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The Possibility of the Same Form of Specific Interaction for All Nuclear Particles

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Abstract

Experimental evidence pointing to a universal form of interaction for all nuclear particles is summarized. The form given by Eq. (1) is discussed. It is found to be satisfactory for H^3, He^3, He^4 considered together with proton and neutron scattering. In order that heavy nuclei should not be too stable it is necessary to require the inequality (7.2) and in order that there be no very heavy and electrically neutral nuclei the inequality (7) has to be satisfied. The form of the interaction energy restricted by these conditions is as satisfactory as nuclear theories postulating different forms for like and unlike particles. The simplest form, satisfying all present requirements is obtained from Eq. (1) by letting $g_1 = g_2 = 0$.

The interaction between protons and protons which acts in addition to the Coulombian repulsion is generally supposed to be the same as that between neutrons and neutrons. The evidence for this lies in the fact[2] that the mass defect of He^3 is smaller than that of H^3 by approximately the amount required by the Coulombian repulsion between the protons in He^3. Further strong support for regarding the specific like-particle interaction as the same for protons and neutrons is found in the close agreement[3] of the

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[2]E. Feenberg and J. Knipp, Phys. Rev. **48,** 906 (1935); S. S. Share, Phys. Rev. **50**, 488 (1936).

[3]G. Breit, E. U. Condon and R. D. Present, Phys. Rev. this issue; Fermi and Amaldi, Ricerca Scient. **1,** 1 (1936); Fermi, ibid., July (1936).

values obtained for its magnitude from experiments on the scattering of protons in hydrogen[4] with that derived from the mass defect of H^3.

The accuracy of the determination of the like-particle interaction from the proton-proton scattering experiments is great. Numerical comparison of its magnitude with that which follows from experiments on the scattering and recombination of slow neutrons in hydrogen shows that in ^1S states the specific interaction between all nuclear particles are the same to within the accuracy of present experiments. The equality of the interactions is obtained independently of what range is used for the spacial extension of the force within wide limits and is, therefore, probably real rather than accidental. It thus becomes reasonable to attempt to regard the specific nuclear forces as the same between all nuclear particles independently of whether they are neutrons or protons. In the present note the qualitative consequences of this view are discussed.

In the early development of nuclear theory it was supposed that the principal interactions within the nucleus take place between neutrons and protons because by doing so the approximate equality of the number of neutrons with the number of protons automatically received a simple explanation. According to present evidence, mentioned above, this explanation does not apply and a new one must be made. This is offered by the exclusion principle which favors the addition of unlike particles analogously to the way in which the building up of electronic shells in an atom proceeds by adding electrons with opposite spin directions. In addition it is favorable for stability to have proton-neutron pairs in symmetrical $(S, D, G, \ldots)$ triplet states because the attraction is then greater than in symmetrical singlet states. For like particles the exclusion principle rules out the symmetrical triplet state. This circumstance is important in the building up of the first shell up to He^4.

The universal interaction energy between any pair of heavy elementary particles will be assumed to be of the form

$$V_{ij} = \left\{(1 - g - g_1 - g_2)P_{ij}^M + gP_{ij}^H + g_1 \cdot 1 + g_2 P_{ij}^S\right\} J(r_{ij}), \tag{1}$$

where g, g_1, g_2 are constants and P^M, P^H are the Majorana and Heisenberg exchange operators while $P^S = P^M P^H$ is the operator proposed by Bartlett which exchanges the spins without affecting the coordinates[5]. For this interaction

[4] M. A. Tuve, N. P. Heydenburg and L. R. Hafstad, Phys. Rev. this issue.

[5] Another formulation of the symmetrical Hamiltonian using a fifth "character" variable is given by Condon and Cassen in this issue.

Table I. *Two-particle systems.*

Even L		Odd L	
triplets	singlets	triplets	singlets
${}^1S,\ {}^3D,\ldots$	${}^1S,\ {}^1D,\ldots$	${}^3P,\ {}^3F,\ldots$	${}^1P,\ {}^1F,\ldots$
$J(r)$	$(1-2g-2g_2)J(r)$	$(1+2g_1+2g_2)J(r)$	$(-1+2g+2g_1)J(r)$
Absent for equivalent like particles			Absent for equivalent like particles

the potential energies in different states are as in Table I where L is the orbital angular momentum in units of $\hbar$. Two isolated like particles moving in each other's field cannot have different principal quantum numbers and are in equivalent states if L is evert and the spins are parallel or when L is odd and the spins are antiparallel. Triplets of even L and singlets of odd L are thus ruled out by the exclusion principle. In proton-proton scattering experiments only states with $L = 0$ matter and, therefore, the effective interaction energy is that corresponding to ${}^1S$ namely $(1-2g-2g_2)J(r)$. In the scattering of slow neutrons by hydrogen a similar ${}^1S$ state is responsible for the large collision cross section according to Wigner's hypothesis. Both sets of scattering experiments thus have to do with $(1-2g-2g_2)J(r)$ while $J(r)$ determines the binding energy of the deuteron.

1 Light Nuclei

In H^3, He^3, He^4 like particles are in nearly spacially symmetric states with respect to each other and the effective interaction potential is thus again $(1-2g-2g_2)J(r)$. This is consistent with the agreement between the values of the like particle interaction energy as obtained from the mass defects of the isotopes of hydrogen and helium with that derived from the scattering of protons in hydrogen. Unlike particles are partly in triplet and partly in singlet states with respect to each other. However, the space wave function is nearly symmetric and thus the effective interaction potential between protons and neutrons is $[1-\frac{1}{2}(g+g_2)]J(r)$ which is the weighted mean of the potentials in singlets and triplets with statistical weights $\frac{1}{4}$ and $\frac{3}{4}$. In these questions the sum of $g+g_2$ occurs as a whole and one cannot distinguish between g and g_2 separately. The calculations on mass defects that have been made without using g_1, g_2 thus apply directly to (1) and are consistent with it. The way in which this comes about may be seen for H^3 as follows. The wave equation is

$$(T+\Sigma_{i>j}V_{ij})\psi = E\psi,$$

where T is the operator representing the sum of kinetic energies of the three particles. Since ψ is antisymmetric in the neutrons it is of the form

$$\psi = u((12_+),3)S^0((12_-),3) + v((12_-),3)S'((12_+),3),$$

where the two neutrons are denoted by 1, 2 and a plus suffix indicates that the function is symmetric in the particles while a minus suffix similarly indicates antisymmetry. The functions u, v contain only the Cartesian coordinates while the functions S^0, S' contain only the spin coordinates. Considering the state with total spin angular momentum $\frac{1}{2}$ in the z direction one may use

$$\begin{aligned} S^0 &= 2^{-\frac{1}{2}}[(+-+) - (-++)], \\ S' &= 6^{-\frac{1}{2}}[(+-+) + (-++) - 2(++-)], \end{aligned}$$

where the $+$ and $-$ signs correspond to positive and negative orientations of the spin axis in an arbitrary fixed direction. Each () stands for a product of spin functions referring to the particles in the order 1, 2, 3. One obtains

$$(H_0 + H' - E)\begin{pmatrix} u \\ v \end{pmatrix} = 0, \tag{2}$$

where

$$H_0 = T + \begin{pmatrix} 1-2g-2g_2, 0 \\ 0, -1+2g_1+2g_2 \end{pmatrix} J_{12} + \begin{pmatrix} 1-\frac{1}{2}(g+g_2), 0 \\ 0, 1-(3/2)(g+g_2) \end{pmatrix}(J_{13}+J_{23}), \tag{2.1}$$

$$H' = \begin{pmatrix} X, Y \\ Y, Z \end{pmatrix}, \tag{2.2}$$

$$X = (1-\frac{1}{2}g - g_1 - g_2)[J_{23}(P_{23}^M - 1) + J_{13}(P_{13}^M - 1)], \tag{2.3}$$

$$Y = 3^{\frac{1}{2}}2^{-1}[(gP_{13}^M + g_2)J_{13} - (gP_{23}^M + g_2)J_{23}], \tag{2.4}$$

$$Z = (1-(3/2)g - g_1 - g_2)[J_{23}(P_{23}^M - 1) + J_{13}(P_{13}^M - 1)]. \tag{2.5}$$

Neglecting H' one obtains an approximation to ψ,

$$\psi_0 = \begin{pmatrix} u \\ 0 \end{pmatrix}, \tag{3}$$

with u defined by

$$\{T + (1-2g-2g_2)J_{12} + (1-\frac{1}{2}g - \frac{1}{2}g_2)(J_{13}+J_{23}) - E\}u = 0 \tag{3.1}$$

In the approximation of using ψ_0 the energy can be calculated without exchange operators by using $(1-2g-2g_2)J$, $(1-\frac{1}{2}g-\frac{1}{2}g_2)J$, respectively, for the potential energies between like and unlike particles. The function u is symmetric in the neutrons but has, in general, no symmetry property for interchange of a proton and a neutron. If, however, $g+g_2$ is small then u becomes symmetric in all three particles. In this case the first-order perturbation due to H' vanishes as is obvious from (2.3). The second-order perturbation may be estimated as $-(\psi_0|H'^2|\psi_0)/E_m$ where E_m is an average energy of perturbing levels above that of the normal state. According to (2.2) and (3)

$$(\psi_0|H'^2|\psi_0) = (u|X^2+Y^2|u).$$

The calculation of this matrix element is readily made using (2.3) and (2.4) and gives for a completely symmetric u

$$(\psi_0|H'^2|\psi_0) = (3/2)(g+g_2)^2 \int u^2(J_{13}^2 - J_{13}J_{23})d\tau_1 d\tau_2. \tag{3.2}$$

Thus the estimate of the second-order perturbation energy depends only on $g+g_2$ and one may expect the energy to depend on $g-g_2, g_1$ only in a secondary way. Since $g+g_2$ is the only combination of the g's which matters for the mass defect of the deuteron and the present proton-proton and proton-neutron scattering experiments the inclusion of g_1 and g_2 makes little practical difference in the comparison of the mass defect of H^3, H^2, He^3 with the scattering experiments.

Similarly in He^4

$$H_0 = T + \begin{pmatrix} 1-2g-2g_2, 0 \\ 0, -1+2g_1+2g_2 \end{pmatrix}(J_{12}+J_{34}) +$$

$$+\begin{pmatrix} 1-\frac{1}{2}g-\frac{1}{2}g_2, 0 \\ 0, 1-(3/2)g-(3/2)g_2 \end{pmatrix}(J_{13}+J_{23}+J_{14}+J_{24}), \tag{4.0}$$

$$H' = \begin{pmatrix} X, Y \\ Y, Z \end{pmatrix} \tag{4.1}$$

$$X = [J_{13}(P_{13}^M-1)+J_{14}(P_{14}^M-1)+J_{23}(P_{23}^M-1)+J_{24}(P_{24}^M-1)](1-\frac{1}{2}g-g_1-g_2),$$

$$Y = 3^{\frac{1}{2}}2^{-1}[J_{23}(gP_{23}^M+g_2)+J_{14}(gP_{14}^M+g_2)-J_{13}(gP_{13}^M+g_2)-J_{24}(gP_{24}^M+g_2)], \tag{4.2}$$

$$Z = (1-(3/2)g-g_1-g_2)X/(1-\frac{1}{2}g-g_1-g_2).$$

These equations differ from Eqs. (2.1) to (2.5) essentially only through the presence of more terms in sums over like and unlike particles. The wave Eq. (2) is the same in form but now.

$$
\begin{aligned}
S^0 &= \frac{1}{2}[(+-+-)+(-+-+)-(+--+)-(-++-),\\
S' &= 12^{-\frac{1}{2}}[2(++--)+2(--++)-(+-+-)-(-+-+)-\\
&\quad -(-++-)-(+--+)].
\end{aligned}
$$

Particles 1 and 2 are alike and so are 3 and 4. With a completely symmetric u

$$(\psi_0|H'^2|\psi_0) = 3(g+g_2)^2(u|J_{13}^2 + J_{13}J_{24} - 2J_{13}J_{23}|u). \tag{4.3}$$

Thus here also only $g + g_2$ enters in the calculation of the normal state. With wave functions $u = \exp\{-(\nu/2)(r_{12}^2 + r_{13}^2 + r_{23}^2)\}$ for H^3 and $\exp\{-(\nu/2)(r_{12}^2 + r_{13}^2 + r_{14}^2 + r_{23}^2 + r_{24}^2 + r_{34}^2)\}$ for He^4 and with

$$J(r) = Ae^{-\alpha r^2} \tag{4.4}$$

one obtains for H^3

$$(\psi_0|H'^2|\psi_0) = \frac{3}{2}(g+g_2)^2A^2\left[\left(\frac{3\nu}{3\nu+4\alpha}\right)^{\frac{3}{2}} - \left(\frac{\nu}{\nu+\alpha}\right)^{\frac{3}{3}}\left(\frac{3\nu}{3\nu+\alpha}\right)^{\frac{3}{2}}\right] \tag{4.5}$$

and for He^4

$$
\begin{aligned}
(\psi_0|H'^2|\psi_0) = 3(g+g_2)^2A^2 &\left\{\left(\frac{\nu}{\nu+\alpha}\right)^{\frac{3}{2}} + \left(\frac{2\nu}{2\nu+\alpha}\right)^3\right.\\
&\left. -\frac{128\nu^3}{[3(\nu+\alpha)^2 + 10\nu(\nu+\alpha) + 3\nu^2]^{\frac{3}{2}}}\right\}.
\end{aligned} \tag{4.6}
$$

The contributions to (3.2) and (4.3) in the present approximation are due entirely to Y^2 and thus, according to (2.4) and (4.2) originate in interactions between unlike particles. They are thus present even if the form of the interaction is not made to be the same for like and unlike particles. Formulas (4.5) and (4.6) are then correct provided for A the value corresponding to the neutron-proton potential in the ${}^3S$ state is used. The magnitude of the perturbation can be estimated using the approximation

$$\Delta E = -(\psi_0|H'^2|\psi - 0)/E_m, \tag{4}$$

where E_m is a properly taken mean of the perturbing energy levels. For lack of better knowledge of the relative importance of different levels one may use the binding energy for E_m and thus obtain most probably an overestimate of the absolute value of ΔE. Using the units $\hbar(mM)^{\frac{1}{2}}c^{-1}$ for length and mc^2 for energy and letting $\alpha = 16$ the values of ν which minimize $(0|H_0|0)$ are $\nu \sim 10$ for both H^3 and He^4. Eq. (5) gives then $\Delta E \sim -mc^2$ for H^3 and $-0.4mc^2$ for He^4. It should be noted that these estimates refer only to the effect of Y and do not represent the total effect of terms in $(g+g_2)^2$ in an expansion of the energy starting from a value corresponding to a symmetric wave function. Thus if the Hamiltonians

$$\begin{aligned}&T + (1-g-g_2)(J_{12}+J_{13}+J_{23}),\\ &T + (1-g-g_2)(J_{12}+J_{13}+J_{23}+J_{24}+J_{34}),\end{aligned}$$

are used respectively for H^3 and He^4 in order to define unperturbed energy values and wave functions then the values of $(\psi_0|H'2|\psi-0)$ given by Eqs. (4.5) and (4.6) should be multiplied by 2.

Calculations have already been made for Hamiltonians having different symmetries[6]. According to these the difference in energy values of H^3 corresponding to

$$\begin{aligned}H &= T + J^*_{23} + J^*_{13}\\ \text{and}\quad H &= T + \frac{2}{3}(J^*_{13} + J^*_{13} + J^*_{23})\end{aligned}$$

is about $2mc^2$. The quantity $\frac{2}{3}J^*_{12} - \frac{1}{3}J^*_{23} - \frac{1}{3}J^*_{13} = H''$ may be considered as a perturbation which when applied to the symmetric form gives the unsymmetric form. For a symmetric initial wave function

$$(0|H''^2|0) = \frac{2}{3}(0|J^{*2}_{13} - J^*_{13}J^*_{23}|0).$$

Equating $2mc^2$ and $(0|H''^2|0)/E'_m$ it is found that $E'_m \sim 100mc^2$. The energy levels which matter in this connection are those corresponding to pure singlet states in neutrons while the effect of Y is concerned with perturbations by states in which the neutrons are in a triplet condition. Nevertheless it is probable that there is a qualitative similarity between E_m and E'_m Using $E_m = E'_m$ the estimate of perturbations due to triplet neutron states in H^3 drops to $0.14mc^2$. It is thus seen that the second-order perturbations which depend essentially only on $(g+g_2)^2$ are themselves likely to be small. Ordinarily the effect of these perturbations is neglected and the calculations are made using $(1-\frac{1}{2}g)A_{\pi\nu}e_{-\alpha r^2}$ for the average neutron-proton interaction and $A_{\nu\nu}e^{-\alpha r^2}$ for the interaction between like particles. The value obtained for

[6] E. Feenberg and S. S. Share, Phys. Rev. **50,** 253 (1936); E. Feenberg, Phys. Rev. **49,** 273 (1936).

$A_{\nu\nu}$ agrees with that from proton-proton scattering experiments and is equal to $(1-2g)A_{\pi\nu}$. The mass defects of the isotopes of hydrogen and helium are thus in agreement with the interaction energy (1), but give at present practically no information about g, g_1, g_2 except for determining $g+g_2$.

2 Heavy Nuclei

Calculations of any exactness are difficult for heavy nuclei. It is, however, possible to establish some inequalities which are necessary for stability of existing nuclei by means of the statistical model. The density matrices for protons and neutrons will be written $\rho_\pi(x,x')$ and $\rho_\nu(x,x')$. These functions are supposed to involve only the space coordinates. For simplicity it will be supposed that each space state is filled by two like particles having opposite spin directions. The density matrix contains therefore two identical terms for each occupied space state. For diagonal elements the abbreviations

$$\rho_\pi(x) = \rho_\pi(x,x), \quad \rho_\nu(x) = \rho_\nu(x,x)$$

will be used.

For the estimates made below the wave function is approximated by a product of two determinants, one for the neutrons and the other for the protons. Substitution of such a wave function into the variational integral gives an upper limit to the energy. The expression thus obtained is

$$E = T + W^\nu + W^\pi + W^{\pi\nu} \tag{5}$$

where T is the average value of the kinetic energy operator (a sum of the kinetic energies of the individual particles). The quantities W represent the contributions due to (1). The interactions between neutrons give W^ν, those between protons give W^π, and those between neutrons and protons give $W^{\pi\nu}$. It is found that

$$W^\nu = (1-\frac{1}{2}g-(3/2)g_1-2g_2)E^\nu_{ex} + (1-\frac{1}{2}-\frac{1}{2}g+(3/2)g_1+g_2)E^\nu, \tag{6.1}$$

$$W^\pi = (1-\frac{1}{2}g-(3/2)g_1-2g_2)E^\pi_{ex} + (-\frac{1}{2}-\frac{1}{2}g+(3/2)g_1+g_2)E^\pi, \tag{6.2}$$

$$W^{\pi\nu} = (2-g-2g_1-2g_2)E^{\pi\nu}_{ex} + (2g_1+g_2)E^{\pi\nu}, \tag{6.3}$$

where

$$E^{\alpha\beta} = \frac{1}{2}\int \rho_\alpha(x)J(x-x')\rho_\beta(x')dxdx';$$

$$E_{ex}^{\alpha\beta} = \frac{1}{2}\int \rho_\alpha(xx')J(x-x')\rho_\beta(x'x)dxdx', \tag{6.4}$$
$$E^{\alpha\alpha} = E^\alpha; \quad E_{ex}^{\alpha\alpha} = E_{ex}^\alpha; \quad \alpha, \beta = \mu \text{ or } \nu.$$

If the density of particles is so high that a large number of them are on the average within the range of J then the quantities E_{ex} show saturation while the quantities E are proportional to the square of the number of particles. This circumstance may lead to instability. Thus consider W^ν. If the coefficient of E^ν in it is positive, the nuclear energy can be lowered by increasing the number of neutrons N. In particular it would be possible to have nuclei consisting entirely of large numbers of neutrons. For such nuclei the kinetic energy would be proportional to $N^{5/3}r_0^{-2}$ where r_0 is the nuclear radius while the potential energy would vary as N^2 and would be independent of r_0, if r_0 is sufficiently smaller than the range of force. Keeping r_0 constant the kinetic energy varies more slowly with N for high N than the potential energy and therefore a negative potential energy will lead to infinite stability for very high N. A positive coefficient of E^ν in Eq. (6.1) would require, therefore, the existence of infinitely heavy nuclei of high stability. This is contrary to experience and hence[7]

$$1 + g \geq 3g_1 + 2g_2. \tag{6}$$

For a nucleus in which the number of neutrons N is equal to the number of protons Z the density matrices ρ_ν, ρ_π may be taken to be equal. Then

$$W = W^{\pi\nu} + 2W^{\nu\nu} = 4(1 - \frac{1}{2}g - (5/4)g_1 - (3/2)g_2 E_{ex} + (-1 - g + 5g_1 + 3g_2)E, \tag{7.1}$$

where the superscripts π, ν on the E's are dropped. The trial wave function can be made to correspond to an r_0 smaller than the range of J. By increasing N and Z simultaneously, the governing term in W becomes that due to E which varies as N^2. The only other term which varies as rapidly is that due to the Coulomb energy. Since in the present case $N = Z$ the whole energy expression can be considered as a function of N. In order that it be impossible to have stable nuclei of this type with very high N and Z it is necessary to require that the coefficient of N^2 in the energy expression be positive. This condition is

$$1 + g + \frac{6e^2}{5r_0|J(0)|} \geq 5g_1 + 3g_2. \tag{7.2}$$

[7] Condition (7) is equivalent to requiring that for like particles the interaction be of the form $(a + bP^M)J$ where $b \geq 2a$ as is clear from the fact that Eq. (1) is equivalent to an interaction energy $(1 - g - g_1 - 2g_1)P^M J + (g_1 - g)J$ for like particles.'

The nuclear radius ro must be chosen smaller than the range of nuclear forces which is of the order e^2/mc^2. If r_0 is made equal to e^2mc^2 the last term on the left side of (7.2) is 1/58 for $|J(0)| = 70mc^2$. Since r_0 must be made still somewhat smaller in order to bring about the full activity of $J(0)$ this term should be considered as somewhat greater than 1/58, say 1/20. It is nevertheless numerically insignificant. The conditions (7.1) and (7.2) may be summarized, using the empirical value of $g + g_2 = 0.2$, as

$$1.2 = 1 + g + g_2 \geq 3(g_1 + g_2);$$

$$1.2 = 1 + g + g_2 \geq 5g_1 + 4g_2. \qquad (7.3)$$

On these grounds therefore the interaction in triplet states of odd L can be expected to lie between $-0.2J$ and $-J$ for $g + g_2 = 0.20$ and between $0.1J$ and $-J$ for $g + g_2 = 0.25$ (provided that $g_1 + g_2$ is not negative). It is these states that matter in the photoelectric disintegration of the deuteron by high energy γ-rays. For high particle densities with $N > Z$ the value of the variational integral is

$$E = t - N(1 - \frac{1}{2}g - (3/2)g_1 - 2g_2)|J(0)| - Z(3 - (3/2)g - (7/2)g_1 - 4g_2)|J(0)|$$

$$+\frac{1}{4}(N^2 + Z^2)(1 + g - 3g_1 - 2g_2)|J(0)| - NZ(g_1 + \frac{1}{2}g_2)|J(0)| + \frac{3}{5}\frac{Z^2e^2}{r_0}. \qquad (7.4)$$

In order that it be impossible to obtain in this expression contributions varying quadratically with N and Z for a given ratio N/Z it is necessary to require that

$$1 + g \geq 3g_1 + 2g_2 + \frac{2NZ}{N^2 + Z^2}(2g_1 + g_2) - \frac{12}{5}\frac{Z^2}{Z^2 + N^2}\frac{e^2}{r_0|J(0)|}. \qquad (7.5)$$

For $Z = 0$ this gives (7) and for $Z = N$ it gives (7.2). Neglecting the relatively insignificant Coulomb energy all conditions implied by (7.4) are contained in (7.3). These conditions are obtained above by considering a higher particle density than that which exists in actual nuclei. This circumstance does not normally spoil the argument because the energy is lower than the value of the variational integral. The above discussion does not say much about the reaction rate or the values of N and Z at which transformations would occur. It does not make much difference, however, if the possibility of very heavy neutral nuclei is allowed but instead it is required that there should be no neutral nuclei of mass 200 having a greater stability than a collection of equal numbers of protons and neutrons in the form of alpha-particles. The kinetic energy is statistically $11.3mc^2N^{5/3}(e^2/mc^2r_0)^2$. The requirement is then

$$1+g-3g_1-2g_2 \leq -\frac{45e^4N^{-1/2}}{|J(0)|r_0^2mc^2}+\frac{4}{N}(1-\frac{1}{2}g-(3/2)g_1-2g_2)-\frac{54mc^2}{N|J(0)|}. \quad (7.6)$$

By using $|J(0)| = 70mc^2$ and $r_0 = e^2/mc^2$ the right side is -0.09. This condition differs very little from (7). If instead of requiring a smaller stability than that corresponding to groups of alpha-particles one requires a smaller stability than that corresponding to $17mc^2$ mass defect per particle the right-hand side of the last inequality is changed only by -0.001. Similarly if instead of (7.2) it is required that a nucleus with $N = Z$ should not have a greater stability than that corresponding to a mass defect of $17mc^2$ per particle, which corresponds to the stablest nuclei, one obtains

$$1+g-5g_1-3g_2 \geq -\frac{45e^4N^{-1/2}}{|J(0)|r_0^2mc^2}+\frac{8}{N}\left(1-\frac{1}{2}g-\frac{5}{4}g_1-\frac{3}{2}g_2\right)$$
$$-\frac{6e^2}{5r_0|J(0)|}-\frac{68mc^2}{N|J(0)|}. \quad (7.7)$$

For nuclei with $Z = N = 100$ this condition is only slightly weaker than (7.2) since the right side of the last inequality is -0.09 using the same constants as previously. The inequality (7.2) is thus also nearly the condition for not having too much stability in nuclei with ordinary numbers of particles.

Actual heavy nuclei are far from being in a state of high particle density such as was just used. For Hg^{200} E^ν/E^ν_{ex} can be estimated to be approximately 4 using the statistical model and $r_0 = 0.8 \times 10^{-12}$ cm with $\alpha^{-1/2} = 2.2 \times 10^{-13}$ cm. For a very high particle density this ratio would be $N/2 = 60$. Eq. (7.4) is thus not a guide for the actual energy dependence. A too large numerical value of a negative coefficient of E in (7.1) would be harmful to stability but it is impossible to tell without further calculation whether higher approximations reduce the energy sufficiently to allow appreciable positive values of $1 + g - 5g_1 - 3g_2$. If $g_1 = g_2 = 0$ and if $E/E_{ex} = 4$ the potential energy occurring in (7.1) is $-6gE_{ex}$ and is positive. Nevertheless this does not exclude the possibility of a simple theory with $g_1 = g_2 = 0$ because (1) the energy corresponding to an assumed interaction is lower than the value of the variational integral with an approximate wave function; (2) E^π/E^π_{ex} is < 4; (3) exact values of E/E_{ex} are hard to obtain since the uncertain nuclear radius is involved.

In all of the above discussion it was supposed that $J(r)$ has the same sign throughout and complications arising from a reversal of sign near $r = 0$

were not taken up. In proton-proton scattering experiments the kinetic energy is of the order of 1 Mev while in the nucleus the kinetic energy of individual particles is of the order of 30 Mev. The kinetic energy inside the "potential well" representing the interaction of two particles is changed, however, only from ~ 20 Mev to ~ 35 Mev or perhaps ~ 50 Mev in the comparison of scattering experiments with conditions inside the nucleus. This corresponds to a decrease of the wave-length inside the deep part of the "well" by a factor of about 1.3 which existing that existing heavy roughly the same features of $J(r)$ come into consideration in existing existing heavy nuclei as in proton-proton scattering experiments. In the deduction of the inequalities (7.3), however, the size of the nucleus was taken to be smaller than that of an actual nucleus and, therefore, the kinetic energy per particle was increased to a maximum of about $(8 \times 10^{-13}/2.8 \times 10^{-13})^2$ 30 Mev ~ 200 Mev for $N = 100$. Since some of the nuclear particles collide while traveling in opposite directions the relative kinetic energy is changed by a factor of about 10 in the deep part of the "well" which corresponds to a factor 3 in the wave-length. A reversal of sign of J within about 1/3 of e^2/mc^2 will thus begin to affect the conditions (7.6) and (7.7) without seriously affecting the scattering experiments and the mass defect calculations of light nuclei. The conditions (7.3) for infinite N and Z are changed to their opposites for such a reversal.

Although it appears from the above that there is nothing against considering the main part of the interaction of nuclear particles as being the same in form for all particles it is not likely that the interaction law is identical in all approximations. Spin-orbit interactions and spin-spin interactions involve the magnetic moments which are different for protons and neutrons. In such, essentially relativistic approximations it appears necessary to consider the interactions to be different between different kinds of nuclear particles. It is also unlikely that the Coulombian interaction which already destroys the symmetry has no more intimate connection with the specific forces than that given by an additive term in the Hamiltonian.

For the sake of simplicity the same form of $J(r)$ was used in Eq. (1) for $P^M, P^H, 1, P^S$. It is obviously possible to use different space functions as multipliers of these operators. There appears to be at present no call for such a generalization.

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