

A TEACHER'S DESCRIPTION of the CUISENAIRE-GATTEGNO APPROACH TO THE TEACHING OF MATHEMATICS

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At the time of writing it, Mr. Trivett was Head of the Mathematics Department, Hengrove Comprehensive School, Bristol and was, shortly afterwards, appointed to a professorship in a United States University. He had already done pioneer work there, lecturing on the Cuisenaire-Gattegno approach and demonstrating the scope and value of the material in experimental lessons with school-children of all ages.

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THE COLOURED STICKS

These are the simple tools of a revolutionary method of mathematics teaching which is steadily gaining ground in British schools. The method shows that we are really much more capable than we think, and that all children can thrive on and create their own mathematics

by JOHN V. TRIVETT

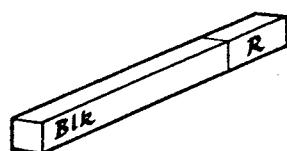
IN a growing number of schools in Britain today a strange kind of lesson can be seen. Six-year-olds or sixteen-year-olds can be seen sitting at their desks, in apparently formal lessons, manipulating small lengths of coloured wood, of which they have a plentiful supply, into various patterns, heaps, arrangements, tidy and untidy. Should one listen carefully or glance at any written work present, one will see that the children are creating their own mathematics. Not that the coloured

sticks are always present. At other times the pupils may be moving lengths of string or wire, sitting with eyes shut, or fitting elastic bands on to nails protruding from rectangular pieces of flat boarding.

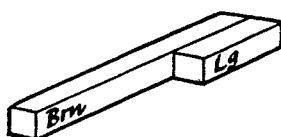
What is the significance of these queer lessons?

A few years ago, quietly and unobtrusively, there began in Belgium a trend in schools, involving a method and an attitude of teaching which may well outlast all the debate on public schools,

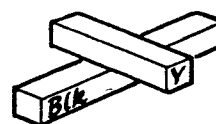
comprehensive and tripartite systems, and show that yearnings for smaller classes and better-behaved children are not the real panacea for successful education. Georges Cuisenaire is a Belgian schoolmaster who for more than fifteen years worked within the compass of his own classroom eager to improve his teaching, particularly in arithmetic. At heart he is a musician who wondered why it is that young children love and memorize tunes so easily, yet find number work neither enjoyable nor easy

THE COLOURED STICKS *continued*

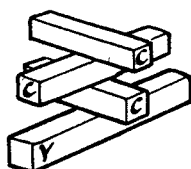
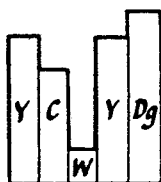
For adding



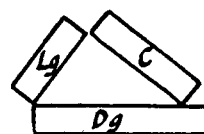
Subtracting



Multiplying

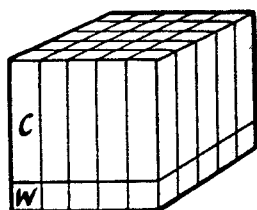
Powers, Logarithms,
Indices

Graphs

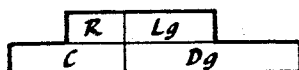


Triangle work

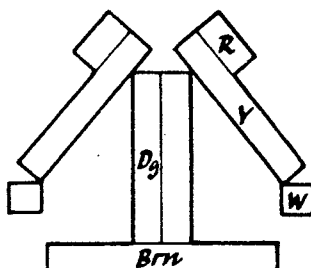
$$(a+b)^2, (a+b)^3$$



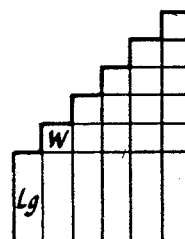
Blk					
Dg					W
R	R		Lg		
W	Dg				
Lg		W	W	W	W
Y				R	
W	R	W	Lg		
C			Lg		

Number dynamics
Factors, multiples

Ratios



Symmetry



Series

KEY: W=white, R=red, Lg=light-green, C=crimson, Y=yellow, Dg=dark-green, Blk=black, Brn=brown.

The top row could give, according to the choice of measuring-rod,

for adding

$$\begin{array}{ll} 7+2 & 2+7 \\ 3\frac{1}{2}+1 & \frac{1}{2}+1\frac{3}{4} \\ 1\frac{2}{5}+\frac{2}{5} & b+r \text{ etc} \end{array}$$

subtracting

$$\begin{array}{ll} 8-3 & 2-\frac{3}{4} \\ 1-\frac{3}{8} & 4-1\frac{1}{2} \\ A-B & x-p \text{ etc} \end{array}$$

multiplying

$$\begin{array}{ll} 5 \times 7 & 7 \times 5 \\ y \times b & 1\frac{1}{4} \times 1\frac{3}{4} \\ x(y+z) & \text{etc} \end{array}$$

to remember. He devised one of the simplest pieces of apparatus possible: pieces of wood one centimetre square in cross-section, of ten different lengths. The shortest, 1 cm. long, is uncoloured, while that of 2 cm. is dyed red, 3 cm. is light-green. The others are as follows: 4 cm., crimson; 5 cm., yellow; 6 cm.,

dark-green; 7 cm., black; 8 cm., brown; 9 cm., blue; 10 cm., orange.

It is neither accidental that these are based upon the primary colours, that the red family consists of the red, crimson and brown, the blue family of light-green, dark-green and blue, nor that the 2, 4, and 8 cm. rods belong to the red

family, the 3, 6, 9 to the blue family, and the 5 and 10 to the yellow family. Each child has access to dozens of each of the coloured rods.

By 1953 the pupils in Belgium using these rods had so amazed the education-alists with their mathematical progress—admittedly shown mostly by their

arithmetic skill in conformity with the strict European type of syllabus—that Dr. C. Gattegno, mathematics lecturer at the London Institute of Education, and secretary of the International Commission for the Study and Improvement of the Teaching of Mathematics, was asked to make an assessment of their value. Dr Gattegno visited Thuin and the classes of M. Cuisenaire, saw the children at work, and recognized at once that the Belgian teacher had stumbled upon the practical implementation of an idea which may well revolutionize our entire attitude to the teaching of children. Since then the coloured rods have been accepted by an increasing number of teachers in Britain. Edinburgh has introduced them into the majority of its primary schools; secondary schools are working with them using the implications in all mathematical fields, including sixth form work. They have found their way into schools for the deaf, for educationally subnormal children, and for the blind, who of course distinguish them not by colour but by length. They are being introduced in other European countries, in New Zealand and in Africa, and their widespread use in the United States and Canada is probably only a matter of time. All teachers who are having success in this way are learning that hitherto we have known very little in general about the mathematical education of primary or secondary pupils, be they “grammar”, “technical” or “modern”.

On what grounds can such claims be made? Firstly the Cuisenaire rods show what we should have known all the time: that children think in mathematical terms as a natural concomitant of their minds, though we may not usually call it such. Watch five-year-olds at play with the rods and see the complicated symmetries they display, unconscious of their mathematical importance or relevance. They quickly recognize that a crimson rod and a dark-green added end-to-end gives the same length as the orange rod, and that dark-green added to crimson is also equivalent to orange. In university circles this is known as the commutative law as applied to addition: here are children showing us they understand this in its algebraic significance! With what they see they use such ideas as “greater than”, “less than”, and groups of equal numbers are selected. These “games” reflect the mathematical patterns of the

children’s thought processes—as does most of their living. For we all use mental substitutes for real objects, children no less than adults, perhaps even more so since it is after childhood that more real things are acquired. A child knows that a piece of planking is not an aeroplane, but he experiments with plank and box as virtual abstract substitutes for the real thing. This is one of the hallmarks of mathematical thought. Day by day the boy or girl tries to overcome and conquer his or her environment using concentrated thought-play, dealing with relationships—relationships between names and people, colour and feeling, smells and food, up and down, north and south, shapes of wood. Mathematics has justly been defined as the perception of relationships.

Secondly, the rods supply a long felt need. In too many schools we do not teach mathematics, call it what we will. We teach a long, protracted, highly developed rearrangement of notational tricks so that children in five years of schooling are persuaded to believe, if they remember at all, that this—6—is a number, that 16 “take away” 1 is 15, that area is “length times breadth”, that to multiply a number by ten we add a nought, and that the real aim of arithmetic is to get right answers and be tidy. Yet all of these are, in fact, false. Some teachers, of course, have known this for a long time, but generally they have not been able to avoid being drawn into the conventional approach, arguing that perhaps the children were not able enough, or that abstract thought is only for the sixth form of a grammar school. Others frankly admit that they have no solution.

We have been unable to change the situation for two reasons: very few have believed that potential ability in the academic field, the world of “thinking”, was other than is reflected by current fashions in grading, “eleven plus”, grammar and modern schools, academic and practical. In addition we have just not had any method for teaching and interesting children in abstract thought. There has, of course, been a very welcome advance in recent years in the presentation of mathematics. Going are the drab, dull, machine-like inculcation of tables and sums. In their place we have practical applications of number work. The Infants use their cans and water to experience elementary ideas of capacity and volume, Juniors use more

advanced “shopping” projects, time-tables, measuring apparatus, calculating machines. Their older brothers and sisters learn the elements of surveying and navigation, and arithmetic is geared to its practical use. But we must ask whether any of this *teaches* mathematics, as that is an affair of the mind, though all of it may *use* the mathematics, little or more, of which the individual child is already aware. By all means let us continue to use that which we already have and by doing so reinforce it, give it its social and practical significance. But is this all, and is it what we should be content in doing?

The truth is that apparatus and symbols usually obscure the ideas the teacher is trying to communicate. Of course, the coloured rods are also examples of apparatus, and used without understanding they too can conceal the mathematics, becoming then just another visual aid. But they have these great advantages:

(a) They are so simply cut that they cause none of the distractions caused by more complicated shapes. Shaped as cuboids they lend themselves to being fitted together end-to-end, side-by-side, on top of each other to form larger solids, in steps, all of these having much significance in the development of topics such as multiplication, powers, logarithms, directed numbers, series, harmonic conjugates, symmetry, area, congruence, ratio, trigonometrical ratios, simple and compound interest, to mention a few.

(b) Number as a relationship is stressed from the outset. It is not a rod which represents a number, but the relationship of one rod with another. Thus by changing the “measuring-rod” one rod can give rise in turn to members of a whole set of fractions, and all this is recognized on sight. The colours help with immediate recognition and children show themselves very susceptible to their influence and the meanings we attach to them. Should the measuring-rod be implicit, rather than stated, the work is more obviously of an algebraic nature, and in fact children of four years of age can operate with the rods algebraically before they learn specific number relationship and develop their arithmetic.

(c) The rods are unmarked, so no fixed visual image is associated with a number. Instead, a number is understood as a concept, not as a figure 5 or a word “five”, which become therefore

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only two conventional ways of recording the concept.

These advantages are all combined in one form of apparatus like no other material aid yet discovered, and can only be appreciated in full when the rods are seen in use. When that is done it is not surprising at all to see children of junior school age adding fractions like

$$\frac{a+x}{b-y} + \frac{3xy}{7}$$

because these are only ways of reporting the simple gestures with the rods which the children have performed with confidence many times. They can show too, for example, that if

$$\frac{a}{b} = \frac{c}{d}$$

then

$$\frac{a}{b} = \frac{a+c}{b+d} = \frac{ma+nc}{mb+nd}$$

that

$$a(b-c) = ab-ac$$

or give the factors of

$$14a^2b + 2ab^3.$$

Their mathematical vocabulary can be correct and based upon real understanding. A line they will know as a set of points and not as a pencil mark on paper; "2 shillings add 6 pence" for them will not be an addition sum in the same sense as is "8 pence add 6 pence". They know the difference between subtraction and "taking away", and that division is an operation evolved from addition. Algebra will be a matter of discovering what can be done with numbers, not a question of using letters,

for the figures in arithmetic are not the numbers, so why should different marks on paper give rise to any change of viewpoint?

Such phrases will probably appear strange to many, for the greatest difficulty in the acceptance of the Cuisenaire-Gattegno ideas and procedures is the prejudice of ourselves, the adults. It may be no fault of ours; indeed, to worry about assigning responsibility in this is a waste of time and energy. And it has nothing to do with our love or otherwise for the children or our job. Our loyalties, intentions or energies are not being criticized. But we also need wisdom and knowledge. We have to be perfectly clear of the truth of what we are trying to teach. Yet many teachers admit only too easily with the rest of the population that they "can't do maths" and that mathematicians are people with special kinds of brains.

The Cuisenaire rods and the Gattegno extensions in the fields of geometry and algebra show us that we are really much more capable than we think, and that *all* children can thrive on and create their own mathematics, acquiring powers of thought and problem-tackling which none will believe until such classes are seen at work. It is known, for example, that most children can *master* fractions by the age of nine and that young children, or so-called backward pupils, relish work on infinite sets, discuss subtle variations of meanings in mathematical terms, calculate mentally in surprisingly quick ways. It becomes evident that

algebra can and should be taught before arithmetic; geometry treated dynamically gives a breadth of mathematical vision which those reared on Euclid can only marvel at; algebraic equations which are not common even in university courses can be solved by 11-year-olds. There is here no new theory to be inflicted on hard pressed teachers; its instant success with children by those who understand is its greatest attraction.

The importance is not confined to the teaching of arithmetic and mathematics as such: it has far-reaching implications. Children learn to read by becoming aware of relationships of experience to sounds and then to shapes of ink marks on paper; they learn to write by relating similar experiences to hand and muscle movements. Geography depends upon the relationship man has to his ethnological environment, history upon that of man to man and man in time. The conscious development of a child's awareness of relationships will affect all his learning. At present we ignore this; we allow him only to express the awareness he may have accumulated incidental to the lesson or the consciously given instruction. Yet on such expression do we base our very definitions of intelligence, dullness, academic and non-academic, for most tests we use to help us in these matters involve relationships.

What will happen once *all* children are consciously and deliberately helped in school to develop the embryonic mathematical relationships which are inborn?