This wave consists entirely of magnetic disturbances, the direction of magnetization being in the plane of the wave. No magnetic disturbance whose direction of magnetization is not in the plane of the wave can be propagated as a plane wave at all.

Hence magnetic disturbances propagated through the electromagnetic field agree with light in this, that the disturbance at any point is transverse to the direction of propagation, and such waves may have all the properties of polarized light.

(96) The only medium in which experiments have been made to determine the value of k is air, in which $\mu=1$, and therefore, by equation (46),

$$V=v. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

By the electromagnetic experiments of MM. WEBER and KOHLRAUSCH [*],

$$v=310{,}740{,}000 \text{ metres per second}$$

is the number of electrostatic units in one electromagnetic unit of electricity, and this, according to our result, should be equal to the velocity of light in air or vacuum.

The velocity of light in air, by M. FIZEAU's [†] experiments, is

$$V=314{,}858{,}000 ;$$

according to the more accurate experiments of M. FOUCAULT [‡],

$$V=298{,}000{,}000.$$

The velocity of light in the space surrounding the earth, deduced from the coefficient of aberration and the received value of the radius of the earth's orbit, is

$$V=308{,}000{,}000.$$

(97) Hence the velocity of light deduced from experiment agrees sufficiently well with the value of v deduced from the only set of experiments we as yet possess. The value of v was determined by measuring the electromotive force with which a condenser of known capacity was charged, and then discharging the condenser through a galvanometer, so as to measure the quantity of electricity in it in electromagnetic measure. The only use made of light in the experiment was to see the instruments. The value of V found by M. FOUCAULT was obtained by determining the angle through which a revolving mirror turned, while the light reflected from it went and returned along a measured course. No use whatever was made of electricity or magnetism.

The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws.

(98) Let us now go back upon the equations in (94), in which the quantities J and Ψ occur, to see whether any other kind of disturbance can be propagated through the medium depending on these quantities which disappeared from the final equations.

[*] Leipzig Transactions, vol. v. (1857), p. 260, or POGGENDORFF's 'Annalen,' Aug. 1856, p. 10.
[†] Comptes Rendus, vol. xxix. (1849), p. 90.   [‡] Ibid. vol. lv. (1862), pp. 501, 792.

If we determine χ from the equation

$$\nabla^2\chi = \frac{d^2\chi}{dx^2} + \frac{d^2\chi}{dy^2} + \frac{d^2\chi}{dz^2} = J, \qquad \dots \qquad (73)$$

and F', G', H' from the equations

$$F' = F - \frac{d\chi}{dx}, \quad G' = G - \frac{d\chi}{dy}, \quad H' = H - \frac{d\chi}{dz}, \quad \dots \quad (74)$$

then

$$\frac{dF'}{dx} + \frac{dG'}{dy} + \frac{dH'}{dz} = 0, \qquad \dots \qquad (75)$$

and the equations in (94) become of the form

$$k\nabla^2 F' = 4\pi\mu\left(\frac{a^2 F'}{dt^2} + \frac{d}{dxdt}\left(\Psi + \frac{d\chi}{dt}\right)\right). \quad \dots \quad (76)$$

Differentiating the three equations with respect to x, y, and z, and adding, we find that

$$\Psi = -\frac{d\chi}{dt} + \varphi(x, y, z), \qquad \dots \qquad (77)$$

and that

$$\left.\begin{aligned}
k\nabla^2 F' &= 4\pi\mu\frac{d^2 F'}{dt^2}, \\
k\nabla^2 G' &= 4\pi\mu\frac{d^2 G'}{dt^2}, \\
k\nabla^2 H' &= 4\pi\mu\frac{d^2 H'}{dt^2}.
\end{aligned}\right\} \quad \dots \quad (78)$$

Hence the disturbances indicated by F', G', H' are propagated with the velocity $V = \sqrt{\frac{k}{4\pi\mu}}$ through the field; and since

$$\frac{dF'}{dx} + \frac{dG'}{dy} + \frac{dH'}{dx} = 0,$$

the resultant of these disturbances is in the plane of the wave.

(99) The remaining part of the total disturbances F, G, H being the part depending on χ, is subject to no condition except that expressed in the equation

$$\frac{d\Psi}{dt} + \frac{d^2\chi}{dt^2} = 0.$$

If we perform the operation ∇^2 on this equation, it becomes

$$ke = \frac{dJ}{dt} - k\nabla^2\varphi(x, y, z). \qquad \dots \qquad (79)$$

Since the medium is a perfect insulator, e, the free electricity, is immoveable, and therefore $\frac{dJ}{dt}$ is a function of x, y, z, and the value of J is either constant or zero, or uniformly increasing or diminishing with the time; so that no disturbance depending on J can be propagated as a wave.

(100) The equations of the electromagnetic field, deduced from purely experimental evidence, show that transversal vibrations only can be propagated. If we were to go beyond our experimental knowledge and to assign a definite density to a substance which

we should call the electric fluid, and select either vitreous or resinous electricity as the representative of that fluid, then we might have normal vibrations propagated with a velocity depending on this density. We have, however, no evidence as to the density of electricity, as we do not even know whether to consider vitreous electricity as a substance or as the absence of a substance.

Hence electromagnetic science leads to exactly the same conclusions as optical science with respect to the direction of the disturbances which can be propagated through the field; both affirm the propagation of transverse vibrations, and both give the same velocity of propagation. On the other hand, both sciences are at a loss when called on to affirm or deny the existence of normal vibrations.

Relation between the Index of Refraction and the Electromagnetic Character of the substance.

(101) The velocity of light in a medium, according to the Undulatory Theory, is

$$\frac{1}{i}V_0,$$

where i is the index of refraction and V_0 is the velocity in vacuum. The velocity, according to the Electromagnetic Theory, is

$$\sqrt{\frac{k}{4\pi\mu}},$$

where, by equations (49) and (71), $k=\frac{1}{D}k_0$, and $k_0=4\pi V_0^2$.

Hence
$$D=\frac{i^2}{\mu}, \qquad \dots \dots \dots \dots \dots \quad (80)$$

or the Specific Inductive Capacity is equal to the square of the index of refraction divided by the coefficient of magnetic induction.

Propagation of Electromagnetic Disturbances in a Crystallized Medium.

(102) Let us now calculate the conditions of propagation of a plane wave in a medium for which the values of k and μ are different in different directions. As we do not propose to give a complete investigation of the question in the present imperfect state of the theory as extended to disturbances of short period, we shall assume that the axes of magnetic induction coincide in direction with those of electric elasticity.

(103) Let the values of the magnetic coefficient for the three axes be λ, μ, ν, then the equations of magnetic force (B) become

$$\left.\begin{array}{l} \lambda\alpha = \dfrac{dH}{dy} - \dfrac{dG}{dz}, \\[2mm] \mu\beta = \dfrac{dF}{dz} - \dfrac{dH}{dx}, \\[2mm] \nu\gamma = \dfrac{dG}{dx} - \dfrac{dF}{dy}. \end{array}\right\} \quad \dots \dots \dots \dots \dots \quad (81)$$

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The equations of electric currents (C) remain as before.

The equations of electric elasticity (E) will be

$$\left.\begin{array}{l} P=4\pi a^2 f, \\ Q=4\pi b^2 g, \\ R=4\pi c^2 h, \end{array}\right\} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (82)$$

where $4\pi a^2$, $4\pi b^2$, and $4\pi c^2$ are the values of k for the axes of x, y, z.

Combining these equations with (A) and (D), we get equations of the form

$$\frac{1}{\mu\nu}\left(\lambda\frac{d^2F}{dx^2}+\mu\frac{d^2F}{dy^2}+\nu\frac{d^2F}{dz^2}\right)-\frac{1}{\mu\nu}\frac{d}{dx}\left(\lambda\frac{dF}{dx}+\mu\frac{dG}{dy}+\nu\frac{dH}{dz}\right)=\frac{1}{a^2}\left(\frac{d^2F}{dt^2}+\frac{d^2\Psi}{dxdt}\right). \quad \cdot \quad \cdot \quad (83)$$

(104) If l, m, n are the direction-cosines of the wave, and V its velocity, and if

$$lx+my+nz-Vt=w, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (84)$$

then F, G, H, and Ψ will be functions of w; and if we put F′, G′, H′, Ψ' for the second differentials of these quantities with respect to w, the equations will be

$$\left.\begin{array}{l} \left(V^2-a^2\left(\frac{m^2}{\nu}+\frac{n^2}{\mu}\right)\right)F'+\frac{a^2lm}{\nu}G'+\frac{a^2ln}{\mu}H'-lV\Psi'=0, \\[2mm] \left(V^2-b^2\left(\frac{n^2}{\lambda}+\frac{l^2}{\nu}\right)\right)G'+\frac{b^2mn}{\lambda}H'+\frac{b^2ml}{\nu}F'-mV\Psi'=0, \\[2mm] \left(V^2-c^2\left(\frac{l^2}{\mu}+\frac{m^2}{\lambda}\right)\right)H'+\frac{c^2nl}{\mu}F'+\frac{c^2nm}{\lambda}G'-nV\Psi'=0. \end{array}\right\} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (85)$$

If we now put

$$\left.\begin{array}{l} V^4-V^2\frac{1}{\lambda\mu\nu}\left\{l^2\lambda(b^2\mu+c^2\nu)+m^2\mu(c^2\nu+a^2\lambda)+n^2\nu(a^2\lambda+b^2\mu)\right\} \\[2mm] \qquad +\frac{a^2b^2c^2}{\lambda\mu\nu}\left(\frac{l^2}{a^2}+\frac{m^2}{b^2}+\frac{n^2}{c^2}\right)(l^2\lambda+m^2\mu+n^2\nu)=U, \end{array}\right\} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (86)$$

we shall find

$$F'V^2U-l\Psi'VU=0, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (87)$$

with two similar equations for G′ and H′. Hence either

$$V=0, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (88)$$

$$U=0, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (89)$$

or

$$VF'=l\Psi', \quad VG'=m\Psi' \text{ and } VH'=n\Psi'. \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (90)$$

The third supposition indicates that the resultant of F′, G′, H′ is in the direction normal to the plane of the wave; but the equations do not indicate that such a disturbance, if possible, could be propagated, as we have no other relation between Ψ' and F′, G′, H′.

The solution $V=0$ refers to a case in which there is no propagation.

The solution $U=0$ gives two values for V^2 corresponding to values of F′, G′, H′, which

are given by the equations

$$\frac{l}{a^2}\mathrm{F}' + \frac{m}{b^2}\mathrm{G}' + \frac{n}{c^2}\mathrm{H}' = 0; \quad \cdots \cdots \cdots \quad (91)$$

$$\frac{a^2 l\lambda}{\mathrm{F}'}(b^2\mu - c^2\nu) + \frac{b^2 m\mu}{\mathrm{G}'}(c^2\nu - a^2\lambda) + \frac{c^2 n\nu}{\mathrm{H}'}(a^2\lambda - b^2\mu) = 0, \quad \cdots \cdots \quad (92)$$

(105) The velocities along the axes are as follows:—

Direction of propagation . . . . . .

Direction of the electric displacements

	x	y	z
x		$\dfrac{a^2}{\nu}$	$\dfrac{a^2}{\mu}$
y	$\dfrac{b^2}{\nu}$		$\dfrac{b^2}{\lambda}$
z	$\dfrac{c^2}{\mu}$	$\dfrac{c^2}{\lambda}$	

Now we know that in each principal plane of a crystal the ray polarized in that plane obeys the ordinary law of refraction, and therefore its velocity is the same in whatever direction in that plane it is propagated.

If polarized light consists of electromagnetic disturbances in which the electric displacement is in the plane of polarization, then

$$a^2 = b^2 = c^2. \quad \cdots \cdots \cdots \cdots \quad (93)$$

If, on the contrary, the electric displacements are perpendicular to the plane of polarization,

$$\lambda = \mu = \nu. \quad \cdots \cdots \cdots \cdots \quad (94)$$

We know, from the magnetic experiments of FARADAY, PLÜCKER, &c., that in many crystals λ, μ, ν are unequal.

The experiments of KNOBLAUCH* on electric induction through crystals seem to show that a, b and c, may be different.

The inequality, however, of λ, μ, ν is so small that great magnetic forces are required to indicate their difference, and the differences do not seem of sufficient magnitude to account for the double refraction of the crystals.

On the other hand, experiments on electric induction are liable to error on account of minute flaws, or portions of conducting matter in the crystal.

Further experiments on the magnetic and dielectric properties of crystals are required before we can decide whether the relation of these bodies to magnetic and electric forces is the same, when these forces are permanent as when they are alternating with the rapidity of the vibrations of light.

* Philosophical Magazine, 1852.

Relation between Electric Resistance and Transparency.

(106) If the medium, instead of being a perfect insulator, is a conductor whose resistance per unit of volume is ϱ, then there will be not only electric displacements, but true currents of conduction in which electrical energy is transformed into heat, and the undulation is thereby weakened. To determine the coefficient of absorption, let us investigate the propagation along the axis of x of the transverse disturbance G.

By the former equations

$$\frac{d^2 G}{dx^2} = -4\pi\mu(q')$$

$$= -4\pi\mu\left(\frac{df}{dt} + q\right) \text{ by (A),}$$

$$\frac{d^2 G}{dx^2} = +4\pi\mu\left(\frac{1}{k}\frac{d^2 G}{dt^2} - \frac{1}{\varrho}\frac{dG}{dt}\right) \text{ by (E) and (F).} \quad \ldots \ldots \quad (95)$$

If G is of the form

$$G = e^{-px}\cos(qx + nt), \quad \ldots \ldots \ldots \ldots \quad (96)$$

we find that

$$p = \frac{2\pi\mu}{\varrho}\frac{n}{q} = \frac{2\pi\mu}{\varrho}\frac{V}{i}, \quad \ldots \ldots \ldots \ldots \quad (97)$$

where V is the velocity of light in air, and i is the index of refraction. The proportion of incident light transmitted through the thickness x is

$$e^{-2px}. \quad \ldots \ldots \ldots \ldots \ldots \quad (98)$$

Let R be the resistance in electromagnetic measure of a plate of the substance whose thickness is x, breadth b, and length l, then

$$R = \frac{l\varrho}{bx},$$

$$2px = 4\pi\mu\frac{V}{i}\frac{l}{bR}. \quad \ldots \ldots \ldots \ldots \quad (99)$$

(107) Most transparent solid bodies are good insulators, whereas all good conductors are very opaque.

Electrolytes allow a current to pass easily and yet are often very transparent. We may suppose, however, that in the rapidly alternating vibrations of light, the electromotive forces act for so short a time that they are unable to effect a complete separation between the particles in combination, so that when the force is reversed the particles oscillate into their former position without loss of energy.

Gold, silver, and platinum are good conductors, and yet when reduced to sufficiently thin plates they allow light to pass through them. If the resistance of gold is the same for electromotive forces of short period as for those with which we make experiments, the amount of light which passes through a piece of gold-leaf, of which the resistance was determined by Mr. C. HOCKIN, would be only 10^{-50} of the incident light, a totally imperceptible quantity. I find that between $\frac{1}{500}$ and $\frac{1}{1000}$ of green light gets through

such gold-leaf.   Much of this is transmitted through holes and cracks; there is enough, however, transmitted through the gold itself to give a strong green hue to the transmitted light.   This result cannot be reconciled with the electromagnetic theory of light, unless we suppose that there is less loss of energy when the electromotive forces are reversed with the rapidity of the vibrations of light than when they act for sensible times, as in our experiments.

Absolute Values of the Electromotive and Magnetic Forces called into play in the Propagation of Light.

(108) If the equation of propagation of light is

$$F = A \cos \frac{2\pi}{\lambda}(z - Vt),$$

the electromotive force will be

$$P = -A \frac{2\pi}{\lambda} V \sin \frac{2\pi}{\lambda}(z - Vt);$$

and the energy per unit of volume will be

$$\frac{P^2}{8\pi\mu V^2},$$

where P represents the greatest value of the electromotive force.   Half of this consists of magnetic and half of electric energy.

The energy passing through a unit of area is

$$W = \frac{P^2}{8\pi\mu V};$$

so that

$$P_* = \sqrt{8\pi\mu VW},$$

where V is the velocity of light, and W is the energy communicated to unit of area by the light in a second.

According to POUILLET's data, as calculated by Professor W. THOMSON[*], the mechanical value of direct sunlight at the Earth is

83·4 foot-pounds per second per square foot.

This gives the maximum value of P in direct sunlight at the Earth's distance from the Sun,

$$P = 60,000,000,$$

or about 600 DANIELL's cells per metre.

At the Sun's surface the value of P would be about

13,000 DANIELL's cells per metre.

At the Earth the maximum magnetic force would be ·193 [†].

At the Sun it would be 4·13.

These electromotive and magnetic forces must be conceived to be reversed twice in every vibration of light; that is, more than a thousand million million times in a second.

[*] Transactions of the Royal Society of Edinburgh, 1854 ("Mechanical Energies of the Solar System").

[†] The horizontal magnetic force at Kew is about 1·76 in metrical units.

PART VII.—CALCULATION OF THE COEFFICIENTS OF ELECTROMAGNETIC INDUCTION.

General Methods.

(109) The electromagnetic relations between two conducting circuits, A and B, depend upon a function M of their form and relative position, as has been already shown.

M may be calculated in several different ways, which must of course all lead to the same result.

First Method. M is the electromagnetic momentum of the circuit B when A carries a unit current, or

$$M = \int \left(F\frac{dx}{ds'} + G\frac{dy}{ds'} + H\frac{dz}{ds'} \right) ds',$$

where F, G, H are the components of electromagnetic momentum due to a unit current in A, and ds' is an element of length of B, and the integration is performed round the circuit of B.

To find F, G, H, we observe that by (B) and (C)

$$\frac{d^2F}{dx^2} + \frac{d^2F}{dy^2} + \frac{d^2F}{dz^2} = -4\pi\mu p',$$

with corresponding equations for G and H, p', q', and r' being the components of the current in A.

Now if we consider only a single element ds of A, we shall have

$$p' = \frac{dx}{ds}ds, \qquad q' = \frac{dy}{ds}ds, \qquad r' = \frac{dz}{ds}ds,$$

and the solution of the equation gives

$$F = \frac{\mu}{\varrho}\frac{dx}{ds}ds, \qquad G = \frac{\mu}{\varrho}\frac{dy}{ds}ds, \qquad H = \frac{\mu}{\varrho}\frac{dz}{ds}ds,$$

where ϱ is the distance of any point from ds.   Hence

$$M = \iint \frac{\mu}{\varrho}\left(\frac{dx}{ds}\frac{dx}{ds'} + \frac{dy}{ds}\frac{dy}{ds'} + \frac{dz}{ds}\frac{dz}{ds'} \right) dsds'$$
$$= \iint \frac{\mu}{\varrho}\cos\theta\, dsds',$$

where θ is the angle between the directions of the two elements ds, ds', and ϱ is the distance between them, and the integration is performed round both circuits.

In this method we confine our attention during integration to the two linear circuits alone.

(110) Second Method. M is the number of lines of magnetic force which pass through the circuit B when A carries a unit current, or

$$M = \Sigma(\mu\alpha l + \mu\beta m + \mu\gamma n)dS',$$

where $\mu\alpha$, $\mu\beta$, $\mu\gamma$ are the components of magnetic induction due to unit current in A,

S' is a surface bounded by the current B, and l, m, n are the direction-cosines of the normal to the surface, the integration being extended over the surface.

We may express this in the form

$$M = \mu \Sigma \frac{1}{\varrho^2} \sin\theta \sin\theta' \sin\varphi \, dS' ds,$$

where dS' is an element of the surface bounded by B, ds is an element of the circuit A, ϱ is the distance between them, θ and θ' are the angles between ϱ and ds and between ϱ and the normal to dS' respectively, and φ is the angle between the planes in which θ and θ' are measured. The integration is performed round the circuit A and over the surface bounded by B.

This method is most convenient in the case of circuits lying in one plane, in which case $\sin\theta = 1$, and $\sin\varphi = 1$.

111. *Third Method.* M is that part of the intrinsic magnetic energy of the whole field which depends on the product of the currents in the two circuits, each current being unity.

Let α, β, γ be the components of magnetic intensity at any point due to the first circuit, α', β', γ' the same for the second circuit; then the intrinsic energy of the element of volume dV of the field is

$$\frac{\mu}{8\pi}\left((\alpha+\alpha')^2 + (\beta+\beta')^2 + (\gamma+\gamma')^2\right)dV.$$

The part which depends on the product of the currents is

$$\frac{\mu}{4\pi}(\alpha\alpha' + \beta\beta' + \gamma\gamma')dV.$$

Hence if we know the magnetic intensities I and I' due to unit current in each circuit, we may obtain M by integrating

$$\frac{\mu}{4\pi}\Sigma\mu \, I \, I' \cos\theta \, dV$$

over all space, where θ is the angle between the directions of I and I'.

Application to a Coil.

(112) To find the coefficient (M) of mutual induction between two circular linear conductors in parallel planes, the distance between the curves being everywhere the same, and small compared with the radius of either.

If r be the distance between the curves, and a the radius of either, then when r is very small compared with a, we find by the second method, as a first approximation,

$$M = 4\pi a \left(\log_e \frac{8a}{r} - 2\right).$$

To approximate more closely to the value of M, let a and a_1 be the radii of the circles, and b the distance between their planes; then

$$r^2 = (a - a_1)^2 + b^2.$$

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We obtain M by considering the following conditions:—

1st. M must fulfil the differential equation

$$\frac{d^2M}{da^2} + \frac{d^2M}{db^2} + \frac{1}{a}\frac{dM}{da} = 0.$$

This equation being true for any magnetic field symmetrical with respect to the common axis of the circles, cannot of itself lead to the determination of M as a function of a, a_1, and b.   We therefore make use of other conditions.

2ndly. The value of M must remain the same when a and a_1 are exchanged.

3rdly. The first two terms of M must be the same as those given above.

M may thus be expanded in the following series :—

$$M = 4\pi a \log \frac{8a}{r}\left\{1 + \frac{1}{2}\frac{a-a_1}{a} + \frac{1}{16}\frac{3b^2 + (a_1-a)^2}{a^2} - \frac{1}{32}\frac{(3b^2 + (a-a_1)^2)(a-a_1)}{a^3} + \&c.\right\}$$

$$- 4\pi a\left\{2 + \frac{1}{2}\frac{a-a_1}{a} + \frac{1}{16}\frac{b^2 - 3(a-a_1^2)}{a^2} - \frac{1}{48}\frac{(6b^2 - (a-a_1)^2)(a-a_1)}{a^3} + \&c.\right\}.$$

(113) We may apply this result to find the coefficient of self-induction (L) of a circular coil of wire whose section is small compared with the radius of the circle.

Let the section of the coil be a rectangle, the breadth in the plane of the circle being c, and the depth perpendicular to the plane of the circle being b.

Let the mean radius of the coil be a, and the number of windings n; then we find, by integrating,

$$L = \frac{n^2}{b^2c^2}\iiiint M(xy\, x'y')dx\, dy\, dx'\, dy',$$

where $M(xy\, x'y')$ means the value of M for the two windings whose coordinates are xy and $x'y'$ respectively; and the integration is performed first with respect to x and y over the rectangular section, and then with respect to x' and y' over the same space.

$$L = 4\pi n^2 a\left\{\log\frac{8a}{r} + \frac{1}{12} - \frac{4}{3}\left(\theta - \frac{\pi}{4}\right)\cot 2\theta - \frac{\pi}{3}\cos 2\theta - \frac{1}{6}\cot^2\theta \log\cos\theta - \frac{1}{6}\tan^2\theta \log\sin\theta\right\}$$

$$+ \frac{\pi n^2 r^2}{24a}\left\{\log\frac{8a}{r}(2\sin^2\theta + 1) + 3\cdot45 + 27\cdot475\cos^2\theta - 3\cdot2\left(\frac{\pi}{2} - \theta\right)\frac{\sin^3\theta}{\cos\theta} + \frac{1}{5}\frac{\cos^4\theta}{\sin^2\theta}\log\cos\theta\right.$$

$$\left. + \frac{13}{3}\frac{\sin^4\theta}{\cos^2\theta}\log\sin\theta\right\} + \&c.$$

Here $a =$ mean radius of the coil.

„ $r =$ diagonal of the rectangular section $= \sqrt{b^2 + c^2}$.

„ $\theta =$ angle between r and the plane of the circle.

„ $n =$ number of windings.

The logarithms are Napierian, and the angles are in circular measure.

In the experiments made by the Committee of the British Association for determining a standard of Electrical Resistance, a double coil was used, consisting of two nearly equal coils of rectangular section, placed parallel to each other, with a small interval between them.