Mass
& *Energy*

by
Q. ter Spill

English corrected by
P. Chesmond

's Gravesande
Institute of Physics Education

Jan van Houtkade 26a, 2311 PD Leiden
Netherlands

tel+fax+answ.m. 071 5142099

CONTENTS

§ 1

Mass and Energy

INTRODUCTION

"Mass disappears and energy appears" is found as an explanation of $E = m \cdot c^2$ in secondary schoolbooks. This is an incorrect statement. The released energy has mass of itself, exactly the mass which seemingly disappeared. The formula $E = m \cdot c^2$ has been explained wrongly up until now.

In September 1905 the first publication on the relation between mass and energy appeared in the form in which it has become generally known: $E = m \cdot c^2$. The author was Einstein.

Even before 1905 some scientists started to suspect that energy, at least in some of its forms, should have mass. They calculated that the electrical field of a charged body should have inertia and therefore mass. This implies a relationship between mass and energy, because an electrical field is thought to be the residence of energy. These calculations finally revealed some inconsistencies in the theory of electromagnetism but did not lead to the correct relationship between mass and energy, although they were not too far off the mark.

At this time Lorentz too came to realize that energy possesses inertia. He had concluded that in certain interactions between charged particles Newton's third law could no longer be valid, unless mass is attributed to the electrical fields.

Einstein was the first to give the proper energy-mass relation. From a thought-experiment, an object that emits two light waves of equal frequency and energy in two opposite directions, he derived $E = m \cdot c^2$.

Nowadays this formula has become one of the best known in physics and can be found in every secondary schoolbook on physics.

Yet $E = m \cdot c^2$ is incorrectly explained. One takes m to be the rest mass and one subsequently arrives at confusing and inconsistent statements, which will give the discerning user of $E = m \cdot c^2$ less rather than more insight into the processes in which this formula plays a role.

Actually the point is that one attributes to rest mass a leading role in these processes, whereas relativistic mass is seen as . . . . well, as what in fact? As a mathematical trick perhaps. As an abstract notion, which at best aids the mathematics somewhat without representing a physical reality. A striking example of this concept is to be found in an article in Physics Today (*The concept of mass*, june 1989, page 31), written by Lev Okun, head of the laboratory of elementary-particle theory at the Institute of Theoretical and Experimental Physics, in Moscow, Russia. In this article Okun states, the m in $E = m \cdot c^2$ represents the rest mass and certainly not the relativistic mass. This point of view is most unfortunate, to such a degree that I would call it wrong. It is precisely the relativistic mass, which plays the leading role in relativistic processes and which we have to substitute in $E = m \cdot c^2$.

An argument often heard in favour of the importance of rest mass is its invariance when Lorentz transformations are applied. Opposed to this is the fact that the sum of relativistic masses in a system is constant (relative to an inertial system) as long as no work is done on the system by the outside world. It should be noted that in that case the mass of the fields (we shall later see that fields possess mass also, namely their energy divided by c^2) should also be added to these relativistic masses. I will therefore from now on define these fieldmasses as relativistic masses. Thus it can be stated: if the total relativistic mass of a system is conserved, so is its total energy. This is again a powerful argument in favour of the relativistic mass (and against Okun's statement).

"Rest mass is important, when we want to ascertain the identity of an elementary particle" is often heard as an argument. However, the relativistic mass at a given speed can serve this purpose just as well. This amounts to the same thing as using rest mass, because this is nothing other than the relativistic mass at speed zero.

Restmass is a special case of relativistic mass and the latter is the only successor to the classical mass, that is to say the mass *which obeys Newton's three laws*, and which can be used as a measure of the quantity of matter and which is conserved for a closed system.

The aim of the argument that now follows is to make plausible that:

1[e] statements as "mass can be converted into energy" are not true in their generality.

2[e] in $E = m \cdot c^2$ the m represents relativistic mass and not the rest mass.

3[e] the relativistic mass obeys Newton's three laws, so the Newtonian and relativistic dynamics appear to be one and the same, as soon as we admit fields to have relativistic mass too.

*

§ 2

$E = mc^2$

misleadingly explained in secondary education

From now on I will indicate rest mass by m_{rest} and relativistic mass by m_{rel}. Only in the formula $E = m \cdot c^2$ for the time being no index has been given to *m*, because one disagrees about which *m* we are dealing with here. The intention in this paper is, amongst other things, to make clear that we are, in fact, dealing with m_{rel}.

A clear example of the misleading explanation of $E = m \cdot c^2$ which is given in Dutch secondary schoolbooks, goes as follows: '. . . when mass disappears energy appears. If on the other hand energy disappears, mass is created.' Other books suggest a similar thing, although mostly not so explicitly stated. The problem with this statement is that the author nowhere says whether he means rest mass or relativistic mass. But in both cases the statement is incorrect, at least in certain situations.

For the following argument it is important to distinguish clearly between the various kinds of mass known in the theory of relativity. Three kinds of mass are defined:

Definition 1: *Restmass of a body is the mass measured in an inertial system moving along with the centre of mass of the system.*

Definition 2: *Invariant mass of a system is the mass of a system consisting of relative to each other moving parts, measured in an inertial system moving along with the centre of mass of the system.*

Definition 3: *Relativistic mass relative to an inertial system is restmass, or invariant mass, times* $g_{\mathbf{M}}$. *Herein* $g_{\mathbf{M}}$ *stands for* $1/\sqrt{1 - v_{\mathbf{M}}^2 / c^2}$, $v_{\mathbf{M}}$ *being the velocity of the centre of mass with respect to that inertial system.*

When measuring rest mass velocities should be used of magnitudes approaching zero in order to avoid relativistic effects.

In the case of a system of free particles which we can regard as point masses, we can calculate the invariant mass of this system from the rest masses of the material particles and the photons;

$$m_{\mathbf{invar}} = \Sigma\ m_{\mathbf{rest},i} \cdot g_i + \Sigma\ E_{\mathbf{photon},j} . c^{-2}, \quad \textbf{with} \quad g_i = 1/\sqrt{1 - v_i^2 / c^2} \qquad (1)$$

in which v_i and $m_{\text{rest},\ i}$ are respectively the velocity and the rest mass of the i-st material particle, and $E_{\text{photon},\ j}$ the energy of the j-st photon, all measured with respect to the centre of mass of the system.

Definitions 1, 2 and 3 demand a definition of the position of the centre of mass. It will be given here only for a system of free particles, which can be considered as point masses:

Definition 4:

$$\vec{r}_{\mathbf{M}} = \frac{\Sigma m_{\mathbf{rest},i} \cdot g_i \cdot \vec{r}_i + \Sigma E_{\mathbf{photon},j} \cdot c^{-2} \cdot \vec{r}_j}{\Sigma m_{\mathbf{rest},i} \cdot g_i + \Sigma E_{\mathbf{photon},j} \cdot c^{-2}}$$

where $\vec{r}_i$ and $\vec{r}_j$ are the positions of the i-st material particle and the j-st photon respectively.

If on the system no forces from the outside world are exerted two conservation laws are valid with respect to a arbitrary inertial system:

1e *the invariant mass is conserved.*
2e *the velocity of the centre of mass does not change.*

The last law can be deduced from the law of conservation of momentum.

Definition 1 is familiar to everyone who knows something about the theory of relativity. Normally the restmass (of a body considered as a point mass) is simply measured by weighing. The relativistic mass of a body considered as a point mass, can be defined by definition 3, that is to say as the restmass times g . These definitions are in agreement with Mach's definition of classical mass. Mach defined classical mass in the following way. The unknown mass and a known testmass, both at rest, are placed side by side with a compressed spring of negligible mass between them (of course, the spring can be replaced by any force interaction between the two masses, for instance by the coulomb force). Forces from the outside world are absent. Now the spring is released and both masses move away from each other in opposite directions. After an arbitrary time has elapsed, their distances from their common point of departure are measured simultaneously. When we define the ratio of their masses as being equal to the ratio of their distances, the unknown mass is defined as well. This procedure can be carried over literally to the theory of relativity in order to define the rest mass of a point shaped body. A necessary extra precaution is to choose the force of the interaction to be so small, that velocities stay in the non-relativistic region. For the relativistic point mass, Machs definition can also be used. Now a grazing collision between the unknown relativistic point mass (which is moving at full speed) and the standard mass (which is at rest) must be arranged. Afterwards, the distances of the masses from the carrier line of the initial velocity must be measured. In this case only for the test mass care has to be taken, that it does not acquire a relativistic velocity. For the rest, the procedure is the same as that described for the rest mass.

For the invariant mass and the relativistic mass in the case of a system of free particles, Mach's method can also be used by applying it to each particle individually and then calculating the position of the centre of mass before and after all the collisions with the formula of definition 4 (see page 6).

Of course in practice, Mach's method is too difficult. However, it is important to point out that the definitions of rest mass, invariant mass, relativistic point mass and relativistic non-point mass are in agreement with Mach's definition. In practice, the restmass of a point shaped body is measured by weighing. The relativistic mass of the same object at speed v is calculated by multiplying its restmass with g .

The invariant mass of a system of free point masses can be found in practice by weighing each point mass at rest and then calculating the invariant mass by formula (1). The invariant mass of a coherent system such as a solid object or an ion still having some electrons, can also be found by weighing it at rest (and this time there is of course no need for formula (1)).

The essence of the invariant mass is reflected by definition 3. The relativistic mass of the system when the centre of mass has velocity v_M can be calculated by imagining the whole of the invariant mass as being concentrated in the centre of mass. So the relativistic mass is found by multiplying the invariant mass by $g_{\mathbf{M}}$. The correctness of this statement can be proven for a system of free point masses out of the linearity of the Lorentz transformation. Then, for the invariant mass of this system, we find the expression on the right hand side of formula (1) on the previous page. Formula (1) can also be used as a definition of invariant mass. Note that in formula (1) the relativistic mass again plays an important role; we find the invariant mass by adding the relativistic masses, as measured relative to the centre of mass of the system. If photons are also involved, we have to add for each photon a term E_{photon}/c^2, or $h.f/c^2$, with f measured relative to the centre of mass of the system. Because of this I will in future call E_{foton}/c^2 the relativistic mass of the photon, so we can say: "The invariant mass is the sum of the relativistic masses measured with respect to the centre of mass."

Definition 4 and formula (1) together seem to give a circular definition; for the position of the centre of mass you need the relativistic masses and for the relativistic masses you need the velocity, that means the time derivative of the position, of the centre of mass. However, the relativistic masses in definition 2 are measured relative to the centre of mass, whereas the relativistic masses in definition 4 are measured relative to an arbitrary inertial system. Therefore there is no question of a circular definition.

Notice that relativistic mass in the definition of the relativistic centre of mass plays exactly the same role as classical mass in the definition of the classical centre of mass.

An important objection to definition 4 seems to be, that especially for photons, the Heisenberg uncertainty relation is so important that positions of photons are far from being defined exactly. To follow through this thought would lead to quantum mechanics, to the Einstein Podolsky Rosen "paradox", to the theorem of Bell and to the experiment of Aspect. This is not my intention. Suffice it to mention what Aspect has demonstrated for a system of one atom and two photons. When detecting one of the particles, so at the moment of collapse of its wavefunction, the wavefunctions of the other particles collapse simultaneously (for *all* observers) in such a way that all conservation laws remain valid. This means therefore, that experimental determination of the centre of mass is meaningful, that is to say it is in agreement with the theory.

It is clear that definition 1 only for the theoretical case of a point mass does not raise questions. In all other cases there is the problem that one never can say with certainty that in each infinitely small scale there are no moving parts in a body. Nevertheless one often speaks of "the rest mass of an atom", whereas in an atom the electrons are certainly not at rest with respect to the nucleus. One should speak here of the invariant mass of an atom. The atom is seen here as a point mass. In fact there is nothing wrong with that, as long as it is justified by our measuring apparatus, as long as an atom looks like a point to our measuring system. In that case rest mass and invariant mass are the same. When a system is involved with dimensions significantly greater than our measuring accuracy, there are two possibilities: first, the body has a centre of mass with a fixed position with respect to fixed and recognizable points on the body. Take for example a solid body (although a solid body also consists of parts moving relative to each other, merely because of the thermal movement of the molecules, talking of fixed points on the body nevertheless implies no contradiction, because we then look at a much larger scale than the molecular one, so the thermal movement averages zero. Besides, we assume that stresses in the body are not so great that the body is noticeably deformed). In this case the centre of mass can be determined by calculation if the density function is known. The invariant mass can be determined in the same way as the rest mass, the covered distance of the unknown mass after the collision being that of its centre of mass.

Secondly, the body is less coherent, the centre of mass having no fixed position with respect to fixed, recognizable points of the body. For instance, a number of nuclei flying away from each other after a nuclear reaction. Then the invariant mass could be determined with the method given on page 7, line 9 to line 12, applied on all point masses in the system. Again, in practice almost impossible to perform, but theoretically not impossible.

Compare a single point mass and a system-consisting-of-several-point-masses-moving- relative-to-each-other, the separate point mass and the centre of mass of the system being at rest. Suppose we take the single point mass as well as the centre of mass of the system to obtain a velocity v. This velocity is obtained not by exerting a force on *them*, but on *us*, the observers. After our relative acceleration has provided the relative velocity v, the force is brought back to zero and we can consider ourselves again as inertial observers. Then m_{rel} of the point mass is a factor $g(v)$ times its rest mass, whereas Σm_{rel} of the system, so the sum of the relativistic masses of its point masses, appears to be *also* a factor g times its *invariant* mass. Therefore it is meaningful to call $\Sigma m_{\mathbf{rel}}$ the (total) relativistic mass of the system. So in imagination we can replace even such a system of point masses, no matter how big their relative freedom of movement is, by one point mass as far as the relativistic mass increase is concerned, provided we imagine this point mass to be localized in the centre of mass and take as the total relativistic mass the sum of the relativistic masses of its point masses. Suppose we accelerate the two systems and not ourselves. Are the dynamics analogous in both cases, is $\vec{F}_{\mathbf{total}} = \mathbf{d}(m \cdot \vec{v}) / \mathbf{d}t$ valid for both single point mass and the centre of mass of the system of point masses? In general the answer is no. See the appendix I, page 53 to 63. The complex system will generally see its invariant mass changed. However, if the complex system is a solid and the internal stresses are not to great, I assume that the answer is yes. When an ionised atom or molecule is accelerated, to all likelihood its centre of mass obeys this equation as long as no internal degrees of freedom are excited.

If we look at it this way the rest mass (of a point mass) is nothing else other than a special case of the invariant mass and the relativistic mass of a point mass is nothing else other than a special case of the relativistic mass of a system of parts moving relative to each other.

If we are only interested in the behaviour of the centres of masses and if the internal energy of the body is not changed, not one single essential difference can be pointed out between rest mass and invariant mass on the one hand, and the relativistic mass of a point mass and relativistic mass of a composed system on the other hand.

As far as the dynamics of centres of masses is concerned we can use "rest mass" and "invariant mass" as synonyms.

Let us now look at an atomic nucleus at rest, which is about to undergo fission. Let us call the point where the nucleus sits P. Let us assume that it splits into two fission products (lighter nuclei) and that no other particles such as photons are produced (the fact that photons *are* produced, is not an essential objection to our reasoning: Taking photons into account doesn't change our argument, it merely complicates it). For the sake of simplicity, we shall assume that both fission products have the same mass. After the fission, the fission products fly away at high speed in opposite directions: both have kinetic energy. Their masses can be measured after they have come to rest. Now, the sum of their rest masses appears to be *less* than the rest mass of the original nucleus. "The difference is converted into energy", students learn in secondary school. I consider this a misleading statement. Look again at the situation immediately after the fission, when the fission products are at full speed, so before they have been slowed down (such as by collision with surrounding molecules). The fission products have kinetic energy. According to the secondary school books, this energy would have been created by the "disappearance" of mass suggesting that the fission products together have less mass than the original nucleus. This is indeed true if one looks at the rest masses of the fission products separately, but is not the case if one considers their relativistic masses. When the relativistic masses are added, one obtains the mass of the original nucleus. This follows from the fact that the invariant mass does not change, because no work is done on the system by the outside world. It can be seen in a different way also, as the following example will illustrate. In figure (a), see below, the original nucleus is denoted by *A*. An additional particle (such as an atom), remaining at rest and undergoing no changes is denoted by *D*. For simplicity's sake, I assume *A* and *D* to have equal rest masses. Now consider figure (b), the situation immediately after nucleus *A* has fissioned into the two fragments *B* and *C*. By symmetry, the center of mass of *B* and *C* taken together has to remain at point *P*. Because there is a mass associated with each center of mass point, we can ask here, which mass? It must be the mass of *A*. This can be seen by considering in figure (a) the center of mass of *A* and *D*. This must lie at the geometrical centre between *A* and *D*, since they have the same rest mass. Now, if in figure (b), the (invariant) mass associated with the centre of mass of *B* and *C* would differ from the mass of *A*, then the centre of mass of *B*, *C*, and *D* would have moved from the midpoint between *P* and *D*. This cannot be the case, according to the principle we stated on page 6, line 16 (statement 2[e]) and 15 from bottom, since no forces external

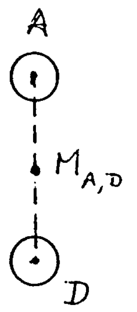


fig. (a)

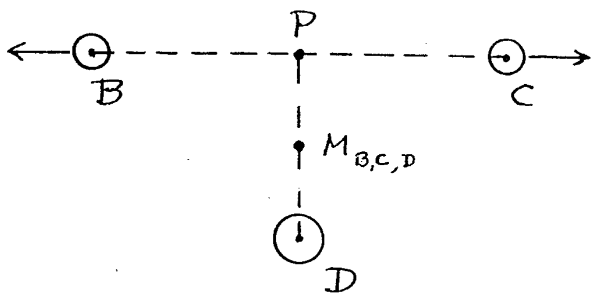


fig. (b)

the system of *A*, *B*, *C*, and *D* have been involved. Clearly, the mass associated with the center of mass of *B* and *C* cannot be the sum of their rest masses: this is less than the rest mass of *A*, because the difference is "converted" into the kinetic energies of *B* and *C*. If, on the other hand, we consider the relativistic masses of *B* and *C*, we see that their sum is equal to the rest mass of *A*. This has been verified both experimentally and theoretically. In simple terms, the mass associated with the kinetic energy has to be taken into account.

We have just seen that rest mass is not an absolute notion. The system of *B*, *C*, and *D* has a centre of mass at rest, so it is meaningful to call this a system at rest. Yet, it is the relativistic mass of the individual particles (of *B*, *C* and *D*), not their rest mass, that we must add up to obtain the (rest) mass of the system. The kinetic energy of the motion of the individual particles relative to the centre of mass of the system contributes to the systems rest mass.

Now consider the following excerpts from a course on the theory of relativity for students of secondary school by a professor of physics at Leiden University (1993, prof. Nienhuis):

. . . The enormous quantities of energy which can be released in

> nuclear reactions originate from a disappearance of a part of the mass. Mass is converted into energy (ref. 9, page 52).

And:

> In the old physics, conservation of mass was considered a fundamental property of matter. Whatever happened in collisions, during combustion or explosions, the total mass could not change. This expressed the imperishableness of matter. Matter could change in nature, it could convert from solids into gases, and *vice versa*, but the amount of matter, as expressed in the total mass, always remained the same. This idea now has to be abandoned. Matter can come into existence and can perish. Not matter is conserved, but energy. Therefore the concept of energy acquires a more fundamental importance than that of matter.
> **Summary:** Mass is energy in the rest frame. Energy is conserved, mass is not. Mass can come into existence and can perish.

At the bottom of the page in small letters:

> Note: If desired, we can define the mass of an arbitrary physical system as the energy divided by c^2. If we do so, then the mass is, of course, again a conserved quantity. The consequences, however, are far reaching. Firstly, the mass of a particle is no longer independent of its motion. [. . .] An object becomes heavier as it moves faster, and the mass becomes arbitrarily large as the speed of light is approached. Furthermore, we must attribute mass also to light, or to an electrical or magnetic field, because they represent energy.

The first two quotations of Prof. Nienhuis apparently refer to the rest mass, but which rest mass? For we have just seen that rest mass is not an absolute notion. If reference is made to rest mass attributed to the centre of mass of a quantity of radioactive gas under low pressure, which is completely isolated (both thermally and otherwise) from the outside world, and which warms itself up by its own radioactivity, then the statement that this heat energy originates "from a disappearance of a part of the mass" is incorrect. The (invariant) mass, which this generated heat possesses, is equal to the decrease in the rest mass of the constituent atoms (I assume a monatomic gas). Because the heat together with its mass remains in the system and contributes in this way to the rest mass of the system, the total rest mass does not change.

A similar example can also be given for a solid body, although things then become a bit more complicated; a piece of uranium warms itself up by its own radioactivity, while it is in a container of thermally isolating material and while radiation particles are prevented from escaping (by a thick layer of lead for instance). In this case too there is a decrease of rest mass if we compare the rest mass of the uraniumnucleus with the sum of the rest masses of the nuclei of its fission products (the energies of the electrons can be neglected). Now again there is a release of (internal) energy, but this time that energy consists not only of kinetic, but also of potential energy, namely that of the intermolecular forces. Both kinds of energy contribute to the restmass of the piece of uranium-with-container. The kinetic energy of each atom contributes $m_{\mathbf{rest}}\{g(v) - 1\}$ to the restmass, in which *v* is the velocity of the atom with respect to the centre of mass. The potential energy of each atom contributes E_{pot}/c^2 to the restmass. When all these (relativistic) masses are added up, we find exactly the mass which we see disappear when we look at those separate restmasses of uraniumnuclei and fissionproducts. In fact we only *thought* to see restmass disappearing. It didn't disappear in reality. We moved in thought subsequenty along with every nucleus apart, because we looked at its restmass and then added all those restmasses. Then it *looked* like mass having disappeared. Of course, we (Lorentz)transformed away the kinetic energy of the thermal movement and therefore its mass also. To make things worse, we didn't take into account that potential energies also contribute to the restmass. Checking a conservation law is like book-keeping while the foregoing is like fraud. If we wanted to do a thing like that in classical mechanics loud protests would arise, as the following example clarifies: " In contrast to what has been said for a long time, energy is not a conserved quantity. Launch a sled on an air-cushion track with a spring. There is no friction anywhere. The potential energy of the spring has disappeared completely and has not been converted into other forms of energy. This becomes clear when we start moving along with the sled; this then has no kinetic energy." The evident deceit of this reasoning is again in the use of a (galilean) transformation. We have transformed away the kinetic energy by starting to move along as an observer. Nobody will take this reasoning seriously. The reasoning just mentioned, in which the same mistake is made, remarkably enough is never contradicted.

We should be well aware of the fact that conservation laws of energy and impulse are valid only as long as we do not step over to another inertial frame. $E = m.c^2$ is about such a conservation law, and in fact tells us that the law of conservation of energy and of mass are one and the same. When working with this law we should not change the inertial system.

The example of the piece of uranium is so gratifying because it apparently involves the quantity which we call restmass. In fact I have been reproached several times for confusing rest- and invariant mass. When dealing with the example of the radio-active gas I was told: "But you are looking at the invariant mass, you wrongly call this the restmass." As remarked earlier, it is the other way round. Everybody calls the mass of such a piece of uranium the restmass, while in fact we should call it the invariant mass. One overlooks the thermal movement. As already said on page 8, directly under the white line, in practice rest- and invariant mass are one and the same.

Suppose the two quotations of prof. Nienhuis of page 10 and 11 refer to the separate restmasses of the gas atoms in our example. Then the above mentioned mistake in book-keeping is made. But apart from that the question arises, why the restmass of a gas is seen as the sum of the restmasses of the *atoms* separately. Why not look inside the atom? This holds the kinetic (and also the potential) energy of the electrons, which are indeed not at rest with respect to the nucleus. Looked at in this way, we should take the restmass of the electrons, add it to the restmass of the nucleus and consider the kinetic and potential energy of the electrons separately; those energies do not contribute to the restmasses of the elementary particles. But do protons and neutrons in the nucleus not have kinetic and potential energy also? The same can be said perhaps for neutrons and protons on their own. There are indications that quarks also have kinetic energy within the baryon. Those who argue that it is logical to compare atoms (or atomic nuclei) before the reaction with atoms (or atomic nuclei) after the reaction, that it is not obvious or even wrong to compare atoms before the reaction with elementary particles after the reaction, forget that this is always done at the fission of an uranium nucleus; one nucleus (that of uranium) splits into two nuclei plus two or three neutrons. The "mass converted into energy" is calculated by comparing the restmasses of the fission product nuclei *and the neutrons* (elementary particles) with the restmass of the uranium nucleus. Whoever argues that you have to see as one entity those particles which are bound to each other and have to compare their restmasses with one another, is missing the point also; the whole fission process could be happening in a container with thick lead walls, out of which neither neutrons nor nuclei could escape. In this way all particles are bound particles. Do you have to see an alpha particle in the uranium nucleus as bound or not? After all it can escape. The notion "bound" is too vague to justify statements such as "you have to compare only nuclei with nuclei". The notion of restmass therefore without further specifications is not unambiguous either. For it is impossible to state wether, on an infinitely small scale, a body is not composed of parts moving relative to each other. Therefore the view discussed above doesn't seem to me to be a fruitful one. It seems to me more justified to state that everything we called energy earlier appears to have mass and contributes to the total mass of the system, in which that energy is lodged, to the restmass (=invariant mass) if the centre of mass of the system is at rest, to the relativistic mass if the centre of mass is in motion. Moreover, what is called restmass here in fact is no restmass, but something what I call with a neologism the restmass sum. Further on I will demonstrate, that the restmass sum is not a relativistic invariant.

The third part of the quotation leans towards our view, yet the author seems unwilling to make the whole step. He foresees far reaching consequences if mass is attributed to light and electromagnetic fields. From my point of view it is the other way round: the consequences would be far reaching if we didn't do that. If the gas mentioned above starts glowing because of its radioactivity, light will be permanently exchanged between its atoms. The mass of this light must contribute to the rest mass of the gas. Must, because otherwise we could present the same argument about a displacing centre of mass as in figures (a) and (b).

Another example: an atom initially at rest that emits a photon. The atom experiences a recoil. The centre of mass of the atom and photon must remain immobile. If we

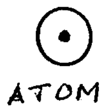


fig. (c)

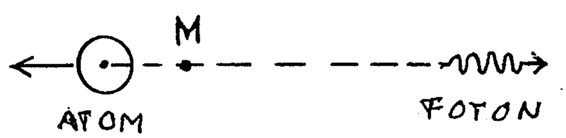


fig. (d)

don't attribute mass to the photon, the centre of mass would travel along with the atom. Because the system does not experience forces from the outside world, this is impossible. Thus, the photon must have mass, see figures (c) and (d).

Mass also has to be attributed to a static electric field. In this case we can safely speak of rest mass, contrary to the case of separate photons. To see this, let us replace the gas by a charged capacitor. An electrical field exists between the plates. If we discharge the capacitor through a long conducting wire (considered to form part of our isolated system), heat is generated. This thermal energy has mass which contributes to the total rest mass of the system. The rest mass has to remain the same, so an equal amount of mass must have disappeared as appeared in the form of heat. The disappearance of mass can only be attributed to the disappearance of the electric field. Thus, one can safely state that a static electric field has a rest mass.

Returning to the statement quoted in the first line of § 1, page 3, "Mass disappears and energy appears": if we assume that the relativistic mass is meant, then the statement is incorrect in the example of the splitting nucleus, since the total relativistic masses of *B* and *C* are the same as that of *A*. If rest mass is intended, then the statement is wrong in the example using the capacitor (more correctly: here the statement "Energy disappears and mass appears" is incorrect).

Another quotation, from a Dutch Secondary school book, the book "*Scoop*", ref. 16 on page 52, states: "According to that law, mass is one of the forms in which energy can exist." This suggests the following: energy can exist in many forms such as kinetic energy (flying bullet), chemical energy (gunpowder), potential energy (a stretched spring) and as mass. It seems quite clear to me that rest mass is meant here. Then this quotation suggests, wrongly, that the first three forms of energy *don't* represent mass. Wrongly, because a compressed spring has more rest mass than the same spring in a released state (after vibrations are damped out and internal friction energy has escaped in the form of heat). A quantity of gunpowder has a bigger rest mass than its combustion products (after the energy of the explosion has been dissipated). Even modern measuring techniques cannot reveal this small mass difference, but it does exist. The difference in mass is the mass of the energy contained in the stretched spring or the gunpowder. When we shoot off a bullet with the spring or the gunpowder, this energy, together with its mass, leaves the spring or gunpowder and adds to the bullets' relativistic mass (which before the shot is equal to its restmass). The bullet's mass is thereby increased to the relativistic mass corresponding to the speed acquired by the bullet.

If we follow "*Scoop*", and consider the rest mass when the bullet is shot off by a spring, we are saddled with an unattractive description: the spring energy has mass. As soon as it has passed over to the bullet it suddenly has no mass anymore.

The argument from "*Scoop*" can also be found in "*Systematische Natuurkunde*", another Dutch Secondary schoolbook which states that $E = mc^2$ should be replaced by $\Delta E = -\Delta m.c^2$. It would be something like $\Delta E_{\mathbf{kin}} = -\Delta E_{\mathbf{pot}}$, or in other words, the increase in kinetic energy equals the decrease of potential energy and *vice versa*. So, $\Delta E = -\Delta m.c^2$ would mean: the decrease in mass equals the increase of other forms of energy and *vice versa*. Probably Einstein would have written it this way if this was in his mind.

To me it seems more likely that Einstein meant: energy and matter are identical, up to now we have made an unjustifiable distinction between the two. To make that clear, it is not sufficient to know that all energy has mass. This only means that energy and matter have a property in common, namely they have mass. We must also show that all matter represents energy and *vice versa*. I have introduced the term matter here. This is necessary, since pre-relativistic physics (prior to the first publication of $E = mc^2$ on 27 September, 1905) considered the terms matter and mass as interchangeable. We have just seen that energy has mass too, so the difference between matter and energy temporarily requires redefinition. Let us define matter as everything made of atoms and their constituents (electrons, protons, and neutrons). The definition of energy is: the ability to do work. Now, electrons, protons and neutrons all have their so called anti-particles. If, for example, a proton collides with an anti-proton, both particles are destroyed and photons are created. Even in pre-relativistic physics, it was already clear that photons (light) could be converted into all the other forms of energy known to us, and *vice versa*. Also, photons can be converted to particle anti-particle pairs. Indeed, matter and energy can be freely converted from one to the other (provided we consider anti-matter to be included in the definition of matter). The definition of matter just given for the sake of argument is too limited. The only meaningful definition of matter is: everything that has mass. But now it has become impossible to point out any essential difference between matter and energy: they are identical. Mass can be taken as a measure of both.

Why do practically all physicists mainly think in terms of rest mass? This is probably because the importance of rest mass is emphasised so much by university lecturers. Some of them even suggest that relativistic mass is not a useful concept. This emphasis comes mainly from the fact that the rest mass is invariant under Lorentz transformations. In other words: the rest mass plays an important and elegant role in the mathematics of relativity theory. But this role is in no way disturbed by my view of relativistic mass.

My conclusion is as follows: it would be better to say that mass and energy are identical, that $E = mc^2$ must be seen as an identity, not as a reaction equation. Consequently, we can use the same unit for mass and energy. By force of habit we still express them using different units, namely kilograms for mass, and joules for energy, but we must be able to convert these units into each other. This can be done with $E = mc^2$. Compare this with the time calories were still in use. It also came from a time when a distinction was wrongly made between two forms of energy: heat (measured in calories) and energy (measured in joules). Until quite recently, the formula 1 calorie = 4.18 joule was common in school physics books. One could just as well have written: $E = 4.18(\text{J/cal}) * Q$, with E for energy (in joules) and Q for heat (in calories). The 4.18 plays the same role as the c^2 in the formula $E = mc^2$.

The law of conservation of mass, whose validity is no longer recognised in the course on relativity theory by prof. Nienhuis (page 10 and 11), can now be restored. We can say: The law of conservation of mass, as formulated amongst others by

Lavoisier, implied that when matter is locked in a vessel and none of it can escape, then its total mass will remain the same, irrespective of its physical (for instance melting or evaporating) and/or chemical (for instance burning) changes. Lavoisier did not think it necessary to prevent energy from passing through the walls of the vessel as well. He did not know energy possesses mass too. Now we know this, the old law of conservation of mass can retain its validity, provided the container is closed to flows of mass in all its forms (including the ones we were used to call energy until recently).

*

§ 3

$E = mc^2$

and Poyntings theorem

Poyntings theorem assigns the EM-field as the resort of the EM-energy, expresses the energy density in the electrical fieldstrength E and the magnetic induction B and gives an equation of continuity, that is to say it suggests that the decrease per second of the EM-energy in a certain volume is equal to the energy flux that passes the boundary of that volume in an outward direction plus the EM-energy that, through the work done by the Lorentz forces, is converted into other forms of energy:

$$\iiint_V \left(\vec{f} \cdot \vec{v}\right)\mathbf{d}V = -\iiint_V \frac{\P U}{\P t}\,\mathbf{d}V - \oiint_A \left(\vec{S} \cdot \mathbf{d}\vec{A}\right) \qquad \text{Poyntings theorem} \quad (2)$$

$$\text{with} \quad U = \tfrac{1}{2}e_0\left(\vec{E}^2 + c^2\vec{B}^2\right) \quad \text{and} \quad \vec{S} = e_0 c^2\left(\vec{E} \wedge \vec{B}\right)$$

Where $\vec{f}$ is the Lorentz force density on an infinitesimal part of an electrically charged system, $\vec{v}$ is the velocity of that part, U is the energy density of the EM-field and $\vec{S}$ is the so called Poynting vector. This vector is the energy stream density of the EM-field. So the theorem states, work done by the EM-field on a charge in a certain volume V (left hand side of (2)) equals the decrease of EM-energy that is located in the volume plus the EM-energy possibly entering the volume by its boundary. This equation is a relativistic covariant, so we can take it over unmodified in the theory of relativity. Its left hand side is the work done on an electrically charged system; note that in the theory of relativity too work is equal to force integrated to displacement. Work equals the energy supplied to a system, so work divided by c^2 is the mass supplied to a system. Which mass? Evidently the relativistic mass, as will become clear in the following. Only if we take for m and E from $E = mc^2$ the relativistic mass respectively the total energy, everything matches. If we divide equation (2) by c^2 we get the statement: *the relativistic mass increase of a charged system being accelerated by an EM-field is equal to the mass decrease of the EM-field in its immediate vicinity plus the mass of the EM-field that possibly streams from elsewhere towards that vicinity.* This because if U is the EM-energy density, then U/c^2 is the EM-mass density. If $\vec{S}$ is the EM-energy stream density, then $\vec{S}/c^2$ is the mass stream density. So:

$$\iiint_V c^{-2}\left(\vec{f} \cdot \vec{v}\right)\mathbf{d}V = -\iiint_V c^{-2}\frac{\P U}{\P t}\,\mathbf{d}V - \oiint_A c^{-2}\left(\vec{S} \cdot \mathbf{d}\vec{A}\right) \qquad (3)$$

That the left hand side of (3) is indeed equal to the relativistic mass increase, can be proved for the case of a pointcharge as follows: first *state* the left hand side is equal to the increase of m_{rel}. Then this increase can be proven to be equal to $m_{\mathbf{rest}} .g - m_{\mathbf{rest}}$, which means the statement to be correct;

$$c^{-2}(\vec{F} \circ \vec{v}) = \frac{\mathbf{d}m_{\mathbf{rel}}}{\mathbf{d}t} \quad \Rightarrow \quad \left(\frac{\mathbf{d}m_{\mathbf{rel}}\vec{v}}{\mathbf{d}t} \circ \vec{v} \right) = c^2 \cdot \frac{\mathbf{d}m_{\mathbf{rel}}}{\mathbf{d}t} \quad \Rightarrow$$

$$\Rightarrow \tfrac{1}{2} m_{\mathbf{rel}} \frac{\mathbf{d}v^2}{\mathbf{d}t} = \left(c^2 - v^2\right) \frac{\mathbf{d}m_{\mathbf{rel}}}{\mathbf{d}t} \quad \Rightarrow \quad \frac{1}{2\left(c^2 - v^2\right)} \cdot \frac{\mathbf{d}v^2}{\mathbf{d}t} = \frac{1}{m_{\mathbf{rel}}} \cdot \frac{\mathbf{d}m_{\mathbf{rel}}}{\mathbf{d}t}$$

integrating from a moment at which $v = 0$ (and therefore $m_{rel} = m_{rest}$) till a moment at which $v = v_{end}$ (and so $m_{rel} = m_{rel}(v_{end})$) gives:

$$\left[-\tfrac{1}{2} \log\left(c^2 - v^2\right) \right]_0^{t_{end}} = \left[\log m_{\mathbf{rel}} \right]_0^{t_{end}} \quad \text{from where: } m_{\mathbf{rel}}\left(v_{\mathbf{end}}\right) = m_{\mathbf{rest}} \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

$$\text{so} \quad m_{\mathbf{rel}} = m_{\mathbf{rest}} .g$$

We obtain the correct formula for relativistic mass (so also the correct one for relativistic mass increase) of a point mass without internal degrees of freedom, therefore in this case the left hand side is indeed equal to the mass increase. It is reasonable to suppose that this is also true for more complex systems, such as systems with internal degrees of freedom. These can enlarge not only their relativistic, but also their invariant mass, for example by absorption of EM-radiation (their mass increase is no longer, as for point masses, a function of their velocity alone).

As Formula (3) makes completely clear, *the relativistic mass increase (for point masses as well as for invariant masses) caused by EM-forces is not merely an abstract mathematical description, it is caused by an incoming flow of mass stored in the EM-field.* From formula (3) we can see that it follows the law of conservation of mass (relativistic mass) for electrodynamics if we integrate it over time and choose surface A in such a way, that the circuital integral over it is zero, that is to say if we take a closed system [1].

Let us assume that for each type of force (so not only for the electromagnetic force) an equation like (3) can be written, then it can be said that for any system, each relativistic and/or invariant and/or field mass increase always goes hand in hand with an equal decrease of relativistic and/or invariant and/or field mass in the outside

---

[1] Equation (3) is also useful in another case. The *B*-field of, for instance, an electromagnet at rest and the *E*-field of an electrical charge at rest in the vicinity give a poynting vector field, which indicates that energy is circulating in closed orbits in this field. Now in such an EM-field an amount of angular momentum appears to be present (see Feynmans paradox on this, *Feynmans Lectures on physics, part II, 17-4*). This becomes clear when we switch off the current through the electromagnet. Then the charge will be brought into motion as a result of the electromagnetic induction and will acquire a certain amount of angular momentum with respect to the electromagnet. This angular momentum appears at first sight to come out of nothing. Looked at it this way we have a paradox. However, as is clear from equation (3), such an EM-field has an angular momentum, because it tells us that a quantity of mass is rotating in closed orbits. Now the paradox is solved, because it is evident that the angular momentum obtained by the system was stored in the EM-field before.

world. This supposition, namely that something like (3) is true for all types of forces is completely justified because it is nothing other than the relativistic principle. This states not only that electrodynamic forces, but also all other types of forces, can be brought into relativistic covariant form, an indispensable postulate in the theory of relativity.

It now becomes clear that statements like 'ascribing relativistic mass to a body gives a false picture' and 'rest mass is the only mass you must work with, this we have to call *the* mass' are completely wrong. The only disadvantage of speaking of the relativistic mass of a body is that you have to say at the same time in which inertial system the mass is measured. On the other hand we have a big advantage here; as long as we don't switch to another inertial frame and no work is done on the system by forces from the outside world, the total relativistic mass does not change (the mass of the fields I also call relativistic mass, see page 7, line 13 from bottom and page 13, text between the two blank lines). Thinking in rest masses, we get on the contrary a confusing image of rest masses being converted into kinetic energy and vice versa, rest masses being "converted" into potential energies or vice versa (in which case one mostly forgets potential energy has rest mass too, think of the example with the capacitor), of a total rest mass that is different each time someone decides to split up the system in a different way in supposed "pointmasses" (billiardballs as point masses, or molecules, or elementary particles, or fundamental particles). A much more logical approach is to call the relativistic mass of a system *the* mass and to see rest mass as a special case of *the* mass, namely the mass at $v_M = 0$, with m as centre of mass of the system. Note how much clearer the first phrase of the previous paragraph then becomes: . . . . then it can be said that for any system, each mass increase always goes hand in hand with an equal decrease of mass in the outside world.

*

§ 4

Gravitational mass

Another argument in favour of the relativistic mass is, that in one and the same inertial frame the sum of the relativistic masses of a system is equal to its gravitational mass. He who adds up the *rest masses* of, for example, the separate atoms of the system, definitely doesn't find the gravitational mass.

This can be shown as follows. We place a miniature nuclear reactor on one of the scales of a balance and bring it in equilibrium. 'The reactor is capable of converting rest mass into thermal energy. So rest mass disappears, a part of the rest mass of, for example, uranium nuclei.' The reactor has walls impermeable to everything, including heat, so all energy and matter stay inside the reactor. Now if gravitational mass should disappear, the balance would dip. There is no law of nature forbidding the 'conversion of this thermal energy via a fusion reaction back again into a rest mass increase (of atomic nuclei).' Suppose the reactor does so. Then the balance would regain its equilibrium. From this movement, energy could be extracted, whereas in the reactor no net change of rest mass has taken place. This is in contradiction of energy-mass conservation. The gravitational mass therefore is not equal to the sum of the rest masses of the nuclei, but is equal to the sum of the relativistic masses. For only then does the paradox disappear, because the thermal energy has exactly the mass (relativistic mass), which the nuclei have lost in their fission, so the balance maintains its equilibrium.

An obvious criticism of this reasoning is that the second law of thermodynamics blocks the depicted process. However, the second law forbids nothing. At most it states that such a cyclic process is highly unlikely.

*

§ 5

Rest mass

Rest mass (or invariant mass, I will use them as synonyms) is usually seen as the most meaningful notion of mass, the relativistic mass is usually seen as of minor importance. To account for this, it is pointed out that *c* times the rest mass is equal to the length of a four-vector, namely the energy-impulse vector. The energy-impulse vector is $(E/c,\, \vec{p})$. The length of this four-vector is, by definition, $(E^2/c^2 - p^2)^{1/2}$. This is another way of saying that rest mass is invariant for Lorentz transformations, because if we switch to an inertial system with a different velocity the length of a four-vector remains unaltered. I don't think this makes rest mass into something more than the relativistic mass at speed zero. Whoever says: "The rest mass is invariant" thinks thereafter "And relativistic mass is not so". But in this way one compares mass at a given speed (namely zero) to a mass at variable speed and therefore one compares incompatible objects. To me "rest mass is invariant for Lorentz transformations" would only become something special, if relativistic mass *at a given speed* would not be so, if the relativistic mass at a speed of, for example, four fifths of *c* would not be relativistically invariant. But of course it is, because g then is five thirds, so m_{rel} is then always five thirds of the rest mass. So $m_{rel}(v{=}0{,}8c)$ is also a Lorentz invariant. The same can be said for each speed between zero and *c*.

The obvious thing to say would then be: "The length of the energy-impulse vector is $m_{rest}c$" and not: ". . . . is $m_{rel}(0{,}8c)c3/5$", but that is merely a choice for simplicity, not a real distinction.

I think the most important role of the rest mass lies in something different; if the rest mass of a body changes, then its internal energy changes. If a body has no internal degrees of freedom, its rest mass will always be the same. A free electron cannot absorb a photon. If on paper we let an electron absorb a photon and calculate its final velocity out of energy conservation (=relativistic mass conservation), impulse is not conserved. So in this example we calculate *v* from $m_{\mathbf{rest}}.g = m_{\mathbf{rest}} + E_{\mathbf{photon}}.c^{-2}$. Notice we ourselves put in the rest mass of the electron as unaltered. This becomes clear by the notation, we give rest mass in both sides the same symbol.

If the rest mass of an electron could increase, absorbtion would be possible. This can easily be calculated, but can be seen without calculation also. Transform to the centre of mass system of photon + electron. In this the electron is at rest after the absorbtion, so the photon energy must have been "converted" to an increase of rest mass of the electron. So far this process has never been observed and therefore we suppose for the time being that the electron has no internal degrees of freedom.

An atom can absorb a photon. It does have internal degrees of freedom, so it can increase its internal energy and therefore its rest mass.

Rest mass plays an important role in defining kinetic energy, amongst other things, as we know it in collisions. To illustrate this we first look at the classical, so the non relativistic, kinetic energy.

In classical mechanics the definition in formula form of the classical kinetic energy for a point mass is: $E_{kin} = 0{,}5mv^2$. If we look at a system composed of several point masses, things get more complicated. What do we mean by *the* kinetic energy? What do we mean with: "An inelastic collision is a collision by which *the* total kinetic energy diminishes"? We mean that the macroscopic kinetic energy diminishes, so the kinetic energy of the centres of masses, not the macroscopic plus the thermal kinetic energy (we could arrange a collision during which a chemical reaction in the bodies causes a temperature rise, but we would never add here the involved thermal kinetic energy to the total kinetic energy of the colliding bodies). If we speak without further specification of *the* kinetic energy of a body, we practically always mean $0{,}5m_M v_M^2$, in which m_M is the mass of the body and v_M is the velocity of the centre of mass. A careful definition of the (centre of mass-) kinetic energy of a body composed of point masses is: *It is that part of the kinetic energy of the point masses, that can be transformed away by stepping over to the centre of mass reference frame.* The transformation being of course Galilean, we are still dealing here with non relativistic mechanics. In this way it can be proven that the maximum part of the kinetic energy (kinetic energy as the sum of the kinetic energies of the separate atoms) is transformed away. With all other transformations a smaller part of the kinetic energy disappears.

Note that the kinetic energy of a point mass can be defined in a similar way as above: *It is the form of energy that becomes zero, if we step over to the CMS of the point mass.*

The relativistic kinetic energy of a point mass can be defined in exactly the same way as the classical kinetic energy. So for the relativistic kinetic energy we have: $E_{\mathbf{kin}} = m_{\mathbf{rest}}(g - 1)c^2$. This formula follows from the verbal definition, from $E = mc^2$ (provided we read for m the *relativistic* mass) and from the fact that a Lorentz transformation leaves the rest mass unaltered.

The relativistic kinetic energy (of the centre of mass) of a system of point masses can again be defined in exactly the same way as in the preceding paragraph for the classical kinetic energy: $E_{\mathbf{kin}} = m_{\mathbf{rest}}(g - 1)c^2$. For the v in g we must fill in the velocity of the centre of mass. I use m_{rest} to mean invariant mass. As already mentioned on page 9, last phrase before the blank line, these two masses are the same in practice.

Now again the total kinetic energy is minimal in the centre of mass system. So you can say that rest mass (=invariant mass) is the minimum value of the relativistic mass, you can give a system by performing a Lorentz transformation on it.

A Lorentz transformation cannot be the cause of a rest mass change. This is because a Lorentz transformation leaves the system unaffected, it changes only the state of motion of the observer. A calculation of the rest mass from the relativistic mass is therefore none other than the observer mentally switching to the center-of-mass-inertial-frame of the system. This mental act can of course never be the cause of input or output of energy to or from the system in the rest frame (and so cannot be so in any other inertial frame, as long as you do stay in that frame). If in the CMS no energy is supplied to or withdrawn from the system, this cannot change its rest mass. Rest mass changes if, and only if, the total energy of the internal degrees of freedom of the system changes.

An atom absorbing a photon changes its rest mass. A free electron in rest hit by a photon doesn't change its rest mass. The relativistic mass however, does increase. The photon's relativistic mass decreases via the mechanism of the Dopplereffect.

Notice, if energy *is* supplied to or withdrawn in the CMS, *two* possibilities arise: rest mass changes (and relativistic mass can change, but can remain the same as well), or rest mass doesn't change. But then the relativistic mass has to change, and then in accordance with $m_{\mathbf{rel}} / g$ = *constant*, in which *constant* = m_{rest}. When $v = c$ in fact something similar is valid for photons, then $m_{\mathbf{rel}} / g = 0$; in other words, when we supply or withdraw energy to or from a photon, it changes its relativistic mass (*without* changing its speed).

The quotient of m_{rel} and g is again a constant, namely zero. [2]

To understand better why there are two possibilities, we can look at collisions. These can be divided into completely elastic and not completely elastic collisions. These classes of collisions can be defined in almost the same manner as in classical mechanics:

Definition (5): *a completely elastic collision is a collision between a number of bodies, in which the sum of the kinetic energies of their centres of mass is conserved and in which their rest masses remain unaltered.*

And non completely elastic collisions are collisions which don't obey definition (5).

A collision between a photon and a free electron is, by definition, a completely elastic collision, for the rest mass of the photon is and remains zero and the rest mass of the electron does not change either. In such a collision for all the bodies involved, $m_{\mathbf{rel}} / g$ is equal to a constant.

A collision between a photon and an absorbing atom is by definition a non completely elastic collision, for the internal energy of the atom increases. This means a decrease of the kinetic energies of the centres of mass.

If a system absorbs energy and increases its rest mass, then there is a degradation of energy, that is to say the total entropy increases. If the rest mass remains the same the process is isentropic.

---

[2] Subtracting a part of the energy of a photon can be achieved by letting it fall onto a receding mirror. Reflection causes the photon to experience a Doppler shift in its frequency, resulting in a decrease in energy.

When photons are absorbed by a system, there is an increase in entropy. When photons are reflected by a system, the energy transfer is isentropic.

A remarkable phenomenon in relation to this is betatron radiation. Here (almost) free electrons emit photons, whereas the reverse, i.e. absorbtion of photons by free electrons is impossible. One could ask why. The answer lies in 'almost'. The electrons in a betatron are not completely free. They are surrounded by an EM-field, that can exchange energy and impulse with the outside world. In such a case it is possible photons are being emitted and at the same time energy- and impulse-conservation remain valid.

Of course it is useful to know the rest mass and to use it as a means of identification of particles which, in our present state of knowledge, have no interior degrees of freedom or are mostly in their ground state, such as atomic nuclei.

The aim of this paragraph is to make clear that rest mass remains a useful thing to look into. It has not become a superfluous notion.

*

§ 6

Within the same inertial system Newtonian and relativistic dynamics are identical

An electrically charged point mass with a non-timedependent rest mass and moving under the influence of the Lorentz force alone obeys the relativistic equation of motion:

$$\vec{F} = \frac{\mathbf{d}m_{\mathbf{rel}}\vec{v}}{\mathbf{d}t} \qquad (4)$$

on closer examination, the relativistic equation of motion (4) appears to be in concordance with classical dynamics, provided we keep in mind that each form of energy has mass and as long as we stay in the same inertial frame. With an example I will try to make this plausible.

We envisage an alpha particle, moving straight towards an atomic nucleus N at rest. On neither particle is a force acting from the outside world. As the alpha particle approaches the nucleus, it is more and more decelerated by the repulsive electrical force. At a certain moment the alpha particle is stopped by the nucleus and subsequently accelerates in the opposite direction. Take $t = 0$ at the moment of zero velocity. So from $t = 0$ on, the alpha particle is accelerated by the electrical field of N. We take N to be so heavy, that its acceleration can be neglected. Now we are going to follow the events, starting from $t = 0$, in more detail; at $t = 0$ the electrical fields of N and the alpha particle overlap each other. Next we invoke the theorem of Poynting, see § 3, page 17 as far as 19. This states amongst other things, that electrical energy is stored the field of such a system and that we can calculate the energy density in every point of the field. The energy density U_{pot} from our example is given by:

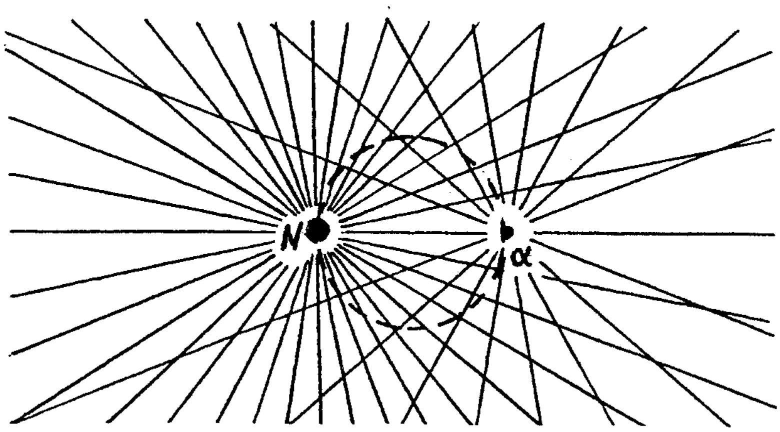
$$U_{\mathbf{pot}} = e_{\mathbf{0}}\left(\vec{E}_{\mathbf{N}} \circ \vec{E}_{a}\right) \qquad (5)$$

In this $\vec{E}_{\mathbf{N}}$ and $\vec{E}_{a}$ are the electrical fields of N and a , is their inner product and U_{pot} is the energy density resulting from the overlap of the fields of N and a , see figure (e) on this page. [3] This energy density signifies, according to $E = mc^2$, also a mass density r :

$$r = \frac{e_0}{c^2}\left(\vec{E}_{\mathbf{N}} \circ \vec{E}_{a}\right) \qquad (6)$$

When we integrate r over space we find the total mass of the potential energy at $t = 0$. This I will call $m_{\text{rel, pot}}$.

fig. (e)



In each point of space around N and the alpha particle there is a mass density present as a consequence of the overlap of the fields of N and the alpha particle (notice, inside the dotted sphere r is negative and outside positive).

---

[3] That the formula $e_0\left(\vec{E}_{\mathbf{N}} \circ \vec{E}_{a}\right)$ at $t = 0$ represents the electrical energy density of nucleus and alpha particle with respect to each other, can indeed be deduced from the expression for the total energy density of the field at $t = 0$; $U = \frac{1}{2}e_0\vec{E}^2_{\mathbf{tot}} = \frac{1}{2}e_0\left(\vec{E}_{\mathbf{N}} + \vec{E}_{a}\right)^2 = \frac{1}{2}e_0\vec{E}^2_{\mathbf{N}} + e_{\mathbf{0}}\left(\vec{E}_{\mathbf{N}} \circ \vec{E}_{a}\right) + \frac{1}{2}e_0\vec{E}^2_{a}$. The first and last term represent the energy densities of the nucleus and alpha particle respectively, for the case in which they are not in each other's neighborhood. So they represent the self energy of the fields. The middlemost term is the energy density caused by the overlap of the two fields and represents the (electrical) potential energy that the two particles have with respect to each other. This term divided by c^2 gives the mass density of the potential energy of the system. In this further paper I will demonstrate that it is this mass which the alpha particle absorbs during its acceleration.

We must consider this mass as being at rest at $t = 0$. [4] The relativistic mass of the alpha particle will increase during its acceleration.No forces from the outside world are exerted on the system of the masses of N, the alpha particle and $m_{\text{rel, pot}}$, so its total energy is conserved. But then the sum of the relativistic masses, $m_{\mathbf{rel},\,a} + m_{\mathbf{rel,\,N}} + m_{\mathbf{rel,\,pot}}$, is conserved too. And because N does not accelerate, the increase of the relativistic mass of the alpha particle must be equal to the decrease of $m_{\text{rel, pot}}$. In formula:

$$-\,\mathbf{d}m_{\mathbf{rel,\,pot}} = \mathbf{d}m_{\mathbf{rel},\,a}$$

Inevitably $-\text{d}m_{\text{rel, pot}}$ traverses the space from its original position to the alfa particle. It cannot be, that $m_{\text{rel, pot}}$ decreases "on the spot" and $m_{\mathbf{rel},\,a}$ increases "on the spot", without $\text{d}m_{\text{rel, pot}}$ travelling through the intermediate space; then there would be a non-local conservation law. Such a law would only be valid in one class of inertial systems. As soon as one switches to an inertial system of different velocity the Lorentz transformation shows that the moments of appearing and disappearing of the masses in question no longer coincide. A non-local conservation law is not Lorentz transformation-proof, so relativistic conservation laws have to be local. This means an equation of continuity has to be valid for $m_{\text{rel, pot}}$. As I showed in § 3, this equation is obtained by dividing the Poynting theorem by c^2. In this way it can be said, that $\mathbf{d}m_{\mathbf{rel},\,a}$ and $-\,\mathbf{d}m_{\mathbf{rel,\,pot}}$ represent one and the same mass. From now on I will call this mass $\text{d}m_{\text{rel}}$. This mass should be accelerated from rest to $\vec{v}$, the velocity of the alfa particle. For this a certain force is needed, in addition to the force which causes the acceleration of the alpha particle. So during the acceleration *two* forces are acting: one to accelerate the alpha particle and one to accelerate $\text{d}m_{\text{rel}}$. Equation (4) can be written as:

$$\vec{F}_{\mathbf{lor}} = m_{\mathbf{rel}} \frac{\mathbf{d}\vec{v}}{\mathbf{d}t} + \vec{v} \frac{\mathbf{d}m_{\mathbf{rel}}}{\mathbf{d}t} \qquad (7)$$

From now on I will call the first term on the right hand side of (7) $\vec{F}_{\mathbf{point\,mass}}$, the second term $\vec{F}_{\mathbf{pot}}$. I see $\vec{F}_{\mathbf{point\,mass}}$ as the force accelerating the alpha particle and $\vec{F}_{\mathbf{pot}}$ as the force accelerating $\text{d}m_{\text{rel}}$. One sees immediately that $\vec{F}_{\mathbf{point\,mass}}$ obeys Newton's

[4] At this point I often got criticised, 'because one cannot say of the energy of an EM-field' (more exactly of an EM-source field, because in relation to radiation fields I never got this criticism) 'whether it is in motion or not.' I am convinced it can be said. The Poynting vector serves as the criterion. When this is zero, the velocity of the field energy is zero. When the velocity is not zero, it can be found by dividing the Poynting vector by the energy density. Namely, the Poynting vector is equal to the energy stream density and so is equal to energy density times velocity. If we divide it by energy density, we get the velocity of the energy. In the preceding phrase we can replace energy everywhere by mass (relativistic mass), because they are equal, except for the scale factor c^2. Now it is completely clear that we can attribute a velocity to an amount of EM-energy; for we could attribute velocity to mass all the time. Moreover, only in this way is the solution to Feynmans paradox in footnote 1 on page 18 completely clear.

second law. However, this is true for $\vec{F}_{\mathbf{pot}}$ also, because:

$$\vec{S} = \Delta\vec{p} \quad \Rightarrow \quad <\vec{F}>\cdot\Delta t = \Delta m\cdot\vec{v} - \Delta m\cdot\vec{0} \quad \Rightarrow$$

$$<\vec{F}> = \frac{\Delta m}{\Delta t}\cdot\vec{v}$$

In the limit for $\Delta t \to 0$ and applied to our case we get:

$$<\vec{F}_{\mathbf{pot}}> = \vec{v}\,\frac{\mathbf{d}m_{\mathbf{rel}}}{\mathbf{d}t}$$

So equation (7) represents a case that can be described with Newton's second law, because the relation between impulse and quantity of movement can be deduced from the second law. [5]

I suppose in more general cases also (for example initial velocity unequal to zero, nuclear forces instead of EM-force) this explanation for equation (4) can be given.
So equation (4) is deducible from Newtons second law.

Because Newton's first law is nothing other than a special case of Newton's second law, we can state that relativistic dynamics obey Newton's first and second law.

Now the question arises, if Newton's third law is valid in relativistic dynamics as well. The answer is yes. This can be understood by looking at the so called Maxwellian stress theorem. It states:

$$\oiint_A \left(\left(\bar{\bar{T}} - \bar{\bar{1}}U\right)\circ \mathbf{d}\vec{A}\right) = \vec{F} + \iiint_V c^{-2}\,\frac{\P\vec{S}}{\P t}\,\mathbf{d}V \qquad (8)$$

With $\bar{\bar{1}}$ as the 3×3 matrix of unity.

It states that the EM-force can be seen as an elastic stress in a solid. Just as from the stresses in a propeller shaft of a ship the amount of force transferred per squared centimetre from motor to propeller can be calculated, the same can be done for the EM-force; the left hand side of (8) is the total EM-force on volume *V* and is transmitted by the surface *A* of *V* to *V*'s content. $\vec{F}$ is the force on a possible electrical charge in *V*. In the case of the ship's shaft, while accelerating its rotation, a part of the stresses are necessary to accelerate the shaft itself. In the same manner in equation (8)

$\iiint_V c^{-2}\,\frac{\P\vec{S}}{\P t}\,\mathbf{d}V$, together with some other terms, which can be split off from the left hand side of (8), represent the force necessary to accelerate the mass of the EM-field

---

[5] A good classical analogy of such an accelerating particle is a certain type of fire-fighting aircraft, which takes up water by opening flaps under the fuselage and by skimming over a lake. Consider the situation in which the plane accelerates while also scooping water. The thrust force of the propellers can be compared to F_{lor} (we could just as well say F_{tot}) on the alpha particle. This thrust force has to increase the speed of the plane *and* accelerate the scooped water from zero velocity to the velocity of the plane. So the thrust force is equal to *m*a plus v**d*m*/d*t*, with *m* as the instantaneous mass of the plane and d*m* as the mass of the scooped water.

inside *V*. We can apply this to the example of nucleus and alpha particle. Consider the nucleus surrounded by a volume V_N and the alpha particle by a volume V_a, bordering on each other with a plane lying between *N* and a and extending to infinity. The other boundary planes are chosen entirely at infinity. It can be proven that at infinity the integral $\oiint_A \left(\left(\vec{\vec{T}} - \vec{\vec{1}}\right) \circ \mathbf{d}\vec{A}\right)$ is zero.

This integral is only significant over the common boundary plane. Each infinitesimal part of the surface d*A* has a normal vector $\mathbf{d}\vec{A}$, which points to the outside of the corresponding volume and is equal in magnitude with d*A*. So $\mathbf{d}\vec{A}_{\mathbf{N}} = -\mathbf{d}\vec{A}_a$, these vectors are each other's opposite. Now if we calculate $\mathbf{d}F_{x,a}$, the *x*-component of the force on $\mathbf{d}A_a$, that is the inner product of $\mathbf{d}\vec{A}_a$ and the first row of $\vec{\vec{T}} - \vec{\vec{1}}U$, then we find the opposite number of what we would have found, if we had performed the same calculation with d $F_{x,\ N}$. Because this is so for each arbitrary d*A* of the common boundary plane, this will be valid also for both integrals over the plane. Both integrals over the surfaces at infinity are zero, so both integrals over A_N and A_a are each other's opposite. In other words, the total $F_{x,\ N}$ is the opposite of the total $F_{x,a}$. Because the same reasoning can be applied to the two other components of the forces, we can say action is minus reaction. Now we can conclude that *all three laws of Newton are valid in relativistic dynamics as well.* All this is true provided the observer doesn't step over to another inertial frame.

I think several aspects of the relativistic mechanics for a system consisting of point masses become clearer by taking my point of view:

1st by giving the principal role in the dynamics to m_{rel}, the relativistic mass of the system as a whole and not to m_{rest} of the separate point masses of the system, the total mass (relativistic mass) of the system remains constant as long as no work is done on it by the outside world. The predominant habit of giving the principal role to the *sum* of the rest masses, $\Sigma m_{\mathbf{rest},i}$, of the separate point masses of the system has as a consequence that this sum is continuously changing, dependent on the fact whether (a part of) $\Sigma m_{\mathbf{rest},i}$ is "converted into energy" or vice versa. Worse, as said before, this sum is dependent on the way one decides to divide the system into what one considers as point masses. For example: does one call the energy of an electrostatic field divided by c^2 rest mass or not? Or: speaking of "the rest mass" of gas in a container, does one mean the sum of the separate rest masses of its molecules, or does one mean its invariant mass, thus the sum of the relativistic masses of its molecules with respect to its centre of mass? Often one talks about the "rest mass" of a system without stating in what way one has subdivided the system into "point masses". But without this information "the rest mass" is not uniquely defined (we will see in the next paragraph that the rest mass sum is not a real rest mass). All these difficulties disappear as soon as one looks at the relativistic mass and one considers the rest mass as a special case of the relativistic mass: the rest mass of a point mass is its relativistic mass at zero velocity. The analogon of rest mass for a non-point mass, the invariant mass, is defined in the same way, with the understanding that "velocity" here means the velocity of the centre of mass.

As the relativistic mass of a point mass of the system increases as a result of the Lorentz force, we no longer have to see this as an abstract mathematical description. The increase is caused by a flow of fieldmass towards the point mass, a clearer point of view than the usual one.
It is only a small step to suppose that this is true for all kinds of forces.

2nd Equation (7), page 29, becomes easier to understand. For instance, quite often people stress with some amazement a consequence of equation (4), page 27, namely that the total force on a body and the acceleration of that body do not

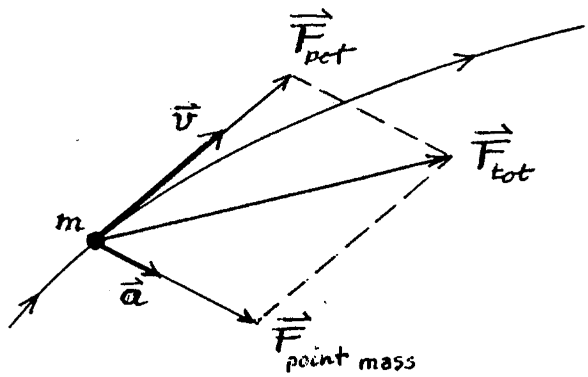


fig. (f)

nessesarily have the same direction. In my view this is not so amazing. $\vec{F}_{\mathbf{pot}}$ can very well have a different direction than $\vec{F}_{\mathbf{point\,mass}}$. $\vec{F}_{\mathbf{pot}}$ has the direction of the velocity of the point mass, $\vec{F}_{\mathbf{point\,mass}}$ has the direction of the acceleration. When acceleration and velocity have different directions, which is quite possible in Newtonian dynamics, so do both forces. Then $\vec{F}_{\mathbf{total}}$, the vector sum of both forces also has a direction different from that of the acceleration. See figure (f).

3rd Problems from the RT can become clearer from my point of view. As an example I give a variant on a thought experiment from Einsteins first publication on $E = m{*}c^2$. A point mass at rest emits simulteaneously in opposite directions two identical photons with frequencies f. The point mass will remain at rest, will undergo no acceleration. Next we step over to an inertialsystem in which the point mass has velocity v. This velocity has the same direction as one of the photons and as the x-axis of the new coordinate system. As a consequence of the Doppler shift the photons get the frequencies:

$f_{forward} = (1 + v/c)/(1 - v^2/c^2)^{1/2}$ en $f_{backward} = (1 - v/c)/(1 - v^2/c^2)^{1/2}$

The problem here is to prove, that in this inertial frame the body does not change its velocity either. He who imagines that before the emission of the photons none of their properties existed, unnecessarily complicates matters for himself. Because, as a result of the differences in frequencies the impulses of the photons are different as well, and one could start thinking that the absolute value of the impulse experienced by the point mass in the forward direction is smaller than that in backward direction. Then the point mass would change its velocity, which is in contradiction with the state of affairs in the rest frame.
If, however, we look at the relativistic mass (so not at the rest mass), everything becomes simple. We find the relativistic mass of the photon by dividing its total energy by c^2. So we find hf/c^2. Before the creation of such a photon its energy, so its relativistic mass as well, was already located in the point mass. This mass therefore had the same velocity as the point mass (which is at least reasonable to suppose). For the forward photon there is no question of a momentum $hf_{forward}/c$ appearing out of nothing, but there is a change in momentum:

$$\Delta p_{\mathbf{forward}} = h.\,f_{\mathbf{forward}} / c - h.\,f_{\mathbf{forward}} . v / c^2$$

Equally:

$$\Delta p_{\mathbf{backward}} = -h.\,f_{\mathbf{backward}} / c - h.\,f_{\mathbf{backward}} . v / c^2$$

Thus it can easily be shown that:

$$\Delta p_{\mathbf{forward}} = -\Delta p_{\mathbf{backward}}$$

So the total impulse on the point mass is zero and so is the change in velocity.

4th The mass defect originating when an atomic nucleus is formed out of unbound nucleons can now be explained more clearly. The fields of the nucleons (their EM-fields and their nuclear force fields) are going to overlap each other more and more. The energy of the EM-fields, and thus their mass, increases, while the energy (mass) of their nuclear force fields decreases (because here we have attractive forces). Finally the decrease overrules the increase, so there is a net mass decrease, a mass defect. Of course no mass (relativistic mass) has disappeared. It has been radiated away in the form of photons, which are always released in an exothermic fusion reaction. So during the creation of the nucleus the fields have radiated away a part of their (relativistic) mass.

5th The idea expressed by Professor Nienhuis (line 10 to 15 of page 11) and living at the back of the minds of many others, that 'the notion of energy has a more fundamental character than the notion of matter', because mass can be seen as a form of energy, but energy cannot be seen as a form of mass (this clearly is the underlying way of thinking in his argument, if we ignore his reluctant footnote), appears to be totally wrong. My view stresses that each form of energy has mass. If we go along with Prof. Nienhuis and we refuse to ascribe mass to electrical energy, we would be just as little entitled to appoint mass to the electrical field of an electron. But an electron's rest mass consists not only of the rest mass of its "hard kernel", but also of the mass of its electrical field in the rest frame. These masses are still not separately known. So indirectly Prof. Nienhuis asks us to consider the electron's rest mass as an unknown. This apparently is an unfruitful way of thinking.
Mass and energy are identical.

Of course relativistic dynamics are no longer the same as the Newtonian when we step over to another inertial frame which is in motion with respect to us. Then time dilatation, Lorentz contraction and a changed synchronisation of clocks give effects which are no longer Newtonian (or rather Galilean). In addition, all relativistic masses and relativistic mass densities change, because the system obtains a different kinetic energy. However, in this new inertial frame that which has just been said, holds again: all energy has relativistic mass and this mass acts in accordance with Newtonian dynamics.

*

§ 7

Conservation, Invariance en Additivity

In this paragraph again invariant mass will be called rest mass.

Invariance and conservation are two different things, which must be distinguished carefully.

A physical quantity of a system is *invariant* when it has the same value for observers with different (constant) velocities.

A physical quantity of a system is *conserved* when before, during and after physical changes in the system (which always imply the system or parts of it undergo accelerations) it has the same value for an observer who doesn't change his own velocity.

When a quantity is conserved, this is expressed in a conservation law. Classical dynamics distinguishes four conservation laws, i.e. of energy, of momentum, of angular momentum and of mass. All four are valid, if and only if, the system is isolated from the outside world in respect to the conserved quantity. Take as an example the conservation of energy. It is valid if and only if the work done by the outside world on the system is zero. All these four conservation laws have as a property, that if we drop the condition of isolation then, even though the quantity may no longer be conserved in the system, it *is* conserved together in the original system and the outside world as a second system. Take again, for instance, the energy; each joule the system looses is gained by the outside world.

Differently stated, if we define several systems, that are in contact with each other but with nothing else, the four classical conservation laws are still valid. This means that such a conserved quantity is indestructible and uncreatable (this neologism will appear useful shortly). When, for example, the quantity diminishes in the system, it increases in an equal amount outside the system.

Another example is found in thermodynamics: the quantity entropy in a reversible process. In adiabatic reversible processes, the entropy of the system is conserved. In diabatic reversible processes, the entropy acts as an indestructible and uncreatable

quantity. Take, for instance, a system performing a Carnot cycle. During this process the entropy of the system alternately increases and decreases, but at the same time there is an equal respective decrease and increase of the entropy in the outside world. Each joule per kelvin that leaves the system enters the outside world and vice versa.
This seems to be self-evident, but in fact is not. [6] An example of another course of events is an irreversible process in thermodynamics. Now the total entropy increases. Though entropy can very well decrease for some time in the system, there will at the same time be a bigger increase in the outside world. A process in which the total entropy decreases is never found, at least not for processes with large numbers of particles involved. So entropy is conserved in a system that goes through an adiabatic reversible process, it is indestructible, however it is in general not uncreatable.

Now can we conceive of a quantity that is conserved when the system is isolated, that is destructible and creatable?
The answer is yes. It is rest mass. The rest mass of a system of point masses (so the invariant mass) is a conserved quantity in the RT, provided no work is done by the outside world on the system. If we drop this condition, then rest mass, for example, can disappear from the system without submerging in the same rate in the outside world. This becomes clear when we consider the following example. Consider a system of two protons which move in opposite directions at equal speeds with respect to an inertial observer, see fig. (g). The right-hand proton collides completely

fig. (g)

elastically and centrally with a helium nucleus initially at rest with respect to the observer. We see the helium nucleus as part of the outside world. The relativistic mass of the helium nucleus increases, so that of the proton decreases. The centre of mass of the system (the system is the proton pair) is going to move to the left (this follows from momentum conservation). It can be calculated that the speeds of both protons relative to the system's centre of mass have diminished. So the rest mass (=invariant mass) of the system has decreased. The rest mass of the outside world however remains the same. So we see, that although for one single system rest mass can be conserved, rest mass is not indestructible.
This is equivalent to non-additivity of rest mass; the total rest mass of a system is in general not equal to the sum of the rest masses of its parts.

Rest masses of systems are additive only, if their centres of masses are at rest relative to each other.

[6] Angular momentum is only indestructible if, for the system and the outside world, the same point of rotation (the point in reference to which the angular momenta are calculated) is taken *or* if the centres of masses of the system and the outside world are at rest relative to each other.

The following two schemes give a survey of classical as well as relativistic dynamics as far as conservation and invariance is concerned.

<u>In classical dynamics</u> . . .

1c the total energy of a system on which no work is done by the outside world, is conserved.
2c the total energy of a system is **not** Galilei invariant.
3c the total mass of a system, which doesn't exchange mass with the outside world, is conserved.
4c the total mass of a system, which doesn't exchange mass with the outside world, is Galilei invariant.

<u>In relativistic dynamics</u> . . .

1r the total energy of a system on which no work is done by the outside world, is conserved.
2r the total energy of a system is **not** Lorentz invariant.
3r the total relativistic mass of a system, on which no work is done by the outside world, is conserved.
4r the total relativistic mass of a system is **not** Lorentz invariant.
5r the rest mass of a point mass is conserved. [7]
6r the rest mass of a point mass is Lorentz invariant.
7r the rest mass of a system on which no work is done by the outside world, is conserved.
8r the rest mass of a system on which no work is done by the outside world, is Lorentz invariant.

Point 2c is evident. An observer who changes his velocity, will see the system getting a different energy.
The explanation of 2r is similar, albeit more difficult to comprehend because of the potential energies in the system.
Statement 3r is in fact the same as 1r, because of the equivalence of mass and energy (relativistic mass and total energy, different from Okuns view); if we change the kinetic energy by a Lorentz transformation, we also change the relativistic mass.
4r and 2r are also the same.

Why do physicists, anyway Okun (in his article, see ref. 10 of the list of literature on page 52 of this article), Taylor and Wheeler (in *Spacetime Physics,* see ref. 15, page 52), and in fact Einstein too (in a letter to Lincoln Barnett, 19 june 1948, see the article of Okun) choose rest mass as *the* mass?
I think they believe mass has to be both conserved *and* invariant, as indeed rest mass is. In my opinion they have this idea, because they stick too much to the prerelativistic dynamics, in which these properties with respect to mass go hand in

[7] By definition I assign no internal degrees of freedom to a pointmass.

hand. In my opinion they are not, or at least not enough, aware of that important difference between the conservation law for rest mass and the four classical conservation laws; rest mass, in contrast to classical mass, classical energy, classical momentum and classical angular momentum, is not additive.

If we start calling rest mass *the* mass all these confusions and mistakes arise, precisely because of the non additivity.

If on the other hand we start seeing relativistic mass as *the* mass, we have a quantity that is conserved, that is additive and in which the rest mass is already included; the rest mass of a system is simply the relativistic mass at zero speed of the centre of mass (or if one wishes, the length of the energy-momentum vector divided by c^2).

Up until now I haven't seen a single valid argument as to why one should not see relativistic mass instead of rest mass as *the* mass. In *Spacetime Physics* Wheeler and Taylor name two arguments to call rest mass *the* mass: "The concept of "relativistic mass" is subject to misunderstanding. That's why we don't use it. First, it applies the name mass - belonging to the magnitude of a 4-vector - to a very different concept, the time component of a 4-vector. Secondly, it makes increase of energy of an object with velocity or momentum appear to be connected with some change in internal structure of the object. In reality, the increase of energy with velocity originates not in the object but in the geometric properties of space-time itself."

In the first argument Wheeler and Taylor make an apparent error of thought; they want to decide which mass, rest or relativistic, deserves most to be called Mass. Then they suggest: It is the mass belonging to the length of the 4-vector (this is the rest mass), because it had this name already. This is no argument, it was indeed the first to be called thus, but that was not the question. The question was, whether it deserved that name. Here I think a certain mystification plays a role. It is not for nothing that Taylor and Wheeler speak of "the mass, belonging to the length of a 4-vector" instead of simply "the rest mass". One sees the four-space like something "really existing" (what does that mean, really existing?). Anyway, many people see the 4-space as something without which "the picture of the theory of relativity is not complete" (professor Berends, Rijksuniversiteit Leiden, in a letter to a scientific journalist of the NRC Handelsblad, a Dutch newspaper). One is then rather tempted to see the length of such a 4-vector as something more important than a notion derived from the "ordinary" threedimensional space. Therefore one sees rest mass not simply as the relativistic mass at speed zero, but as something that is derived from a more fundamental concept. This idea is definitely wrong, the SRT can fully be described and understood without the formalism of the 4-space, beautiful as it may be. We can conceive the 4-space as merely a mathematical aid. And rest mass we can see as nothing more than the relativistic mass at speed zero.

As far as the second argument is concerned, Wheeler and Taylor have something in mind which I cannot follow. Why should I have to think that relativistic mass increase would mean a change of the internal structure of a body? With an internal change in structure of an object I imagine a change in the arrangement of the atoms and/or the elementary particles of that object. With that mass increase I, on the other hand, imagine that each infinitesimal part of the object undergoes a similar mass increase and for that I need not presume that that arrangement changes. The structure changes in so far, that Lorentz contraction takes place. But if that were not admitted, we would have to abandon the RT.

This second argument cannot be correct for two other reasons also. First: applied to the proper time versus an observer time it would mean that only the proper time deserves the name Time. "If travelling by rocket we pass an observer, who determines our speed with his clocks, we would not be allowed to see his clocked times as values of *the* time;" (so far nothing really incorrect has been said) "for they are read from clocks which have a slowed down pace for us and then it seems as if those clocks have had a change in their internal structure." And: "In reality the slowed down pace originates not in the clock but in the geometric properties of timespace itself". The conclusion should then be that we only have the right to call proper time *the* time and a time of an inertial observer moving in respect to the body in question would be subject to misunderstanding, therefore should rather be avoided. This conclusion is obviously false, so Wheeler's and Taylor's reasoning is false as well.

Secondly: remarkable is their view of that relativistic mass increase. This would "originate from the geometric properties of spacetime itself". Evidently they think of an object which obtains a bigger relativistic mass for us as observers because we step over to an inertial frame with another velocity. They completely ignore the possibility that an object increases in relativistic mass with respect to us because it undergoes an acceleration itself, while we as observers remain in the same inertial system. In this last case it is practical/customary to regard the relativistic mass increase as a consequence of the supply of energy. It can on no account be seen as "originating in the properties of spacetime itself". So Wheelers and Taylors second argument moreover has no general validity.

In the following passage mistakes, which are made in relation to rest mass and which arise from overlooking the non-additivity of rest mass are discussed. Because there is a great deal of confusion on the notion of rest mass, good definitions should, first of all, be given for some notions concerning rest mass;

1 We have the theoretical case of rest mass of a point mass.

2 We have the rest mass of a system of parts moving relative to each other, defined by equation (1) on page 6, which one has redundantly named "invariant mass". This I call the rest mass of a system (in contrast to the rest mass of a point mass).

3 We have something which is carelessly called "rest mass", namely the sum of a number of rest masses of a number of bodies, which can be seen as point masses and which are in motion relative to each other. For example, the sum of rest masses of the fission products flying away from each other after a nuclear reaction. This "rest mass" I will name "rest mass sum".

Moreover it is important to note that the sum of rest masses (for a number of systems interacting with each other but not with the outside world) is **not** conserved. Is the rest mass sum Lorentz invariant? No, it isn't either. This is clarified by the following (in which A and B are two inertial observers in motion with respect to each other): A looks at the fission products of a uranium nucleus flying apart. He starts looking *after* the moment of fission. He therefore sees no change in the rest mass sum (I mean the sum of the separate rest masses of the fission products). Now we transform to B. We have chosen B's velocity and position with respect to A in such a way that, in B's *local* time, the fission has not yet taken place. Then B finds another rest mass "sum" (a bigger one, namely the rest mass of the uranium nucleus. So a "sum" of one term). Looked at it this way, the rest mass sum is not invariant under Lorentz transformations. Of course this is caused by the non-conservation of the rest mass sum *and* the shift in time in a Lorentz transformation. So we see here that invariance and conservation are interconnected as in a Gordian knot, and are in fact inseparable. Because a number of readers of my article found the above argument unclear, I added appendix III, page 69.

An example of an obvious mistake: consider an isolated system of a uranium nucleus on the verge of fission. Even after the fission, the system has no interaction with the outside world, so the fission products are not slowed down, but remain at full speed. It is often said that rest mass disappears in the fission, which is incorrect. This way one compares the rest mass of the uranium nucleus with the *sum* of the rest masses of the fission products, *not* with the rest mass of (the system of) the fission products. The rest mass of (the system of) the fission products, that is to say the sum of their relativistic masses with respect to their joint centre of mass, is exactly the same as the rest mass of the uranium nucleus. Calling the rest mass sum of the fission products "rest mass" is suggesting that rest mass is an additive quantity. That this is often not only a matter of sloppy language becomes clear when one adds: "Rest mass is Lorentz invariant". The rest mass sum is certainly not Lorentz invariant.

Another mistake: to speak of the rest mass sum of a system without adding in what way one divides the system in objects of known rest masses. The rest mass of a hydrogen atom is not the same as the sum of the rest masses of an electron and a proton. The rest mass of the fission products in the example above is equal to the rest mass of the uranium atom. The rest mass sum of the fission products is less than the rest mass of the uranium atom. This mistake makes the quantity of "mass converted to energy" appear to be dependent on this partition. But this partition is arbitrary, so the quantity of mass "converted to energy" would be arbitrary too, which is absurd.

People who consider rest mass as the only true mass can never say that the mass and energy of a system are really equivalent, because the rest mass is a Lorentz invariant, while total energy is not. Then they don't belong together in $E = mc^2$, for in such a relativistic equation the left and right hand side should transform in the same manner. These people have to construct an elaborate scheme describing which kinds of energy do and which don't have mass; for example, the kinetic energy of a system "doesn't have mass", because it is proportional to the difference of relativistic mass (*not* invariant) and rest mass (invariant). From the linearity of the lorentz transformation it follows then that the difference of the two is not invariant either.
Another example; a *B*-field of itself has no mass, because "you cannot state as a matter of fact whether such a field is at rest or not, so you cannot determine its rest mass", but nevertheless it contributes to the rest mass when it is part of a bigger system.
These people are also inclined to make the following mistake: they deny the fact that

photons (for they exist merely out of kinetic energy), and consequently other types of EM-field in the system, contribute to the rest mass (=invariant mass) of the system. But then things definitely go wrong. Consider a system of two atoms exchanging a photon. One atom emits the photon, the other one absorbs it. Before and after the exchange, the rest mass of the system is the same to an inertial observer *A*. Now transform to an inertial observer *B*, for whom the photon is just passing over from atom to atom. Then the rest mass for *B* would be all of a sudden smaller than that for A. From this point of view rest mass would, besides being non-conserved, also be non-Lorentz invariant. Once again, professor Nienhuis makes this mistake (page 11 of this paper). At the same time he suspects it is wrong, which is apparent from the footnote.

Note that Okun says explicitly that mass has to be *invariant*, page 31 of Okun's article, from the 8-th to the 4-th line from the bottom. He nowhere says that mass has to be conserved. He apparently sees invariant mass as the only mass.

Another mistake made just as often is the supposition that there is no measure (in relativistic dynamics) for the amount of matter. In classisal dynamics it was mass, in relativistic dynamics nothing would have taken over its role. In *Spacetime Physics* Wheeler and Taylor literally say: 'Nature does not offer us any such concept as "amount of matter." History has struck down every proposal to define such a term. Even if we could count number of atoms or by any other counting method try to evaluate amount of matter, that number would not equal mass. First, mass of the specimen changes with its temperature. Second, atoms tightly bonded in a solid weigh less - are less massive - than the same atoms free. Third, many of nature's atoms undergo radioactive decay, with still greater changes of mass. Moreover, around us occasionally, and continually in stars, the number of atoms and number of particles themselves undergo change. How then speak honestly? Mass, yes; "amount of matter," no.'

This supposition is really wrong. We must look at the relativistic mass, that is the one which is conserved and additive. It is indeed not invariant, but that's no major objection. We can say therefore, *in a given inertial frame* that relativistic mass is the measure of total amount of matter (in a system, in the universe). We simply have to abandon the idea that the total amount of matter has to be an invariant for some reason. It is conserved, not invariant. The non-invariance is only a small price to pay for the preservation of a criterion of the amount of matter. Completely to drop such a criterion is certainly a much higher price.

I propose to name relativistic mass *the* mass and the related conservation law *the* law of conservation of mass. Do we have to add to this law: . . . "for an observer who doesn't change his velocity"? Yes and no. No, because it goes without saying more or less. To the classical law of conservation of energy we don't add it either, although it is a necessary condition. Yes, because nothing appears to be so confusing as notions in the theory of relativity. For all clarity it is advisable, at least for the time being, to add it.

It takes a while to get used to the idea that the total mass in the universe is not an invariant, but depends on the state of motion of the observer. On the other hand this step is not such a big one. The total energy in the universe also depends on the state of motion of the observer. This is not only so in the theory of relativity, this was already so in classical mechanics.

*

§ 8

Problems I have been unable to solve

The notion that an electrical field has mass is not new. At the second last turn of the century the idea even came up that the *whole* mass of the electron could be electrical field mass. This idea was worked on by, amongst others Lorentz, Abraham, Pauli and von Laue. One started representing the electron as a small, electrically charged, hard kernel with its charge homogenically divided over its surface (or sometimes otherwise). If the radius of the electron is made small enough, the field near the surface gets so strong that the entire electron mass is finally located in the field. One calculated that radius, which is now known as the classical radius of the electron. For a short period it seemed as if the phenomenon of mass and its behaviour could be deduced from electrodynamics. Newton's laws would then loose their status as first principles.

This soon appeared to be impossible. When we try to deduce from the EM-field the momentum of an electron with the classical radius and velocity $\vec{v}$, things go wrong. Instead of $m_{\mathbf{rest}}g\,.\vec{v}$ we find $\frac{4}{3}m_{\mathbf{rest}}g\,.\vec{v}$. To my knowledge this problem has never been solved satisfactorily. Poincaré pointed out in one of his publications (Circ. Mat. Palermo 21, 129 of 1906) that there has to be another force-field holding the electron together, preventing it from being torn apart by the repellant electrical forces. If these new forces are taken into consideration, the right momentum would be obtained according to Poincaré. I haven't read this publication, but apparently it has not convinced everybody, for there have been later publications of others with different explanations. One of them is by Rohrlich (*Self-energy and Stability of the Classisal Electron*, F. Rohrlich, Department of Physics and Astronomy, State University of Iowa, Iowa City Iowa, 12 feb. 1960). Unfortunately I only possess a photocopy of it and therefore don't know the name of the scientific periodical from which it comes. Rohrlich claims that in calculating the momentum the integration is wrongly performed. According to me his claim boils down to having to integrate in three dimensional space over the fields not at the same but at different times for the different volume elements. I cannot think of one reason why this should be done thus. I therefore don't think this is a real solution. I think here the opinion plays a role, that the special theory of relativity can only be understood completely in four dimensional spacetime. This is not so. Spacetime is, in contradiction to relativistic mass, only a mathematical construct (I refer to the special theory of relativity, whereas of the general theory of relativity I have little or no knowledge). As a matter of fact it is possible in the STR to imagine that the aether does exist and that Lorentz contraction and time dilatation (=slowed down pace of all time processes) are physical effects of the aether wind. It is just not possible to say in respect to which observer the aether is at rest. In fact, each observer can imagine with the same right that the aether is at rest with respect to hìm (or is moving with an arbitrary velocity). To me this was a discovery. From the very moment that I comrehended this, I didn't make a single mistake anymore in solving the standard problems given by the teaching books on STR (in sharp contrast to before). Yet I keep on hearing: "But the aether doesn't exist, this is a proposition of the TR", or ". . . that would include, something like 'absolute movement' exists, but this is not so" or ". . . Einstein has demonstrated, aether doesn't exist".

Einstein never demonstrated that. He has merely proposed to abandon the idea of an aether, because we cannot measure with respect to which inertial frame the aether is at rest, and to see the Lorentz transformation as an abstract, mathematical procedure. One often says "as a transformation of space itself". This of course is no more than play on words. We replace "measured distances and times" by "space".

The aether simply is a handy mental model. Imagine the aether is at rest with respect to yourself. Then all clocks moving with respect to you are running too slowly (slowed down with a factor g), all measuring rods having a certain velocity relative to you are too short (contracted with a factor g in the direction of their velocity) and Einstein-sychronicity is then nothing more than an incorrect synchronising of the clocks, namely without taking into account the aether wind. In this way you can completely think in three (spacial) dimensions, because time, your time, has become absolute again (and keep in the back of your mind the fact that each other observer can reason in just the same way, with the aether at rest with respect to him).

Looked at in this way, Rohrlichs explanation is completely unsatisfactory. Integrating in three spacial dimensions while considering time as a constant *should* give the right momentum. It is not true that in three dimensions we get an incomplete picture of a relativistic system.

I myself think that something in the EM-theory is wrong. When I calculate for an electron with a constant velocity $\vec{v}$ which velocity the field mass has in each point of the field (this I calculate again with the method of dividing the Poynting vector by the energy density, see footnote 4 on page 29), I would expect to find $\vec{v}$. Instead I find a complicated stream pattern, in which the field mass continually overtakes the electron in a curved path around it, and is subsequently overtaken by the electron.

So the EM-part of the electron mass has never been found and the problem of the factor 4/3 has never been solved. Does this undermine my view on relativistic mass? I don't think so. The problems manifest themselves in the so called source terms, thus in the terms describing the field of an electrical charge at the points where the charge is located. My formula (6), on page 28, is not a real source term. It doesn't contain the square of the charge's own field, but the product of its own field and the external field. I therefore cherish the hope such terms behave themselves properly.

For some time I have thought that it would be possible to deduce the term $v{*}\mathrm{d}m/\mathrm{d}t$ from the Maxwellian stress theorem. The Maxwellian stress theorem forms, together with the theorem of Poynting, a relativistic covariant four by four tensor. The Maxwellian stress theorem has already been given on page 30. Here it is again:

$$\oiint_A ((\vec{\vec{T}} - \vec{\vec{\mathbf{1}}} \cdot U) \cdot \mathbf{d}\vec{A} = \vec{F} + \iiint_V c^{-2} \cdot \frac{\P\vec{S}}{\P t} \cdot \mathbf{d}V \qquad (8)$$

As already explained, it states that the EM-force can be seen as an elastic stress in a solid. Just as from the stresses in a propeller shaft of a ship the amount of force transferred per square centimetre from motor to propeller can be calculated, the same can be done for the EM-force; the left hand side of (8) is the total EM-force on volume V and is transmitted by the surface A of V to V's content. $\vec{F}$ is the force on a possible electrical charge in V. In the ship's shaft, while accelerating its rotation, a part of the stresses is necessary to accelerate the shaft itself. In the same manner in equation (8) $\iiint_V c^{-2} \cdot \frac{\P\vec{S}}{\P t} \cdot \mathbf{d}V$, together with some other terms, which can be separated from the left hand side of (8), represent the force necessary to accelerate the mass of the EM-field inside V. I have tried to prove $\iiint_V c^{-2} \cdot \frac{\P\vec{S}}{\P t} \cdot \mathbf{d}V$ and the other terms are equal to $\vec{v} \cdot \frac{\mathbf{d}m}{\mathbf{d}t}$. I did not succeed. Later I found out by way of a thought experiment these expressions cannot be equal. What $\vec{v} \cdot \frac{\mathbf{d}m}{\mathbf{d}t}$ represents is something which takes place on the boundary layer between field and charge. I am now inclined to think this is a region in which the Maxwell equations no longer apply, so the Poynting vector cannot be of help either. I think Poincaré was right, i.e. that stresses of a non EM-nature are at work here. As soon as we know these, it is possible to calculate the stream field of the mass in this region.

For a simple case I have been able to reason out that the velocity of the field mass streaming toward the charge is perpendicular to the velocity of that charge at the places where that mass is entering the charge (that is in the boundary layer). That is in agreement with the idea that the field mass has to be accelerated form speed zero to the velocity of the charge; the component of the mass' velocity in the direction of movement of the charge is zero.

*

§ 9

Relativistic mass is *the* mass

As soon as we consider relativistic mass as being *the* mass, a considerable number of things become simpler. That is what I have tried to make clear and I will resume it here. I will do so in a question and answer form, like Taylor and Wheeler do in *Spacetime Physics*, but with the important difference that in their dialogue relativistic mass falls short in respect to rest mass. Of course I will make the opposite happen. Besides, I don't give only (in bold type) my own arguments/answers in favour of relativistic mass, but subsequently also (in italics) the arguments/answers of the rest mass adherents. It is important to realize in the bold text the word 'mass' is seen as the relativistic mass and in the italic text 'mass' is seen as the rest mass.

1. Does the equivalence of energy and mass, $E = m.c^2$, mean that the notions of energy and mass, except for their historically grown units, are identical?

Yes.

No, not identical. The kinetic energy of the system as a whole has no mass. On the other hand, when a system is built up of moving point masses, their kinetic energy relative to the rest inertial frame of the system contributes to the total mass of the system.
EM-fields themselves have no mass, but belonging to a bigger system they can contribute to the mass of the system. The equivalence means that mass can be converted to energy and vice versa. Mass is a form of energy. 'In that way the notion of energy gets a more fundamental character than the notion of matter' (I am quoting professor Nienhuis).
With $E = m.c^2$ *we can, for example, calculate the energy released during the*

explosion of a hydrogen bomb. Here, part of the sum of masses of the nuclei is converted to energy. The mass of the system as a whole does not change as long as it does not interact with the outside world. In the words of Wheeler and Taylor: "Thus part of the mass of the constituents has been converted to energy; but the mass of the system has not changed."

2. Can we say of a single photon that its mass is equal to its energy divided by c^2?

Yes.

No, a photon has no mass, it has only energy. This is a subtle point; a massless photon may "transfer" nonvanishing mass (this is stated by Okun on page 34 in his article, see ref 10, page 52).

3. In classical mechanics the momentum of a pointmass is defined as its mass times its velocity. Can we maintain this definition in STR in such a way that again a law of conservation of momentum is valid and apply it also to photons?

The answer to both questions is yes.

No, material bodies in the STR have a momentum which is g times the classical momentum. Photons have a momentum equal to their energy divided by c. For momentum of material bodies we need another definition than for momentum of photons.

4. Can we, if we merge two systems into one, add both their masses to find the mass of the combined system?

Yes.

*No, mass isn't additive. This is only possible when 'two non-interacting objects move freely and in step, side by side' (I cite Wheeler and Taylor, page 247 of "Spacetime Physics", second edition. Notice, they don't say: "With equal velocities". **The** velocity for a non-point mass has a cumbersome definition, unless you simply define it as the velocity of the centre of mass. But in a moment we will see that the centre of mass cannot be defined in the rest mass view).*
We have to determine again the length of the energy-momentum vector of the combined system and divide it by c.

5. Can we say that the energy density of the EM-field, as long as we don't take into account the self energy of the source terms with that factor 4/3, has mass density, can we attribute velocity to it and can we say, this velocity times this mass density is equal to impulse density, just as in classical hydrodynamics?

To all three questions the answer is yes.

No, to the energy of any EM-field we cannot attribute mass density, because then we should do the same to an EM-radiation field and next we would have to attribute mass to photons. To photons we can attribute velocity, but to different forms of EM-field energy, or to any other form of field energy, we cannot do so. We have indeed no criterion for the "velocity" of the energy density of a field.

6. Can an inertial observer (who of course by definition doesn't step over to an inertial system with a different velocity) use mass as a measure for the amount of matter, for matter in the sense of a conserved, indestructible quantity?

Yes.

I repeat the quotation (page 41) in this paper of Taylor and Wheeler: 'Nature does not offer us any such concept as "amount of matter". History has struck down every proposal to define such a term. Even if we could count number of atoms or by any other counting method try to evaluate amount of matter, that number would not equal mass. First, mass of the specimen changes with its temperature. Second, atoms tightly bonded in a solid weigh less - are less massive - than the same atoms free. Third, many of nature's atoms undergo radioactive decay, with still greater changes of mass. Moreover, around us occasionally, and continually in stars, the number of atoms and number of particles themselves undergo change. How then speak honestly? Mass, yes; "amount of matter," no.'

7. Can the centre of mass of a system of particles with non-zero rest mass and photons in the STR have the same definition as in classical mechanics?

Yes.

The notion centre of mass in fact has taken French leave in the STR. We prefer to speak of the rest inertial frame, this is the frame in which the total momentum is zero. That is nothing more than doing one's poor best, for this frame only tells us with what velocity the CM moves, not where it is situated. This poor best however is unavoidable, because EM-fields do have momentum, but no mass. If then we would calculate the CM in the classical way, we could not *take the EM-fields into consideration, because their mass is zero. But then their momentum is also missing on the roll-call, so we would find an incorrect value for the total momentum linked with the CM, thus also an incorrect value for the state of movement of the CM. That Einstein with his "box of Einstein" used the CM and found in this way a correct deduction of* $E = m.c^2$ *has to be seen as an historical lucky dip, for it is an 'inconsistent conclusion' (the latter is said by Okun, page 34 of his article, see ref. 10).*

8. Are relativistic dynamics the same as Newtonian?

Yes. See the explanation in § 6, page 27 to 34.

No. Consider a rectilinear accelerated movement of a mass. The magnitude of the total force is more than we would expect on the ground of $\vec{F} = m \cdot \vec{a}$.

9. Is the formula for centripetal force in the STR the same as in classical dynamics, namely $|F_c| = m.w^2.|r|$?

Yes.

No, in the STR it becomes $|F_c| = m.g.w^2.|r|$

10. Which quantity is equal to the length of the energy-momentum vector?

Rest mass times *c*.

Mass times c.

***

Recommended literature:

ref. 1 H.Dryden, *Hydrodynamics*, Murnahan Bateman. Treats mathematics of hydrodynamics in a transparent way. Important for understanding the Maxwellian stresses.

ref. 2 A. Einstein, *Does inertia of a body depend upon its energy-content?*, English translation of the article of 27 sept. 1905 in Annalen derPhysik 17, 639, copy available at Institute for Physics Education **'s Gravesande**.

ref. 3 R. Feynman, *Lectures on Physics, Volume II*, Addison - Wesley, Massachusetts. See 17-4 of the *Lectures*, in which is given the paradox mentioned in the footnote on page 18 of this article. See also 27-5 en 27-6. In 27-5 Feynman treats the case of a permanent magnet and a charge, around which the Poynting vector field is non-zero, but represents a stationary flow in closed orbits. In 27-6 he remarks, the Poynting vector divided by c^2 is equal to momentum density. One could just as well say that it is the density of mass stream, then the solution of the paradox of 17-4 is clear at once; in the EM-field an amount of mass was stored, which was flowing in loops around the apparatus and therefore possessed an amount of angular momentum. That amount has been transferred to the apparatus, which starts it turning.
My idea as depicted in this article I owe in fact to the reading of these two paragraphs, especially the last three sentences: "This mystic circulating flow of energy, which at first seemed so ridiculous, is absolutely necessary. There is really a momentum flow. It is needed to maintain the conservation of angular momentum in the whole world." Everything became clear to me by reading in this text 'mass' instead of 'energy'.

ref. 4 French, *Special Relativity*, W. W. Northon & Company, Inc., New York 1968.

ref. 5 Jeans, *The mathematical theory of electricity and magnetism*. The Lorentz library at Leiden, Netherlands has a copy. Gives a good picture of how mechanical stresses and accelerations (waves) in a solid caused thereby can be described using tensors (page 142). The aim of it is to show that the Maxwellian stress tensor gives exactly such a description of EM-forces, namely like stresses in the aether, and that the aether itself has to be seen as a body with mass, which can undergo accelerations.

ref. 6 J. Kokkedee, *Klassieke Mechanika en Relativiteitstheorie*, sept 1992, Technical University of Delft, faculty of Theoretical Physics. Only available in Dutch.

ref. 7 B. Kwal, *Les expressions de l'énergie et de l'impulsion du champ* *électromagnétique propre de l'électron en mouvement.* J. Phys. et Rad. 10 (1949), 103.

ref. 8 Mason and Weaver, *The EM-field*. The Lorentz library in Leiden, the Netherlands, has a copy. Educational book on electromagnetism, extremely obsolete and therefore interesting. On page 270 there is an eloquent plea for the view that electron mass is of a purely electromagnetic nature.

ref. 9 G. Nienhuis, *Inleiding tot de relativiteitstheorie, cursus voor 5 VWO*, Huygens Laboratorium Rijksuniversiteit Leiden. Only available in Dutch. On page 32 he gives his interpretation of $E = m{*}c^2$.

ref. 10 L. Okun, *The concept of mass*, Physics Today, june 1989, page 31. Copies available at the Institute for Physics Education **'s Gravesande**.

ref. 11 A. Pais, *"Subtle is the Lord . . ."*, Clarendon Press Oxford, ISBN 0-19-853907-X. See above all page 148 et. seq.

ref. 12 Panofsky en Phillips, *Electricity and Magnetism*. Chapter 21 gives a treatise on the radiation reaction force on an accelerating electron.

ref. 13 Rohrlich, *Self-energy and Stability of the Classisal Electron*, F. Rohrlich, Department of Physics and Astronomy, State University of Iowa, Iowa City Iowa, 12 feb. 1960, copies available at the Institute for Physics Education **'s Gravesande**.

ref. 14 Rohrlich, *Classical charged particles*. The Lorentz library at Leiden, the Netherlands has a copy.

ref. 15 E. F. Taylor en J. A. Wheeler, *Spacetime physics, introduction to special relativity* 2^e edition, Freeman and Company, New York, ISBN 0-7167-2327-1. A popular scientific book, but then on a high level. In particular Wheeler is an authority on the field of TR. The book is extremely female-friendly. In the chapters with odd numbers, inertial observers are referred to with female pronouns, in the even ones with male pronouns. A book of uncontested scientific level and a high entertainment value.

ref. 16 H. Biezeveld en L. Mathot, *Scoop, Natuurkunde voor de bovenbouw 5 havo*, 2^e edition, November 1991, ISBN 90 01 07636 X.

***

APPENDIX I

Conservation of momentum and the centre of mass

CONSERVATION OF MOMENTUM IN CLASSICAL MECHANICS

The law of conservation of momentum for n point masses in classical mechanics states: **If *n* point masses experience forces exerted by each other but not by the outside world, the sum of their momenta is conserved**.
The law of conservation of momentum in classical mechanics can be deduced from Newton's laws:
For particle i we can write:

$$\sum_{j=1}^{n} \vec{F}_{i,j} = m_i . \frac{\mathbf{d}\vec{v}}{\mathbf{d}t} \qquad (\textbf{with } \vec{F}_{i,i} = 0)$$

If we choose the masses so that they are not a function of time, as is usual in classical mechanics (it is really a matter of choosing, think of a rocket burning fuel as a counter example), m_i can be carried through the d of the differential quotient. If we then sum over i as well we get:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \vec{F}_{i,j} = \sum_{i=1}^{n} \frac{\mathbf{d}m_i . \vec{v}}{\mathbf{d}t} \qquad (9)$$

Now on the left-hand side all forces cancel in pairs, because of Newton's third law. Integrating the equation to time yields the classical law of conservation of momentum for n pointmasses:

$$\sum_{i=1}^{n} m_i . \vec{v}_i = \textbf{quantity independent of time} \qquad (10)$$

Notice that, contrary to relativistic mechanics, classical mechanics permits forces to be of the type actio in distans; there is no need for the action and reaction forces $\vec{F}_{i,j}$ and $\vec{F}_{j,i}$ to be contact forces.

Suppose external forces do act on the particles. Then on the left hand side of equation (9) the summation $\sum_{i=1}^{n} \vec{F}_{i,ext}$ appears. If this summation is zero, then the total momentum of the system is still conserved. So in classical mechanics the total momentum of a system is also conserved when external forces do act on some or all particles of the system, provided the sum of these forces is zero. It is important to realize that these forces don't have to sum up to zero for each individual particle. In a moment we will see that two extra conditions have to be added if we want the relativistic total momentum to be conserved in every inertial system.

CONSERVATION OF MOMENTUM IN RELATIVISTIC MECHANICS

If in equations (9) and (10) classical mass is replaced by the relativistic mass equation (10) becomes the relativistic law of conservation of momentum for pointmasses *in a given inertial system* and equation (9) the essential step in its derivation; the derivation is completely analogous to the classical case. If, however, we want the law to be valid for all inertial systems, two extra conditions must be added. The first condition concerns the internal forces. Neither action nor its reaction force, so neither $\vec{F}_{i,j}$ nor $\vec{F}_{j,i}$, should be of the type actio in distans. They should be contact forces, which are exerted for an infinitesimal amount of time during the collisions of the point masses. In other words, the particles can only collide if their position vectors are the same. If this were not demanded, a Lorentz transformation whould cause a violation of Newton's third law, because when action and reaction forces are not exerted at the same point in space, it whould mean that after a Lorentz transformation these forces are not necessarily opposite to each other because of the time difference caused by the transformation (after a Lorentz transformation $\vec{F}_{i,j}$ could start to act sooner than $\vec{F}_{j,i}$ for instance). So the first condition has to be that the internal forces are contact forces which, just as in classical mechanics, obey Newton's third law. The second condition concerns external forces. Suppose there are external forces whose sum is zero in the given inertial system and which are acting on different particles at different positions. In this case, again, the momentum is only conserved *in the given inertial system*. In certain other inertial systems the Lorentz transformation would cause the forces to start and end at different moments, and they would therefore no longer compensate for each other's effects (same argument as above with the internal forces). Only when each external force is compensated by one or more external forces acting on the same particle of the system can we expect relativistic momentum conservation to be valid also in all other inertial systems. I will call these forces locally compensating forces. So the second condition is that all external forces are locally compensating forces.

Because in relativistic mechanics force is defined as:

$$\vec{F} \equiv \frac{\mathbf{d}m_{\mathbf{rest}}.\vec{v}}{\mathbf{d}t} \qquad (11)$$

the derivation of conservation of the total relativistic momentum of a system of *n* point masses in one and only one inertial system is completely analogous to that in the classical case given above. The two extra conditions we just posed are necessary to ensure that momentum conservation will also to be valid in any other inertial system. The derivation remains true if photons are involved (of course the forces involved are infinite, because in the relativistic case we assume the collisions to take place in an infinitesimal amount of time. These forces should be represented by Dirac delta functions. This poses no real problem. Moreover, we can avoid talking about the forces altogether and simply state that during a collision the increase of momentum of one particle is equal to the decrease of momentum of the other particle(s). This argument is correct for collisions between photons and rest mass particles as well as for collisions between rest mass particles among each other).

Suppose that an electromagnetic field is part of the system, for instance because some of the point masses in the system are electrically charged. This would not change the above reasoning. The system could now be split up into an infinite number of infinitesimal volumes, each touching its neighbours at its boundary surface. Imagine these volumes to move along with the energy that they contain (see again footnote 4 on page 29). These volumes can be seen as point masses which, according to Maxwell's stress tensor, exert only contact forces on each other. Now Maxwell's stress tensor is a complete analogy for the classical stress tensor for stress forces in a solid or a liquid. And the stress forces in a solid or a liquid across each surface are contact forces that obey Newton's third law (see again page 30, from formula (8) to page 31, the first blank line). So these volumes can be considered as point masses exerting Maxwellian contact forces on each other which also obey Newton's third law. This means that the classical deduction of the law of momentum conservation is also valid for an electromagnetic field. The question is, what will be the interaction between these infinitesimal EM-field volumes and the "material" point masses (after all, the material point masses can at all times absorb and create the masses in these infinitesimal EM-field volumes). If Poincaré was right (see page 43, line 15 to 19), then in this process stress forces of a non-electromagnetic nature are at work. I simply suppose that these stress forces are also contact forces and that they too obey Newton's third law. Then the proof of momentum conservation in the classical case can be carried over word for word into the relativistic case for a system consisting of point masses and EM-fields.

According to the relativistic principle one can assume that the same can be said for all other force fields (a gravity force field for instance). **So for each system consisting of point masses and force fields the total relativistic momentum is conserved in each inertial system, provided there are no external forces acting on the system, other than locally compensating forces, and provided all internal forces are contact forces obeying Newton's third law.** However, the total momentum is not the same in all inertial systems, so momentum is *not* invariant. Furthermore, several external forces with zero sum and acting on the *same point* of the system are allowed (these are what I call locally compensating forces); they will leave momentum conservation unaffected in each inertial system.

Because conservation of relativistic momentum is always experimentally confirmed, this indirectly confirms the validity of Newton's third law in relativistic mechanics.

NEWTON'S SECOND LAW AND THE CENTRE OF MASS IN CLASSICAL MECHANICS

Consider a system of n point masses exerting forces on each other (not necessarily contact forces) while some or all are experiencing an individual force from the outside world, the individual forces not necessarily being the same. The masses don't change in time. Then for point mass i Newton's second law can be written as:

$$\sum_{j=1}^{n} \vec{F}_{i,j} + \vec{F}_i^{\mathbf{external}} = m_i . \frac{\mathbf{d}^2 \vec{r}_i}{\mathbf{d}t^2} \qquad (\mathbf{with}\ \vec{F}_{i,i} = 0) \qquad (12)$$

In classical mechanics it can be shown that the total external force on a system of point masses, the total mass and the acceleration of the centre of mass obey Newton's second law. When equation (12) is summed over i and the time differential operator is put in front of the sum on the right hand side of the equation (allowed, because m_i is not time dependent) the result is:

$$\sum_{i=1}^{n} \left(\sum_{j=1}^{n} \vec{F}_{i,j} + \vec{F}_i^{\mathbf{external}} \right) = \frac{\mathbf{d}^2}{\mathbf{d}t^2} \sum_{i=1}^{n} m_i . \vec{r}_i \qquad (13)$$

The internal forces cancel again. We write $\vec{F}_{\mathbf{total}}^{\mathbf{external}}$ for the sum of the external forces and M for the sum of the masses.We define the position of the centre of mass M by:

$$\vec{r}_{\mathbf{M}} \equiv \frac{\sum_{i=1}^{n} m_i . \vec{r}_i}{M} \qquad (14)$$

Because M is independent of time, we can write:

$$\vec{F}_{\mathbf{total}}^{\mathbf{external}} = M . \vec{a}_{\mathbf{M}} \qquad (15)$$

which is Newton's second law for the centre of mass in classical mechanics.

THE ROLE OF THE CENTRE OF MASS IN RELATIVISTIC MECHANICS

Because relativistic mass is the legitimate successor of the classical mass it is a logical step to define the centre of mass of a relativistic system using the same formula as is used for the classical case, that is formula (14), with the classical masses being replaced by the relativistic ones, this time with $n + m$ point masses, n non-zero rest mass particles and m photons. So:

$$\vec{r}_{\mathbf{M}} \equiv \frac{\sum_{i=1}^{n} m_{i,\,\mathbf{rest}} \cdot g_i \cdot \vec{r}_i + \sum_{j}^{m} \frac{E_{\mathbf{photon},j}}{c^2} \cdot \vec{r}_j}{M}$$

$$\left(\textbf{with} \quad M \equiv \sum_{i=1}^{n} m_{\mathbf{rest},i} \cdot g_i + \sum_{j}^{m} \frac{E_{\mathbf{photon},j}}{c^2} \right) \qquad (16)$$

Now the question is, whether this relativistic centre of mass has any meaning in the sense, that it obeys Newton's laws. The answer is, it obeys Newton's first law if the total external force on each separate point mass is zero; several forces on *different* point masses, even if their sum is zero, are not allowed. Why this is so, is explained in the following example, see figures (h) and (i). Figure (h) represents a system consisting of two ions (A and B) of opposite charge and one atom (C), all three bodies being at rest. Then their centre of mass is also at rest. The system is in a homogeneous electrical field. This field is so strong, that the interaction forces between A and B are very small compared with the forces on A and B caused by the external field. All fields, also the ones of A and B, are considered as not being part of the system. Now A and B are allowed to accelerate, while C remains at rest. The force on A is minus the force on B, so the total external force on the system is

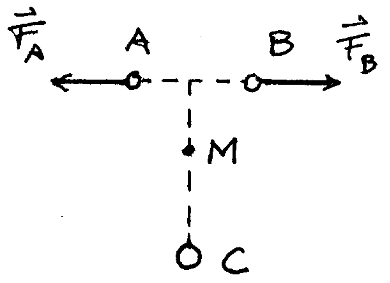


fig (h)

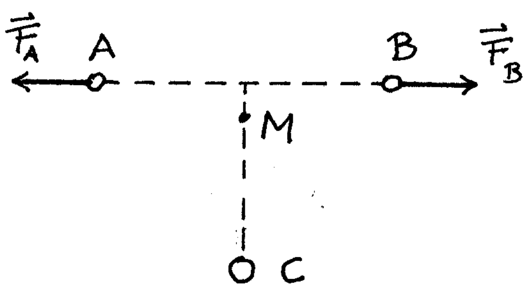


fig (i)

zero. Yet the relativistic masses of A and B increase, so the centre of mass of the system moves, in figure (i) in an upward direction. The centre of mass therefore undergoes a change in velocity while the total external force on the body is zero. So in relativistic mechanics the centre of mass does not always obey Newton's second law, nor does it obey the first if external forces with sum zero are present. However, in the absence of external forces, it does obey Newton's first law. Consider a system of n point masses, not experiencing forces from the outside world and only exerting contact forces on each other, so the forces are only acting during possible collisions between some of the particles, while at the moment of collision the colliding particles can be said to be at the same position. In these conditions it can be shown that the centre of mass of the system described above does not change its velocity. Take the time derivative of equation (16). Because M is not time dependent one can write:

$$\vec{v}_{\mathbf{M}} \equiv \frac{1}{M}\cdot\frac{\mathbf{d}}{\mathbf{d}t}\left(\sum_{i=1}^{n} m_{\mathbf{rest},i}\, g_i \cdot \vec{r}_i + \sum_{j=1}^{m}\frac{E_{\mathbf{photon},j}}{c^2}\cdot\vec{r}_j\right)$$

or:

$$M.\vec{v}_{\mathbf{M}} = \sum_{i=1}^{n}\frac{\mathbf{d}m_{\mathbf{rest},i}\, g_i}{\mathbf{d}t}\cdot\vec{r}_i + \sum_{j=1}^{m}\frac{\mathbf{d}E_{\mathbf{photon},j}\, c^{-2}}{\mathbf{d}t}\cdot\vec{r}_j +$$

$$+\sum_{i=1}^{n} m_{\mathbf{rest},i}\, g_i \cdot\frac{\mathbf{d}\vec{r}_i}{\mathbf{d}t} + \sum_{j=1}^{m} E_{\mathbf{photon},j}\, c^{-2}\cdot\frac{\mathbf{d}\vec{r}_j}{\mathbf{d}t} \qquad (17)$$

The sum of the first two summations in (17) is zero, because for particles not in collision the time derivative of their relativistic mass is zero, while for particles in collision the $\vec{r}_i$'s are the same. This means that $\vec{r}_i$ can be placed outside brackets, while inside these brackets the terms $\frac{\mathbf{d}\left(m_{\mathbf{rest},i}\cdot g_i\right)}{\mathbf{d}t}+\frac{\mathbf{d}\left(E_{\mathbf{photon},j}\, c^{-2}\right)}{\mathbf{d}t}$ cancel each other out, because of energy conservation. Strictly speaking, the time derivative of the photon mass, E_{photon}/c^2, is not defined; the appearance/disappearance of a photon is a discontinuous process. Formally this process could be described with the Heavyside step function. However, I have avoided this in order to make the notation as transparent as possible. A similar remark can be made about the time derivative of the mass of the material particles, when we consider their collisions to take place in an infinitesimal amount of time. The essence of the argument doesn't suffer from this. When, for example, a photon is absorbed by an atom, the disappearance of a relativistic mass of E_{photon}/c^2 is simultaneous with the appearance of an equal increase in the relativistic mass of the atom. This happens at the same $\vec{r}$, because we consider the particles as point masses.

In the last summation in (17), the differential quotients $\frac{\mathbf{d}\vec{r}_j}{\mathbf{d}t}$ are equal to $c\cdot\vec{e}_j$, in which $\vec{e}_j$ represents the unit vector in the direction of movement. So this last summation is the sum of the momenta of the photons. Therefore the sum of the third and the last summation is the total relativistic momentum of the system. The total relativistic momentum therefore can be written as the relativistic mass of the system (see definition 3 on page 5) times the velocity of the relativistic centre of mass. Because the relativistic mass of this system as well as its total momentum are independent of time, so is the velocity of the centre of mass. The relativistic centre of mass obeys Newton's first law and in this aspect is meaningful. However, as already pointed out above, work-performing external forces the sum of which is zero and which are acting on different particles in different locations are not allowed, this in contrast with the classical case.

Suppose that EM fields are part of the system. This would not change the above reasoning. The system could again be split up in an infinite number of infinitesimal volumes in the same way as was done on page 55. These volumes can be seen as relativistic point masses which, according to Maxwell's stress tensor, exert only contact forces on each other.

Then exactly the same reasoning and conclusion as just given above are valid. Therefore it can be stated that in relativistic mechanics a system of electrically charged point masses and EM fields experiencing no external forces has a centre of mass obeying Newton's first law.
According to the relativistic principle, one can assume that for other force fields the same can be said. Then one can say that **any system in relativistic mechanics has a centre of mass obeying Newton's first law** (on condition that the external force on each separate point mass is zero; it is not sufficient that the sum of the external forces on different particles is zero in a given inertial frame).

In one case it can be shown that the centre of mass does obey Newton's second law. This is the case of a system consisting of point masses, some of which experience an external force perpendicular to their velocity, while others experience no external force at all, but none experiences a force not perpendicular to its velocity. These forces do not necessarily have to be equal, either in direction or in magnitude. In this case the relativistic masses of the bodies are not changed by the external forces, so the proof given above starting from equations (12) to (15), can also be applied to the relativistic case.

See page 57, the text around figures (h) and (i). Here a system is described that does not obey Newton's second law and that has a non-constant invariant mass. Could it be that each system whose centre of mass obeys Newton's second law has a constant invariant mass? No, the counter example is the following. Imagine a system of two point masses, one electrically charged, the other not. They move with equal initial velocities in a *B*-field, not parallel to the *B*-field lines. Neither the *B*-field nor the overlap field (see footnote 3, page 28) are seen as part of the system. The charged point mass will describe a helix, whereas the velocity of the uncharged point mass is unaltered. Now according to the preceding paragraph the centre of mass obeys Newton's second law. However, the invariant mass changes, because the point masses are no longer immobile with respect to each other (most of the time).

Could it be that each system of free point masses, which undergo the same acceleration at the same moment, has a centre of mass obeying Newton's second law? I tried to prove this but wasn't able to. I suspect it is so, when all particles have the same acceleration in their momentaneous rest system. As already pointed out, all (relativistic) mass obeys Newton's third law.

The position vector of M, the centre of mass, in general does *not* transform under a Lorentz transformation like a position vector of a point mass. This is the easiest to prove for the *y*- or *z*-coordinate. Look at a system of two point masses with equal *x*-coordinates at moment t and different velocities in the *x*-direction in coordinate system S. Then:

$$y_{\mathbf{M}} \equiv \frac{g_1 . m_{\mathbf{rest},1} \cdot y_1 + g_2 . m_{\mathbf{rest},2} \cdot y_2}{g_1 . m_{\mathbf{rest},1} + g_2 . m_{\mathbf{rest},2}} \qquad (18)$$

Consider a Lorentz transformation from S to S* (with their relative velocities along their *x*-axes). It can be shown that $y_{\mathbf{M}}$ does *not* transform in the same manner as the position vector of a single point mass, that is to say, $y_{\mathbf{M}} = y^*_{\mathbf{M}}$ is not true.

For convenience, in equation (18) all $g.m_{\mathbf{rest}}$-s will be replaced by the corresponding *E*. Now the question is, whether the right hand side of equation (18) is equal to:

$$\frac{E_1^*.\bar{y}_1^* + E_2^*.\bar{y}_2^*}{E_1^* + E_2^*} \qquad (19)$$

It is not. <u>Proof:</u>

First, the transformation of the energies is needed:

$$E^* = g.E - gb.cp_x$$

With this, (19) can be rewritten:

$$\frac{E_1^*.y_1^* + E_2^*.y_2^*}{E_1^* + E_2^*} = \frac{\left(g.E_1 - gb.p_{1,x}\right).y_1 + \left(g.E_2 - gb.p_{2,x}\right).y_2}{g.E_1 - gb.p_{1,x} + g.E_2 - gb.p_{2,x}} \qquad (20)$$

Now $p_{1,x} = E_1.c^{-2}.v_{1,x}$ and $p_{2,x} = E_2.c^{-2}.v_{2,x}$. These *p*-s can be substituted in equation (20). Then the *E*-s in the nominator can be placed outside the brackets. A similar pair of brackets can be created in the denominator. But because $v_{1,x}$ is not equal to $v_{2,x}$, the right hand side of (20) cannot be reduced to that of (18). *The time-space vector of the centre of mass is not relativistically covariant.* This does not contradict the fact that the centre of mass obeys Newton's first law in a given inertial system.
Notice that, if $v_{1,x}$ and $v_{2,x}$ had been equal, y_1* and y_1 would indeed have been equal. It can be shown, that if all point masses of a system have equal velocities, the time-space vector of the centre of mass *does* transform as the position vector of a single point mass. I leave the proof to the reader.

It is worthwhile having a closer look at the relativistic conservation law of energy and momentum and its covariance. As was pointed out on page 54, if internal forces in the system of point masses were of the type of actio in distans and if momentum conservation were valid in this system of point masses in a given inertial system, there would always be other inertial systems in which momentum would not be conserved. Of course, we expect momentum conservation to be valid in every inertial system. In other words, we want the law of conservation of momentum to be relativistically covariant. Now in several courses on relativity I saw deductions being made with respect to this covariance which were more or less incomplete. Take, for instance, the teaching notes of Mr. Hesselink, who gives a course on special relativity at the Vrije Universiteit van Amsterdam. It is a copy of a part of the book *A course in modern Physics*, by Brehm and Mullin, published by John Whiley & Sons, Inc. The following text concerns an isolated system of point masses colliding with each other. **P** is the total momentum:

DDD

$$\Delta\mathbf{P} = \mathbf{P}_{\mathbf{after}} - \mathbf{P}_{\mathbf{before}}$$

and

$$\Delta E = E_{\mathbf{after}} - E_{\mathbf{before}}$$

The transformation rules in Equations (1-32) can then be used to deduce the corresponding differences determined by an observer in another Lorentz frame *S* ' in motion relative to *S*. The relations are

$$\begin{aligned} \Delta P_x' &= g\left(\Delta P_x - b\,\frac{\Delta E}{c}\right), \\ \Delta P_y' &= \Delta P_y, \\ \Delta P_z' &= \Delta P_z, \\ \frac{\Delta E'}{c} &= g\left(\frac{\Delta E}{c} - b\,\Delta P_x\right). \end{aligned} \qquad (1\text{-}40)$$

Next, we suppose that the observer in *S* confirms momentum conservation by finding $\mathbf{P}_{\text{before}}$ to be the same as $\mathbf{P}_{\text{after}}$ so that $\Delta\mathbf{P} = 0$. We also assume that the observer in *S'* agrees, finding the momentum to be conserved in the form $\Delta\mathbf{P}' = 0$. Equations (1-40) then imply that $\Delta E = 0$ in *S* and that $\Delta E' = 0$ in *S'*. Thus, both observers agree that the total relativistic energy is conserved because they have already agreed that the total momentum is conserved.

DDD

Firstly, it is remarkable that nowhere is the premise explicitly stated, that the internal forces are not of the actio in distans type. Undoubtedly, this is tacitly assumed by the author while he thinks of an interaction of bare point masses, which are free as long as they don't collide. Now he assumes momentum in *S* as well as in *S'* to be conserved. This implies that there is no actio in distans, at least when *S'* can be chosen at random. Then it can be concluded that $\Delta\mathbf{P}'$ is zero indeed. The author fails, however, to demonstrate this. He even gives me the impression that I should think that $\Delta\mathbf{P}' = 0$ is equivalent to momentum conservation in *S'*, irrespective of the presence of actio in distans. It is not. Let us take as an example a system of two particles which collide. Then $\Delta\mathbf{P}'$ is in fact a shorthand notation for $\boldsymbol{p}'_{1,\text{ after}} + \boldsymbol{p}'_{2,\text{ after}} - \boldsymbol{p}'_{1,\text{ before}} - \boldsymbol{p}'_{2,\text{ before}}$. Of course we can write out $\Delta\mathbf{P}$ (without prime) in a similar way. It is important to realise that the ***p'***-s and the ***p***-s are functions of time (for they change at the moment of collision). Now if we want $\Delta\mathbf{P} = 0$ to represent the law of conservation of momentum in *S*, then all the ***p***-s in it are, have to be, observed at the same time. Observing them at different times is only allowed if, in the corresponding time interval, no collisions have taken place. Now the ***p'***-s are obtained from the corresponding ***p***-s by a Lorentz transformation. Because the particles to which the ***p***-s belong can be at different positions (at the same time), the ***p'***-s can be the momenta in *S'* at different times. In that case we measure $\boldsymbol{p}'_{1,\ \text{after}}$ at one moment and $\boldsymbol{p}'_{2,\ \text{after}}$ at another. If in the time interval between those moments the particle has changed momentum (due to a collision), our measurement of the total momentum is false. Can such a false measurement happen? Only when there is actio in distans. See figure (j), the actio in distans collision (the contact force collision is discussed later). In the unprimed system the momentum is assumed to be conserved, $\Delta\mathbf{P} = 0$. The horizontal line at ct_1 intersects the two world lines. The momenta found at the intersection points always have the same sum, no matter how much the line is moved up or down (remaining horizontal, so parallel to the *x*-axis). The transformed momenta, however, are given at *different* times, namely ct'_1 and ct'_2. Their sum stays constant when the horizontal line is moved up or down, because $\Delta\mathbf{P}' = 0$ (a consequence of the fact that $\Delta\mathbf{P} = 0$ and the linearity of the Lorentz transformation). But this is an incorrect way of checking momentum conservation in the primed system. The momenta should be measured at the *same* time. So the correct way is to perform the same procedure with a line parallel to the *x'*-axis, for instance with the line ct'_2. When this line is moved over the bent

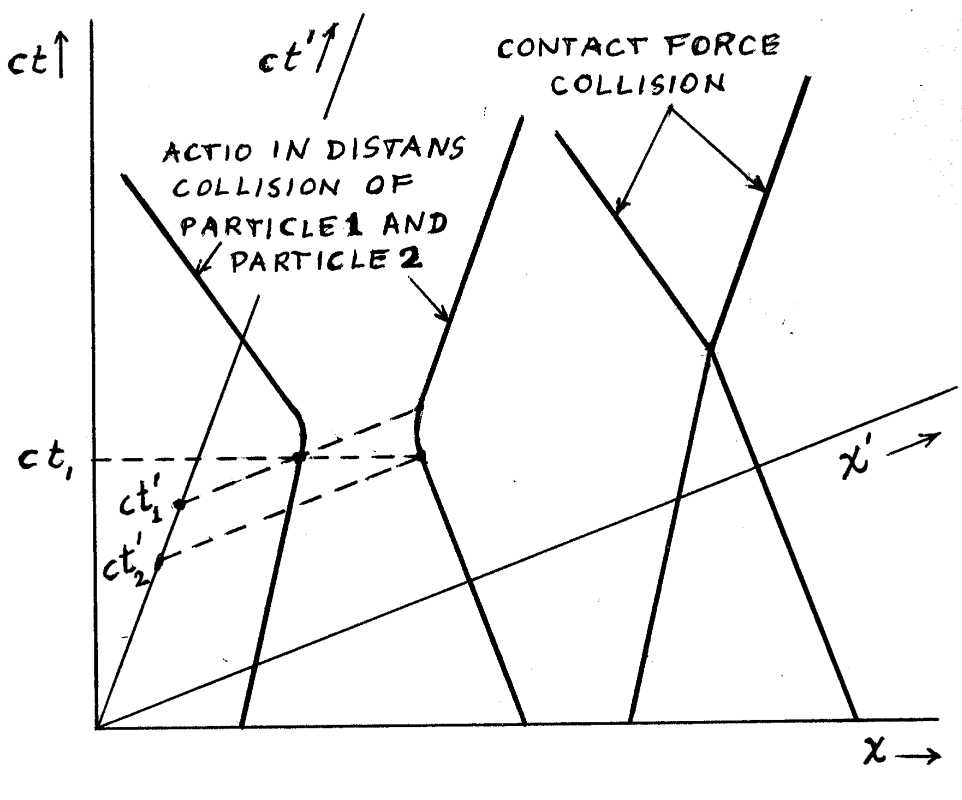


fig. (j)

parts of the world lines (keeping it parallel to the *x'*-axis), we see that momentum 2 is changing earlier than momentum 1. So momentum is not conserved in *S'*.
When we repeat this procedure for the contact force collision in figure (j), there is no problem. Now we do find that the momentum is conserved in the primed system. This demonstrates, once more, that in relativistic mechanics there is no place for actio in distans.

This knowledge gives us a more elegant theoretical treatment of the conservation laws of relativistic momentum and relativistic energy:

1) Relativistic momentum conservation in one inertial system can be proven from Newton's third law, a law from classical mechanics also valid in the RT (see page 54).

2) Relativistic energy conservation in one inertial system is identical with mass conservation, also known in classical mechanics; mass conservation in classical mechanics, together with the non-classical addition that energy also has mass, gives us the conservation law of relativistic energy.

3) From 1), from 2), from the exclusion of actio in distans and from the linearity of the Lorentz transformation, it can be proven that both laws are then also valid in any other inertial system.

The proof of 3) is as follows (underscore means four vector):
Statements 1) and 2) in formula form give: $\Delta \underline{P} = \underline{0}$. Now transform to another inertial system: $\Delta \underline{P}' = \Lambda \Delta \underline{P}$. Because Λ is a linear transformation, $\Rightarrow$ $\Delta \underline{P}'$ has to be $\underline{0}$ too. Because Λ^{-1} exists and because the inverse of a linear transformation is linear too, we can write: $\Delta \underline{P} = \underline{0} \Leftrightarrow \Delta \underline{P}' = \underline{0}$. Again, $\Delta \underline{P}' = \underline{0}$ does *not* yet tell us that energy and momentum are conserved, because the energies and momenta of the different particles are measured at different times. We must accept only forces that are contact forces. Then the reasoning with the contact force collision in the Minkowski diagram (see previous page) can also be applied to an energy-momentum four-vector like $\Delta \underline{P}'$. It then follows that momentum and energy are conserved in any inertial system.
As said before, because the mass stored in the EM field can also be seen as a collection of infinitesimal point masses exerting contact forces on each other, the above reasoning is also valid for a system of point masses and EM-fields together. According to the relativistic principle it can be supposed to be true for systems of point masses together with all kinds of fields.

-*-

APPENDIX II

An example of rest mass sum being confused with rest mass

The following text is a part of a course 'Relativiteitstheorie' given by prof. Terwiel in 1995, University of Leyden, Netherlands (translation is given below):

DDD

Voor een collectie vrije deeltjes {i} geldt $\underline{P} = \sum_i \underline{p}_i$ (additiviteit van de vierimpuls).

Voor de relativistische totale massa *M*, gedefinieerd volgens - zie (3.4) -

$$Mc \equiv (\underline{P} \cdot \underline{P})^{1/2} \tag{3.20}$$

houdt dit in dat *M* in het algemeen niet gelijk is aan $\sum_i m_i$, want de vierlengte van een som van viervectoren is in het algemeen niet de som van de vierlengtes van die viervectoren. Als niet alle $\underline{p}_i$ lichtachtige vierimpulsen in dezelfde richting zijn is $\underline{P}$ tijdachtig, en dus $M > 0$. Er is dan een inertiaalsysteem, het 'Center of Momentum'- of CM-systeem S^{CM}, waarin het ruimtelijk deel van $\underline{P}$, de totale impuls $\vec{P} = \sum_i \vec{p}_i$, $\vec{0}$ is. In het S^{CM} heeft $\underline{P}$ de componenten $(E^{\mathbf{CM}} / c,\ \vec{0})$, dus

$$\boxed{Mc^2 = E^{\mathbf{CM}} = \sum_i m_i c^2 + \sum_i T_i^{\mathbf{CM}}} \tag{3.21}$$

Naast de massa's m_i dragen dus de kinetische energieën $T_i^{\mathbf{CM}}$, gedeeld door c^2, tot *M bij.*

(3.18) en (3.21) laten zien dat het relativistische massabegrip <u>niet-additief</u> is (in tegenstelling tot het niet-relativistische massabegrip, waarvoor $m^{\mathbf{NR}}_{(a,b)} = m_a + m_b$, $M^{\mathbf{NR}}_{\{i\}} = \sum_i m_i^{\mathbf{NR}}$ gepostuleerd wordt).

3. Voor een interactieproces $\{i\} \to \{f\}$ houdt de relativistische energiebehoudswet

$$cP^0_{\textbf{voor}} = \sum_i (m_i c^2 + T_i) = cP^0_{\textbf{na}} = \sum_f (m_f c^2 + T_f)$$

In dat

$$\Delta T \equiv T_{\textbf{na}} - T_{\textbf{voor}} = c^2 \left(\sum_i m_i - \sum_f m_f \right) \tag{3.22}$$

waaruit men ziet dat $\sum_f m_f$ in het algemeen niet gelijk is aan $\sum_i m_i$: in het algemeen is er sprake van een omzetting van rustenergie ('massa-energie') in kinetische energie ('thermische energie') en omgekeerd.

Translation:

For a collection free particles $\{i\}$ the expression $\underline{P} = \Sigma_i \underline{p}_i$ holds (additivity of the four momentum). With respect to the relativistic total mass M (*the author means the invariant mass or rest mass of the system, Q.t.S.*) , defined by - see (3.4) -

$$Mc \equiv \left(\underline{P} \cdot \underline{P} \right)^{1/2} \tag{3.20}$$

this implies that in general M is not equal to $\Sigma_i m_i$, because the four length of a sum of four vectors in general is not equal to the sum of the four lengths of those four vectors. If not all $\underline{p}_i$ are light-like (*I never met an English equivalent of 'lichtachtig'. It means the length of the four vector is zero. 'Tijdachtig' and 'ruimteachtig', meaning the length being positive respectively negative, could be translated by 'time-like' and 'space-like', QtS*) four momenta in the same direction, then $\underline{P}$ is time-like, and therefore $M > 0$. There is thus an inertial system, the 'Centre of Momentum' or CM system S^{CM}, in which the spacial part of $\underline{P}$, the total momentum $\vec{P} = \Sigma_i \vec{p}_i$, is equal to $\vec{0}$. In S^{CM} the components of $\underline{P}$ are $(E^{\textbf{CM}} / c,\ \vec{0})$, so

$$Mc^2 = E^{\textbf{CM}} = \sum_i m_i c^2 + \sum_i T_i^{\textbf{CM}} \tag{3.21}$$

So besides the masses m_i the kinetic energies $T_i^{\,\mathrm{CM}}$, divided by c^2, contribute to *M*.

(3.18) and (3.21) demonstrate that the relativistic notion of mass (*here the author does not mean relativistic mass, he means here the only notion that can be called mass in his view, namely the invariant mass, Q.t.S.*) is non-additive (in contrast to the notion of non-relativistic mass, for which $m^{\mathbf{NR}}_{(a,b)} = m_a + m_b$, $M^{\mathbf{NR}}_{\{i\}} = \sum_i m_i^{\mathbf{NR}}$ is postulated).

(*I left away the foregoing text with formula* (3.18), *because it is not important for my argumentation. It speaks about binding energy. The second last formula refers to* (3.18), *QtS*)

3. For an interaction process $\{i\} \rightarrow \{f\}$ the relativistic energy conservation law

$$cP^0_{\mathbf{before}} = \sum_i (m_i c^2 + T_i) = cP^0_{\mathbf{after}} = \sum_f (m_f c^2 + T_f)$$

implies that

$$\boxed{\Delta T \equiv T_{\mathbf{after}} - T_{\mathbf{before}} = c^2 \left(\sum_i m_i - \sum_f m_f \right)} \qquad (3.22)$$

(*in formula* (3.22) *the i and the f are used both as dummies and as non dummies. I would have preferred* $m_{i,\,\mathrm{before}}$ *and* $m_{i,\,\mathrm{after}}$, *Q.t.S.*)
from which it can be seen that $\sum_f m_f$ is not, in general, equal to $\sum_i m_i$: in general we can speak of a conversion of rest energy ('mass energy') into kinetic energy ('thermal energy') and vice versa.

DDD

In the text under **3.** shown above, the author confuses rest mass and rest mass sum. This becomes apparent from the last underlined sentence and from (3.22). He uses the expression 'rest energy' in the sense of the rest mass *sum* times c^2. Rest mass sum is not a relativistic invariant. See page 39, from point 3 to the end of the first paragraph on page 40, and appendix III. Let us take here as an example of such an interaction process, a uranium nucleus at rest. The interaction process is the spontaneous fission of the nucleus. While the fission products are flying away at high speed, their total rest mass is given by *M*, as given in (3.21) in prof. Terwiels text, not by $\sum_i m_f$. This mistake is frequently made by authors of texts on RT and illustrates the great confusion about mass in relativistic physics. Each time I asked questions about this to one of these authors, including prof. Terwiel, they answered that 'the rest mass has to be the only mass and that in $E = m{*}c^2$ the m has to represent the rest mass, because it is a relativistic invariant'. The fact that they use a relativistic non-invariant quantity like the rest mass sum and substitute it into the relation $E = m{*}c^2$, undermines their argument and proves that they are on the wrong track.

Another remarkable point in the above text is the careful way in which referring to the 'centre of mass system' is avoided. The author speaks of the 'Centre of Momentum' system. Many textbooks on RT do the same. The name seems to be inspired by the Centre of Mass System, or CMS employed in classical mechanics. In the CMS the centre of mass is a clearly defined point in space. But what is the 'centre of momentum'? Is it a point in space? I never saw a definition of it. Mr. Hesselink, who gives a course on special relativity at the Vrije Universiteit van Amsterdam, told me that the centre of momentum is the point of collision. I don't think this can be a meaningful definition, because if we don't have a single projectile and a single target, but a system of particles in which several collisions take place, how would this 'centre of momentum' behave? Suppose these collisions whould happen in rapid succession, or some of them even at the same time. This 'centre of momentum' would jump through space in an irregular way and sometimes even split into more than one point. This cannot be a meaningful definition and I cannot think of any other meaningful way of defining such a 'centre of momentum'. Then what could be the physical meaning of this point? In classical (non-relativistic) mechanics the physical meaning of the centre of mass is clear; our view of a system of point masses is simplified by the centre of mass, because the relationships between its position, the mass of the system and the total external force are described by Newtons three laws in exactly the same way as for a point mass. But in what way does 'the centre of momentum' simplify our view on the system? And why is this centre never defined? Of course, the *system* in which the total momentum is zero, is clearly defined. But this '*centre* of momentum' never is. Worse still, I have never found an explicit statement that the centre of mass in the RT in general is useless and why it is useless (except in a letter from prof. G. 't Hooft, in which he states that the centre of mass in the RT cannot be defined in a meaningful way when EM fields are part of the system).
I think the name 'centre of momentum' just expresses the confusion physicists feel at this point. It looks to me like a half way effort to replace something that has become lost for unclear reasons. As I have already demonstrated, the unwillingness to use relativistic mass (including that of the EM fields) has led to the wrong idea that the centre of mass in relativistic physics has no significance. The centre of mass has significance in the RT, see my proof on pages 56 to 59.

Look again at the underlined sentence in prof. Terwiels text. He speaks of mass energy and kinetic energy. This suggests that kinetic energy has no mass. This whould mean mass and energy are not equivalent. Obviously wrong, mass and energy are equivalent.

-*-

APPENDIX III

Rest mass sum is not a relativistic invariant

First of all it is important to be clear about definitions. The time *t,* as measured by a certain inertial observer I will call the observer time, this in contrast with t , the proper time. In a certain inertial system I define rest mass, invariant mass, relativistic mass and rest mass sum each as a function of the *observer time* belonging to that inertial system. This seems logical to me, because the RT strives to describe all physical phenomena with quantities belonging to a certain inertial system. I mean to say, that it does not seem obvious to me to describe the rest mass sum, for instance, as a function of the proper times, so of the t_i -s of the different particles. True, sometimes we use the proper time to describe things. If I would have to make a relativistic invariant out of the rest mass sum, I see no other option than trying to describe the system with just as many proper times as there are particles. Because the number of particles can increase or decrease, this would yield an unattractively complicated function of time. I therefore stick to my definition, that rest mass sum is a function of the observer time.

As a definition of a relativistic (or Lorentz) invariant quantity I take: *A relativistic invariant quantity is a quantity to which all different inertial observers attribute the same value as measured at the time of the same event.* The event can be anything. A red light that gives a flash, or one object passing another one. When we accept these definitions, the conclusion that the rest mass sum is not a relativistic invariant is unavoidable. The proof is as follows:

See figure (j), situation I; a system consists of two atomic nuclei, P and Q, observed in an inertial system IS, at time t_1. Q is at rest (for convenience on the *x*-axis), P has a velocity v_P

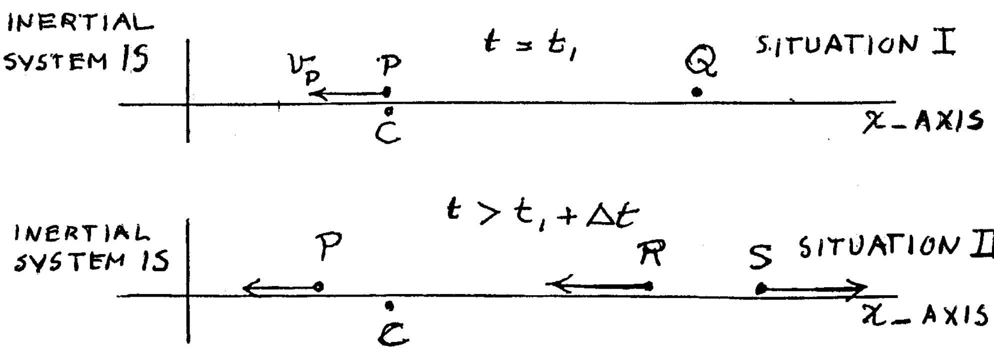


fig. (j)

(P moves along the *x*-axis) and at time t_1 passes an object C, situated close to the *x*-axis. This is situation I. Q splits into the nuclei R and S at time $t_1 + \Delta t$ (I take Δt positive). Also in figure (j), but now on a moment later than $t_1 + \Delta t$, a situation after the fission is rendered.

This is situation II. Now $m_{rest,\,R} + m_{rest,\,S} < m_{rest,\,Q}$. The distance $x_Q - x_C$ I call Δx (and I choose it to be positive, see figure (j)). I choose the interval $(\Delta ct, \Delta x)$ so, that it is space-like. Herein Δct is of course c times the time lapse between the "passing-by event" and the fission. Now a Lorentz transformation is performed on to an inertial system IS*, which has a velocity of v with respect to IS. When we calculate Δt^* with the Lorentz transformation we find: $\Delta t^* = g\,.\Delta t - g\,.b\,.c^{-1}.\Delta x$

It is possible to choose b in such a way, that Δt^* is negative. This is the case when $b > c.\Delta t\,/\,\Delta x$. Because b is always less than 1, $c.\Delta t\,/\,\Delta x$ has to be less than 1 too, if we want to find values for b obeying the unequality. This condition is fulfilled, because the interval was chosen to be space-like. That Δt^* is negative, means that in IS* the fission *precedes* the passing by-event.

At the moment t_1 the situation in IS is that Q has not yet split. So if we see the rest masses as a function of the observer's time, then in IS at t_1 the rest mass sum is equal to $m_{rest,\,P} + m_{rest,\,Q}$. So if we perform a Lorentz transformation to IS* on the event (ct_1, x_C), then in IS* we find the event happens at t_1*, a moment at which the fission of Q has already taken place and so the rest mass sum is equal to $m_{rest,\,P} + m_{rest,\,R} + m_{rest,\,S}$. A Lorentz transformation of this event causes a change in rest mass sum. The conclusion is, that rest mass is not a relativistic invariant. In figure (k) a Minkowski diagram of the thought-experiment is given as to illustrate the statement above.

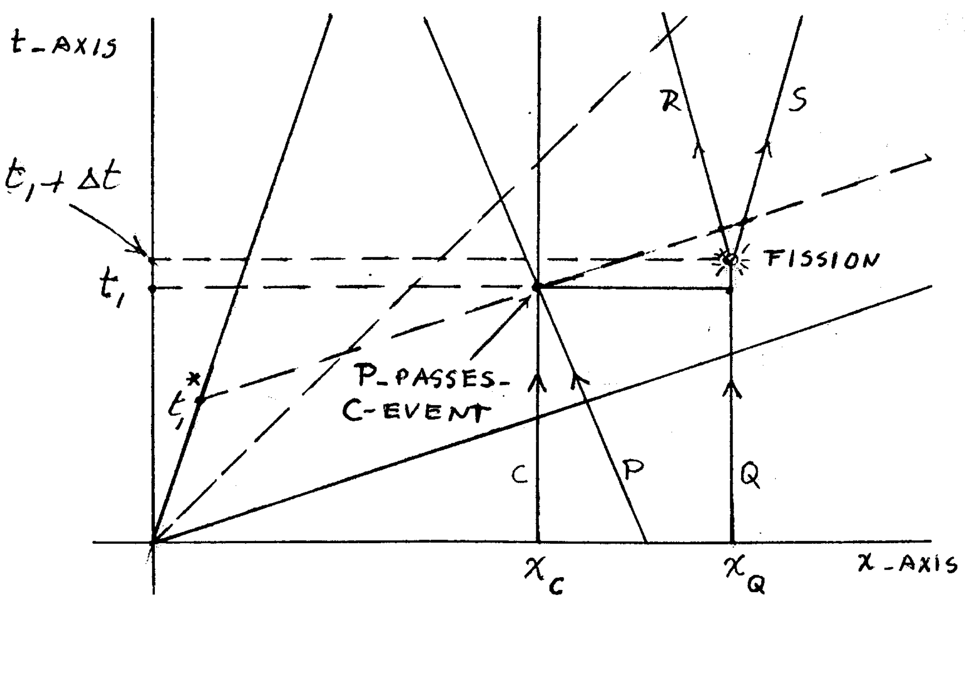


fig. (k)

-***-