) translated in just a year the following works (from the words of Juan de Herrera):"The eleventh and twelfth books of Euclid, the *Perspectives* and *Reflexions* printed at his own expense, the *Sphaerica* of Theodosius, the

[1] ESTEBAN PIÑERO, M. 2003. Las Academias Técnicas en la España del siglo XVI. Quaderns d'Història de l'Enginyeria Volum V 2002-2003.

Equilibria of Archimedes, and he is in the process of finishing another book entitled *Apollonius of Perga*".

So, the main knowledge on which Roman surveyors relied, as well as the present-day surveyors, was the part of mathematics known as trigonometry.

Trigonometry relates the angles of a triangle to its sides. It can be said that it was started by Hipparchus (fl. 146-127 B.C.). Others followed him until Ptolemy took over. Then, as now, engineers and physicists use many of their tools in their day-to-day work.

The two main branches of trigonometry are plane trigonometry, dealing with shapes in a plane, and spherical trigonometry, dealing with the triangles that go to make up the surface of a sphere. Both aspects were well mastered in antiquity, since this branch of knowledge goes back to Egyptian and Babylonian mathematicians, the Egyptians being the first to measure angles in degrees, minutes and seconds. However, let us here examine the work in this subject by the scientists of antiquity of whom we have knowledge.

2.1.-Forerunners

Thales of Miletus

It is probable that his work has had the greatest influence on the science of measuring in the West. He was born around the year 624 B.C. and he died between 548 and 545 B.C. As a philosopher he is remembered mainly for his *Cosmology*, based on water being the essence of all matter, and for his forecasting the eclipse of the sun, which must have occurred on 28th May, 585 B.C.

No writings of Thales survive, nor contemporary sources on him, that could serve as references. This makes it exceedingly difficult to know the extent of what he achieved. It is even more difficult, because in ancient Greece it was usual to ascribe many discoveries to people upheld as experts, without their having been involved.

Thales was considered to be a disciple of the Egyptians and the Chaldeans, since he travelled to Egypt and Mesopotamia. Lost documents would have shed more light on his work; but writing paper easily rots. Thus a student of Aristotle, called Eudemos of Rhodes (320 B.C.), referred to the science that Thales had gained from the Egyptians in a work entitled *History of mathematics*. This work has been lost; but, before this happened, a summary was made that disappeared later as well. Information about his summary appears in the fifth century in the *Commentary* of the philosopher Proclus on the *First book of the Elements of Euclid*. There, after referring to the origins of geometry in Egypt, he speaks of Thales and says, "... first he went to Egypt and then introduced this study to Greece. He discovered most of the propositions and instructed his followers on the underlying principles of many others. His method was to deal more generally with some and more empirically with others."

Further on in his *Commentary*, and quoting Eudemos, he states that Thales established four theorems:

1.-The circle is bisected by its diameter.

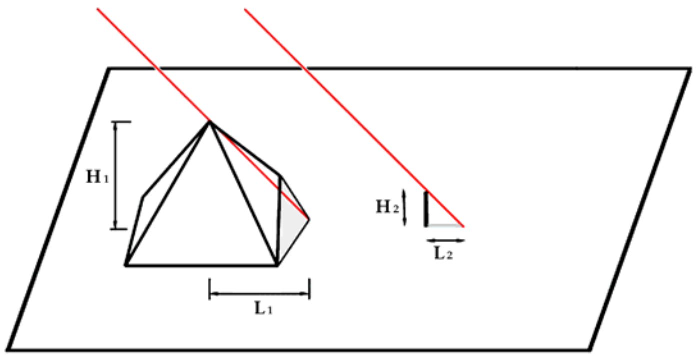
2.-The angles at the base of an isosceles triangle are equal.

3.-Opposite angles of intersecting straight lines are equal.

4.-If two triangles are such that two angles and one side of one are equal to two angles and one side of another, the triangles are congruent.

A fifth theorem is traditionally added to this list. It states: "The angle inscribed in a semicircle is a right angle." It is currently thought that his theorem had its true origin in Babylon and Thales later introduced it to Greece.

Part of this legend ascribes to Thales the use of his knowledge of geometry to measure the sizes of the Pyramids of Egypt and to calculate the distance of a ship from the shore. Thus Diogenes Laërtius (3rd century A.D.), with Pliny (*N.H.* xxxvi.12) and Plutarch state that the measurement of the height of the Pyramids was effected by determining when the length of the shadow produced by a stick stuck vertically in the ground was equal to its height. For this the sun's rays need to have an inclination of 45 degrees. Due to the Pyramids' being located at Gizeh, at 30 degrees latitude in the Northern hemisphere, there are only two possible days of the year when Thales could have made this measurement, the 21st November and the 20th January.



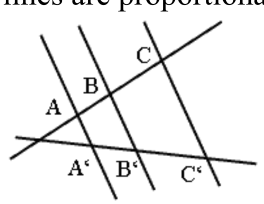
L1/L2=H1/H2

To measure the distance of ships at sea from the shore, legend has it that Thales was the first to use the relationship between the sides of similar triangles. There are doubts about this, seeing that these had currency much earlier in Egypt and Mesopotamia, where Thales lived for some time. It is very possible that his role is not so much of inventing as intelligently interpreting, organizing and copying these logical constructions.

More remarkable was his prediction of the solar eclipse that halted the battle between Alyattes (king of Lydia, 609-560 B.C.) and Cyaxares (king of Media, ruled about 624-584 B.C.) in 585 B.C. Modern experts are convinced that Thales lacked the knowledge to predict precisely the locality where it could be observed or its character, and his estimates must have been very approximate. Herodotus refers to a prediction with only a year's notice. It is probable that the fact that the eclipse happened to be total and that the locality affected happened to coincide with an important battle contributed enormously to Thales' reputation as an astronomer.

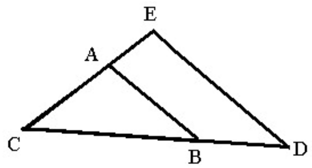
Some consequences of Thales' famous theorems are as follows:

<u>Similar triangles</u>: Segments of sides created by parallel straight lines on two intersecting straight lines are proportional.



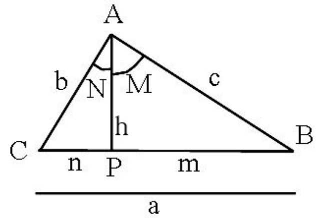
$$\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{AC}{A'C'}$$

Hence, any line parallel to one side of a triangle creates with the other two sides a new triangle similar to the first.



Two triangles are similar if they have two angles equal or three sides in proportion or two sides in proportion and the angle contained between them equal.

Theorem of height and of the side of a right-angled triangle adjacent to the right angle:



Triangles PCA and PBA are similar; since they have a right angle, and M and C are both complements of B and hence equal. Therefore b/c=h/m=n/h; $h^2 = m.n$
That is, in a right-angled triangle, the height is in mean proportion to the segments it makes on the hypotenuse.
Triangles PCA and ACB are similar, since they have three angles the same, namely a right angle and B and N, both complements of C, are equal. Therefore b/a=h/c=n/b; $b^2 = a.n$
By the same reasoning, triangles PAB and ACB are also similar because they have a right angle and angles C and M, both complements of B are therefore equal. Therefore c/a=m/c=h/b; $c^2 = a.m$
That is to say, in a right angled triangle, the side adjacent to the right angle is proportional to the hypotenuse and a line dropped from the right angle onto the hypotenuse.

Pythagoras

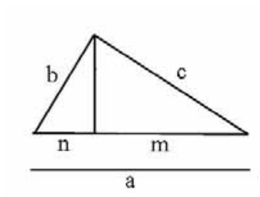
Pythagoras was born in the 6th century B.C. (probably 589) on the island of Samos (Greece), and died in the 5th century B.C. in Crotona (Italy). Pythagoras was educated in the teachings of the first Ionian philosophers, Thales of Miletus (640-546 B.C.), Anaximander (¿611-546 B.C) and Anaximenes (c.380-320 B.C.).. It is said that Pythagoras had been condemned to exile from Samos on account of his opposition to the tyrant Polycrates (C.353-515 B.C.). Towards 530 B.C. he set up home in Crotona, a Dorian colony of the South of Italy where he founded a movement with religious, political and philosophical tenets, known as Pythagorism. Pythagoras' philosophy is known only through the work of his disciples.
He established his famous theorem to demonstrate that, in a right-angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides. This is how Vitruvius (Marcus Vitruvius Pollio c.60/70-c.25 B.C.)[2] explains Pythagoras' discovery: *"Pythagoras created a T-square that did not require the work of craftsmen, and that solely by means of much diligence and hard work it could be created precisely; it can be created by strictly following his methods and precepts. This is how: take three rules, one three foot long, another four and the third five, and join them end-to-end to form a triangle, thereby making a perfect T-square. Now along the length of each of the three rules describe as many squares: the one on the side measuring three foot will have an area of nine foot, that measuring four will have 16' and that measuring five will have 25'.*

[2] VITRUVIVS. *De Architectura*, 9, *praef.* 6 and 7

Thus the number of square feet contained in the areas of the sides of three and four foot respectively will be equal to the number of feet contained in the area of the square on the side of five foot. This proof, useful to measure dimensions and many other things, is very useful in constructing buildings and particularly staircases to ensure that each tread is in the correct proportion."

Many researchers state that the Egyptians were aware of the properties of a right-angled triangle with sides of 3, 4 and 5 units of length, which proves the relationship $5^2 = 3^2 + 4^2$; but the discovery of the general relationship $a^2 = b^2 + c^2$ for any right-angled triangle and its proof is certainly down to Pythagoras.

This proof can be deduced simply from the theorem of the side of a right-angled triangle adjacent to the right angle:



$b^2 = a.n$
$c^{2\,=} = a.m$
$b^2 + c^2 = a.n + a.m = a\,(n + m) = a^2$
i.e. $a^2 = b^2 + c^2$

Pythagorism was revived in the second century A.D. by Nicomachus of Gerasa (fl. c. A.D. 100) in his *Introductio arithmeticae*, a classical work considered fundamental into the Renaissance period.

Euclid

He probably lived in the third century B.C. His monumental work entitled *Elements* remained unchallenged until the beginning of the 20th century. It contains 13 books dealing with plane geometry, an exclusive study of polygons and circles, relations and proportions, with the concept of similarity, number theory, the simplest irrational algebraic expressions and a final part on space. It is one of the books of reference in mathematics and modern engineering in the 16th century, its study being considered essential to make use of applied mathematics.

Apollonius

Apollonius of Perga, "the Great Geometer" lived at the end of the third and beginning of the 2nd century B.C. in Alexandria, Ephesus and Pergamon. His work *Conics* is composed of eight books of which seven have been preserved, four in Greek and three in Arabic. They deal with: 1. *Cutting off a ratio*; 2. *Cutting off an area*; 3. *Determinate section*; 4. *Tangencies*; 5. *Inclinations*; 6. *Plane loci*.

Conics were known by the names that Apollonius invented: conic section at an oblique angle (ellipse), right-angled conic section (parabola) and obtuse angle conic section (hyperbola).

Apollonius' theorems on conjoined diameters of conic curves with a centre are famous. He discovered what today we call the evolute of the ellipse. He also studied similarities, translations, rotations (i.e. movements) and also similarities in a plane and in space. It is also known that Apollonius knew the stereographic projection of a sphere on a plane.

Archimedes

Archimedes of Syracuse (c.287-212 B.C.), Magna Graecia (Sicily), was a Greek mathematician and physicist renowned for his inventiveness and creativity. He is considered the forerunner of modern engineering.

Archimedes proved that the surface area of a sphere is four times that of its great circles. He calculated areas of spherical surfaces and volumes of spherical segments. He proved that "*the*

area of a spherical cap is equal to the surface of a circle whose radius is a straight line linking the centre of the cap with a point on the basal circumference".
The problem which gained him great importance proves that "*the volume of a sphere inscribed in a cylinder is equal to 2/3 the volume of the cylinder"*. He proved that the surface of a sphere is also 2/3 the surface if its right circumscribing cylinder.
His work on measuring the circle is more interesting. He measured its circumference and area. Archimedes is the first to make a truly positive attempt to calculate the number *pi (π)*, to which he gave the value of between 3-1/7 and 3-10/71. The method he used was to calculate the perimeters of regular polygons inscribed within and circumscribed around the same circle.
He also proved that a circle is equivalent to a triangle with base the circumference and height the radius.
Another work relates to mechanics, in particular to the principles of the lever. His starting point was two fundamental principles that may well be considered as the axioms of mechanics:
1.-A lever with equal weights at each end, is in equilibrium if the fulcrum is in the middle.
2.-A weight can be distributed in two halves working at equal distances from the middle.
Basing himself on these two principles, he founded the laws of the lever. His words on the multiplying forces of the lever are famous: "*Give me a leverage point and I will lift the world*".
But he is most famous for having discovered the method for determining the density of a body, taking that of water as the unit.
However important Archimedes' science and discoveries, perhaps more important are his revelations and inventions in physics. Indeed, besides the principles of hydrostatics, already adequately covered, he invented a system of pulleys, the winch, the gear wheel, the endless screw and a series of at least forty inventions. Among them, of greatest importance for the use made later is the endless screw for pumping water.
In the military field, we owe him the invention of the catapult and of lever-operated grab-hooks for mechanical devices.
He was able to defend Syracuse for three years by his optical techniques when it was being besieged by the Romans. Using "burning mirrors", huge concave reflectors, he succeeded in concentrating the sun's rays on the Roman fleet and setting fire to it. Finally, in 212 B.C., Syracuse fell to the Romans and, at the age of 75, Archimedes was killed by a Roman soldier, even though the consul Marcellus had ordered his life to be spared.

Hipparchus

Hipparchus was born in 190 B.C. in Nicaea, Bythinia (now Turkey) and died in 120 B.C. in Rhodes (Greece). He is looked upon as the first scientific astronomer. Practically all that is known about Hipparchus derives from Ptolemy's *Almagest*.
He made important contributions to trigonometry, both plane and spherical, and introduced into Greece the division of the circle into 360 degrees.
He constructed a table of chords with which he could easily relate the sides and angles of any plane triangle. This is an early example of a trigonometry table similar to the modern sine table, with the aim of devising a method to resolve triangles. The table gave the length of a chord described by the sides of the central angle that it cuts at a circumference with radius *r*.
In astronomy: he discovered the precession of the equinoxes; he described the apparent motion of fixed stars; he calculated the length of the year to an accuracy of about 6½ minutes; he calculated a period of eclipses of 126,007 days and one hour; he calculated the distance to the moon through his observation of an eclipse on the 14th March, 190 B.C., his calculation being between 59 and 67 earth radii, very close to the actual 60 radii.

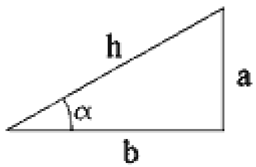
He developed a theoretical model of the motion of the moon based on epicycles. He compiled the first star catalogue containing up to 850 stars differentiated by brightness into six categories or magnitudes. This catalogue was probably used by Ptolemy as the basis of his own.
He had a great influence on Ptolemy in rejecting the heliocentric theory of Aristarchus of Samos (fl. 280 B.C.) and was thus the forerunner of the geocentric works of Ptolemy.

Heron of Alexandria
His dates are not known precisely; but the record that he made use of the eclipse of the sun, that could only have occurred in 62 A.D.[3], to carry out certain measurements of the earth as a sphere placed him in a time very much more recent that did other authors who have him more or less in the third and second centuries B.C.[4] We have before us a man who is key to applied mathematics, mechanics, physics, geodesy, logarithmic curves, and numerical calculating. All this has come down to us in scraps, among which the so-called 'Hero collection'.
Metrica, a work not found until 1896, is devoted to the measurement of flat or curved surfaces, based on a progression of problems. Among the results is his famous formula for the area of a triangle, although recent researchers attribute it to Archimedes:
$\Delta = \sqrt{(s(s-a)(s-b)(s-c))}$
where s is the semiperimeter
$s = (a+b+c)/2$
He is one of the best known inventors of antiquity with devices such as a rudimentary steam engine, and automata, and with various engines of war, such as ballistas and long-range catapults. In the field of surveying, he did very important work in developing the diopter, a very accurate theodolite, with which one could make many varied measurements both terrestrial and astronomical.

Ptolemy
Claudius Ptolomeus lived in the second century A.D., astronomer, mathematician physician and geographer. He was Egyptian, born in Ptolomai Hermii, a Greek city of the Thebaid, and lived in Alexandria.
His *Syntaxis mathematica*, more widely known as the *Almagest* synthesizes and orders the astronomical knowledge of the Greeks, especially of Hipparchus.
Ptolemy carried out a number of calculations related to the earth's sphere: he gave two methods of determining he inclination of the equinoxes; he calculated the height of the earth's pole and the length of the day in different parts of the globe; he completed tables of the angles and arcs formed by the intersection of the ecliptic with the meridian and the horizon. He explained the irregularities in the apparent motion of the sun by means of the hypothesis of motion along an eccentric circumference. He completed Hipparchus' moon theory and discovered the annual variation in the eccentricity of its orbit. He used the epicycle to explain the apparent motion of the moon. Ptolemy described the astrolabe, explained the parallax method to find the distance of the moon. He described Hipparchus' method for calculating eclipses and extended his predecessor's catalogue to a total of 1,022 stars.
His most original contribution was the theory of planetary motion. He remarked that the planets (or "wandering stars") are situated between the moon and the fixed stars. He attempted to explain their complicated motions in a way similar to what he had done in the case of the moon; but, instead of attributing to the centre of the epicycle a uniform motion about the eccentric deferent,

[3] NEUGEBAUER. 1938: *Ubre eine Methode zur Distanzbestimmung Alexandria-Rom bei Heron*. Copenhague.
[4] HEIBERG, J L. 1912: *Heronis Alexandrini Opera quae supersunt omnia* (Leipzig).

he introduced what he called the "equant", an even smaller circle from which the motion of the planet appears uniform. With his *Almagest* Ptolemy brought ancient astronomy to its peak and its end, where it stayed until the end of the Renaissance.
Ptolemy made capital contributions to the plane and spherical trigonometry created by Hipparchus. He incorporated in his great astronomical work, the *Almagest* a table of chords at 1/2 degree intervals of angle from 0 degrees to 180 degrees using the sexagesimal system invented by the Babylonians, with an error of less than 1/3,600 of a unit. He also explained his method for compiling this table of chords, and throughout the book he gave many examples of how to use the table in calculating the unknown elements of a triangle starting with the known.



$a = h.\text{chord } \alpha$
i.e.: $\text{Chord } \alpha = 2.\sin \alpha$
He expounded the theorem that bears his name, concerning a quadrilateral inscribed in a circle, giving the formula that relates the sine of an angle to the sine of its half, using a method of interpolation and in general he developed almost all the trigonometry needed for his astronomical calculations without the help of trigonometrical functions. He also revived the mathematical geography invented by Eratosthenes and Hipparchus, which had been abandoned in favour of descriptive geography.
In his *Geographia* he described in great detail the construction of maps using different methods of projection. Ptolemy also dealt with the balance, physical acoustics and geometric and physiological optics. In his *Optics* he made a special study of the phenomena of refraction, compiling tables of values for different transparent media. He stated that rays coming from the stars are refracted by the atmosphere, for which reason their observed direction differs from their actual.
These scientists were the founders of all subsequent trigonometry. At the end of the eighth century, Arab astronomers, who had inherited the legacy of the traditions of Greece and India, preferred to work with the sine function. In the last decades of the tenth century, the sine function was completed together with the other five functions and they discovered and proved various basic trigonometrical theorems for both plane and spherical triangles. The Arab astronomer Albatenius, Al-Battani (Haran, Turkey, 858 - Samarrah, Iraq, 929 A.D.) was fundamental to the construction of the theory of the cosine.

2.2.- The spatial concept
To understand the accuracy of the work in measurement carried out in antiquity, we must first bear in mind that the degree of astronomical knowledge concerning the Earth, its position and movement in space was very precise at that time. It is in fact impossible to say when ancient scientists first knew that the Earth was round: it is thought that all ancient civilizations were aware of it, and that there was no doubt among ancient scientists. This is the basic concept from which, for example, they calculated the degree of error in levelling to channel water and the longest distance possible to lay the channels.
We can be somewhat surer about when the sizes of the terrestrial sphere began to be estimated with some precision, as well as its distance from the other most important celestial bodies, the Sun and the Moon.

Eratosthenes (Cyrene, 276 B.C. -- Alexandria, 195 B.C.) calculated the size of the Earth by means of a trigonometric method, using also the concepts of latitude and longitude which had apparently already been introduced by Dicaearcus, for which he richly deserved the title of father of surveying. According to Cleomedes' account[5], Eratosthenes knew that in Syene (present-day Aswan, in Egypt), at noon at the Summer solstice, objects shed no shadow at all, and the light lit up the bottoms of wells. This indicated that the city was situated precisely on the tropic and its latitude was equal to that of the ecliptic, the existence of which was already known to Eratosthenes.

If it is assumed that Syene and Alexandria had the same longitude (in fact that are about 3 degrees apart), and that the Sun was so far from the Earth that its rays could be assumed to be parallel, he measured the shadow in Alexandria on the same day of the Summer solstice at midday, demonstrating that the zenith of the city was one fiftieth of the circumference (that is 7º12') that of Alexandria.

Later he took the distance between the two cities, which he established at 5,000 stadia, from which he deduced that the circumference of the earth corresponded to 250,000 stadia, which he later adjusted to 252,000 stadia, so that each degree corresponded to 700 stadia. If one assumes that Eratosthenes used the Egyptian stadion (300 cubits of 0.524 m. or 1.7 ft), the polar circumference can be calculated at 39,614.4 km. (24,615.3 miles) as compared with the currently calculated 40.008 (24,850 miles), which is an error of less than 1%.

Poseidonius (Apameia, Syria, 135 B.C. -- Rhodes, 51, B.C.), redid Eratothenes' calculation, reaching a substantially smaller circumference, a value later adopted by Ptolemy (2nd century A.D.), on which Christopher Columbus was to base his justification of the viability of his journey to the Indies by the West, a journey that he would probably never have dared carry out with Eratosthenes' more realistic calculations. It is thought that Columbus had access to the accounts of Strabo as well as those of Ptolemy. Strabo mentions the possibility of getting to India by the West, and he postulated a figure under 200,000 stadia for the circumference of the Earth. Nonetheless, Strabo himself returned to the subject estimating half the circumference of the Earth at 70,000 stadia, which served to increase the confusion as to its reliability.

"*Erastosthenes maintains that the inhabited earth is approximately the shape of a circle, which tends to close on itself, such that if the extent of the Atlantic Ocean did not prevent it, <u>we could go by sea from Iberia to India</u>. All that was needed was to follow the same parallel and travel the remaining section, something more than a third of the total circumference, suggesting a value less than 200,000 stadia (some 36,000 km. – 22,370 miles) with reference to the parallel on which the previous journey from India to Iberia was made.*" (Strabo, I.4.6-7)

"*Eratosthenes also formulated the hypothesis that the approximately 70,000 stadia (some 12,500 km. – 7,767 miles) that represent the longitude of the inhabited world are equal to half the whole circle that lies on that longitude. In other words he maintains that <u>if one sails from the West with an Easterly wind, after an equal number of stadia, one would reach the Indies</u>.*" (Strabo II.3.6)

That the Earth was round, seems, contrary to what is assumed today, never to have been questioned by sailors or Mediaeval scholars. Although there were moments when it was the subject of polemic, recent research[6] indicates that belief that the inhabitants of the Middle Ages believed the Earth to be flat, which is now taught as historical truth, is an invention of a British

[5] CLEOMEDES *De motu circulari corporum caelestium* 1st century A.D.

[6] PÁEZ CANO, R. 2003 *La esfera: la tierra plana medioeval como invención del siglo XIX* [The sphere: the Mediaeval flat earth, an invention of the 19th century], thesis for master's degree in Communications, ITESO, Guadalajara, Mexico, 150 pp. Rubén Páez Cano, a researcher of the Instituto Tecnológico y de Estudios Superiores (ITESO) in Mexico.

historian of the first half of the 19th century, William Whewell (1794-1866).See *History of the inductive sciences* (1840).

In fact, Columbus did not have to convince anybody that the earth was round, and that the maximum size of the circumference of the Earth was less than what was supposed, accepting Strabo's and Ptolemy's figures, so that his journey to India by the West was a possibility. He found, however, a reticence on the part of the scientists of the European courts, including those in the service of the Catholic Monarchs, who supported a very much larger circumference, which was the actual calculated by Eratosthenes[7]. This may mean that the actual size of the Earth has been known for ever since Eratosthenes' calculations and despite Strabo's and Ptolemy's calculations.

Also the movements of the stars were successfully studied by **Aristarchus** of Samothrace, born in Samos, Greece, in the year 310 B.C. and died in 220 B.C. Aristarchus' revolutionary idea consisted of the fact that it was the Earth that revolved round the Sun (the heliocentric theory) and not vice versâ (the geocentric theory). Aristarchus considered the Sun to be a star and the probability that the stars were suns. Furthermore he concluded that the Earth's orbit was inclined.

Archimedes in his *Psammites* (Latin *Arenarius*, English *Sand reckoner*) tells us: "*Aristarchus of Samos published a book based on certain hypotheses and in which it seems that the universe is many times bigger than the one that bears that name. His hypotheses are that the fixed stars and the Sun do not move, that the Earth circles the Sun following the circumference of a circle with the Sun at the centre of the orbit, and that the sphere of the fixed stars, also with the Sun as its centre, is so large that the circle which the Earth is thought to go round has the same proportion to the distance of the fixed stars as the centre of the sphere to its surface*".

Nonetheless, the model in vogue at his time was that the Earth was fixed at the centre of the universe, and that Man was at the centre of Creation. We know from Plutarch (c. A.D. 46-120) that Cleanthes (c.391-232 or 352 B.C.) towards 260 B.C. denounced Aristarchus for lack of piety, in daring to deny that the Earth was the centre of the universe.

Unfortunately subsequent scientists denied this theory, and we had to wait more than 1.700 years, until 1542 A.D., for Copernicus (Nicolaus Copernicus, 1473-1543, *De revolutionibus orbium coelestium*, 1543) to resuscitate the idea, which again met with serious doubts. His detractors soon called for his work to be censored. Luther and Calvin came out against it, and in 1616 the Catholic Church included Copernicus' work on the list of proscribed books. He died a year after the publication of his work.

Specifically, because the Earth is round and its surface is generally very uneven, the ancients realised the need to take plane projection measurements in as many dimensions as possible. They knew perfectly well that any measurement on the ground had to be placed in a projection onto its horizontal plane, not only in the interests of accuracy but of pure justice.

It is often asked by what method one can reduce to a horizontal projection an area of ground that is on a slope. In effect it is as though one were to flatten the higher areas so as to reduce them to a plain.

The nature of seeds themselves prove this method: because the ground on a slope cannot be regarded as horizontal, everything that sprouts from the ground grows vertically into the air, and, as it grows, disregards the slope of the ground, and takes up no more space than if it were to grow from a plain.

If all seeds were to germinate at right angles to the slope, we would measure according to the configuration of the area; but, since the slope contains no greater number of rows of trees than

[7] FISCHER, I, 1975, pp.152-167, *Another look at Erastothenes' and Posidonius' determinations of the Earth's circumference*, Quarterly Journal of the Royal Astronomical Society, vol. 16.

the corresponding surface on flat ground, one can consider the horizontal projection of the ground as that which one must measure[8].

In this way they represented the ground by drawing the survey from their measuring efforts with the same precision as is known to ourselves. Also they drew up their maps and plans in such a way that one can make accurate measurements from them and compare any information taken from them with the ground if it were required or transfer the established limits or lines that are relevant to any topographic work one undertakes.

The totality of the scientific knowledge that they possessed, together with the techniques needed for its application, allowed the Roman engineers to apply excellent surveying abilities. For this they also required instruments of sufficient accuracy, which they assuredly possessed, because otherwise they could not have reached those important achievements that arose from that civilization.

Enormously long aqueducts (over 100 km. - 60 miles long), with very low slopes, carefully calculated along its whole length, and often over continuous lengths. Extraordinarily long straight lines of highways, sometimes more than 50 km. (39 miles) long, big dams made to spill their water at a specific point and at a specific fill level, large flooded areas, sometimes enormous, surface drainage areas, made by breaking the endorheism over a long distance with very costly channels, precisely traced and levelled. None of this could be the result of chance; rather is it the well developed, exact, advanced, surveying knowledge, which was available to the Romans.

3.- INSTRUMENTS

First of all it should be said that little certainty exists at present as to the construction and accuracy of Roman surveying instruments. It should be borne in mind that most have been estimated from descriptions in classical writers, which descriptions have not always been well translated or understood. There is greater certainty from the few facts that have been established from archaeological evidence.

Seldom have ancient authors, in the scarce literature that exists, dealt with instrumentation, and, when it has occurred, the descriptions have not had experts with adequate surveying knowledge to interpret them. Perhaps now is the time to take account of other complementary research methods. From the engineering, technical and practical surveying points of view, starting with an analysis of the Roman engineering texts which have demanded a close investigation from the surveying aspect, we arrive at adequate conclusions as to the absolute basics of Roman methods and instrumentation.

The reconstruction of the ground-plan required to establish the degree of exactness of the Romans can be seen in the precision of the instruments in use, which can give us an idea of the skill needed to fulfill the required objective. For this reason experimental surveying needs to be carried out by professionals who have the required knowledge and ability to reproduce the construction techniques of a work.

We intend here to outline some of the instruments used in Roman surveying that are known to us; we shall describe what we know about them; and we shall explain their most probable use in the light of new experiments that we have carried out.

3.1.- String

This is probably the most rudimentarily simple and ancient measuring instrument. Nonetheless we know from Heron of Alexandria[9] that ancient surveyors made use of this tool. It was not

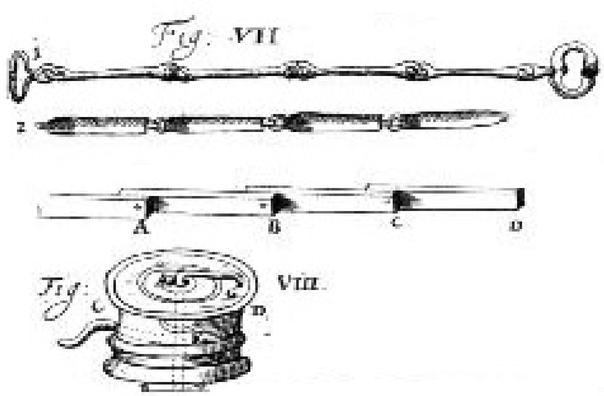
[8] FRONTINUS, *De limitibus*, 26.

[9] HERON. *Automatopoietice* (Automata)

subjected to deformation and changes of length over a long period. This made it much more precise than what would be expected at first thought.
Heron tells us that a mixture of wax and resin was applied to it, and that it was then left suspended for a specified time with a weight on its lower end. *"The cords must not be capable of stretching or shrinking, but must remain the same length as they were to start with. This is done by passing them round pegs, tortioning them tightly, leaving them for some time, and tensioning them again. Repeat this a number of times and smear them with a mixture of wax and resin. It is best then to hang a weight on them and leave them for a longer time. A cord thus stretched will not stretch anomore, or only a very little"* M.J.T. Lewis, *Surveying instruments of Greece and* Rome, Cambridge University Press, 2001, ISBN 0-521-79297-5, p.20, quoting Heron of Alexandria, Automata, 2.4-5. This provided a cord that could be used for measuring with little error and was proof against variations of humidity and temperature.

3.2.- Chains

There is no knowledge of the use of surveying chains in classical antiquity. We should however point out that the chain is a very ancient instrument in any form and that there is therefore a strong possibility that it was used by the Romans.
Since it is simple to make, versatile, easy to gather up and to carry, slow to deteriorate, we know that it has been used for surveying measurements for many centuries.
A chain consists of a set of metal links of uniform size, joined to make a chain of a given length. It usually had handles at the end to facilitate its use
We have seen surveying chains represented in modern surveying books of the 20th century, but there are also identical forms in treatises from the 17th century[10], from which we deduce that they have been in continuous use for measurements that require a certain precision.



Surveyor's chain and other length measuring instruments from a 17th century engraving from a book by Schotto.

3.3.- Decempeda or pertica (perch)

For precise measurements of length an instrument called a *decempeda* (because it was ten foot, about three metres, long) was used. For that reason surveyors or *agrimensores* were commonly called *decempedatores*. It was also known as the *pertica*, and it seems that under both names it was made of wood. [Translator's note: the English traditional measurement of length, the 'perch'

[10] Based on the work of Athanazius Kircher (1601-1680), a German Jesuit acholar, the *Pantometrum kircherianum, hoc est instrumentum geometricum novum a celeberrimo viro P. Athanasio Kirchero, Herbipoli* ['The Kircher measuring instrument, which is a new geometric device by the very famous man Father Athanasius Kircher', Würzburg] 1690.

is 16-1/2 foot (5.03 m.), more than half as much again.]. It should be pointed out that certain woods, when they have undergone special procedures, acquire great strength and resistance to deformation: the Romans were perfectly aware of these techniques. We have seen an explanation of this instrument in use in a 16th century treatise by Giovanni Pomodoro[11], and in recent years we have known of these instruments made of light metals that suffer little distortion.

3.4.- Odometer

We know that Heron built and described an odometer; but we owe to Vitruvius[12] the best known description of this ingenious device, which, in all probability was very much used in antiquity for the measurement of roads and certain distances that did not require accuracy. It consisted of a system of cogs in a box that engaged with another placed in the wheel of a cart, the wheel being built with precision. For each mile traversed a pebble dropped into a specifically placed receptacle.

With slight modifications, and substituting the cart wheel with a paddle wheel under the ship, marine navigation distances could be measured, albeit with a rather lower accuracy.

3.5.- Range poles

Straight alignments were effected by the use of vertical poles, which, in groups of three, served to follow a line and to extend it further by bringing the first pole up to the front. By themselves they were perfectly adequate to follow straight lines, for example on roads; but they were also used to support other measuring instruments, as we shall see as follows, such as the *groma*, the surveyor's

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[11] POMODORO, G. *'Geometria prattica'. Geometria prattica dichiarata da Giovanni Scala sopra le tavole dell'ecc.te mathematico Giovanni Pomodoro, tratte d'Euclide et altri authori: opera per generali da guerra, capitani, architetti, bombardieri e ingegnieri cosmografi, non che per ordinarii professori di misure*. Novamente ristampato -- in Roma, appresso Giovanni Martinelli, 1603. [Practical geometry described by Giovanni Scala based on the tables by the expert mathematiciam Giovanni Pomodoro, Euclid's treatises and other authors: a work for generals in war, captains, architects, artillery officers and cosmographic engineers, as well as for ordinary teachers of surveying. Reprinted in Rome at Giovanni Martinelli press, 1603].

[12] VITRUVIVS. *De architectura*, 10.9.1-4.

***[1]** Transfertur nunc cogitatio scripturae ad rationem non inutilem sed summa sollertia a maioribus traditam, qua in via raeda sedentes vel mari navigantes scire possimus, quot milia numero itineris fecerimus. Hoc autem erit sic. Rotae, quae erunt in raeda, sint latae per medium diametrum pedum quaternûm [et sextantes], ut, cum finitum locum habeat in se rota ab eoque incipiat progrediens in solo viae facere versationem, perveniendo ad eam finitionem, a qua coeperit versari, certum modum spatii habeat peractum pedes XII s<emissemque>.*

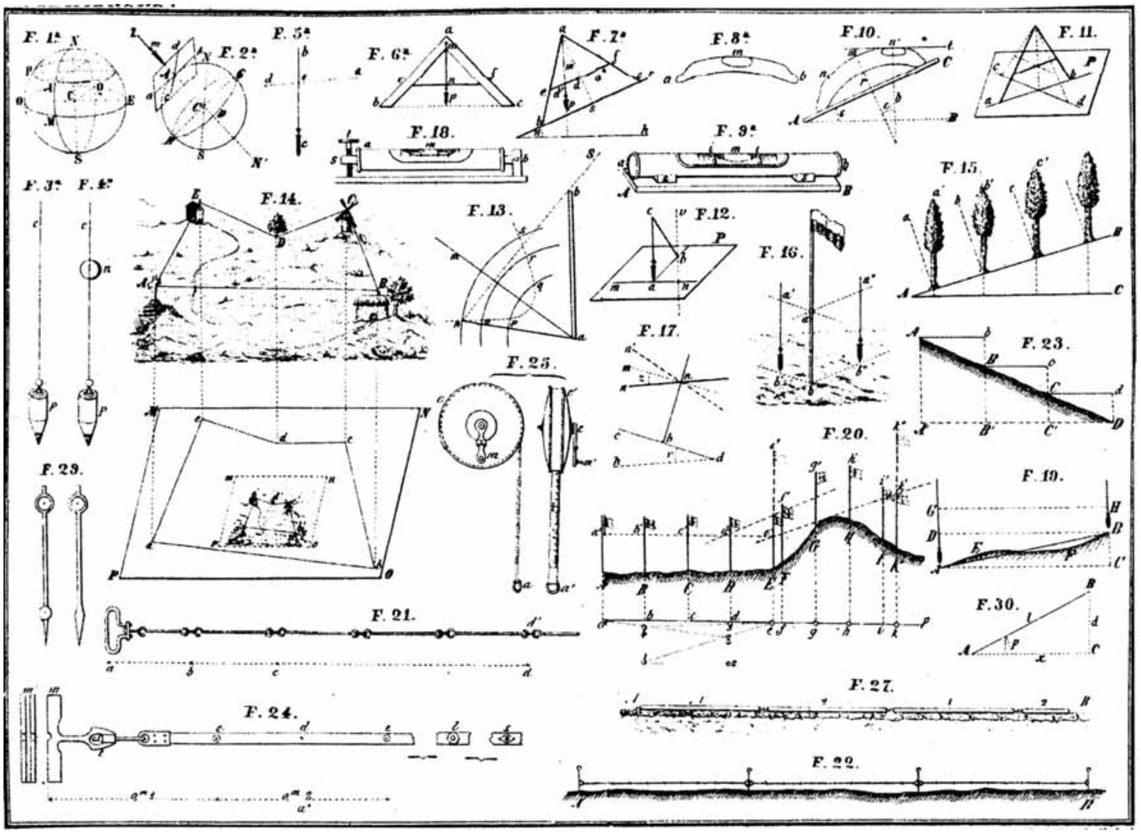
***[2]** His ita praeparatis tunc in rotae modiolo ad partem interiorem tympanum stabiliter includatur habens extra frontem suae rutundationis extantem denticulum unum. Insuper autem ad capsum raedae loculamentum firmiter figatur habens tympanum versatile in cultro conlocatum et in axiculo conclusum, in cuius tympani frontem denticuli perficiantur aequaliter divisi numero quadringenti convenientes denticulos tympani inferioris. Praeterea superiori tympano ad latus figatur alter denticulus prominens extra dentes.*

***[3]** Super autem, planum eadem ratione dentatum inclusum in alterum loculamentum conlocetur, convenientibus dentibus denticulo, qui in secundi tympani latere fuerit fixus, in eoque tympano foramina fiant, quantum diurni itineris miliariorum numero cum raeda possit exire. Minus plusve rem nihil inpedit. Et in his foraminibus omnibus calculi rotundi conlocentur, inque eius tympani theca, sive id loculamentum est, fiat foramen unum habens canaliculum, qua calculi, qui in eo tympano inpositi fuerint, cum ad eum locum venerint, in raedae capsum et vas aeneum, quod erit suppositum, singuli cadere possint.*

***[4]** Ita cum rota progrediens secum agat tympanum imum et denticulum eius singulis versationibus tympani superioris denticulos inpulsu cogat praeterire, efficiet, <ut,> cum CCCC imum versatum fuerit, superius tympanum semel circumagatur et denticulus, qui est ad latus eius fixus, unum denticulum tympani plani producat. Cum ergo CCCC versationibus imi tympani semel superius versabitur, progressus efficiet spatia pedum milia quinque, id est passus mille. Ex eo quot calculi deciderint sonando singula milia exisse monebunt. Numerus vero calculorum ex imo collectos summa diurni <itineris> miliariorum numerum indicabit.*

[text from thelatinlibrary.com].

square and the diopter or alidade. They were used to establish the alignment based on an angle determined by the main instrument.
"If there were a valley outside the line of sight of the surveyor, when poles have been placed next to the groma, he must descend along the line.
Alternatively, if there is a narrow valley, it can be crossed and the difficulty avoided by placing no less than three poles on the further side, which can be aligned by a groma situated on the other side. They can be aligned again with the first poles and the original alignment extended by means of a carefully directed alidade as far as the operation requires"[13].



Reproduction of various simple surveying instruments from an ancient treatise on this subject.

3.6.- Groma

This instrument is probably the best studied and known of those used in antiquity. It has been the subject of varied hypotheses and also of essays on experimental archaeology[14]. Despite all this, its use does not seem to have been interpreted with adequate success.
It is a very rudimentary tool for making alignments at right angles to one another, a surveyor's square as primitive as it is inaccurate. According to some authors, this instrument, already known in Greece, came to Rome through the Etruscan culture, as the origin of the word itself seems to indicate[15].

[13] FRONTINUS *De limitibus* 33, 34.
[14] ADAM, J.P., 1989, pp.9 and ff., 2nd Spanish edition 2002, *La construcción romana: materiales y técnicos* [Roman building: materials and techniques].
[15] RESINA SOLA, P. 1990. *Función y técnica de la agrimensura en Roma (II). Instrumental técnico y sistema de medidas agrarias*. Topografía y Cartografía. Colegio de Ingenieros Técnicos en Topografía.

It is consists of a simple combination of a cross with arms at right angles, at the end of which hang plumb lines, and a vertical foot to adjust this combination in a horizontal plane.
Graphical representations of the *groma* have been found in bas-reliefs[16]. One of them was found almost complete in the excavations of Pompeii[17]. We also find descriptions by classical authors that are sufficiently exact, both in their shape and their use.
Frontinus explains in adequate detail its use to establish boundaries, and for measurements and parcelling in surveying[18]:
Every part of a field no matter how small should be in the power of a surveyor and subject to his requirements in terms of right angle procedures. We must, therefore, especially be prepared to cross any obstacle that may present itself by means of the groma. We must also take care in measuring so that a given movement can achieve a representation as close as possible to the proportion of the length of the sides. The groma should be used firstly to align all the obstacles in line with the alidade. Its lines or strings, drawn taut by bobs, and parallel with each other, can be seen well from the all corners, until only the nearest can be seen, with the other out of sight. Then fix ranging poles, again setting the sight on them, when the groma has meanwhile been taken to the last ranging pole in its same position and prolong the line that has been started to the corner or to the boundary. The right angle from any corner on the perimeter determines the position of the groma.
One must first of all look over the ground to be surveyed, and place at each of its corners markers that can be aligned at right angles from the base line. Then, with the groma placed and well lined up, trace a second line to the larger side and, with ranging poles placed in correlation, take a line to the other side, so that when it reaches the end, is parallel to the first.
This *limitatio* was used for private and public fields, such as cities, colonies, temples and military camps. In the last case, it was called *castramentatio*. All these operations were the object of ritual and were charged with heavy religious meaning[19].
However, as a precision instrument the *groma* leaves much to be desired. In fact it is subject to gross errors from its construction and by the effects of external agents, such as the wind, despite recommendations that have come to us from the like of Heron[20]:
Wooden cylinders can be added to shield the strings from the wind: when the bobs rub against the sides of the tubes, the strings are not precisely at right angles to the horizon. Also, even when the strings are settled and at right angles to the horizon, planes derived from these strings are not necessarily at right angles to one another.
In fact, it can be said that the *groma* is a low precision instrument. Although it is clear from reading the classics that it was used in surveying and in the laying out of military camps and new-build cities, it is of no use for drawing out long alignments or for the outline of for determining the boundaries of properties (*centuriato*). By analysing very exact reproductions of long straight lines in roadworks, and the precision of the outlines of well-known measured properties from aerial photography, one can only conclude that the *groma* was not the instrument that was used.
Perhaps we should limit the *groma* in principle to its ritual use in religion by augurs and surveyors who in the early years of Rome had such an important rôle. At that time every

[16] Such as the sculptures on the tomb of Arbutius Faustus, surveyor of Ivrea (Valle di Aosta) and of the Pompeian Nicostratus.
[17] DELLA CORTE, M., 1922, *groma, Monumenti antichi della reale Accademia dei Lincei*, p.28.
[18] Frontinus, *De limitibus*, 32, 33.
[19] RESINS SOLA, P. 1998, p.386, *Algunas precisiones sobre los campamentos romanos* [Some remarks about Roman emcampments], op. cit., RESINA SOLA, P., 1990, p.12 *Función y técnica de la agrimensura en Roma* (II), op.cit.
[20] HERO, *On the diopter.*

measurement and ground plan of a new building had a clear religious significance with the essential presence of the augurs[21].

The first geometers belonged to the priestly class, where they kept the secrets of numbers, geometry and augural sciences derived from the Etruscan divinations[22]. In the words of Higinius Gromaticus: "*Of all the rites or activities of surveying, tradition has it that the establishement of boundaries is the most important. This measuring method has its origin in the science of the Etruscan haruspices".*[23].

Surveyors lost their priestly character starting with the promulgation of the 'Law of the twelve tables', in respect of the secularization process that the Roman world was undergoing[24]. Probably from this moment forward the *groma* was relegated to what we could call unprofessional surveying, the sharing out of land among families, and the distribution of specific farms or smallholdings.

The *groma* never had a rôle in the layout of roads or hydraulic works, as has often been claimed in the modern texts that are in circulation, and no classical text indicates this happening. In our analysis we have found no indication of this instrument being used for these ends, and moreover there were instruments that were very much more simple, versatile and effective. For reasons that will be revealed in due course, long alignments on highways and *centuriationes* were based on careful previous triangulations, which must necessarily have given very exact readings, as we can tell from the results. The *groma* could not have had a part in them.

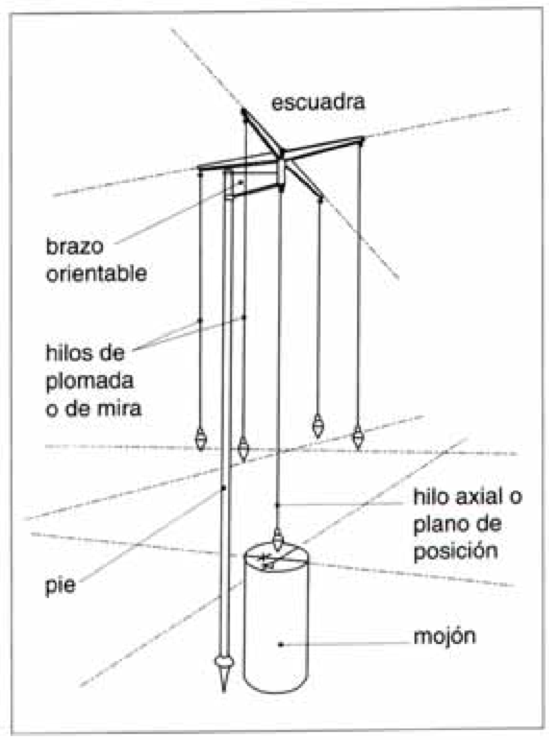
[21] RESINA SOLA, P. 1998, p. 386. *Algunas precisiones sobre los campamentos romanos…* ob. cit.
RESINA SOLA, P. 1990, p. 12. *Función y técnica de la agrimensura en Roma (II)…* ob. cit.

[22] RESINA SOLA, P., 2003, *El agrimensor en Roma* [The surveyor in Rome]>>> In the words of Hygienus Gromaticus: "Of all the rites and operations of surveyors, tradition has it that the most important is establishing boundaries. This measuring system has its origins in the science of Etruscan haruspices".

[23] HIGIENUS GROMATICUS, *De limitibus constituendis* [The establishment of boundaries], 166. <<<Translator's note: Hygienus Gromaticus flourished under the emperor Trajan. The work on castrametation, *De munitionibus castrorum* [Camp defences], ascribed by some to Hygienus, is thought to be of the third century A.D.>>>

[24] RESINA SOLA, P., 2003, p.306, *El agrimensor en Roma*, op.cit.

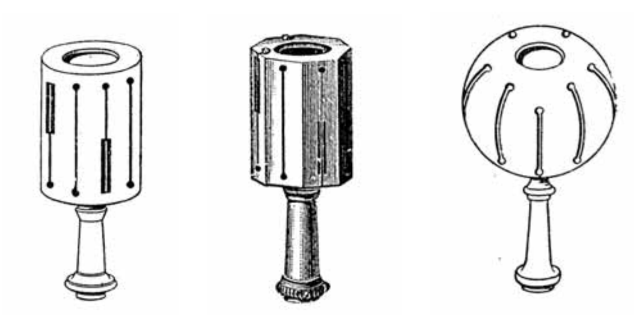
Tombstone of Aebutius Faustus, mensor (priest surveyor?) from Ivrea (Val di Aosta, Italy, Museo Civico P.A. Garda e del Canavese, Ivrea.



Reconstruction sketch of the *groma* in Adam's work. To the right a prototype intended to be published, based on Adam's design.

3.7.- Surveyor's square

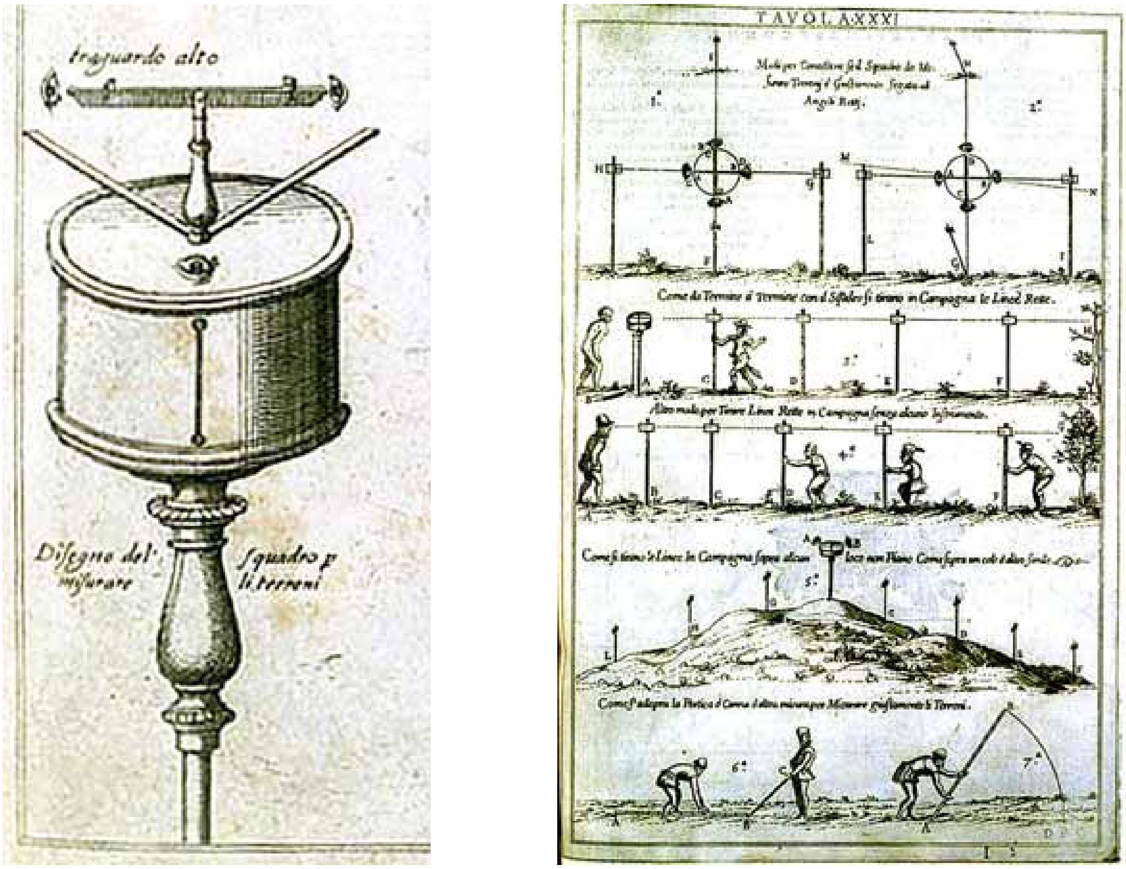
In fact the *groma* is a surveyor's square; but in this section, we wish to look at the instrument that goes under that name today. It consists of a cylinder vertically grooved in such a way that the sight holes made up by the grooves make a precise shape in right-angled planes. They enable alignments at 45 or 90 degrees, as desired.



Different models of modern surveyor's squares.

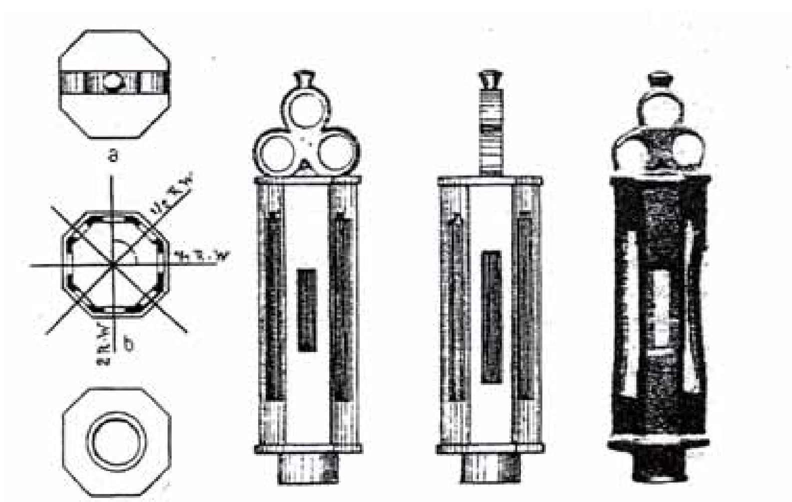
The instrument is known and used in surveying in modern times at least since the 16th century. In that century it is found drawn in practical manuals and its function and use are fully explained[25].

[25] 25. POMODORO, G., 1603, *Geometria prattica…* op. cit.



Surveyor's square from the treatise *Geometria prattica* by Giovanni Pomodoro, with explanation of its use, together with an explanation of the use of the *pertica* [perch].

Nonetheless in the museum of Koblenz (Germany), before the Second World War, there was an item, later lost, that is now identified with the surveyor's square. It consisted of an octagonal object with openings for fins on each of the faces[26].



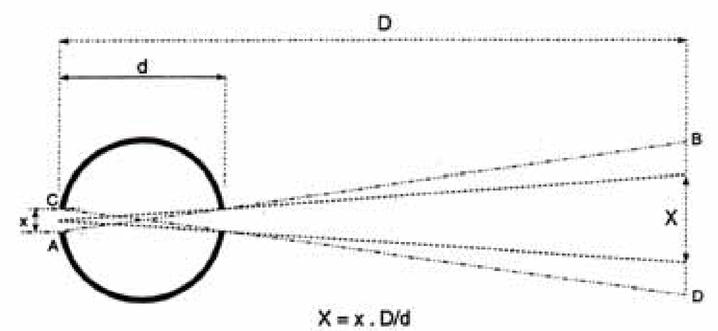
Roman surveyor's square from Koblenz museum (now lost) GREWE. K., 1985.

[26] GREWE, K., 1985, *Planung und Trassierung Römansischer Wasserleitungen dans Schriftenreihe der SEXTUS JULIUS FRONTINUS-Gesellschaft'*, Suppl,mentband I, Wiesbaden.

An unusual case, as yet unequalled, was recorded by the fact that in 1997, during archaeological procedures on the remains affected by the A29 motorway from Amiens to Saint-Quentin, a surveyor's square was discovered in a Roman villa at levels equivalent to the second half of the third century. From all appearances, but for the circumstances in which it was found, it could have been taken for a square from the last century.

Roman surveyor's square from L'Orme-Ennemain (Somme). Photo: Hervé Petitot.

This square that turned up in L'Orme-Ennemain (Somme) was in a very good state of preservation. It allowed some interesting research by which, among other things, the degree of precision of the instrument was determined[27].
The height of this square from L'Orme is 183 mm. (ten Roman inches, 7.2 English inches), and it is 76.5 mm (3") in diameter. It has 16 sights with grooves 0.6 mm (0.024 in.) wide. It could measure angles of 22º30' and its multiples (45º, 90º and 180º).
The incremental inaccuracy of this type of square has always been the main concern of surveyors. The accuracy of construction of one of these has always been the first thing to be checked on acquiring it, for which surveyors carried out a series of accuracy checks[28]. The precision of the surveyor's square is easy to establish by lining up firmly fixed vertical ranging poles. You then take a sight on them of the alignments of the planes that result from the sights. By successive 90º turns around the axis of the square, the successive coincidences can be checked, that is to say the poles placed at 90º with tips on the square, can be seen to be equal to the successive sightings through the sight-holes.
The visual field of the square is given by the following diagram and formula:



In the square from L'Orme,
x = 0.60 mm. (.024")
y = 76.5 mm. (3")
Therefore at 50 metres (164") the field of vision is 39 cm. (15").
In respect of angular error, the tests gave some deviation of as much as 0.4 degrees from construction faults in the sight-holes. One may wonder whether this error could have been less at the time when the square was being used, since the amount of corrosion of the sighting holes and the alterations in shape resulting from the the conditions in which it was buried could have had some bearing.
We should point out here that the model that has disappeared from Koblenz had a much wider use and perhaps accuracy. This square had on all its faces wide gaps for the sights. Upon them would have been fixed fine reference cords, and these, in view of its high quality, gave greater accuracy. Also the instrument could have been calibrated from time to time by means of ranging poles placed at appropriate distances, at 90 and 45 degrees. This very precisely, efficiently and speedily restored and updated the instrument.
We can therefore see that the surveyor's square is a very much more advanced instrument than the *groma*. With it one could get greater accuracy without the problems added by the ever-present action of the wind during its use.

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[27] MARCHAND, G., PETITOT, H. Y VIDAL, L. 2000: *L'équerre d'arpenteur de l'Orme à Ennemain (Somme)*. Autour de la Dioptre d'Héron d'Alexandrie. Centre Jean-Palerne. Université de Saint-Étienne.
[28] DOMINGUEZ GARCIA-TEJERO, F., 1958, p.68, *Topografía general y agrícola* [General and agricultural surveying], Salvat Editores, Madrid.

3.8.- Gnomon

Another basic concept of ancient surveying is the determination of the North, in order to orient precisely the topographical, geodesic, surveying and other works. We are talking of a time when the compass did not exist, and probably, had it existed, it would not have been used in this type of field surveys. In fact the determination of true or astronomic North[29] was in the hands of the scientists of antiquity with very much simpler methods than one would suppose.

In these respects, let us quote the information that Hygienus Gromaticus[30] brings us, as he deals with the errors if it is not carried out with precision:

Many, on account of their ignorance of cosmology, follow the sun, that is its rising and its setting.

And he goes on to describe how to find the North with exactitude:

First one should draw a circle on a flat piece of land. At its centre one should put a gnomon, such that the shadow of its tip falls at some time inside the circle. This is more certain than making a line from East to West. It will be observed that the shadow gets shorter from dawn onwards. Then, when the shadow of the tip touches the circle, one should make a mark on the circumference. In the same way, a mark should be made when the sun leaves the circle.

Once these two points where the shadow enters and leaves have been marked on the circle, a straight line should be drawn through them cutting the circumference, and its mid point indicated. A line should be drawn at right angles from that point from the centre of the circle.

This line will mark the North-South direction exactly.

The gnomon was widely used in antiquity, and the properties of the movement of the Earth through the ecliptic were exploited to establish by means of the gnomon the largest number of measurements possible. Let us recall that Eratosthenes made use of the *scaphium* or *gnomon* in his calculations of the sizes of the Earth according to the information from Cleomedes.

Vitruvius shews us another interesting use in his Book 9, Chapter 7, when whilst describing analemmas, he says [31]: "*Namque sol aequinoctiali tempore ariete libraque versando, quas e gnomone partes habent novem, eas umbrae facit VIII in declinatione caeli, quae est Romae. Idemque Athenis quam magnae sunt gnomonis partes quattuor, umbrae sunt tres, ad VII Rhodo V, ad XI Tarenti IX, ad quinque <Alexandriae> ceterisque omnibus locis aliae alio modo umbrae gnomonum aequinoctiales a natura rerum inveniuntur disparatae*".

During the Spring and Autumn equinoxes, when the Sun is positioned in Aries and Libra (21st March and 21st September respectively), nine parts of the gnomon give eight of shadow at the altitude of the pole (latitude) of Rome. At Athens, four parts of gnomon give three of shadow. At Rhodes seven parts give five. At Taranto eleven give nine. At Alexandria five give three. At other different places we find that the equinoctial shadows are always different, according to the natural setting. For that reason, the equinoctial shadows should always be observed in the place where the clocks are to be made.

The equinoctial shadows of the vertical gnomons played a fundamental rôle in constructing clocks or solar quadrants. By attention and measurement, the latitude of the place could be established with sufficient accuracy to complete these time measuring devices.

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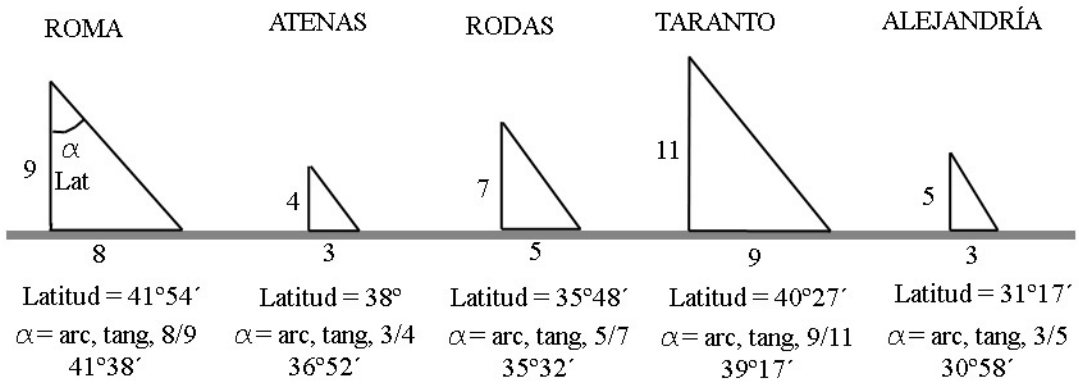
[29] The true North and South poles relate to the points at which its axis cuts the earth. Nonetheless, this rotational axis is not fixed. It undergoes a precession of a period of 25,800 years. The axis of rotation of the Earth at present points to Polaris, the Pole Star. In the year 3,000 B.C. it pointed to Thuban (alpha Draconis), in 14,000 A.D. it will point to Vega (alpha Lyrae), and in 22,800 A.D. it will again point to Thuban.

[30] HYGIENUS GROMATICUS, *De limitibus constituendis*, 188.

[31] VITRUVIUS, *De architectura*, 9.7.1.



It will be seen that the proportions established by Vitruvius are accurate, especially if it is borne in mind that the present-day geographical latitudes referred to in the drawing correspond to the map-making development of the reference ellipsoid defined by the American J.F. Hayford, and not to the actual surface of the Earth, as was believed in ancient times.

By means of the gnomon clocks and calendars were constructed with extraordinary precision in antiquity. They were fully informed on the movement of the Sun and they had fully researched the outlines formed by the projection of a shadow from a point at the same hour of the day and as the days of the year passed by (*analemma*).

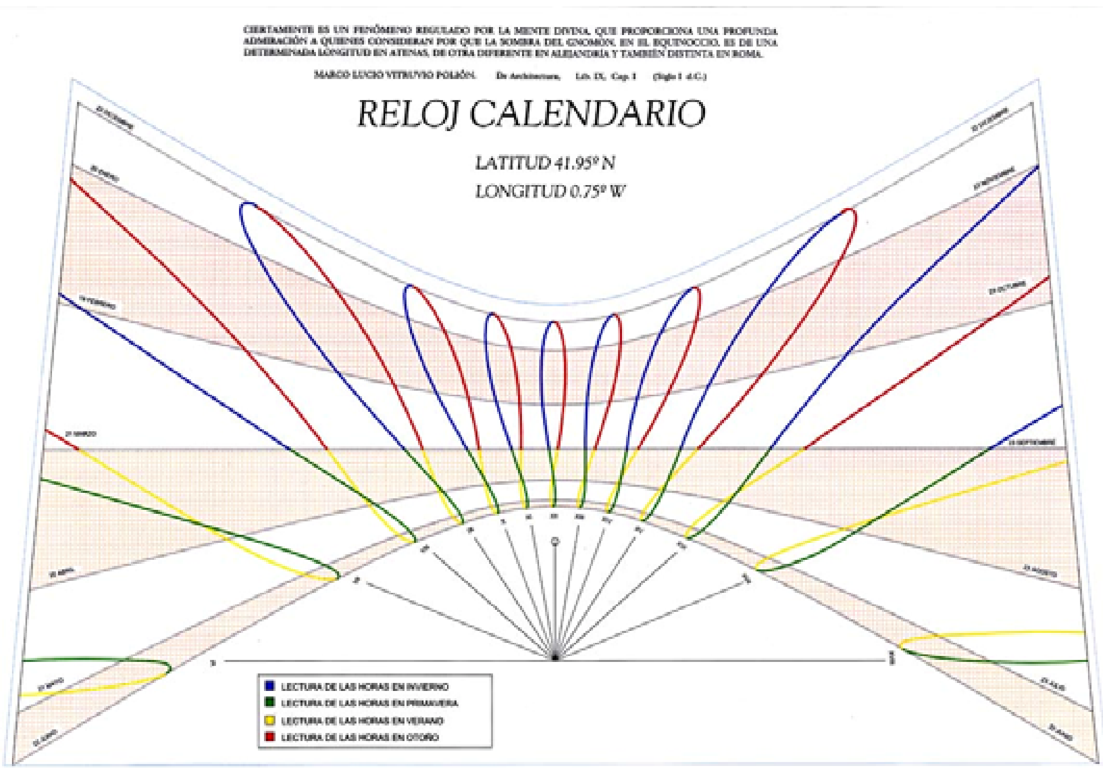
It is clearly a phenomenon regulated by the divine mind. It provokes deep admiration in those who study why the shadow of the gnomon, at the equinox, is at a calculated distance from Athens, at a different one from Alexandria, and also at a different one from Rome, at Placentia [Piacenza] again different, as it is in other parts of the world. This is the reason why clocks cast very different shadows when we refer to one place or another. The length of the shadow at the equinox firmly fixes the lines of the analemma[32].

Vitruvius describes at length the concept and construction of the *analemma*. He then adds:

After describing and explaining the *analemma*, we have used the Winter and Summer *lines, or rather those at the equinoxes and also of the months, we should mark the lines that mark the hours on a plane base, in accordance with the analemma calculations. Many variations can be derived from the analemma, and many types of clocks, simply by making a few technical calculations. The result of these figures and diagrams is always the same -- divide the equinoctial day and the day of the Winter and Summer solstices into twelve parts*[33].

[32] VITRUVIUS, 'De architectura', 9.1.1:
Ea autem sunt divina mente comparata habentque admirationem magnam considerantibus, quod umbra gnomonis aequinoctialis alia magnitudine est Athenis, alia Alexandriae, alia Romae, non eadem Placentiae ceterisque orbis terrarum locis. Itaque longe aliter distant descriptionis horologiorum locorum mutationibus. Umbrarum enim aequinoctialium magnitudinibus designantur analemmatorum formae, e quibus perficiuntur ad rationem locorum et umbrae gnomonum horarum descriptiones. Analemma est ratio conquisita solis cursu et umbrae crescentis ad brumam observatione inventa, e qua per rationes architectonicas circinique descriptiones est inventus effectus in mundo. [text from thelatinlibrary.com]

[33] VITRUVIUS, 9,7.7:
Cum hoc ita sit descriptum et explicatum, sive per hibernas lineas sive per aestivas sive per aequinoctiales aut etiam per menstruas in subiectionibus rationes horarum erunt ex analemmatos describendae, subicianturque in eo multae varietates et genera horologiorum et describuntur rationibus his artificiosis. Omnium autem figurarum descriptionumque earum effectus unus, uti dies aequinoctialis brumalisque idemque solstitialis in duodecim partes aequaliter sit divisus. [text from thelatinlibrary.com]



Actual example of a Sun clock-calendar constructed in monumental dimensions (60 m. by 20 m.) near Zuera (Zaragoza). The *analemma*s are drawn in four colours, one for each season of the year (calculated by Civil engineer Antonio Ramírez).

3.9.- Libra Aquaria

The water leveller or water balance, to which there are vague references in some classical authors[34], has been interpreted and drawn in many forms.

From the proper etymology of the word, we tend to think that the object in question is the classical water level that makes use of linked containers to maintain a constant level at its extremes, subject to a stabilizing balance or counterweight of the liquid.

Any flexible conduit, with small cylindrical transparent glass containers at its ends, can be turned into an excellent and versatile levelling instrument. Also, as we shall see, it complements other more powerful instruments, such as the *chorobate*.

Specifically, the Greek term referring to the conduits in the shape of a U which channel the water, like the inverted siphons in large aqueducts, is *koilia* (*κολία*), literally 'intestine'.

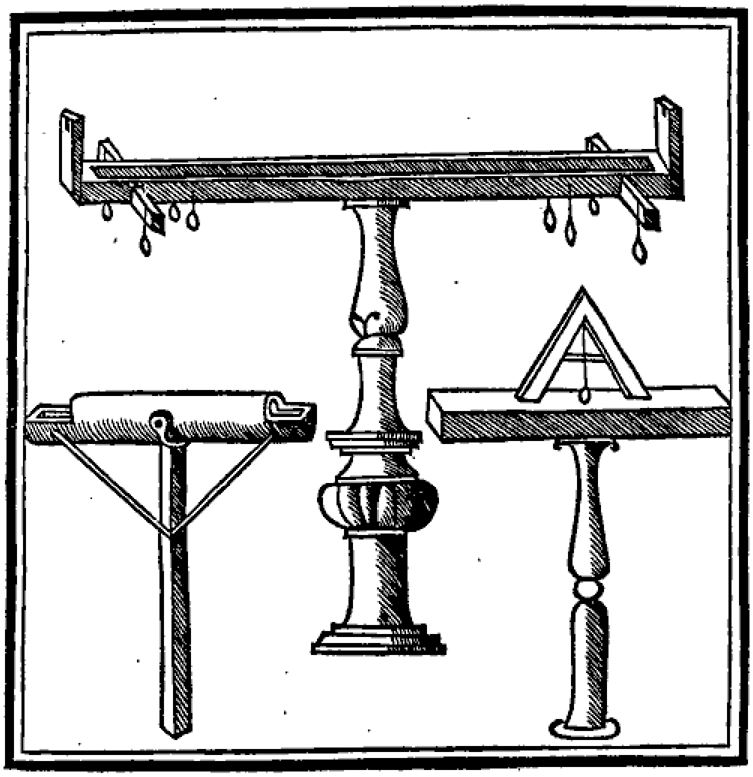
This instrument, so simple but so effective, is what we consider as the 'water balance' to which we refer.

3.10.- Chorobate or leveller

The existence of a precise, flexible and efficient levelling instrument during the time of the Romans can easily be deduced from an analysis of the great works for the canalization of water left by this civilization. However, it is from the information in the works of Vitruvius that we can tell what it looked like.

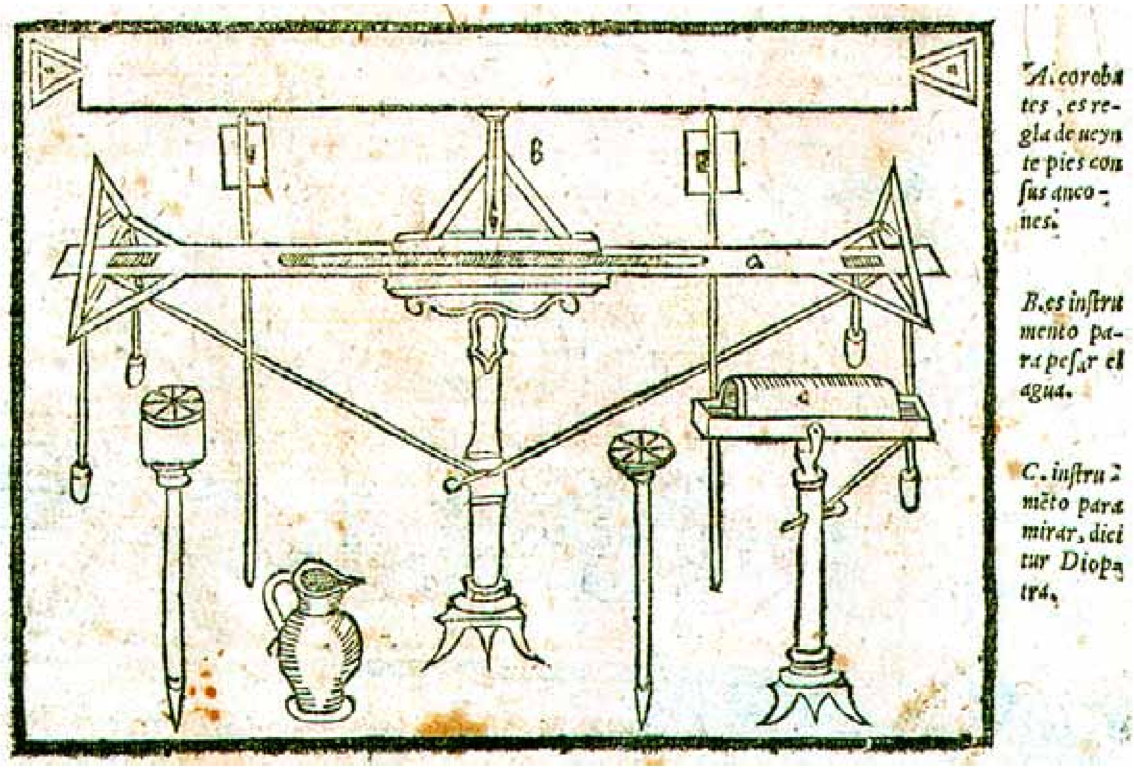
[34] For example VITRUVIUS, 'De architectura', 8.6-3.

Throughout history interpretations as to the shape and working of this instrument differed greatly and were even self contradictory, because, although based on Vitruvius' description, the original illustrations had been lost. Consequently they were to a very large extent basically flawed.
A short historical overview of these interpretations leads us firstly to one of the oldest representations known: the first edition of the Ten Books of Architecture entirely in French in 1547[35].
Somewhat later is the first Castilian-language edition by Miguel de Utrea 1582[36]. From the same century and surprisingly close in the design of the chorobate are the *Veintiún libros de los ingenios y las máquinas* [21 books of engines and machines] named as being by Juanelo Turriano but attributed to Juan de Lastonosa[37].
All these early works contain very similar representations of the chorobate, essentially a ruler on a vertical leg.

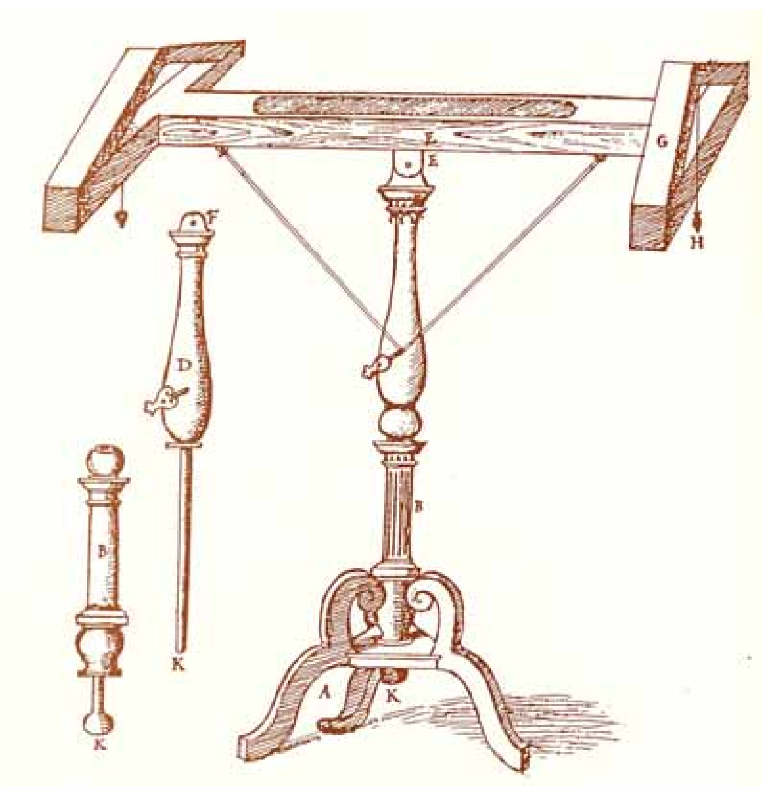


Chorobate and other instruments in the French edition of Vitruvius of 1547 by MARTIN and GOUJON.

[35] MARTIN, Jean (translator) and GOUJON, Jean (illustrator), 1547, 'Architecture ou Art de bien bastir, de Marc Vitruve Pollion' autheur romain antique, mis de latin en Françoys, par Ian Martin Secretaire de Monseigneur le Cardinal de Lenoncourt. Création/Publication Paris: pour la veuve et les héritiers de Jean Barbé et pour Jacques Gazeau.
[36] URREA, Miguel de (translator) 1582 *Marco Vitruvio Pollión, De architectura*, Alcalá de Henares: Juan Gracián, 1582.
[37] GARCÍA TAPIA, N. 1997: *Los veintiún libros de los ingenios y máquinas de Juanelo atribuidos a Pedro Juan de Lastanosa*. Gobierno de Aragón, Zaragoza.



Reproduction of the chorobate and other instruments in the edition of Vitruvius by Miguel de Urrea, printed in Alcalá de Henares in 1582.



Chorobate drawn in the *Veintiún libros de los ingenios y las máquinas*.

Later, in 1673, the Frenchman Claude Perrault translated Vitruvius[38] and made a radically different interpretation, converting the broad strip ending in brackets and supported by a vertical leg of the oldest editions, into a kind of table supported on four legs.
Works published later in Spain[39] adopt the Perrault design without further explanation. Later more modern authors, who are now the archaeological reference sources in this matter, eagerly accept this design[40].
Lastly, Adam[41] popularizes the chorobate table-type, and reduces its size to 1.5 m. (4.5 feet) to make it more manageable[42]. Employing one constructed in this way, he made a series of measurements in the ruins of Pompeii.


On the left chorobate constructed by Adam and tried out in Pompeii. On the right one of the many table-type chorobates of a metre and a half length, Adam's design. This can be seen in modern reconstructions for educational purposes. This one can be seen in the Exhibition Museum at the Pont du Gard (Nîmes, France).

By a detailed analysis of these reproductions and reconstructions that have occurred throughout history, any experienced surveyor will conclude as follows:
1.- The model having one vertical leg, with flat surfaces at each end, with or without reinforcement brackets, as is found in the Martín and Goujon edition, in Urrea and *Los veintiún libros* [‘The 21 books’], will work within certain drawbacks for levelling. One main drawback of this model is that the sight cannot be clearly set on account of the arrangement of the flat surfaces or the wooden triangles that have been drawn over the surfaces.
Other serious unresolved problems in this model are:
- There is no device to swing the chorobate on its longitudinal (its horizontal) axis, so as to set the apparatus correctly in the horizontal plane, although the one drawn in *Los veintiún libros* [‘The 21 books’] shews the plumb line positions better for this purpose.

[38] PERRAULT, Claude (translator) 1673: *'Les dix livres de l'architecture', Diverses rééditions, par exemple Les Librairies Associés*, 1965.
[39] VITRUVIO POLIÓN, Marco. *Los diez libros de Architectura*. En Madrid: En la Imprenta Real, 1787.
[40] COZZO, G. 1928: *Ingegneria Romana*, Mantegazza di Paolo Cremonese, Rome
CHOISY, A. 1899: *Histoire de l’architecture*. Diverses rééditions, par exemple Vincent Fréal et Cie 1964.
KRETZSCHMER, F. 1966: *La technique romaine*. Documents graphiques réunis et commentés, Bruxelles.
[41] ADAM, J. P. 1989, pp. 9 y ss. 2ª ed. esp. 2002: *La Construcción romana. Materiales y técnicas*… op. cit.
[42] At the same time also highly inaccurate, since the error involved is directly proportional to the square of the distance, and any error from levelling with a chorobate of a metre and a half (four and a half feet) is sixteen times greater than with a chorobate 6 metres (19½ feet) in length

- Turning in the vertical axis of the support, so as to bring the chosen level to different points within a determined radius of action. This turning is very important and has to be carried out on an axis below the levelling point, that is to say, not vertically, as is clearly seen in *Los veintiún libros* ['The 21 books'] model, and it seems that in the others where it is not allowed for levelling will be lost as it turns.

2. - The model in the form of a table is the one that archaeology, and research into Roman engineering in general, accepts these days. However, it presents excessive difficulties for efficient and exact levelling:

- It is very difficult to fix, as it requires a difficult, complex folding of three of the four legs to permit leveling.
- No kind of swivelling is possible.
- Once in position it does not present a clean view of the horizontal lines that form the plane to be transferred.
- Any misshaping in the wood of its complicated structure, something inevitable with time and with humidity variations, renders this model more unusable than any other.

Hence one can safely conclude that it is simply useless for levelling.

For this reason, we have undertaken to revisit the origins of the problem, that is, a correct rendering of Vitruvius' text, as we consider that it is the major cause of error in current thinking.

De Architectura libri decem, liber VIII, cap. V, 1-3[43].

1. Nunc de perductionibus ad habitationes moeniaque ut fieri oporteat explicabo. Cuius ratio est prima perlibratio. Libratur autem dioptris aut libris aquariis aut chorobate, sed diligentius efficitur per chorobatem, quod dioptrae libraeque fallunt. Corobates autem est regula longa circiter pedum viginti. Ea habet ancones in capitibus extremis aequali modo perfectos inque regulae capitibus ad normam coagmentatos, et inter regulam et ancones a cardinibus compacta transversaria, quae habent lineas ad perpendiculum recte descriptas pendentiaque ex regula perpendicula in singulis partibus singula, quae, cum regula est conlocata eaque tangent aeque ac pariter lineas descriptionis, indicant libratam conlocationem.

1. I shall now explain methods for getting water to dwellings and to strongholds. The first problem is levelling. It is however, possible to achieve levels with diopters, with water balances or with chorobates; but they are achieved with greater precision by means of chorobates, because diopters and water balances are inaccurate. The chorobate is a ruler about twenty feet (just under six meters) long. It has elbows[44] at both ends constructed in the same fashion. They are fixed at right angles to the ends of the ruler. Between the ruler and the elbows it has hinged struts. These struts have precisely inscribed vertical straight lines on them. It also has plumb-lines hanging from each part of the ruler. When the ruler is positioned and the plumb-lines touch the inscribed lines precisely and simultaneously, the levelling is established.

2. Sin autem ventus interpellavit et motionibus lineae non potuerint certam significationem facere, tunc habeat in superiore parte canalem longum pedes V latum digitum altum sesquidigitum eoque aqua infundatur, et si aequaliter aqua canalis summa labra tanget, scietur esse libratum. Item eo chorobate cum perlibratum ita fuerit, scietur quantum habuerit fastigii.

2. If, however, the wind interferes, and because of their movement the lines cannot make a sure reading, there is a channel in the upper section, five feet long, and inch wide and an inch and a half deep (1,48 x 0,0185 x 0,0278 m), in which water has been poured. If the water touches the upper parts of the channel, one can tell that it has been levelled. Thus, when the chorobate has been levelled in this fashion, one can tell the degree of the

[43] Traducción de C. Andreu en la edición *M. L. Vitruvius De Architectura* (prólogo de Eugenio Montes), Madrid, 1973, pp. 177-178, revisada por Alicia Canto, de la Universidad Autónoma de Madrid.

[44] 'Ancones' can be rendered as ends, arms or tables, but scarcely as feet, as translators who originated the mistake have done.

slope.

3. Fortasse qui Archimedis libros legit dicet non posse fieri veram ex aqua librationem, quod ei placet aquam non esse libratam sed sphaeroides habere schema et ibi habere centrum quo loci habet orbis terrarum. Hoc autem, sive plana est aqua seu sphaeroides, necesse est, ad extrema capita dextra ac sinistra cum librata regula erit pariter sustinere regulam aquam, sin autem proclinata erit ex una parte, qua erit altior non habere regulae canalem in summis labris aquam. Necesse est enim quacumque aqua sit infusa in medio inflationem curvaturamque habere, sed capita dextra ac sinistra inter se librata esse. Exemplar autem chorobatae erit in extremo volumine descriptum. Et si erit fastigium magnum, facilior erit decursus aquae. Sin autem intervalla erunt lacunosa, substructionibus erit succurrendum.

3. Perhaps a reader of Archimedes will say that it is not possible to achieve a reliable levelling with water. Archimedes said that water is not level but has the form of a sphere with its centre in the centre of the earth[45]. Nonetheless, whether water be flat or curved, it necessarily follows that when a straight ruler is levelled on the extreme left and right it will at the same time hold the water level; if, on the other hand, if it slopes one way of the other, where it is higher it will not contain the water up to the top of the channel. It follows that, even if the water has a curved surface in the centre, the left and right extremities will still be balanced. An example of a chorobate is described at the end of the book. If the degree of slope is large, the water will flow more readily. If, however, there are gaps in the slope, it will be necessary to have recourse to constructions beneath.

On this basis we have constructed the instrument with the greatest precision possible, following Vitruvius' description faithfully. We have created an efficient and flexible levelling device, suited to the purposes that Vitruvius describes.

We have fallen back on our experiment with basic instruments that are no longer in use, but were previously normal and practical when it was not easy to use an optical leveller.

When we look at the working of 'levels'[46] and the wise recommendations that technicians at the start of the twentieth century have given us as to its correct usage, we find that this instrument is somewhat like a disassembled chorobate, although no less efficient, manageable and flexible, although different in the way that it is set up. However simple, its precision can achieve that of modern instruments, depending as ever on the eye and skill of the surveyor.

'Levels' have been used in the construction of canals, railways and many, many other engineering works throughout history since time immemorial. In present days we have witnessed its use and its degree of accuracy in comparison with current optical levelling devices.

[45] It should be noted that antiquity was aware of the shape of the earth and the properties of water.

[46] Instrument in the shape of a truncated cross, used in groups of three, supported by a water level instrument to put the first two in a horizontal plane. It works well in irrigation ditches and channels, and consequently ideal for railway and highway inclinations

An actual case of the use of ' levels ' in the construction of a railway in the 19th century.

On the basis of this data and following Vitruvius' instructions, we have placed a beam 20 feet (5.92 m.) long on a sturdy tripod. We placed between them a simple hand-made levelling base that swivels. The beam has small transverse tables at its ends, joined to it by large-gauge pegs to which the tables are fixed.

The shape and joints of all these units allow them to be set up precisely and rapidly, assisted by plumb lines as described by Vitruvius. The chorobate can be efficiently set up and the tables placed in a horizontal plane in a few seconds.

The chorobate can be calibrated with equal speed. By means of the water level (*libra aquaria*) it is quickly placed in the horizontal plane. It only remains to establish the position of the plumb-lines to continue functioning without difficulty. This operation can be carried out as often as is deemed necessary if there is a suspicion that the wood has warped or that it has undergone some change due to circumstances.

The result of the first chorobate that we have made in Zaragoza in May, 2004, surprised everyone from the start. It was able to compete with a modern optical level over relatively long distances of over 50 metres/yards, adequate for the desired objective. After a few details had been refined as we went along, we arrived at the final model, which furnishes a levelling accuracy very close to an optical level over distances of more than 70m. (75 yards), provided that the surveyor has a good eye. In other words, this has an excellent degree of precision in view of the fact that longer visual levellings are not possible due to the error resulting from the sphericity of the earth.

Above, first calibration attempts, setting up and precision of our chorobate, May, 2004.
Below, in competition with a modern optical level, September, 2004.
Patented as: COROBATE ROMANO (26/11/2004 – **Pat. nº 200402837**). © Isaac Moreno Gallo

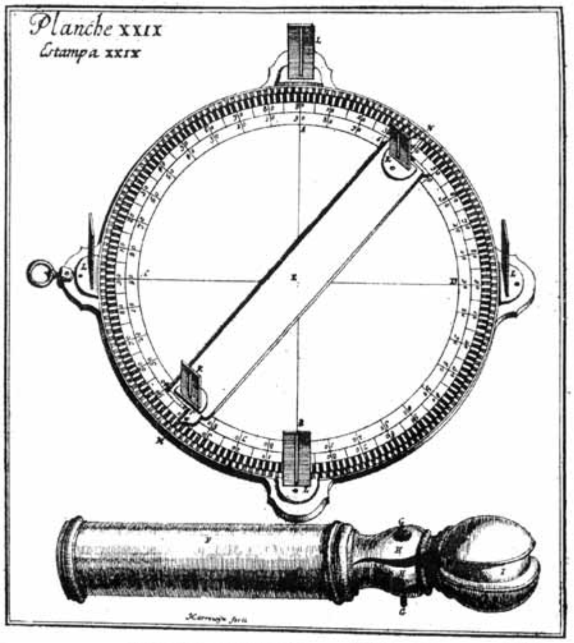
Detail of the plumb-lines indicating horizontality of the apparatus and of the apparatus on site.

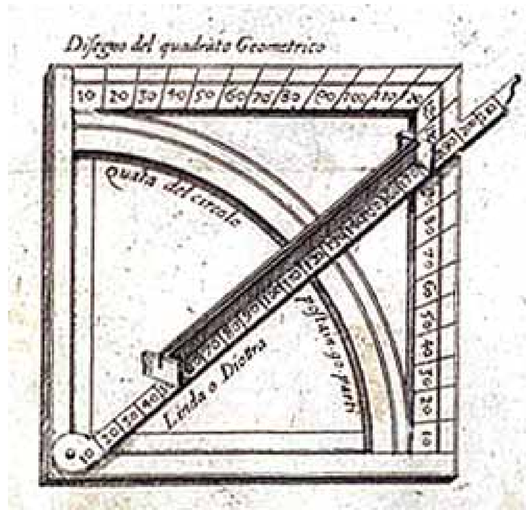
The whole system can be improved through slight modifications and simple devices that allow greater effectiveness, provided it has been correctly built. Thus it is possible to add an alidade with horizontal sighting that permit very exact observations of the reference of the sight available to the operator over the terrain, and serves to fix the resultant level.
The alidade equally allows for observations over very long distances, when this technique needs to be employed. It is often necessary to enhance the device with special visual means, like the slits, that have been always used in long-distance surveying and which we shall deal with next.

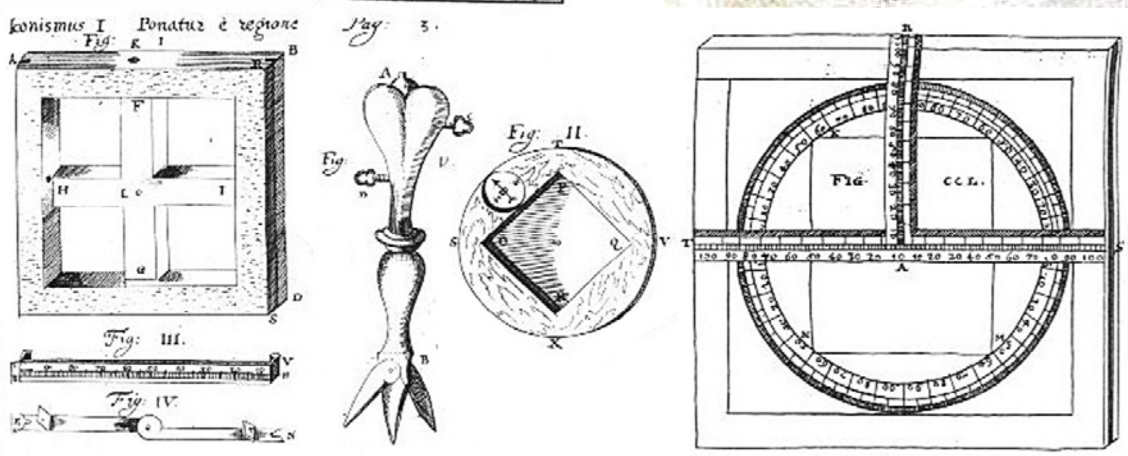
3.11.- Diopter

This instrument (from the Greek δίοπτρα meaning 'seeing through') consisted essentially of an alidade with sight that could be moved on a graduated limb. However, there were different designs over time.
Based on the information provided by Hipparchus (*fl.* 146-127 B.C.), we know that the diopter that this writer was using had one fixed sight and another that slid along a graduated bar four cubits in length (about 1.78 metres or 5½ feet). It is probable that this mechanism was used to allow calculating horizontal distances by indirect methods. The instrument based on this system has been successfully used in modern surveying. But we can say that Hipparchus invented the method, given that his is the first reference that we have for such a use of sights.
Heron of Alexandria wrote on the use of the diopter. This writer ascribed a horizontal and a vertical limb to this instrument. In this way he achieved a proper distance measuring device (theodolite). Its versatility was such that it allowed all the operations that we have been able to carry out in recent years with these instruments. According to Hero's own words[47] it can be used for the construction of plans, for levellings, for measuring fields without the need to enter, for measuring angles, for finding the area of a triangle, for crossing a mountain in as straight line, for measuring distances and heights of places that cannot be got to, etc.
Instruments of the greatest interest, based on the diopter mechanism, were developed in sixteenth century Europe. In *Los veintiún libros de los ingenios y las máquinas* [the '21 books of engines and machines'] stated as by Juanel Turriano but attributed to Juan de Lastanosa[48] several of these instruments are brought together, in particular the ingenious Geometric Quadrant, which was simply a diopter fitted to a squared limb, and which allowed simple operations to be performed by means of the technique of the similarity of triangles.
Giovanni Pomodoro[49] in 1603 has a drawing of it that he calls 'Quadrato geometrico' [Geometric square].
Athanasius Kircher[50] demonstrated how this instrument, the *Pantometrum* (or 'general measurer'), which he had designed, could be used to resolve the main surveying problems of his age. This instrument, certainly complex, half way between the diopter and the geometric quadrant, nonetheless reached greater complexity in it construction and use.
Sebastián Fernández[51] makes a precise drawing of an alidade with sights that revolve on a circular limb, to which he adds other sights on a square, corresponding to one of the present-day developments of the diopter of Heron.

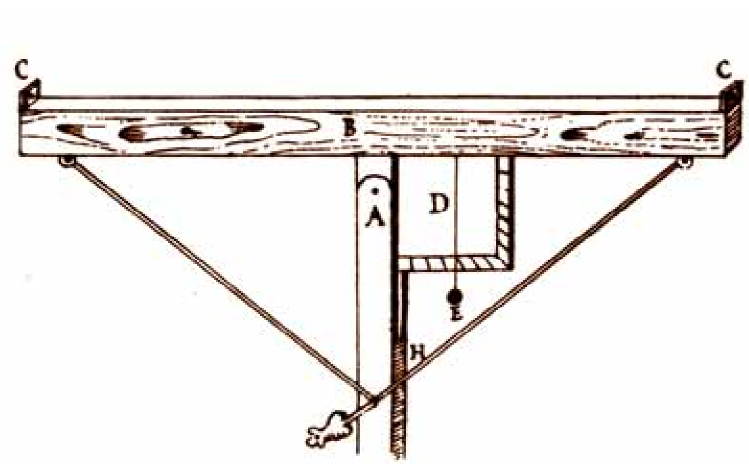
[47] HERON '*On the diopter*'
[48] GARCÍA TAPIA, N. 1997: *Los veintiún libros*… op. cit.
[49] POMODORO, G. 1603. *Geometria prattica*… ob. cit.
[50] SCHOTTO, Gaspare. 1690: *Pantometrum Kircherianum*… ob. cit
[51] FERNÁNDEZ DE MEDRANO, S. 1700: *El architecto perfecto en el arte militar*. Bruselas.





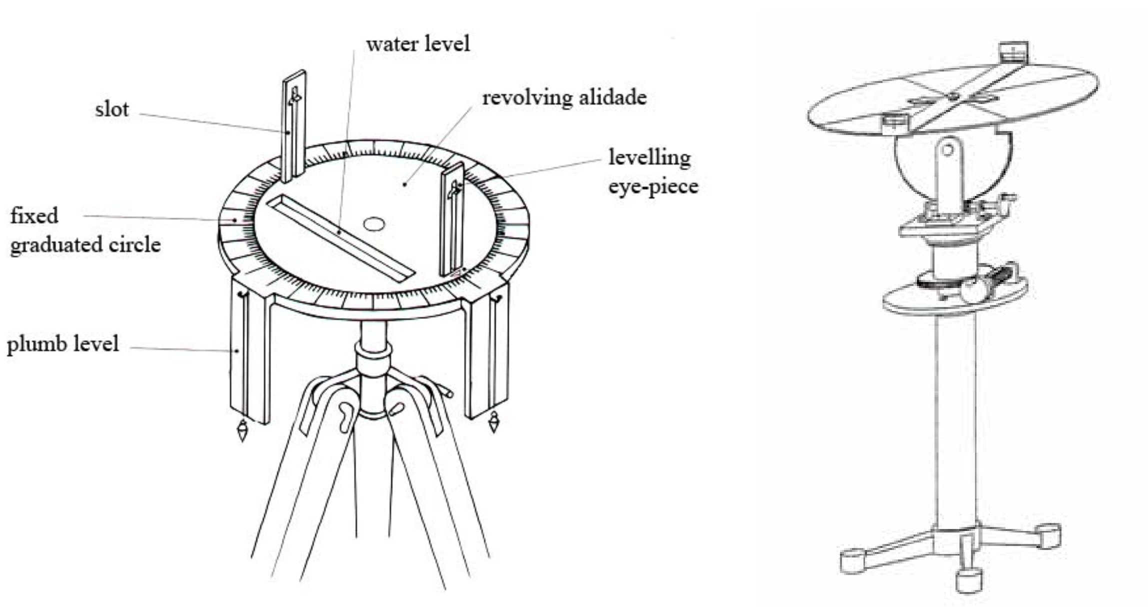


Above left, limb with slots and articulated foot for surveying work, drawn in 1700, the work of Sebastián Fernández. On the right Pomodoro's geometric quadrant, drawn in the 1603 edition.
Below, break-down of the *pantometrum* in the work of Gaspare Schotto.



Diopter with square vertical limb for surveying measurements represented in the *21 libros de los ingenios y las máquinas*.

With regard to the precision inherent in this instruments, we should point out that, to assess the degrees of angle in Sebastián Fernández's limb, alternating black and white divisions were employed that could easily distinguish half degrees, with a need to have recourse to approximations for smaller fractions.
Pedro Núñez (Nonius, 1502-1578), royal cosmographer, for whom a chair of Mathematics was expressly created in Coimbra, invented the *nonius* (i.e. vernier) to meet deficiencies of this kind. The vernier is a device that allows for precise measurement of small angles. Due to the difficulty in its construction, it took a long time for it to be regularly applied to limbs and graduated rulers.
There is no evidence that this type of device for the precise measurement of angles was in use at Rome. It can, however, be pointed out that a bronze calibrator discovered in China, made in the year 9 A.D. is the oldest vernier calibrator known. Judging from its principle, its characteristics and its uses, it is very similar to modern verniers. This calibrator, 14.22 cm. (c. 5½") long, was composed of two rulers, one fixed and the other moveable. In the middle of the fixed ruler there is a groove, whilst there is a bolt such that the moving ruler can run along the groove to right or left. The face of the fixed ruler is graduated in *cun* and *fen*, and on the reverse is the date when the vernier was calibrated. The discovery of this Chinese instrument advances its invention some 1,500 years.
Let us return to the diopter which is the subject of our consideration. We have seen several reconstructions of it in modern specialist texts. Vincent[52] and Schöne[53] proposed models that have gained popularity with other writers who have dealt in this topic.
Adam[54] proposes yet another model: he does not state whether he experimented with it personally.



Left: Adam's proposal for a reconstruction of the diopter. Right: Schöne's reconstruction.

[52] VINCENT, M. 1858 : *Géodesie de Héron de Byzance'. Notices et extraits des manuscrits de la Bibliotheque Impériale*, XIX.2, Paris.
[53] SCHÖNE, H. 1903: *Herons von Alexandria Vermessungslehre und Dioptra*. Leipzig.
[54] ADAM, J.P., 1989, pp. 9 et seqq., 2nd Spanish edition, 2002: *La construcción romana. Materiales y técnicas...* op cit.

From an analysis of models proposed in modern times, it can be seen that it is extremely complicated to set them up and that they depend a great deal on the precision of their construction for their correct functioning. Their overall precision cannot be exact since the designs are inadequate to keep the adjustment after swivelling in the horizontal and vertical planes, even if it has a vertical adjustment.

We, starting from zero and on the basis of the bare descriptions that we have of the instrument, have looked for an instrument that will allow us to obtain an acceptable efficiency with the highest precision.

We know that certain works of surveying, such as work in triangulation and large areas, in tracing long alignments, in drawing precise maps, etc., require ground measuring work of very high precision. To carry out measurements of large areas, efficient equipment capable of successfully carrying out a wide range of operations is required. We are convinced that the Roman engineers had the capability and means for doing this.

When we had constructed an acceptable prototype, after long sessions of draughting and designing, the first results we obtained with the equipment that we are now putting into practice, were satisfactory, and we are sure that the level of efficiency with this land measure will be high.

Nonetheless we shall leave the public presentation of the equipment in question until another occasion. Meanwhile we shall continue carrying out appropriate trials. Our only expectation is that it will look radically different from equipment proposed to date.

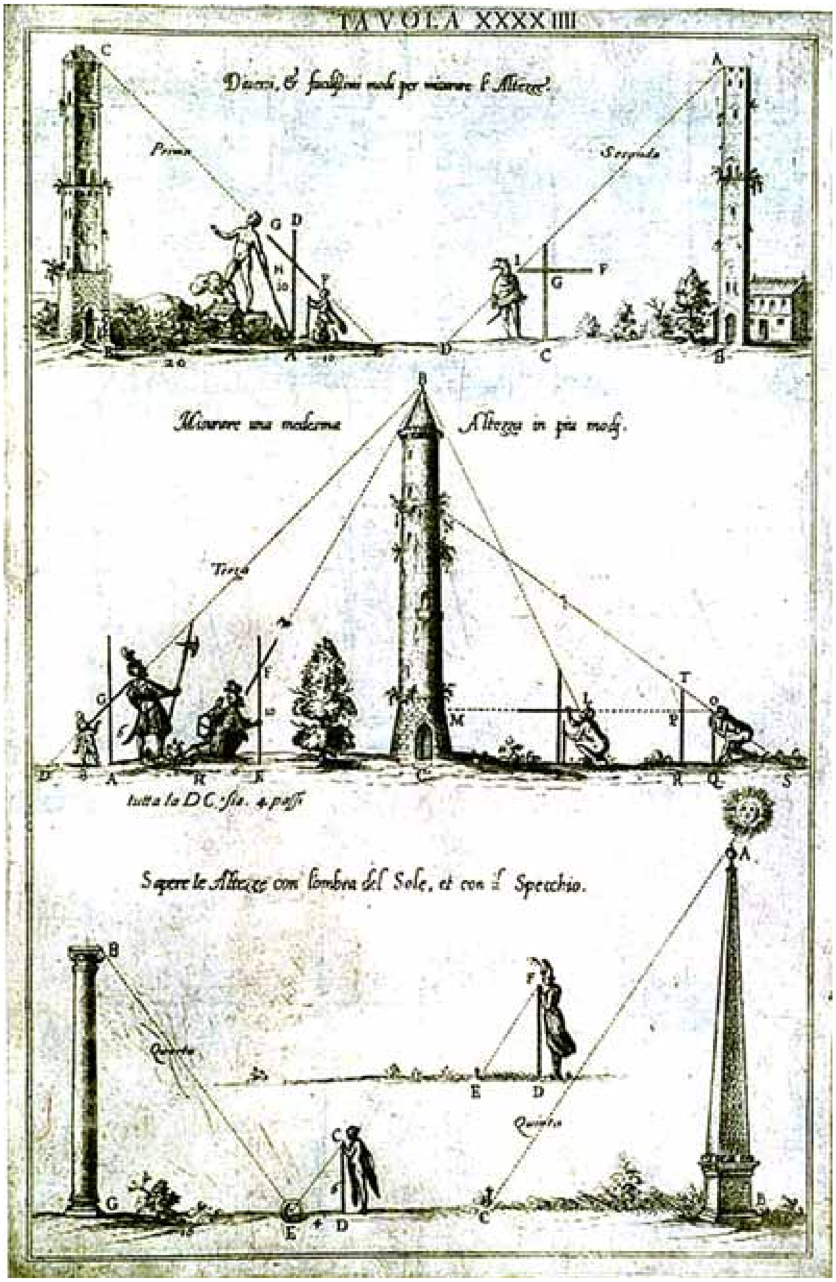
3.12.- Lychnia or lamp-standard

Called Lychnia (λυχνία, i.e. lamp-standard) in antiquity, this was a piece of equipment that was simple but powerful. It consists of a vertical foot with good plumb lines and a graduated horizontal arm that can turn and position itself vertically.

The triangles formed by two readings allow distances from the observed points to be calculated by the principle of similar triangles.

We have seen in the drawings of Pomodoro how, at the end of the 16th century, a simple instrument was known and employed, responding to the same functions as the lamp-standard in antiquity.

The power and versatility of the lamp-standard can be significantly improved by placing sighting slots on the horizontal arm, making possible the indirect calculation of horizontal ranges.



Example of the use of the measuring instrumentation based on the classical Lychnia, visible in Pomodoro's work.

4.- TECHNIQUES

If the knowledge about the most useful and exact surveying instruments used by the Romans is very limited these days, we cannot say that the techniques they used are known any better.

Classical texts are generally scanty in this matter. In other words the amount of writing that has survived from Roman times on these subjects is practically non-existent. Texts of a scientific nature were always held in suspicion within later ways of understanding life, and probably most were destroyed on purpose or were simply not transcribed.

We have seen that the understanding of the science of trigonometry was already quite complete in Roman times, having been largely inherited from what was known in the Greek world. From this point, the formation of easily resolved triangles that lead to the discovery of sizes on the ground was merely a question of talent.

In relation to the difficulty of resolution it is preferable to lay out a right-angled triangle, wherever possible. In triangles the resolution of the third angle is important, but the application of Pythagoras' theorem will solve it without recourse to other more complex formulae: it is easy to inscribe or fit other smaller triangles, from which to establish the similarities of Thales or to apply the theory of the side of a right-angled triangle adjacent to the right angle, and lastly it would be very simple to apply the Ptolemaeic table of chords.

If this is not immediately possible, one can try to reduce the triangles that cannot be resolved with the theorems already known into two rectangles, or apply Thales' similarities translating the angles derived from the appropriated instruments, or applying the sine theorem by means of the table of sines (chords) that are available.

In short, choose the shortest and easiest way in each case.

We can have not the slightest doubt that the engineers employed in the construction of the aqueducts that amaze us today, or in the unending network of roads and the excellent quality that we discover in them, were absolute masters of these and many other techniques. Their works speak for them.

Nonetheless, the first treatises on surveying in Europe of which we have knowledge, from the 16th century, which we have been discussing in this work, and in which these techniques using trigonometrical functions can be seen in some detail, are based on rudimentary but effective trigonometry. Regarding the techniques of resolution based on ancient scientists, one can see little of Thales and Pythagorus. The most complex trigonometrical functions, based on the chords or sines of angles, cosine, tangent, etc., were not applied, despite their being known in the Arab world at least six centuries previously.

These ground measuring techniques carried out by means of triangles, as a constant from the beginnings of modern surveying science, were explained in the works of Lastanosa, Kircher and Pomodoro, as a compendium of the surveying knowledge of the Renaissance. In all of them the right angle was the main technique, although the very useful similarities of Thales were used very often in those days. By using diopters on geometric quadrants or pantometers, which had adequately accurate elements, surveys were carried out that doubtless led to sufficiently accurate maps and detailed plans. At the same time aqueducts were constructed in Europe to supply cities, like the Romans had done, although with certain notable faults because they did not have the outstanding construction expertise of the Romans.

The city of Toledo had a mechanical device designed by Juanelo Turriano, watchmaker and astronomer to Charles V and Philip II, to raise water from the Tagus up to the city. This device, however, which created wide admiration in its time, was far from the effectiveness of water supply of the Romans. Not only was the water that it carried of poor quality, but there was a difference between this and a lasting piece of engineering, with low maintenance and without moving sections, such as was the syphon that overcame the depth of the river cutting to take water up into the city in the time of the Romans.

Going back to Roman engineering, it is difficult to be sure whether the Romans applied systematically the table of sines in resolving triangles. Moreover, at times, its application leads to solution of greater accuracy than other ways that involve the measurement of many more positionings which can lead to so many more errors.

The large scale graphical resolution of triangles, on a planning table or paper, is another method that should not be spurned and which we should consider more than once if we do not wish to make use of complicated trigonometrical formulae, which we cannot be sure were known in Roman times. Such is the case of the formula derived from the cosine theorem, which seem to originate in Islamic culture.

Nonetheless it should be underlined that, by an analysis of the engineering works that are conserved, from the mathematical knowledge that we know that they possessed, and from the known surveying instrumentation, we can deduce the techniques they used to complete some of the known works that were designed so well, some of which would still be a huge challenge to modern engineering.

4.1.- Land measurement, geodesy and triangulation.

Land measurement, either on the flat or in height, has always been a question of the resolution of triangles. From a basic polygon we can proceed to form other polygons and reduce any parcel of ground to triangles.

The first matter encountered is to establish the actual position of places on the surface of the earth. This enables them to be represented to scale on maps.

Thus it is necessary to calculate the straight-line distances of the places to be represented with respect to a known point, and the directions they take, that is the angle with respect to a line previously established. This line may be the one pointing to the North from the point of departure, in which case the angle is termed the azimuth, and is derived from the two known points of departure, that make the base.

There are several reasons for us to suppose that ground triangulation, even over long distances, was a common feature in the Roman world. The precision of long alignments, which have recently been identified with the help of aerial photography, cannot have been achieved without this technique.

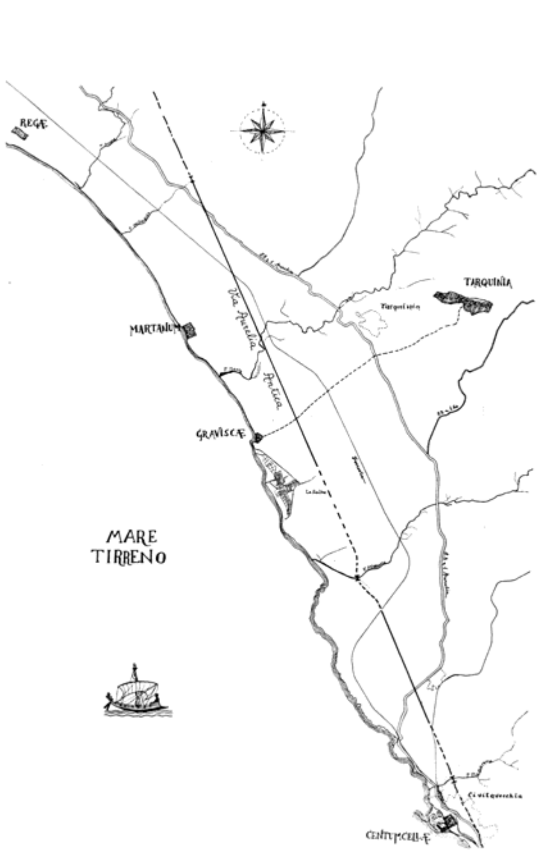
Amazing main road alignments are known: for example the Via Appia between Rome and *Tarracina* (Terracina) over more than 90 km. (36 miles) and also a good length of the Via Aurelia between *Forum Aurelii* (Montalto di Castro) and *Centum Cellae* (Civitavecchia) with its astonishingly straight 55 km. (35 miles). This could only be observed with the aid of aerial photography, using the techniques of the Roman Institute of Ancient Topography[55]. Long alignments are also found on the Via Domitia, in French Provence, in large areas of the plains in the North of Gaul, and in all places where the terrain was appropriate.

The outside boundaries of the large parcels of land (*centuriato*)that are known, are often square of over 50 km. (30 miles) on their longest side, with boundary lines forming perfect right angles[56]. Such precision could not have been achieved from the smaller grid, but, on the contrary, the *centuriatio* will be geometrically perfect if one starts by establishing the outside boundaries exactly.

In the same way, to establish precisely the angle formed between the baseline and that between the start and end points, over any alignment more than 10 km (6 miles), is a tremendously difficult task. It is impossible to achieve with simple alignment methods, which is needed when one wishes to site precisely the outside points of a *centuriatio* to form a perfect right angle.

[55] STERPOS, D. 1970, p.22: La strada romana in Italia [Roman roads in Italy].

[56] FAVORY, F., 1997: *Via Domitia et limitations romaines en Languedoc oriental: la centuriatio Sextantio-Ambrussum* . Voies romaines du Rhône à l'Ebre: Via Domitia et Via Augusta. [Via Domitia and Roman boundary lines in East Languedoc: centuriatio Sextantio-Ambrussum].

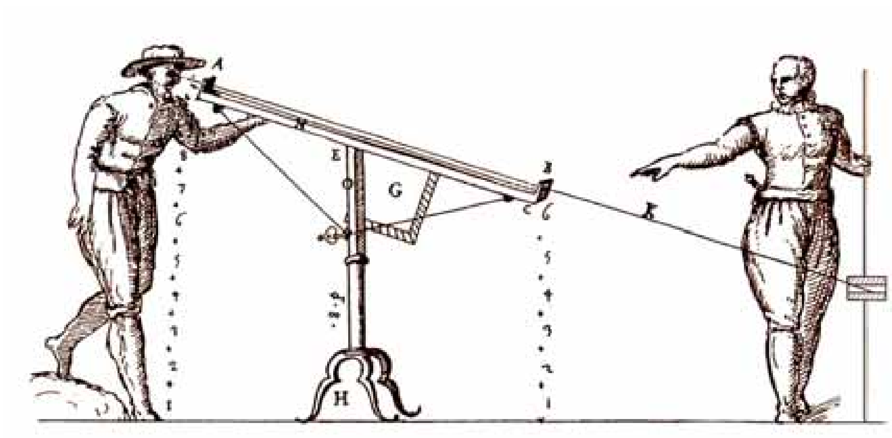


On the left a perfectly straight secion of the Via Aurelia between *Forum Aurelii* (Montalto di Castro) and *Centum Cella* (Civitaveccio), over 55 km. (35 miles). Graphics by D.Sterpos. To the right, *centuriatio* on a side of 80 km. (50 miles on the *Via Aemilia*, to the West of Bologna (Italy), according to Chouquer (1981).

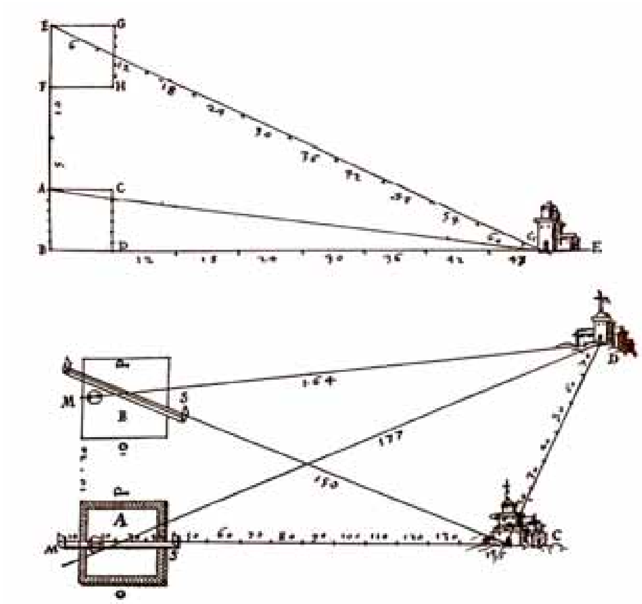
It is also very costly to position precisely the source and destination of a water reservoir, so as correctly to evaluate its viability, its length and especially the total drop of the water that it has to provide. It should be borne in mind that conduits of between 60 km. (37 miles) and 100 km. (52 miles) were common under the Empire.

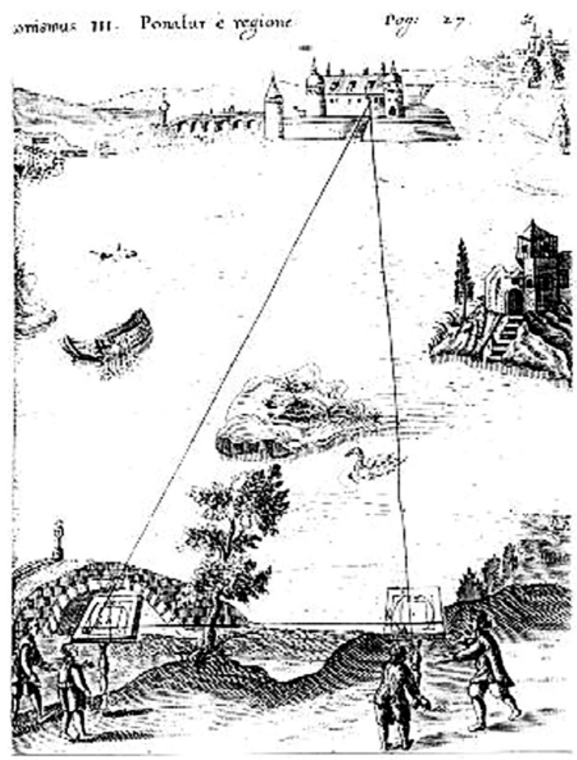
This is all difficult work, comprising a very accurate measurement of the terrain with triangulation readings, which could be useful for several purposes outside those mentioned.

Techniques and instrumentation described in Renaissance times were quite adequate for these tasks; but there are quite a few outstanding works undertaken by the Romans that surpass the short measurements that appear in present-day drawings.

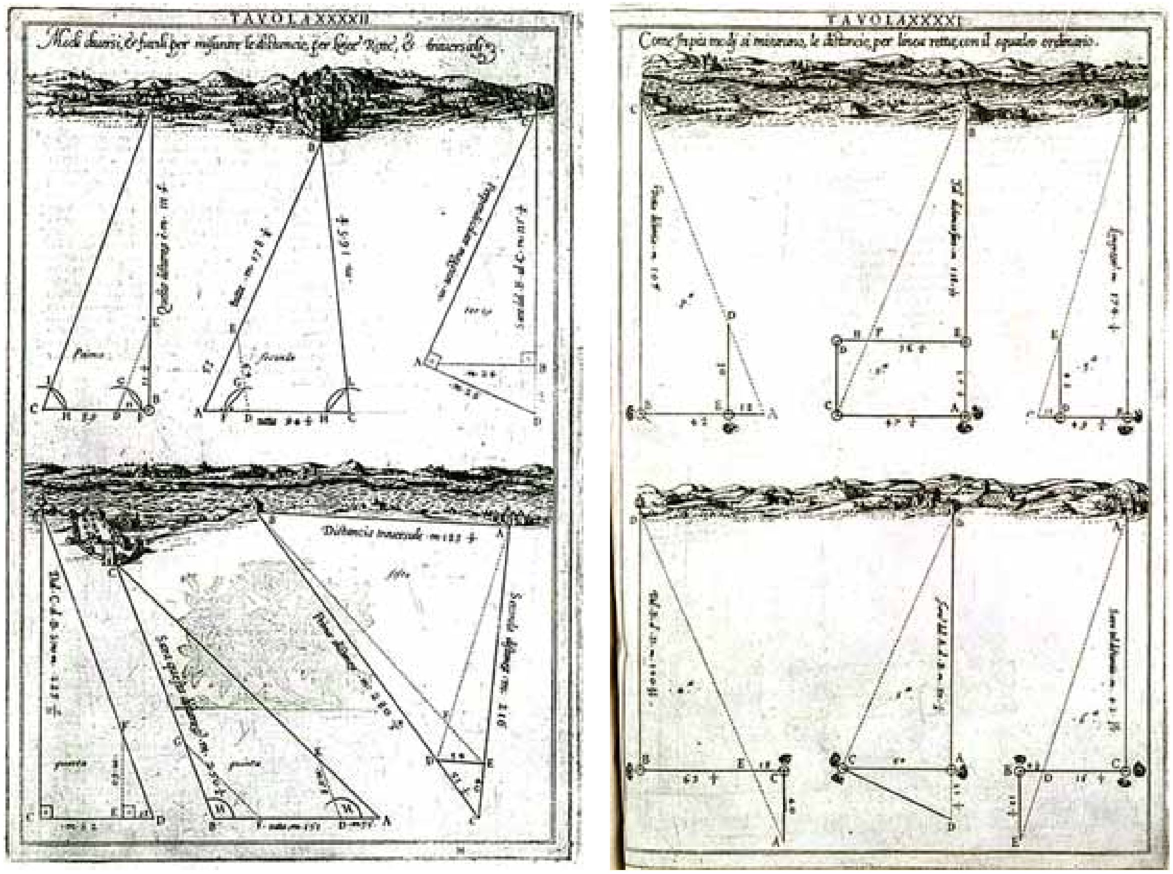


Example of a dioptre with vertical movement from the *Veintiún libros de los ingenios y máquinas.*





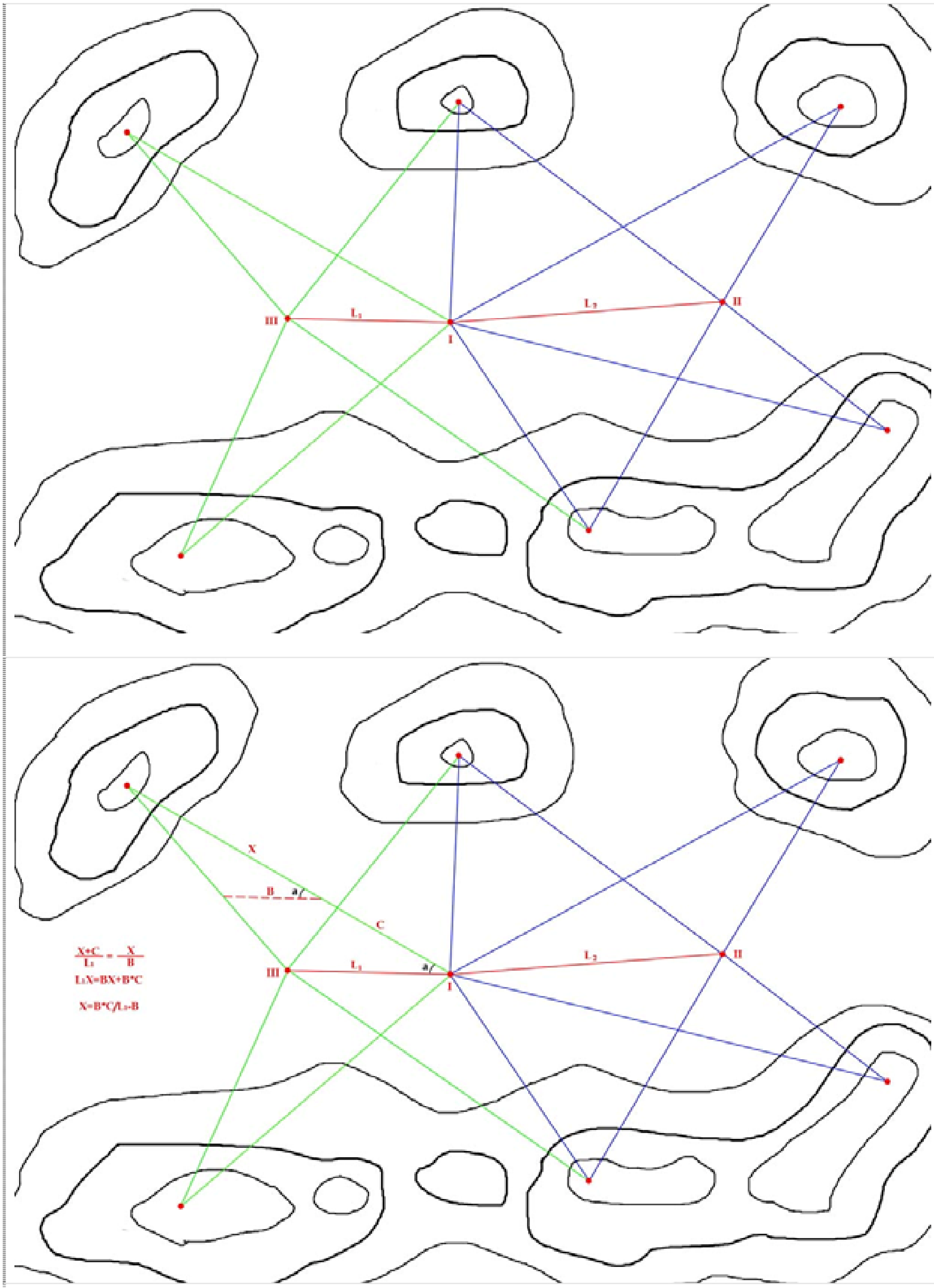
On the left, examples of the use of a geometric quadrant in the *Veintiún libros de los ingenios y máquinas.* On the right, the same technique using the pantometer as explained in the work by Gaspare Schotto.



Examples of calculating distances by means of resolving triangles and using Thales' theorem. Graphics from the work of Giovanni Pomodoro, 1603.

We believe that the more complicated triangulations in the Roman period were carried out with the aid of supplementary lighting elements, signal lanterns such as were used in so often for the transmission of messages. These allowed very long visual sightings by night, under certain atmospheric conditions of more than 10 km. (6 miles). Thus the construction of very big network of triangles with extraordinary precision became possible. Naturally it will be necessary to close the grid coordinates on the same starting base, by repeatedly checking, by sharing the discovered errors between the measured angles and by other basic surveying techniques still used today. We shall not go into them here.

Starting with these networks, it is a simple task to establish the angle of attack with the starting point of an alignment that goes on for tens of miles until it reaches its desired destination, or the exterior rectangle of the largest of the *centuriatio*s. Other intermediate points will be established and confirmed on the lines formed in this way, and the partial stretches will be filled in by means of the simplest alignment methods, such as alignments with ranging poles or any other system.



Examples of triangulation procedures using supplementary base lines. On the right, using the theorem of the similarity of triangles to save lines of sight from elevated points.

4.2.- Surveying

Measuring, fixing and drawing onto maps, of parcels of land is one of the most ancient tasks entrusted to the science of surveying. Its religious character reached its greatest expression in Roman times. We have already seen that the *groma*, despite being an inadequate instrument for such tasks, continues to be associated with Roman surveyors, probably because of the ritual characteristics represented by both the instrument and the surveying process.

The reduction of farmland to measurable polygons is nonetheless an essential procedure to apply justice in the distribution, use and transfer of farms. This is made necessary by the vital economic importance that farming held for mankind from the Neolithic Age. In particular, it was in Roman times that this process acquired its essential character. This is a moment in which advances through conquests and the acquisitions of vast areas of land under the Empire, with their consequential distribution between the big landowners coming from retired army commands, or between colonials of varying kinds, increased the production, the riches, and the power of the Empire to limits never before known.

It is to Frontinus[57] that we owe much of our information on means of allocating land fairly, as well as on other details of this subject. Columella[58] also provides many facts, among them that any surface measurement in Rome was in terms of square feet.

Multiples of the basic surface unit of measurement, the Roman square foot (0.0876 m^2)[59] gave rise to various surface measurements, among which the most common were the *actus* (14,400 sq.ft. = 1,261 m^2), the *jugerum* (28,800 sq.ft = 2,523 m^2), the *haeredium* (57,600 sq.ft. = 5,046 m^2), the *centuria* (5,760,000 sq.ft = 504,576 m^2) and the *saltus* (144,000,000 sq.ft = 12,610,440 m^2).

So, if, starting with an *actus*, a square with sides of 120 Roman feet (35.5 m) and using only a *groma*, we wish to construct a *saltus*, a square with sides of 12.000 Roman feet (3551 m.) (100 × 100 *actûs*), the outcome would certainly not be a square. We would dare to doubt that the outcome would be a square, even if we were to use the most accurate surveyor's square[60]. If we therefore take this problem to the outside boundaries of a *centuriatio* of over 20 *saltûs* on each side, the solution would prove impossible using these methods.

The only answer is to use triangulation methods so as to ensure the precision of the alignments and the angle of the plot. Even if the longer sides were measured with a dioptre, aided by light signals, by night, we would need to close the triangles so that the angle is still 90° at the closing point opposite to the starting point. This is triangulation.

When the outer sides of the parcel have been constructed and divided into *centurias* (side 710 m. = 20 × 20 *actûs*) it is then that the surveyors' squares come into play for the smaller divisions.

A different problem from the difficult construction of a Roman *centuriatio* is the measurement, division or transfer to a plan of relatively small surfaces. In these cases, the surveyor's square is always appropriate. We have seen representations of these operations in modern texts of the 16th

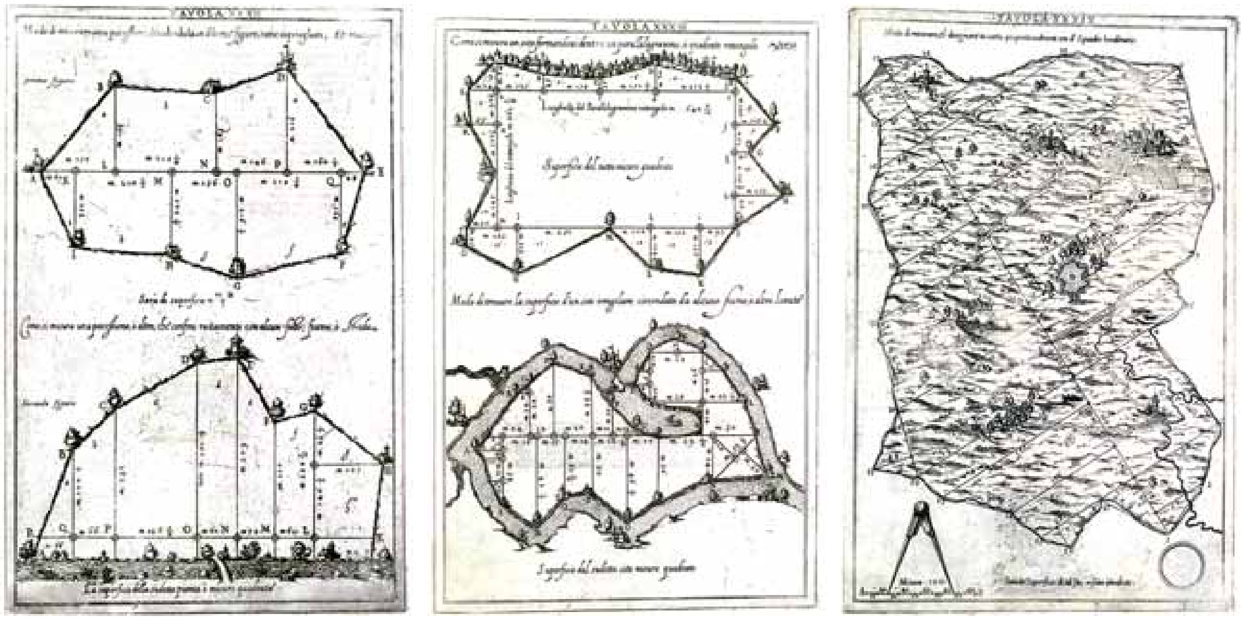
57 Sextus Julius Frontinus (c. A.D. 40-103) was not an engineer, but a high-ranking officer in the civil service, who was charged with very relevant duties throughout the course of his life. He was in the habit of writing up the tasks which he carried out, and it is in this way that he provides us with very valuable data on engineering, law, administration, surveying, etc.

58 COLUMELLA, Lucius Junius Moderatus (fl. middle 1st century A.D.). *De re rustica* [Husbandry] 5,1.

59 RESINA SOLA, P. 1990, p. 24. *Función y técnica de la agrimensura en Roma (II)…* [Functions and methods of land surveys in Rome] op.cit.

60 On the limits in practice of the surveyor's square, see DOMÍNGUEZ, GARCÍA-TEJERO, F. 1958, p.68: *Topografía general y agrícola* [General and field surveying] … op.cit.

and 17th centuries. Pomodoro brings several of these cases together, and he advises, as Frontinus had done, reducing the irregular farm estates to polygons by using the square.



Graphics from the treatise by Giovanni Pomodoro, with examples of measurements of estates by reducing them to regular polygons. On the right, procedure for constructing a plan of an estate. In all cases a surveyor's square is used.

4.3.- Laying out main roads.

When a Roman engineer was planning a road link between two cities, he would wisely choose the best of the routes that provided a cross-section with gentle slopes without excessive costs, and at the same time that did not take him too far off the straight line. We have observed the high degree of success[61] by which this was so often achieved. Far from being the result of chance, it must be attributed to the skill of the technicians who could manipulate the instruments necessary to determine the shape of the ground and the distances and heights of key points for their layout.

In short, it was necessary to have precision maps that would allow the correct choice of the route, and the most suitable alternatives when the uneven ground argued for a departure from the straight line.

To construct such maps, it is necessary to draw networks of triangles to be filled in later with additional information until the totality of the ground is covered. We do not know how the Romans expressed relief on these maps, since it seems that they did not use contour lines; but somehow they drew at least the highest points and hills, with an indication of the different heights.

First the general outline has been established, with the obligatory points of passage across the ground marked, by the use of dioptres and if needs be by techniques of lines of sight over long distances. Then surveying poles are the most versatile instruments to complete the layout on the partial stretches that have been so created.

Thus it does not seem necessary to have recourse to recondite hypotheses of the layout, and still less when using inaccurate instrumentation, such as the *groma*, for the task, or by means of weird

[61] MORENO GALLO, I. 2004: *Vías romanas: Ingeniería y técnica constructiva* [Roman roads: the engineering and techniques of their construction]. Ministerio de Fomento, Madrid.

techniques of reiterative approximation to the definitive alignment[62]. Yet we see this so often in modern texts. It would take several days for a task that, if done efficiently, would not take more than a few hours.
In order to establish slopes so that they may be the smallest possible, uniform and below the maximum recommended, there are instruments of extreme simplicity perfectly suited to this task. In the same way as the *groma* is useless in layout work, the *chorobate* is of no help in road construction planning, since it deals only with points in the horizontal plane, and as such is useless for tracing slopes usual on roads.
Much more efficient are simpler instruments, such as the diopter itself, or more versatile and manageable, such as levellers. Perhaps on account of their simple construction and use, nothing can be discovered as to their origin. Then there is the inclinometer, a simple piece of equipment for measuring slopes: it consists of a graduated vertical limb held by the hand or on a vertical foot, on which is placed a ruler that can be provided with sights and, as it swings on the centre of the limb it reads off the value of the slope.
Levellers are used by placing the first two at a distance of about 6 m. (20 ft.) from each other, and with the slope to be determined between them, the first being positioned at the starting point. The third is moved over the ground, indicating the layout points to be established, and this is achieved when its upper part is level with the line of sight originating from the first two. The system works on the principal of the projection of two horizontal lines located in the same plane, not necessarily horizontal, but always at right angles to the vertical.
The same principal of the projection of lines located in the same plane, but in this case in the vertical, is that used for a ground plan using range poles, which were called poles in previous times. Moreover they are used in groups of at least three, so as to ensure that the line is made to coincide with the vertical plane formed by the two that are indicating the direction.
Thus we see that the planning of roads in both the horizontal and the vertical planes does not require complicated instruments, once one has established precisely the necessary staging points, with a reasonable distance between them.

4.4.- Channelling water and levelling techniques

The same cannot be said of routing channels of water in free fall. Modern technicians are well aware of the extreme precision necessary on the slopes in which the water is to run, so as to ensure success. Not only has the water to arrive perfectly at its destination, but also there must not be difficulties on the channel as a result of inadequate velocity of the liquid on its journey.
Obviously, the ideal speed of the water needs to be known with relation to the type of lining of the channel, and the characteristics of the liquid itself. Sediment and obstructions can lead to as many problems in the life of the channel as erosions. However, once this optimum velocity is established, by means of the relevant techniques already existing, and once the volume is known and the water level, the slope of the channel will be the one that ensures the fulfilment of the desired parameters.
This delicate balancing, laid a lot of responsibility on the engineers engaged on the construction of these works, starting with the surveyors. The Romans, nonetheless knew how to achieve this balancing so magnificently in the great majority of the aqueducts that can be observed even today.
They needed to ensure that the reservoir provided an adequate volume, which is quite difficult to achieve when it was situated some tens of miles away. The also had to adjust the slope at each

[62] LEWIS, M.J.T. 2001, pp.222 et seqq.: *Surveying instruments of Greece and Rome*, Cambridge University Press, ISBN 0521792975

stage of the aqueduct to what was required. The slightest error in the geometry of the fall would be noticed Even when at first the channelling seemed to function correctly, there could be serious consequences at times as a result of the problems to which we have referred, caused by differences in the optimum velocity of the water.

The levelling, therefore, had to be absolutely perfect.

The chorobate was used by projecting the horizontal plane of the starting slope over the whole of the ground that was to support the channelling. Then the slope was to be increased or decreased, dependent on the direction of the geometry with respect to the direction of the flow, in exact proportion to the distance covered by the aqueduct.

In order to achieve this, the instrument needed to be positioned at the points from which the largest proportion of the ground over which the channelling was to be constructed where there are maximum visual sightings. Previously the slopes to be constructed at these points had to have been established with the chorobate itself.

A question of vital importance is to allow for the error caused by the sphericity of the earth. We know that the Romans were perfectly aware of this. It will be enough to transcribe the words of Vitruvius[63]:

Fortasse, qui Archimedis libros legit, dicet non posse fieri veram ex aqua librationem, quod ei placet aquam non esse libratam, sed sphaeroides habere schema sed ibi habere centrum, quo loci habet orbis terrarum.

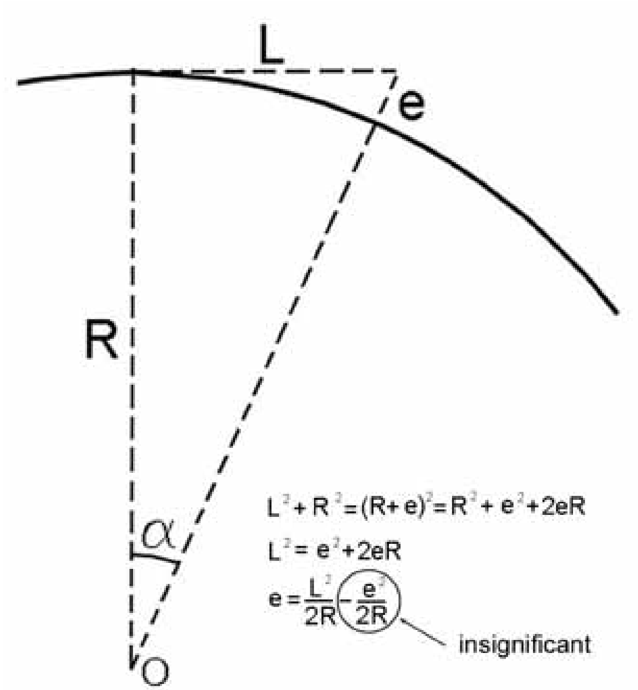
It is probable that someone who has read Archimede’s works will say that one cannot have a true levelling of water. Archimedes holds that water cannot be levelled in a straight line, but takes a spherical form, with its centre at the centre of the Earth.

But the Romans were well acquainted, not only with the characteristics of water flow, but also with the value of the radius of the Earth. We should recall that Eratosthenes of Cyrene around 200 B.C. had calculated the radius of the Earth with admirable accuracy.

Even if we allow for a certain error by the Romans in the value of the radius of the Earth, if one allows for Ptolomey’s adjustment, to calculate the levelling error resulting from the Earth’s sphericity was well within their reach, given its extreme simplicity.

Even if we assume a radius of the Earth around 80% of its real value, a calculation of the sphericity of the earth does not involve variations that could affect the outcome of the final levelling.

[63] VITRUVIUS, Marcus Pollio, *De architectura*, 8.5.3



$e = L^2/2R$

	R real	R =
	6,378,000	80 % real
L (m)	**e (mm)**	**e (mm)**
50	0,20	0,24
80	0,50	0,63
100	0,78	0,98
150	1,76	2,20
200	3,14	3,92
300	7,06	8,82
400	12,54	15,68
500	19,60	24,50
1000	78,39	97,99
2000	313,58	391,97
3000	705,55	881,94

Where:
e = the error in millimetres
L = the horizontal in metres
R = radius of the earth
O = the origin of the earth's sphere.

With lines of sight over long distances the levelling error by this calculation is the greatest that can be produced, but is also the best known. For this reason levelling with the chorobate was not carried out over distances of more than 70-80 m. (75-85 yards). Levellings over these distances are not recommended to be carried out today either.
This matter has been always known to hydraulic conduit engineers. In fact it is mentioned more than once in *Los veintiún libros de los ingenios y las máquinas*, attributed to Juan de Lastanosa, which recommends that levelling should not be undertaken over distances of more than 50 paces[64].
On occasions, however, levelling becomes convenient, and even necessary, over long distances. In those cases, knowing the important results of this error, the Romans made the required corrections.
This is the case, for example, of projection levels derived from bases on a slope opposite to that of the channelling, which although making the surveying task easier, were nonetheless beyond the recommendable distance.
These conclusions arise more frequently when the levellings have to cover long distances over uneven terrain. We shall consider that the advantages that we find in short distance levellings, due to the low error resulting from the Earth's sphericity, are lost when we are forced to position the instrument time and time again to advance the flow. However precise our readings at each reference point, the changes of positioning create unavoidable errors in all surveying work, including, of course, levelling.

[64] *Los veintiún libros de los ingenios y las máquinas* [The twenty-one books of inventions and machines], Book 4, p.57.

This quite basic factor must be perfectly known to any engineer actually responsible for surveying water levels, on account of its importance. However, it is systematically ignored in the majority of the texts in present times that have dealt with ancient surveying. It is most likely to result from the authors having no experience in this science.

Some specialists in this matter have been concerned with the error that the table type chorobate, generally suggested by archaeologists, can cause in levelling work. They mention errors resulting from visual estimations, from refraction, from faults in the construction of the instrument, etc., ignoring the most important of all, that inherent in the sphericity of the Earth. It has been demonstrated that the design of the aqueduct at Nîmes could not have been made with this supposed chorobate. In summing up the effect of all these considered errors, it has been concluded[65]: *levellings by the eye/chorobate method can be affected by reading errors of more than 3·9 cm. over 50 m (1·4 inches over 50 yards), which, by extension becomes about 80 cm. over a kilometre (50 inches over a mile).*

We make two deductions from this: firstly that the table type chorobate is quite inadequate for levelling water; and secondly since the errors do not increase in proportion to the different positionings, the author allows in all seriousness for levellings to be realized at the distance of a kilometre, ignoring in effect the error caused by the sphericity of the Earth.

Nonetheless a consideration of the sphericity error holds even other very interesting applications, that we shall refer to briefly here. For example the possibility of carrying out levellings going over large distances between one and three kilometres (say, half and two miles), using the method of signal lamps illuminated by night, and then applying the corresponding sphericity error to establish the correct slope between these points. Then one can use the reference placed at this point to evaluate the cumulative positioning errors in the levelling at the end of the section, in short distances not over 70 m. (75 yards).

Another very interesting aspect is the determination of the exact level of the long syphons that were so often constructed[66]. Exact levelling going across a valley, making significant descents to go up again, can be a very complex task, and they lead to other very expensive tasks, such as deforestation, etc. The number of positionings can be so great as to make it impossible to estimate the resulting inaccuracy. Just one night sighting, applying the sphericity error, will give us the most exact slope among those that can be obtained by transference from the other end.

For all these operations, it is necessary to calculate the horizontal distance of the point of the transferred slope. For this one has to bring into play other surveying operations to determine the distance. For example, by simply measuring, from a supplementary base-line with positionings at both ends, the angles formed by the lines of sight with the distant point, ensuring that one of these angles is a right angle.

The Romans, in their design of water conduits, would make a last precision reading of the channel, which would serve for the final refinement. Whether the channel were excavated across the ground, or inside a gallery, or installed over arches or sustaining walls, this final reading was carried out with a chorobate, following the route over the actual casing of the channel that had been built, in a forward direction, from which it is very advantageous to work with the instrument and to carry out iterative checks of the cumulative error in level over the various sections, making permanent readings over the channel itself.

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[65] LARNAC, C.2000: *Les limites du système «œil-chorobate» pour l'implantation de l'aqueduc de Nîmes* [Limitations of the 'eye-chorobate' method in the design of the Nîmes aqueduct]. Autour de la dioptre d'Héron d'Alexandrie. [About Hero of Alexandria's dioptre], Centre Jean-Palerne, University of Saint-Étienne.

[66] In very long syphons, although the Roman engineer would have known with the greatest precision the identical slope at the two ends, they in effect deliberately left a very significant water drop of several metres between both ends, since the pressure losses were quite considerable, and over the life of the syphon, losses grew even greater.

One must suppose, given the enormous distances over which the slope was maintained constant as previously decided over each stretch, that small drops for safety were left in the longitudinal profile to allow for corrections in the last phase of levelling. If such safeguards were not needed, these tiny jumps were absorbed by inspection chambers, in which they created a small water jump.

These chambers, in principle designed for purposes of construction and maintenance, performed several functions at the same time. Where a section was broadened, the velocity of flow decreased to increase again at the exit. In fact they sometimes placed sandboxes to contain the sedimentation produced for this purpose. Besides the obvious hydraulic function, they constituted the best way to connect the aqueduct with the outside, marking the position if the underground channelling on the surface.

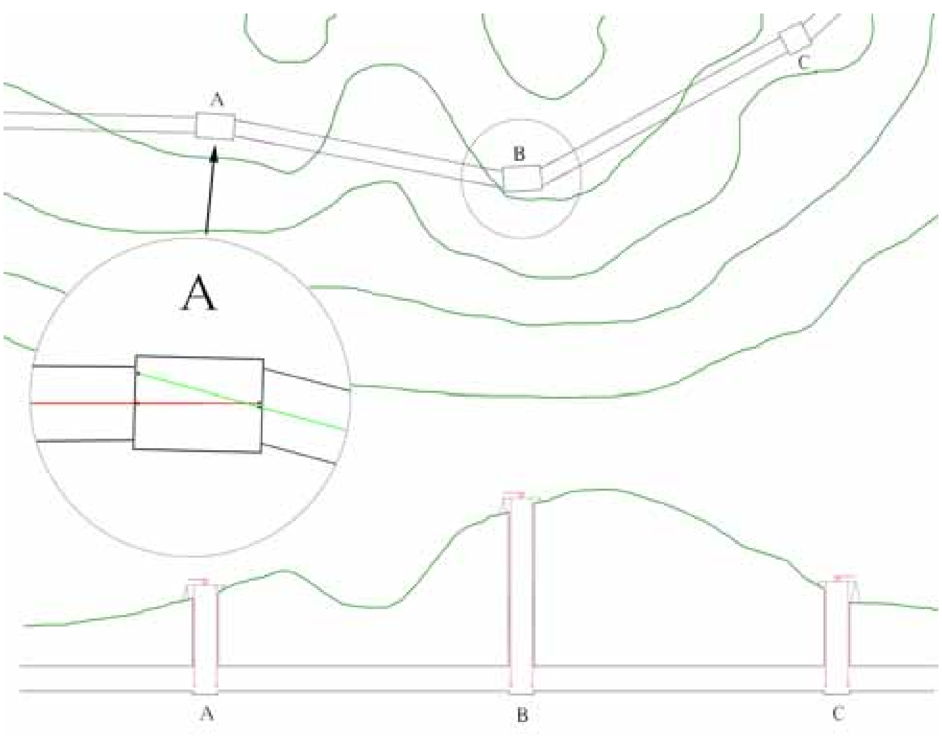
4.5.- Drilling of galleries.

Tunnels were often constructed in Roman public works. Perhaps where the greatest problems were to be found was in the construction of galleries for the flow of water, of which there were many. We should bear in mind that, in an aqueduct, underground conduiting constituted the largest proportion of the work, and was very different from stretches built on the surface or on arches.

The narrowness and small dimensions of the gallery made construction very difficult, as well as its planning on the level and in height.

The routing of these galleries in many cases depended on the ground above. The most usual was to plan the gallery as a projection of the area established on the surface. The gallery could be straight or with breaks, normally in inspection chambers, which always occurred at a given distance.

The course of the gallery was transferred from the surface by means of the chambers. These were always built before the channelling. The channelling slope and the direction it was to take were established in the chambers by using plumb lines.



Procedure methods for routing aqueduct galleries by transferring alignment data and slopes from outside.

Once these data are transferred to the inside of the gallery, it could be excavated from any chamber and in any of the directions. If a cord were fixed to the roof of the gallery held very taught by nails, plumb lines could be hung from it and by artificial light the direction could be perfectly maintained.

Using the water level (*libra aquaria*), one could raise the slope from any of the chambers through the gallery and in any of the directions, applying the increase in slope required by a unit of length appropriate to the slope. Marks on the ceiling or wall can be used to establish the slope, once the definitive profile has been refined.

The water level works well over short lengths, of ten to twenty metres (yards), because over longer distances and with the materials used at the time (animal gut), breaks, leaks and other problems could occur. It is therefore necessary to plan the slope over small sections.

Once a sizeable length of gallery has been excavated, before the floor of the channel is finally completed, a chorobate can be introduced into the inside and used to identify the positionings needed to mark the level with greater precision. Artificial light enables the mobile horizontal reference to be seen at each levelling point inside the gallery.

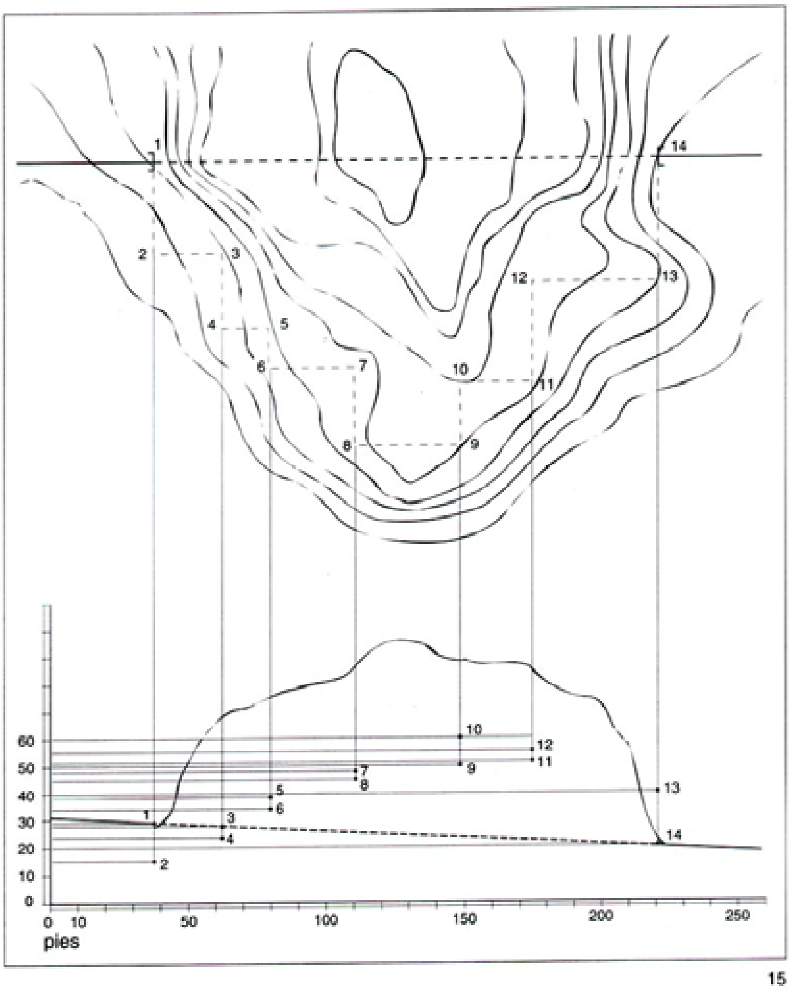
One of the challenges in surveying aqueduct galleries was to plan the boring to occur at the same time from each of the two mouths, which were very inaccessible in mountainous country. It was not possible to be guided by intermediate chambers, which would have needed an enormous height, or they were simply not necessary for the total short length of the gallery.

There are many cases of this kind, in which the excavations reveal traces of the start at the two mounts and their meeting in the central section[67].

Adam[68] put forward a method by which, using a chorobate and a *groma* that had to be positioned very often, he claimed to draw the desired direction and level towards the other mouth. This method requires both instruments to be positioned so often, more often than is normally to be expected in the little hill in his picture, with such an accumulation of resultant errors, that to arrive at the opposite mouth with the necessary direction and slope would have been a miracle. That is without counting the effort of repeatedly siting the table type chorobate that he is using: on certain sides of the hill he would have needed a substantial amount of scaffolding, the work of deforesting the sides to enable lines of sight, and so on.

[67] The tunnel on the island of Samos in Greece and at Briord in France and Bologna in Italy are well-known. Serious problems of galleries starting from both mouths can be observed.

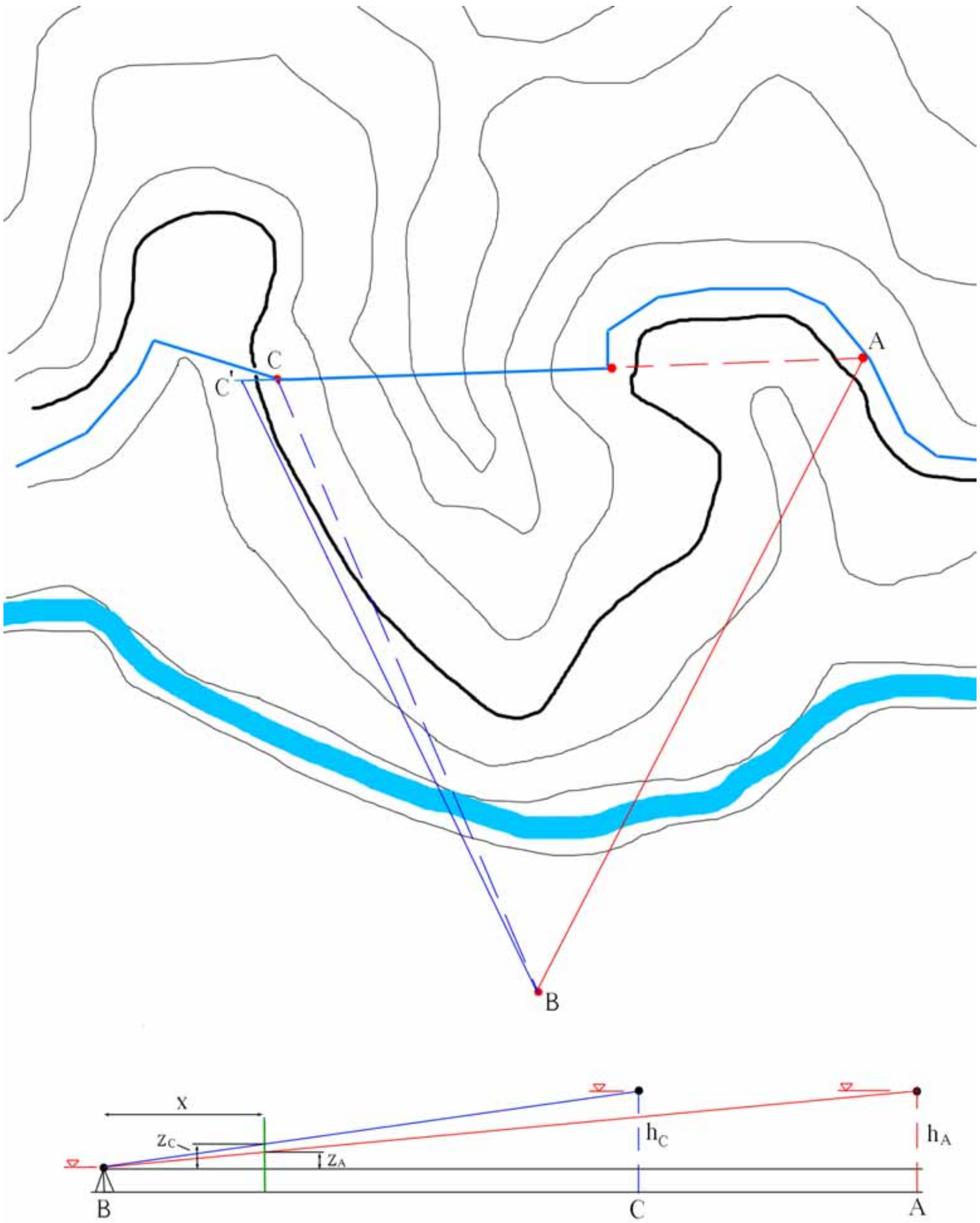
[68] ADAM, J.P. 1989, p.9 et seqq., Spanish edition 2002: *La construcción romana – materials y técnicas*… op.cit.



Adam’s proposal for designing a gallery in very uneven ground whilst nonetheless using a route on the surface itself.

We can, however, be sure that none of the great aqueducts of antiquity would have been constructed by these methods. Roman engineers possessed excellent surveying and mathematical knowledge, as well as instruments sufficiently powerful to complete these works successfully. Confirmation of the degree of perfection can be found in the high geometrical standards of their actual works.

Trigonometry was probably better understood than the Romans have been given credit for, so that there were few problems beyond their capability. As so often occurs in surveying, there is more than one way of arriving at the same solution: nonetheless we have preferred to advance a solution for boring a tunnel, using a procedure by which only elementary trigonometry is needed, with the least possible number of positionings and operations. One of these examples can be explored in the following graphic.



$$h_A = \frac{AB \cdot Z_A}{X_A} \qquad h_{C'} = \frac{C'B \cdot Z_{C'}}{X_{C'}} \qquad Z_C = \frac{X_C \cdot h_C}{BC}$$

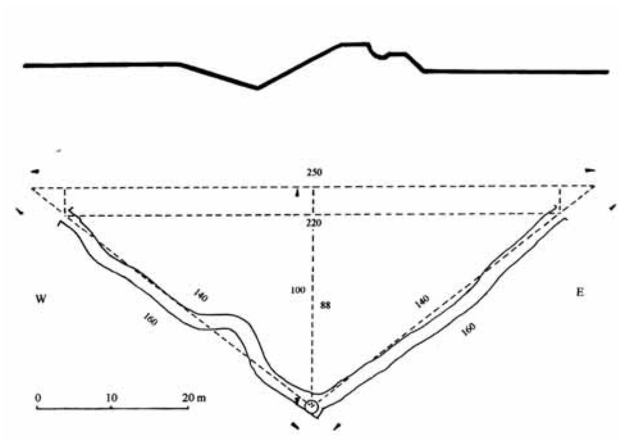
<u>Design of a gallery over very uneven gound, without the possibility of following its route on the surface</u>: it is resolved by means of an outside point B, from which one can see the ends of the tunnel alignment.

Point C' is chosen on the exit side with which we can establish the vertical plane that will include the tunnel axis. Horizontal distances BA and BC' are calculated using any of the indirect methods already mentioned. We can apply the similarity of triangles beginning with other similar triangles formed in the vertical or horizontal plane. We can form a right-angled triangle, of which we wish to measure the longer side adjacent to the right angle, by previously measuring the shorter side on the ground. We shall select for each case the method to use according to the characteristics of the terrain.

We measure the angle at B to resolve the triangle by using these three pieces of information (two sides and angle B). Angles at A and C' will establish the alignments at the ends.

By using a vertical sight and the properties of similar triangles (lower graphic), we calculate the gradient of rise between A and B and then between B and C'. Finally we establish point C, that of the tunnel exit, on its correct alignment and slope.

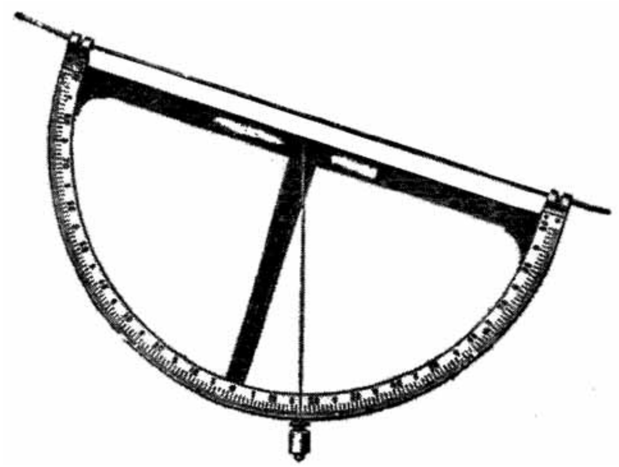
This type of surveying operations, in works where the accuracy of their results is so very important, should preferably be carried out more than once. Different base point should be selected every time and repeated so long as there are significant differences in the results. Observations taken in conserved aqueducts indicate that, during the course of the boring work, routing calculations were repeated periodically inside the gallery, so as to avoid possible deviations. Whenever these deviations, unavoidable through error or through the geological characteristics of the materials, are found[69], the necessary corrections were carried out in this way.



On the left, horizontal line of the tunnel in Samos, Greece, with significant curves in the central section. Until today they have been attributed to surveying difficulties. Below, the line of the tunnel of Chagnon in the aqueduct at Gier, as drawn in the work by Grewe (1998), with odd twists in the route.
On the right, tunnel of a mining supply aqueduct in Llamas de Cabrera, Valle Airoso, in the province of León. Walls of hard quartzite can be seen that have been excavated by a combined technique of fire and water. Such geological formations were often the cause of detours in the routing.

Other simpler aspects of underground routing are to be met in road tunnels, of which we know many. They were usually shorter than in aqueducts and hence much more spacious, which resolved most of the problems.
Lastly we have the case of mining galleries. Normally these galleries followed veins of the mineral being extracted. Unless they were subject to later flooding for the channelling of water that was to cause the *ruina montium* – the destruction of the site after mining --, they did not entail big routing works.
Routing on the surface, whenever it was necessary, was transferred from the outside by means of ventilation shafts or entrance openings. Nonetheless, for safety reasons, it was always necessary to keep the slopes of the galleries within reasonable limits, and for this reason levelling of underground corridors was frequent. For this specific instruments were used, simple for work in the dark, and efficient so as to give results with the few points of reference that existed in the interior of the mine. Suspended inclinometers are most suitable for this purpose.

[69] This subject has been studied little or not at all: it could reveal important clues as to the causes of deviations in some galleries.



Special inclinometer for underground constructions indicating the value of the slope. They are hung from a cable fixed to follow the key line of the gallery, which is the basic guide for underground routing.

5.- CONCLUSION

Surveying in antiquity remains one of the least known disciplines of our time. It is accompanied by engineering itself at the time of the early Empire, of which probably more is unknown than is known. Fundamental aspects remain to be uncovered. This unknowns can and should be approached beginning with the proper analysis of Roman engineering works. In this task professionals and engineers with sufficient knowledge and training to interpret them should be called upon.

It is essential to start from the basis that it is not possible to carry out many of these works with rudimentary methods or without adequate knowledge of hydraulics or roads appropriate to the issue. Such works are not viable without large scale advanced surveying science, which is also necessary for planning, technical methodology used in an acceptable and reasonable manner, minimum required procedures and performance in execution. In short close attention must be given to whatever other technical or skill ability is required by the task.

6.- BIBLIOGRAPHY.

- ADAM, J. P. 1989, ed. esp. 2002: *La Construcción romana. Materiales y técnicas.*
- ATHANASIUS KIRCHER. 1631: *Pantometrum Kircherianum*
- CHOISY, A. 1899: *Histoire de l'architecture*. Diverses rééditions, par exemple Vincent Fréal et Cie 1964.
- CLEOMEDES: *De motu circulari corporum coelestium.* Siglo I d. C
- COLUMELA. *De re rustica* 5, 1.
- COZZO, G. 1928: *Ingegneria Romana*, Mantegazza di Paolo Cremonese, Rome
- DELLA CORTE, M. 1922. *Groma*. Monumenti antichi della reale Academia dei Lincei, 28.
- DOMÍNGUEZ GARCÍA-TEJERO, F. 1958, p 68: *Topografía General y Agrícola*. Salvat Editores. Madrid.
- ESTEBAN PIÑERO, M. 2003. Las Academias Técnicas en la España del siglo XVI. Quaderns d'Història de l'Enginyeria Volum V 2002-2003
- FAVORY, F. 1997: *Via Domitia el limitationn romaines en Languedoc oriental : la centuriation. Sextantio-Ambrussum*. Voies romaines du Rhone à l'Ebre : Via Domitia et Via Augusta.
- FERNÁNDEZ DE MEDRANO, S. 1700: *El architecto perfecto en el arte militar*. Bruselas.
- FISCHER, I. 1975: *Another look at Erathosthenes' and Posidonius' Determinations of the Earth's Circumference*. Quarterly Journal of the Royal Astronomical Society, vol. 16.
- FRONTINO. *De limitibus.*
- GARCÍA TAPIA, N. 1997: *Los veintiún libros de los ingenios y máquinas de Juanelo atribuidos a Pedro Juan de Lastanosa*. Gobierno de Aragón, Zaragoza.
- GREWE, K. 1985: *Planung und Trassierung Römanischer Wasserleitungen dans Schiriftenreihe der Frontinus-Gesellschaft*. Supplémentband I. Wiesbaden.
- HEIBERG, J L. 1912: *Heronis Alexandrini Opera quae supersunt omnia* (Leipzig).
- HERON. *Automata.*

- HERON. *The diopter*.
- HIGINIO GROMATICO. *De limitibus constituendi.*
- KRETZSCHMER, F. 1966: *La technique romaine*. Documents graphiques réunis et commentés, Bruxelles.
- LARNAC, C. 2000: *Les limites du système «œil-chorobate » pour l'implantation de l'aqueduc de Nîmes*. Autour de la Dioptre d'Héron d'Alexandrie. Centre Jean-Palerne. Université de Saint-Étienne.
- LEWIS, M.J.T. 2001.: *Surveying Instruments of Greece and Rome*. Cambridge University.
- MARCHAND, G., PETITOT, H. Y VIDAL, L. 2000: *L'équerre d'arpenteur de l'Orme à Ennemain (Somme)*. Autour de la Dioptre d'Héron d'Alexandrie. Centre Jean-Palerne. Université de Saint-Étienne.
- MARTIN, Jean [Traducteur] et GOUJON, Jean [Dessinateur]. 1547*: Architecture ou Art de bien bastir, de Marc Vitruve Pollion autheur romain antique, mis de latin en Francoys, par Ian Martin Secretaire de Monseigneur le Cardinal de Lenoncourt*. Création / Publication Paris : pour la veuve et les héritiers de Jean Barbé et pour Jacques Gazeau.
- MORENO GALLO, I. 2004: *Vías Romanas. Ingeniería y técnica constructiva*. Ministerio de Fomento. Madrid.
- NEUGEBAUER. 1938: *Ubre eine Methode zur Distanzbestimmung Alexandria-Rom bei Heron*. Copenhague.
- PÁEZ CANO, R. 2003: *La esfera. La tierra plana medieval como invención del siglo XIX*. Tesis de Maestría en Comunicación, ITESO. Guadalajara, México. 150 pág. Rubén Páez Kano, is a researcher in the Instituto Tecnológico y de Estudios Superiores de Occidente (Institute of Technology and Higher Studies of the West) (I.T.E.S.O.), Mexico.
- PERRAULT, Claude (traducteur) 1675: VITRUVE. *Les dix livres de l'architecture.* Diverses rééditions, par exemple Les Libraires Associés, Paris 1965.
- POMODORO, G. 1603 : *Geometria prattica*. Geometria prattica dichiarata da Giouanni Scala sopra le tauole dell'ecc.te mathematico Giouanni Pomodoro tratte d'Euclide et altri authori: opera per generali da guerra, capitani, architetti, bombardieri e ingegnieri cosmografi, non che per ordinarii professori di misure. - Nouamente ristampato. - In Roma : appresso Giouanni Martinelli, 1603
- RESINA SOLA, P. 1990. *Función y técnica de la agrimensura en Roma (II). Instrumental técnico y sistema de medidas agrarias*. Topografía y Cartografía. Colegio de Ingenieros Técnicos en Topografía.
- RESINA SOLA, P. 1998: *Algunas precisiones sobre los campamentos romanos*. Florentia Liberritana. Revista de estudios de Antigüedad Clásica.
- RESINA SOLA, P. 2003: *El Agrimensor en Roma*. En Grecia y Roma: Sus gentes y sus cosas. GARCÍA GONZÁLEZ, J.M., POCIÑA PÉREZ, A. Granada. 2003.
- SCHÖNE, H. 1903: *Herons von Alexandria Vermessungslehre und Dioptra*. Leipzig.
- SCHOTTO, Gaspare. 1690: *Pantometrum Kircherianum, hoc est instrumentum Geometricum novum a Celeberrimo viro P. Athanasio Kirchero. Herbipoli*, 1690.
- STERPOS, D. 1970: *La Strada Romana in Italia.*
- URREA, Miguel de (traductor). 1582: *Marco Vitruvio Pollión, De Architectura*. Alcalá de Henares: Juan Gracián, 1582.
- VINCENT, M. 1858 : *Geodesic de Heron de Byzance*. Notices et extraits des manuscrits de la Bibliotheque Imperiale, XIX 2. Paris.
- VITRUVIO POLIÓN, Marco. 1787. *Los diez libros de Architectura* [Ten books on architecture]. En Madrid: En la Imprenta Real, 1787.