Production of Mesons as a Shock Wave Problem
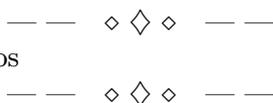
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Translated by Herman Boos

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Abstract

Multi-meson production in two-nucleon collision is described as a shock wave process which is governed by a non-linear wave equation. Since one deals with big quantum numbers, these quantum processes may be approximately described by means of the correspondece principle. Analysing solutions to the non-linear wave equation one can get the energy and angular distribution for different meson sorts.

An analysis of experimental data on the π-meson production obtained in last years tells us that a collision of two nucleons of high energy produces a big number of mesons. The fact that the strong interaction of nucleons with mesons and, in particular, of mesons with themselves results in such a multiple production was established long time ago [4]. In oder to give a quantitative estimation of the energy dissipation in meson field one can compare this process with a turbulence flow [5]. Some other way as Fermi did [3] is to consider the temperature balance in a moment of collision. It allows to calculate the energy distribution of mesons.

The present consideration of the problem is based on the point of view that was suggested by author in 1939 in connection with the Yukawa theory [4]. The meson production is considered as a shock wave process which

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is described by a non-linear wave equation. It will be shown that in this way it is possible to get quantitative results for the spectral and spatial distributions of different meson sorts.

1   *Visual description of a shock wave*

Below the meson production will be considered in the center of mass frame. A transition to the laboratory frame can be easily done [5]. This question will not be discussed here.

a) In the center of mass frame two nucleons move to meet each other in opposite directions (Fig.1) until they cross in some region (shaded area in Fig.1). In this region they strongly interact. The nucleons can be imagined as discs with a thickness which is less than their cross section because of the Lorentz factor $\sqrt{1 - \beta^2}$ (β is a speed of mass center). One can take this cross section to be of order $1.4 \cdot 10^{-13} cm$. In a moment of collision the speed of nucleons change in such a way that in their intersection area the energy is transfered to the meson field. So, in the very beginning when a shock wave just appears the whole energy of the meson field is concentrated in a thin flat layer which was filled by both nucleons in the moment of collision.
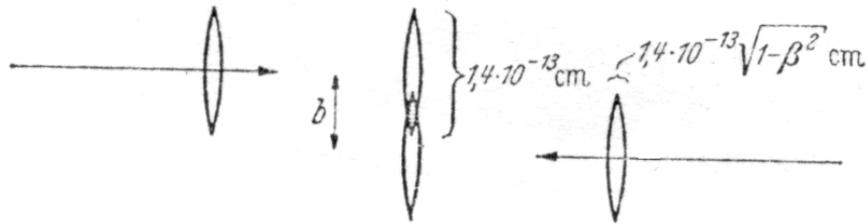

Figure 1:

b) If one could neglect the interaction between the mesons then they would propagate in accordance with a wave equation

$$\Box\varphi - \kappa^2\varphi = 0 \tag{1}$$

(or more complicated linear wave equation that contains different meson sorts). Then also the angular distribution of this wave would not change during it's propagation. These distributions could be determined already in the first moment by means of an expansion of the wave into the Fourier series. Thus we can conclude that in the frequency region from k_0 to $k_0 + dk_0$

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(k_0 is energy of a single meson) the energy of the meson wave does not depend on k_0 up to the frequencies corresponding to such wave lengthes which are comparable with the thickness of the layer where the collision takes place. This thickness is of the order $\frac{\sqrt{1-\beta^2}}{\kappa}$ (κ is the meson mass). When $k_0 > k_{0m} = \frac{\kappa}{\sqrt{1-\beta^2}}$ the intensity abruptly decreases as a function of k_0.

$$d\varepsilon = \text{constant} \cdot dk_0 \quad \text{for} \quad k_0 \leq k_{0m}. \tag{2}$$

For a number of mesons with frequences lying in the region dk_0 one gets

$$dn = \text{constant} \, \frac{dk_0}{k_0} \quad \text{for} \quad k_0 \leq k_{0m}. \tag{3}$$

In Fig.2 is shown the behaviour of the function φ on the axis which is normal to the radiation plane (right after the act of emission). Also shown are functions $\frac{d\varepsilon}{dk_0}$ and $\frac{dn}{dk_0}$ under condition that the equation (1) is fulfilled. The spectrum (3) corresponds to the known spectrum of the Röntgen Bremsstrahlung of the electron. It is also valid in case when the most part of energy of mesons is transfered to the meson field in such a way that the number of outgoing mesons is not too big. It means that the energy of a single meson is approximately $\geq \frac{1}{2}k_{0m}$.
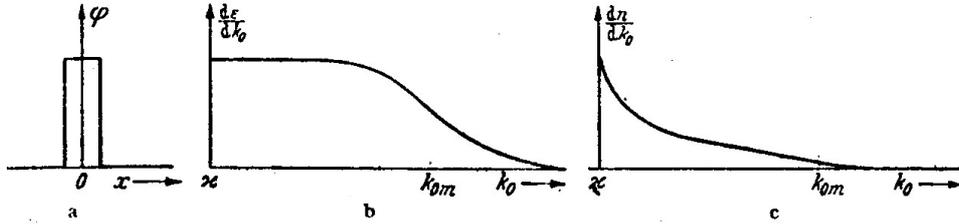

Figure 2: a - c

c) But in reality the interaction of mesons can not be neglected i.e. the wave propagation is going on according to the same non-linear wave equation which only approximately becomes linear in case of a small intensity. As will be shown below the non-linearity results in a slight smoothing the singularity of a wave crest. Due to this, during the wave propagation the energy is transfered from shorter to longer waves. Therefore in the end of this propagation the spectral distribution goes down steeper in comparison with the previous case when the equation (1) is valid. Thus we come to the behaviour shown in Fig.3. The spatial distribution is shown in Fig.4(a-d).
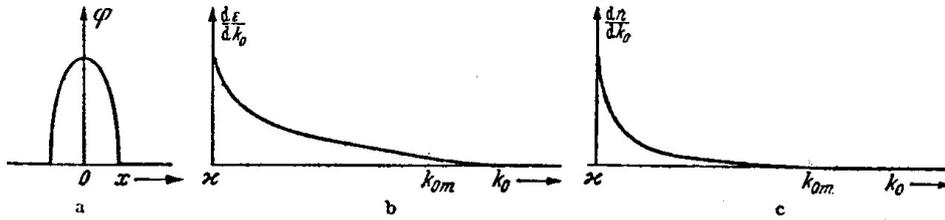
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Figure 3: a - c.

In the moment of collision the whole energy is concentrated in the intersection area of two nucleons (a). After this two shock wave fronts start to propagate to the right and to the left respectively. The most part of the energy is still concentrated in these two fronts. But a wave perturbation also appears in a region between them where the rest of the energy is concentrated (b). When the propagation of the shock wave fronts goes further the perturbation on their wakes captures more and more space which in it's turn becomes a source of a new wave propagation. Then the front energy becomes lower. It is transfered to other wave lengthes, in particular, to longer waves (c). During the further propagation the perturbation in a central area decreases. Thus actually the produced wave moves faster in a direction of the shock wave fronts than in transverse direction. Therefore short waves have a higher group speed. Now perturbation of a very low intensity spreads out into all directions also with the speed of light. The energy of shock wave fronts becomes so low that the non-linearity does not play a big role any more here also. So the further propagation continues according to the usual linear wave equation (d).
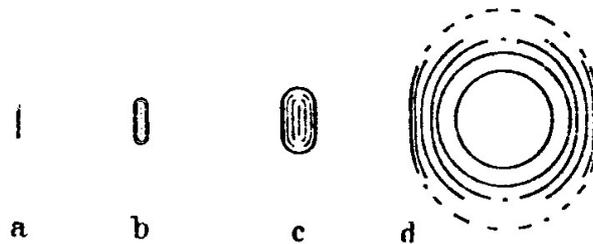

Figure 4: a - d.

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Up to the moment we ignored the quantum effects. This approximation is acceptable because we deal with production of a big number of mesons i.e. with a process with big quantum numbers. How the correspondence principle is applied in quantum theory was described in detail in above mentioned paper [4]. Here it is enough for us to accept some qualitative consideration based on Fig.4d. Namely, the most part of energy is emitted in all directions in form of mesons whose wave lengthes are comparable with the disc cross section i.e. $1/\kappa$. The transverse momentum can be greater than κ only in rare cases so that the Fourier coefficients of that waves are very small. On the other hand, the longitudinal momentum can be greater so that one can face the shorter waves in the shock wave front. The mesons with the energy k_0 will be emitted mostly only into the angle interval of order κ/k_0 with regard to both dominating directions. Also heavier mesons are mostly emitted only into the front of the shock wave.

2   Solution of the shock wave equation

The propagation of the shock wave depends on the form of the non-linear wave equation on which the description of mesons is based. But it is possible to show that there is a special case of "strong" interaction for which one can accept that the spectral distribution of mesons does not depend on the concrete form of the wave equation. The solution of the non-linear equation can be performed by means of a simplification if to be interested only in the spectral and not in the angular distribution. Namely, if the plane where the emission happens tends to be infinite then the layer tends to be infinitely thin. Then because of the Lorentz invariance the function φ can depend only on $s = t^2 - x^2$. Thus the partial differential equation becomes an ordinary differential equation which is simpler to analyse.

Let us discuss two examples of non-linear wave theories:

1. The equation suggested by Shiff [10] and Thirring [12] in connection with description of nuclear forces:

$$\Box\varphi - \kappa^2\varphi - \eta\varphi^3 = 0. \tag{4}$$

2. The wave equation which analogously to earlier works by Born [1] follows from the Lagrange function

$$L = l^{-4}\sqrt{1 + l^4\left[\sum\left(\frac{\partial\varphi}{\partial\kappa_\nu}\right)^2 + \kappa^2\varphi^2\right]} \tag{5}$$

Long time ago Born noted that the non-linear theories of that type have solutions that are less singular in comparison with the solutions to linear equations. That time this fact was used for the description of the electron self-energy. But the same consideration is also acceptable for the multiple production of mesons. In the earlier works [4] the investigation of the meson production was already based on the Lagrange function [5].

To the item 1. The first of two these equations gives for $\varphi = \varphi(s)$:

$$4 \frac{d}{ds}\left(s \frac{d\varphi}{ds}\right) + \kappa^2\varphi + \eta\varphi^3 = 0. \qquad (4a)$$

When $\eta = 0$ one comes again to the linear equation (1) and then it's solution looks as follows:

$$\left.\begin{array}{llll}\varphi = aJ_0\left(\kappa\sqrt{s}\right) & \text{for} & s > 0 \\ \varphi = 0 & \text{for} & s < 0,\end{array}\right\} \qquad (6a)$$

where a is an integration constant; also compare it with Fig.2. For $\eta \neq 0$ one can write down a power series at the point $s = 0$.

$$\left.\begin{array}{ll}q = a[1 - (\kappa^2 + \eta a^2)s + \frac{1}{4}(\kappa^2 + 3\eta a^2)(\kappa^2 + \eta a^2) & s^2 - + \ldots] = 0 \\ & \text{for} \quad s > 0 \\ & \text{for} \quad s < 0.\end{array}\right\} \qquad (6b)$$

Let us note that the equation (4) corresponds to some "weak" interaction which does not change the shock wave front if the wave function is continuous. It is connected with the fact that the theory based on eq. (4) belongs to the group of renormalizable theories because the coupling constant η is dimensionless. It was established in different ways that renormalizable theories describe only "weak" interactions which in general do not result in the multiple production of mesons.

To the item 2.

However the situation with the wave equation that follows from the formula (5) is different. For $\varphi = \varphi(s)$ this wave equation looks as follows:

$$4 \frac{d}{ds}\left(s\varphi'\right) + \kappa^2\varphi = 8l^4 s\varphi'^2 \frac{\varphi' + \kappa^2\varphi}{1 + l^4\kappa^2\varphi^2}. \qquad (7)$$

If $\kappa = 0$ (zero meson mass) then the solution may be readily found:

$$\left.\begin{array}{llll}\varphi & = \frac{1}{a}\lg\left(1 + \frac{a^2}{2l^4}s + \frac{a}{2l^4}\sqrt{4l^4 s + a^2 s^2}\right) & \text{for} & s \geq 0 \\ & = 0 & \text{for} & s \leq 0.\end{array}\right\} \qquad (8)$$

In general case ($\kappa \neq 0$) one can apply again the power series expansion. Let us take

$$\varphi = \frac{1}{\kappa l^2} \, f(\zeta); \qquad \zeta = s\kappa^2 \tag{9}$$

and we write

$$\left.\begin{aligned}
f(\zeta) \; &= \sqrt{\zeta}\left(1 + a\zeta + \frac{27a^2 + 2a - 1}{10} \, \zeta^2 + - \ldots\right) && \text{for} \quad \zeta \ll 1 \\
&\approx \gamma \zeta^{-1/4} \cos(\sqrt{\zeta} + \delta) && \text{for} \quad \zeta \gg 1 \\
&= 0 && \text{for} \quad \zeta \leq 0.
\end{aligned}\right\} \tag{10}$$

The constants γ and δ are completely fixed via the integration constant a. However it's value was not evaluated.

One can note that the non-linearity essentialy changes the character of the solution. Namely, at $s = 0$ the discontinuity of φ disappears. More exactly, φ' has a discontinuity while φ behaves as $\sqrt{s}$ in a small vicinity of $s = 0$.

Let us assume that to a given moment of evolution the function $\varphi(s) = \varphi(x,t)$ is defined by the Fourier integral over the wave number k. Then for the Fourier coefficient $\varphi(k,t)$ with $k \sim k_0 \gg \kappa$ one obtains up to some constant

$$q(k,t) \sim k^{-3/2} t^{1/2} e^{\pm ik_0 t}. \tag{11}$$

It is easy to understand that the multiplier $t^{1/2}$ appears due to the fact that during the propagation process the energy is transfered from the head part of the shock wave to other parts of the wave and so to lower frequences also. Actually the energy potential in the head part of the shock wave is infinite. This is just a consequence of our assumption that the shock wave appears inside an infinitely thin layer. By means of this assumption we have achieved that the solution $\varphi(x,t)$ depends only on combination $t^2 - x^2$ which is invariant under the Lorentz transformation in the x, t-space. Some finite energy-momentum would define some direction in this space. Therefore in this case it could not lead to some invariant solution.

However in reality the shock wave appears in a layer with a finite thickness $\sim \frac{\sqrt{1-\beta^2}}{k}$. Therefore the energy-momentum vector is finite. The increasing of the Fourier amplitudes in (11) starts with some delay. When the energy potential concentrated in the head part of the shock wave is exhausted it comes to the state of rest. Then for a big t the Fourier coefficients as a function of k for $k > k_{0m} = \frac{\kappa}{\sqrt{1-\beta^2}}$ fall down faster than $k^{-3/2}$. Thus

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for the intensity distribution one gets

$$\frac{d\varepsilon}{dk_0} = \text{const} \ \frac{dk_0}{k_0} \quad \text{for} \quad \kappa \le k_0 \le k_{0m} = \frac{\kappa}{\sqrt{1 - \beta^2}} \tag{12}$$

and

$$\frac{dn}{dk_0} = \text{const} \ \frac{dk_0}{k_0^2}. \tag{13}$$

for the same region.

This is the form of the spectrum that was earlier suggested in connection with the multiple production of mesons [4], [5] and it was also shown in Fig.3.

The wave equation (7) borrowed from the Born theory [1] is the typical case of "strong" interaction and describes the multiple production of mesons. Here the coupling constant has a dimention of length in the fourth power.

b) Now it is necessary to show that the spectrum (12) and (13) in general corresponds to the strong interaction independent of a concrete form of the Lagrange function and of the properties of participating particles.

We start with an arbitrary Lagrange function which depends on some scalar wave function φ and it's first derivatives $\frac{\partial}{\partial x_\nu}$. Because of the Lorentz invariance L can depend only on φ and on $\sum_\nu (\frac{\partial}{\partial x_\nu})^2$. For very small values of φ and $\frac{\partial}{\partial x_\nu}$ L should reduce to the Lagrange function of the usual wave equation (1). Now we would like to know the value of $\frac{\partial}{\partial x_\nu}$ in a small vicinity of $s = 0(s > 0)$. For $s \to 0$ $\sum_\nu (\frac{\partial}{\partial x_\nu})^2$ can be either infinite or finite or even tend to zero. First one can exclude the last of these three possibilities because in this case right at the critical point $s = 0$ the non-linearity would not play any role. But this is impossible because for the usual wave equation (1) $\sum_\nu (\frac{\partial}{\partial x_\nu})^2$ can never be equal zero in the critical point but is infinite.

Among the both residual possibilities actually only the second one gives a smooth behaviour of φ at the singular point. Therefore it may correspond to the strong interaction. In the vicinity of $s = 0$ it becomes

$$\sum \left(\frac{\partial \varphi}{\partial x_\nu}\right)^2 = -4s \left(\frac{\partial \varphi}{ds}\right)^2 = \text{const} \ (\ne 0 \quad \text{and} \ \ne \infty). \tag{14}$$

from which one gets

$$\varphi(s) \sim \text{const} \ \sqrt{s}, \tag{15}$$

and also expressions like (7) and (10)-(14).

c) But for (12) and (13) one can give more general reasons that are acceptable also for arbitrary particles with higher spin. Already in item 11a it was mentioned that in that special case where the shock wave appears

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within an infinitely thin layer it's total energy should be infinite. In this case the wave function is invariant with regard to the rotations in the x, t-space. The bigger is the energy dissipation caused by the interaction the steeper falls down the energy spectrum of mesons. Since the spectrum has a form that exactly corresponds to the potential law (and it should be true for a majority of simple wave equations) it can not fall down faster than in (12) and (13). The reason for this is that the total energy is divergent for (namely logarithmically) for $k_{0m} \to \infty$. Thus the spectrum (12) and (13) just corresponds to the special case of the strong interaction. As was mentioned above the Lagrange function (5) that comes from the Born theory gives only one special example of a theory for the strong interaction. But also for many much more complicated Lagrange functions which in the case of a weaker interaction contain as a solution different sorts of mesons the spectrum (12) and (13) remains valid when one deals with a theory of strong interaction.

3 Application to the meson production.

Now the multiple meson production should be quantitatively described under the condition that we deal with the strong interaction.

a) One of the most important values for characterizing the meson showers is the average energy of mesons in the center of mass frame. In a very rough approximation one can take the spectrum (12), (13) as the exact one between $k_0 = \kappa_i$ (the rest meson mass for the corresponding sort of a meson) and $k_0 = k_{0m}$.

Then one has

$$
\left.
\begin{aligned}
\varepsilon_i &= A_i \int\limits_{\kappa_i}^{k_{0m}} \frac{dk_0}{k_0} = A_i \lg \frac{k_{0m}}{\kappa_i} \\
n_i &= A_i \int\limits_{\kappa_i}^{k_{0m}} \frac{dk_0}{k_0} = \frac{A_i}{\kappa_i} \left(1 - \frac{\kappa_i}{k_{0m}} \right)
\end{aligned}
\right\}
\tag{16}
$$

and therefore

$$
\bar{k_{0i}} = \frac{\varepsilon_i}{n_i} = \kappa_i \frac{\lg \frac{k_{0m}}{\kappa_i}}{1 - \frac{\kappa_i}{k_{0m}}} \quad \text{for} \quad k_{0m} > \kappa_i.
\tag{17}
$$

For $k_{0m} \le \kappa_i$ the meson of the corresponding sort would not produced at all.

In reality the spectrum should contain the factor kdk_0 because of the phase space volume. Therefore for small k it does not have the form (12),(13) at all. Moreover for $k_0 > k_{0m}$ the influence of it will not completely disappear but only cause a faster falling down than in (12) and (13). One can try some probably a bit better approximation

$$d\varepsilon_i = A_i \; \frac{kdk_0}{k_0^2 \left(1 + \frac{k_0^2}{k_{0m}^2}\right)}.$$ (18)

Then one gets

$$\left. \begin{array}{l} \varepsilon_i = A_i \left(-1 + \sqrt{1 + \alpha^2}\lg \frac{1 + \sqrt{1 + \alpha^2}}{\alpha}\right) \\[2mm] n_i = \frac{A_i}{\kappa_i} \frac{\pi}{4} \left(1 + 2\alpha^2 - 2\alpha\sqrt{1 + \alpha^2}\right). \end{array} \right\}$$ (19)

$$\bar{k_{0i}} = \kappa_i \frac{4}{\pi} \frac{-1 + \sqrt{1 + \alpha^2} \; \lg \frac{1 + \sqrt{1 + \alpha^2}}{\alpha}}{1 + 2\alpha^2 - 2\alpha\sqrt{1 + \alpha^2}},$$ (20)

where we have put $\kappa_i/k_{0m} = \alpha$.

Both approximations (17) and (20) are shown in Fig. 5 as functions of $\log(1/\alpha)$. The difference between both of these curves shows the uncertainty of the whole estimation.

From these calculations follows that in the special case of the strong interaction the average meson energy increases only logarithmically as a function of the initial energy. Thus the meson number grows almost proportionally to the energy which was transfered to the meson field in the center of mass frame.

b) Nevertheless for the higher energies the above behaviour becomes more complicated because of new meson sorts that come to the game. One can assume that for a big enough value of k_0 ($k_0 \gg \kappa_i$) the relative share g_i of the meson sort κ_i becomes independent of k_0 and depends only on the form of the shock wave equation. Hereby in this region it would be allowed for different meson sorts to appear in general with comparable frequencies. However g_i are not necessarily proportional to the statistical weight of the corresponding meson sort. Let us normalize them as follows

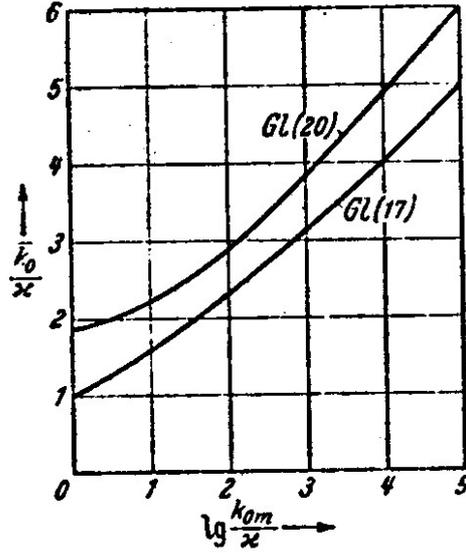$$\sum g_i = 1$$ (21)

and then take

$$A_i = g_i A.$$ (22)

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Figure 5:

Then the following rough approximation for (16) and (17) is valid

$$\varepsilon = A \sum g_i \lg \frac{k_{0m}}{\kappa_i}, \tag{23}$$

$$n = A \sum \frac{g_i}{\kappa_i} \left(1 - \frac{\kappa_i}{k_{0m}}\right), \tag{24}$$

Thus

$$\left.\begin{aligned}
n_i \;\; &= \varepsilon \; \frac{\frac{g_i}{\kappa_i}\left(1 - \frac{\kappa_i}{k_{0m}}\right)}{\displaystyle\sum_l g_l \lg \frac{k_0 m}{\kappa_i}} \quad \text{for} \quad \kappa_i \leq k_{0m} \\[4mm]
&= 0 \qquad\qquad\qquad \text{for} \quad \kappa_i \geq k_{0m}.
\end{aligned}\right\} \tag{25}$$

We get that for the value k_{0m} $(k_{0m} \gg \kappa_i)$ the multiplicities in different meson groups behave in the same way as g_i/κ_i. Therefore when k_{0m} decreases the number of heavier mesons falls down faster than the corresponding number of lighter mesons. Once k_{0m} becomes less than the value κ_i the corresponding meson sort completely disappears. So in the approxi-
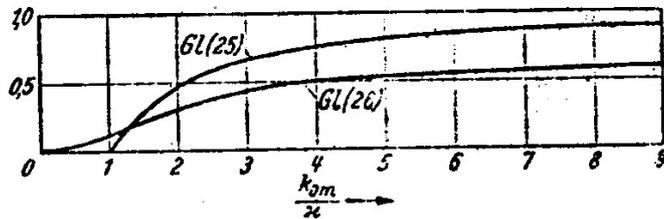
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Figure 6:

mations of eqs. (18)-(20) the following relation

$$n_i = \varepsilon \; \frac{\frac{g_i}{\kappa_i} \frac{4}{\pi} \left(1 + 2\alpha_i^2 - 2\alpha_i \sqrt{1 + \alpha_i^2} \right)}{\sum g_i \left(-1 + \sqrt{1 + \alpha_i^2} \; \lg \frac{1+\sqrt{1+\alpha_i^2}}{\alpha_i} \right)}.$$ (26)

would be valid instead of (25).

The factor before g_i/κ_i which charaterizes the dependence of n_i on k_{0m} in eqs. (25) and (26) respectively is graphically shown in Fig. 6. For the second approximation formula when $k_{0m} < \kappa_i$ the number of remaining mesons of the sort κ_i would become even smaller. One may expect this from the physical point of view.

c) In order to give an estimation of the total number of the emitted mesons one should also know the total energy ϵ of the meson field in (25) and (26) respectively. As a first step one can take only the maximal value. Namely, the energy can not be greater than the kinetic energy of both nucleons in the center of mass frame before the collision.

Since, generally speaking only a part of this energy is transfered to the meson field it would be reasonable to call this part γ a "degree of the inelasticity" of the collision. Then it is valid (M = the nucleon mass)

$$\varepsilon = \gamma \cdot 2M \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right).$$ (27)

where

$$0 \leq \gamma \leq 1.$$

One can expect that in the central collision γ should be close to 1 while for the peripherical collision only a small part of the total energy will be transfered to the meson field.

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Let us take b for the distance between the nucleons in a moment of the collision then one can consider the overlapping integral of the π-meson field for both of the nucleons as a measure of the interaction intensity. As a very rough approximation for the inelasticity degree γ one can just take it proportional to this overlapping integral. Then one gets

$$\gamma = e^{-b\kappa}, \tag{28}$$

where κ is the mass of the π-meson. From this one can get the differential cross section for the interval between γ and $\gamma + d\gamma$

$$d\sigma = 2\pi b db = \frac{2\pi}{\kappa^2} \frac{d\gamma}{\gamma} \lg\left(\frac{1}{\gamma}\right). \tag{29}$$

In order to evaluate the total cross section one should define a minimal value of γ. For instance, in order to determine the total cross section for the multiple meson production one should take for a minimal value of γ the value for which at least two mesons can be produced.

$$\gamma_{\min} = \frac{\bar{k}_0}{M\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right)}. \tag{30}$$

($\bar{k}_0$ corresponds here to the lightest meson sort, namely, to the π-mesons.)

From (30) follows:

$$\sigma = \frac{\pi}{\kappa^2} \lg^2 \gamma_{\min} \tag{31}$$

and

$$\bar{\gamma} = \frac{2}{\lg^2 \gamma_{\min}} \left(1 - \gamma_{\min} + \gamma_{\min} \lg \gamma_{\min}\right). \tag{32}$$

Let us note that the estimation for the γ-distribution which was used in eqs. (28)-(30) does not depend on previous observations for the propagation of the shock wave. Therefore it should be considered as less reliable. The experimental data that we have for the moment are not enough for the experimental determination of the γ-distribution.

In the following Table 1 there are shown the data on the total cross-section, expectation values of γ, n_π and n_κ (the number of π- and κ-mesons respectively),

Table 1

E	10	10^2	10^3	10^4	BeV
σ	0.18	0.49	0.85	1.3	10^{-24} cm^2
$\bar{\gamma}$	0.34	0.19	0.13	0.09	
$\bar{n}_\pi$	3.6 ± 0.7	4.2 ± 0.8	5.2 ± 0.8	8.0 ± 1	
$\bar{n}_\kappa$	–	0.9 ± 0.2	2.0 ± 0.4	3.4 ± 0.6	
$\bar{k}_{0\pi}$	0.25 ± 0.04	0.36 ± 0.04	0.50 ± 0.05	0.67 ± 0.06	BeV
$\bar{k}_{0\kappa}$	–	1.0 ± 0.2	1.4 ± 0.15	2.0 ± 0.18	BeV
$\gamma = 1$ $\left\{\rule{0pt}{12pt}\right.$ $\bar{n}_\pi$	10.7 ± 2	22.1 ± 4	40.3 ± 6	89 ± 12	
$\bar{n}_\kappa$	$--$	4.7 ± 1	15 ± 6	38 ± 6	

their average energy and also the number of mesons in a special case $\gamma = 1$ as the function of the initial energy E (in the laboratory frame). Other sorts of mesons, exept π- and κ-mesons, were not taken into consideration. Then it was assumed that $g_\kappa = 2g_\pi$ i.e. $g_\pi = \frac{1}{3}, g_\kappa = \frac{2}{3}$ in order to take into account a relatively more frequent appearance of κ-mesons in accordance with the recent measurements in Bristol. These values are still to be checked on the basis of more precise measurements. For the mass of the κ-meson it was taken 0.61 BeV. In order to take into account an uncertainty of the theoretical estimation each time (except the two first columns) it was taken the mean-value of the expressions (16), (17) or (18) until (20). For the error the half-difference has been taken.

d) The angular distribution of the emitted mesons follows from a visual consideration of the Section 1. However the details of the angular distribution will also depend on the shock wave equation. Nevertheless generally speaking the transverse to the initial direction component of the momentum of mesons can drastically exceed the value κ only in rare cases. The mesons with the energy k_0 are emitted mostly into the interval of angles of order k/k_0 to the axes. Therefore the distribution of κ-mesons is always anisotropic while the distribution of slower π-mesons in the center of mass frame can be to some extent isotropic.

4 The comparison with the experiment.

So far only some of meson showers without gray or black tracks have been observed. Thus only for these showers it is possible to assume that one deals with the collision of only two nucleons without participation of a bigger nucleus. If the showers with a small number (until 3) of thick tracks will be

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also involved into the experimental consideration then generally speaking the change of shower due to nuclei should be small.

Table 2

E	30	40	40	90	130	1000	2000	30000 BeV
$n_\pi - n_\kappa$	9	18	25	10	18	9	12	21
1/emp	0.51	0.8	1.0	0.38	0.61	0.16	0.17	0.1

However, since the secondary scattering of the emitted mesons on the atomic nuclei may happen the determination of the initial energy from the angle distribution and evaluation of the angle distribution itself become rather uncertain.

The observations of showers that are suitable for comparison with the theory were done in works of Teucher [11], group of Bristol [2], von Shein and coauthors [9], Pickup and Voyvodic [8] and Hopper, Biswas and Derby [6]. If according to the published data one tries to estimate initial energies from the angle distribution (this is in some cases very uncertain) then one can gets the numbers of mesons for the eight observed showers which were showed in the second line of the Table 2 (these numbers may be slightly different because of neutral mesons). Hereby it was assumed that the relation of neutral mesons to the charged ones is $1 : 2$. Assuming that two last lines in the Table 1 are correct one gets some empirical value of γ for each of these showers. These values are shown in the third line of the Table 2.

First one can see that these numbers of mesons are not well-defined functions of the initial energy. The values of γ fluctuate very strongly, as was expected. However they are on average a bit bigger than it could be expected from the Table 1. This can be based on the fact that a small shower is easier to miss than a big one. But this may also mean that the estimation in the equation (28) is still too rough[1]. On the other hand the empirical γ-values themselves in the Table 2 are still rather uncertain because, for instance, the part of κ-mesons is not exactly known. Also Perkins [7] communicated

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[1] *Noted during proof-reading* In the conference in Kopenhagen, June 1952, Le Couteur showed that average value γ in a heavier matter (as, for example, in a photo emulsion) should be essentialy bigger than in the Hydrogen (in the Table 1 are shown the data for the Hydrogen) because peripherical collisions happen only for those nucleons which are placed on the surface of an atomic nucleus. Then Powell has reported about new experiments which indicated that the particles called here κ-mesons decay into the two groups of mesons with masses 0.74 and 0.54 BeV respectively. These mesons have absolutely different properties.

about relatively higher γ-values. But it is still to wait for more experimental data.

Two showers (Teucher [11] and Hopper, Biswas and Derby [6]) could be measured so precisely that the average energy of mesons in the center of mass frame could be found. In the first case (40 BeV, approximately 25 mesons) the average energy of mesons is 0.29 BeV which is comparable with 0.31 BeV from the Table 1. In the second case (1000 BeV, approximately 9 mesons) there is some uncertainty due to a possibility that some of the observed particles can be κ-mesons. This possibility was not taken into account by the authors (from the Table 1 it could be expected that among 9 mesons are to expect about 3 κ-mesons). If one does not take this into consideration then the measured average energy of π-mesons in the center mass frame is 0.44 BeV which is comparable with 0.50 BeV from the Table 1 also. Thus both of this measurements confirm relatively low meson energies from the Table 1. On the other hand Perkins [7] takes the value 1.5 BeV as the average energy of mesons from a group of showers with initial energy from 10^2 untill 10^3 BeV. It is essentialy higher. However one needs to take into consideration some uncertainty in the measurement of the initial energy. Any error of the initial energy in general increases the average energy of mesons because it has a minimum value just in the center of mass frame.

Concerning the frequency of κ-mesons there exists only one result obtained by the Bristol group which tells us that at high energies it is comparable with the frequency of π-mesons [7]. However it is still impossible to determine this ratio from the theory (in the Table 1 it was taken $g_\kappa/g_\pi = 2$ just ad-hoc).

Concerning the angular distribution it is observed that in the center of mass frame the angular distribution is almost isotropic for showers of slow energy. Meanwhile for showers of a higher energy one can clearly observe the accumulation of events in the forward and backward directions. It exactly corresponds to the picture from the item Ic. Actually the mesons at high energy seem to be distributed always anisotropically, in particular, the κ-mesons (Perkins [7]). And also the degree of the anisotropy corresponds to the theoretical estimation.

In general, the impression is that for the "strong" interaction the formulae from the Section III satisfactory describe the experimental data. Thus the interaction of elementary particles at high energy really belongs to the group of "strong" interactions first studied by Born.

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References

[1] Born, M.: Proc. Roy. Soc. Long., Ser. A **143**, 410 (1933); Born, M. and L. Infeld: Proc. Roy. Soc. Long., Ser. A **144**, 425 (1934); **147**, 522 (1934); **150**, 141 (1935).

[2] Camerini, U., P.H. Fowler, W.O. Lock and H. Muirhead; Phil. Mag., (7) **41**, 413 (1950)

[3] Fermi, E.: Progr. theor. Phys. **5**, 570 (1950).–Phys. Rev. **81**, 683 (1951).

[4] Heisenberg, W.; Z. Physik **113**, 61 (1939)

[5] Heisenberg, W.; Z. Physik **126**, 519 (1949)

[6] Hopper, V.D., S. Biswas and J.F. Derby; Phys. Rev. **84**, 457 (1951)

[7] Mr. Perkins has kindly informed us about the results of his recent work before it's publication.

[8] Pickup, E., and L. Voyvodic; Phys. Rev. **82**, 293 (1951); **84**, 1190 (1951).

[9] Schein, M., J.J. Lord and J. Fainberg; Phys. Rev. **80**, 970 (1950); **81**, 313 (1951)

[10] Schiff, L.J.; Phys. Rev., **84**, 1 (1951)

[11] Teucher, M.; Naturwiss, **37**, 260 (1950); **39**, 68 (1952).

[12] Thirring, W.; L. Naturforsch. **7a**, 63 (1952)