Finding the Repeat Times of the GPS Constellation

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Abstract

Single-epoch estimates of position using GPS are improved by removing multipath signals, which repeat when the GPS constellation does. We present two programs for finding this repeat time, one using the orbital period and the other the topocentric positions of the satellites. Both methods show that the repeat time is variable across the constellation, at the few-second level for most satellites, but with a few showing much different values. The repeat time for topocentric positions, which we term the aspect repeat time, averages 246 s less than a day, with fluctuations through the day that may be as much as 2.5 s at high latitudes.

Authors' Note

The official version of this paper was published in *GPS Solutions* in 2006 (Online First, doi 10.1007/s10291-006-0038-4), and is copyright ©Springer-Verlag. After this paper was finalized, we discovered a bug in the `asprep` program which made all the aspect repeat times 1 s too short. The text and figures of this version have been corrected for this error.

Introduction

A feature of the GPS satellites that was well-known to early users was that the constellation would repeat: the same satellites would appear in the same part of the sky with a period slightly less than one day–an important characteristic when the system was incomplete and data had to be collected when the most satellites were visible. The usual rule of thumb was that the repeat time was 4 minutes (240 s) earlier each day, a good approximation to a more exact rule that the repeat time was one sidereal day (235.9 s less than one solar day). Over such a period the satellites would complete exactly two orbits in inertial space, and the Earth one revolution, bringing everything back to the same geometry.

That this repetition could be used to reduce one source of noise was first pointed out by Genrich and Bock (1992). We can write the carrier-phase signal (the electric field, say) at the antenna as (using complex notation)

$$E_t e^{2\pi ft+\phi} = E_d e^{2\pi ft} + E_s(\hat{\mathbf{n}}) e^{2\pi ft+\theta(\hat{\mathbf{n}})} \tag{1}$$

where the left side is the total, which can be broken down into two parts. The direct signal E_d is what would be observed in the absence of any local scatterers, or with a very directional antenna pointed at the satellite, in the direction of the unit vector $\hat{\mathbf{n}}$. E_s is the contribution from all scatterers, both near-field and far-field. If the geometry and electromagnetic properties of the scatterers do not change, the amplitude and phase of E_s will depend only on the direction to the satellite, $\hat{\mathbf{n}}$. Therefore, when the constellation repeats, so will this scattered contribution, which is also known as carrier-phase multipath. Insofar as multipath is a significant source of noise, we should be able to reduce the variation in a time-series of sub-daily positions by differencing observations, or analyzed results, observed at one (or more) repeat cycles.

Genrich and Bock (1992) demonstrated that such differencing, which they called multipath modeling, did in fact reduce the observed scatter on a 260-m baseline. This same method was used by Bock *et al.* (2000) and Nikolaidis *et al.* (2001), again assuming a 4-minute advance, for reducing the noise on longer baselines determined with single-epoch positioning. This was termed sidereal filtering, by analogy with the method used for correcting GPS time series (spatial filtering) for seasonal fluctuations (Wdowinski *et al.*, 1997). (A better term, by analogy with similar procedures used to correct for periodic components, would be sidereal adjustment.) Other workers have used this repetition period to isolate signals presumed to be from multipath, for example Elósegui *et al.* (1995), Wübbena *et al.* (1997), and Park *et al.* (2004b).

However, as Seeber *et al.* (1998) first pointed out (at least in the geodetic literature) the actual orbital period of the GPS satellites is not half a sidereal day, but slightly less. The simple model for repetition that we gave above is correct only if the plane of the orbit is fixed in space, and it is not. The ellipticity of the Earth, as expressed by the C_{20} term in the spherical harmonic expansion of its gravity field, produces a slow rotation of this plane (called the regression of the nodes), at a rate of about $14.66°\mathrm{y}^{-1}$ for the right ascension of the ascending node. (See Airy (1834) for a qualitative explanation, not of course with reference to GPS, and Beutler *et al.* (1998) for a mathematical development). In order to maintain the same ground track, the terrestrial longitude of the ascending node needs to be kept constant, which is accomplished by shortening the period so that the regression of the node in space is canceled by the fact that the earth has rotated slightly less than one revolution when the satellite reaches the equator; the daily regression of the node corresponds to 9.6 seconds of earth rotation.

While Ge *et al.* (2002) observed the shorter repetition period by cross-correlation of measurements intended to determine multipath, the importance of using this period to improve high-rate GPS positioning was first pointed out by Choi *et al.* (2004) in the context of GPS seismometry (Nikolaidis, 2002; Larson *et al.*, 2003; Bock *et al.*, 2004; Langbein and Bock, 2004; Genrich and Bock, 2006). Choi *et al.* (2004) showed that correcting 1-Hz positions using data from the previous day, shifted by the orbital period, gave a lower scatter than using a sidereal day as the correction period.

It should be clear that the use of a sidereal period, while a conventional assumption that was adequate for low sampling rates, is simply incorrect, and should be abandoned. There is no need to try to find the true repeat time from observations of the GPS signal, since it can be computed from the orbital parameters. Our purpose in this note is to provide two programs which do this computation; we use these to describe different repeat times and how they might be used in practice.

Orbital Periods

The easiest computation is to find the true periods of the different satellites. This requires two parameters provided in the broadcast ephemeris: a_s, the square root of the semimajor axis, and n_c, the correction to the mean motion that would be deduced from Kepler's third law. The mean motion n is given by

$$n = \sqrt{GM} a_s^{-3} + n_c \tag{2}$$

where $\sqrt{GM}$ for the Earth is 1.996498×10^7 in SI units. The repeat time T_o is twice the orbital period: $T_o = 4\pi/n$. The program `orbrep` reads a broadcast ephemeris file in RINEX format (Hofmann-Wellenhof *et al.*, 1994) and writes out the repeat times T_o for all the satellites; if multiple ephemerides are given in the file, the values of a_s and n_c are averaged for each satellite.

Figure 1 shows the resulting periods, computed at 5-day intervals, over the last decade. We show both the time needed for two full orbits, and the "daily advance", meaning how much earlier the repeat is per solar day. The upper panel shows the full range of periods, and illustrates that most of the periods fall within a narrow band, which the lower panel shows to be within 5 seconds of a mean value of 86154.4 s for T_o (a daily advance of 245.6 s), which is 9.7 seconds less than sidereal (shown by the dashed line). For most of the time span shown, there have been one or two satellites with very different periods, presumably ones being adjusted into orbit, or failing.

Aspect Repeat Time

While the orbital period is easy to compute and gives the most direct insight into the range of behaviors of the satellites, it is not the repeat time of most interest. Another time, slightly different, would be that required for the satellite to be over the same location (at the same point in its ground track); in astronomical terms this would amount to having the same geocentric place. But, for measurements in a given location, what is most relevant is the period over which the satellite comes closest to occupying the same topocentric place: that is, the period for most nearly having the same direction vector $\hat{\mathbf{n}}$ for a particular point of observation. This is slightly different from the geocentric repeat time because of parallax. To revive an old astronomical term, this time is when the satellite has the same aspect, so we call this the aspect repeat time.

The program `asprep` reads in sp3 format files (Remondi, 1989), which give the satellite position, very conveniently for this application, in Earth-fixed coordinates. The vector $\hat{\mathbf{n}}_0$ is found for a specified time, using polynomial interpolation of the tabulated positions (Schenewerk, 2003), and finding the topocentric position vector for a given place, for each

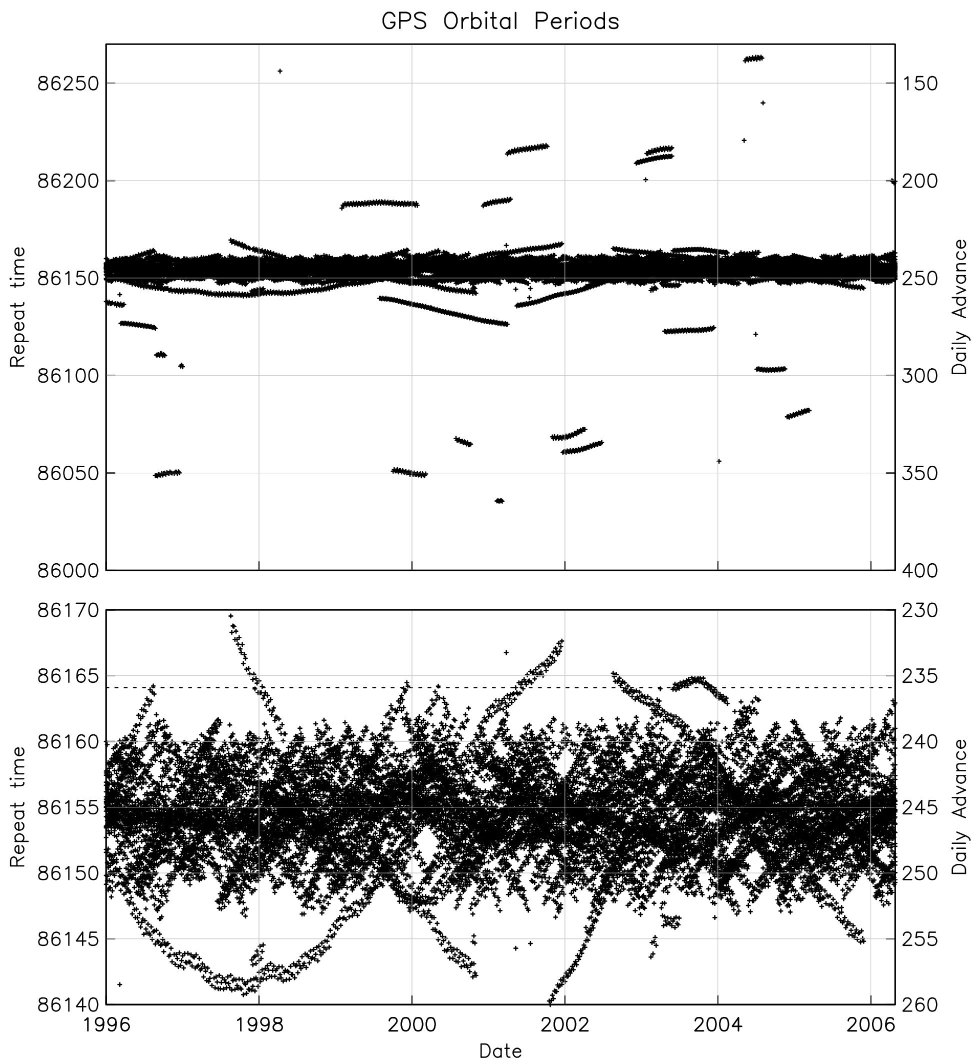

Figure 1: Orbital repeat times (twice the orbital periods) found from the broadcast ephemerides of all GPS satellites from 1996:001 through 2006:120. The top plot shows the full range of periods; the expanded scale on the bottom, the band within which most periods fall. The left axis is the repeat time, the right the daily advance relative to 24 hours; an advance of 4 minutes/day corresponds to 240 seconds on the right axis. Both times are in seconds. The dashed line in the bottom plot is the sidereal-day period.

satellite. The program then steps N days ahead and searches for the time, for each satellite, at which $\hat{\mathbf{n}}(t)$ is closest to $\hat{\mathbf{n}}_0$; this is done using s grid search, followed by inverse parabolic interpolation. If this time is t_r, the repeat time is $T_a = t_r - t_0$. Two byproducts of this method are the difference in direction, $\Delta\theta$, between $\hat{\mathbf{n}}_0$ and the nearest $\hat{\mathbf{n}}(t)$, and the angular velocity of the satellite on the celestial sphere, $\dot{\theta}$. The ratio $t_s = \Delta\theta/\dot{\theta}$ gives the difference in angle in terms of time, which is a useful parameter for deciding how close the actual repetition is.

Table 1 gives the results from these two programs, for the location and time examined by Choi *et al.* (2004). As it happens, at this time there were no satellites with significantly different periods from the nominal: the most discrepant is SV 23, about 10 seconds away from the nominal value. This table shows that the orbit repeat periods and aspect repeat times, while similar, can differ by as much as 3 seconds, with the aspect repeat time being, usually, shorter.

Of course, part of the aspect is whether or not the satellite is actually observable, and so `asprep` also gives the elevation and azimuth. To find the appropriate aspect repeat time for adjusting a position series at a given location and time, we would take only those satellites above a given elevation (or, given data, those actually observed), and find the mean aspect repeat time, ignoring satellites with very discrepant periods to avoid biasing the mean. (Such satellites do not in fact repeat their sky paths well in any case, so it is not clear that the multipath signals from them will repeat). Figure 2 shows the results of such a computation for a range of latitudes and the existing constellation. The aspect repeat time has an average value over the day corresponding to a daily advance of 246.8 s, but fluctuating around this by several seconds, especially at the higher latitudes. This value is what was found by Ge *et al.* (2002) from observation; while Park *et al.* (2004b) argued for a sidereal period based on the annual repetition of the time of maximum correlation, their plot is consistent with a repetition over 350 days, which is what would be expected for the repeat time found here. For daily sampling, this repeat period will alias to a frequency of 0.0028565 cycles/day, or 1.04333 cycles/year; variations at this frequency (and its harmonics) have been seen in stacks of global GPS data (J. Ray, pers. commun.).

It is perhaps worth noting that the sky tracks of the satellites do not in fact repeat exactly over short times. The mean value of $\Delta\theta$ (again ignoring the discrepant satellites) after N days is about $0.15N$ mrad. Given the extremely small angular size of multipath fluctuations observed by Park *et al.* (2004a) (in the only direct measurement of these yet made), such an increase in day-to-day separation may account in part for the observation that the multipath signal is most similar only on adjacent days, becoming different as the time separation increases.

To conclude, we see that the idea of a "GPS repeat time" is not as simple as it might seem, at least when we are looking at sampling at high rates. Correcting for multipath effects on the assumption that they repeat "every day" requires some care in defining what a day really is. We have described two ways of finding this, and given average results. Since we find that the aspect repeat time (the most relevant) varies among the different satellites of the constellation, it seems likely that the best corrections for multipath will be obtained by processing data from each satellite separately, rather than working with time series of positions.

Table 1: Aspect and Orbit Repeat Times

SV	$86400 - T_o$	T_o	T_a	$\Delta\theta$	Elev.	Az.	$\dot{\theta}$	t_s
	s	s	s	mrad	°	°	mrad/s	s
1	247.05	86152.95	86152.06	0.016	-38.8	139.0	0.0937	0.2
2	239.98	86160.02	86160.93	0.346	-41.2	97.6	0.1029	3.4
3	246.42	86153.58	86152.98	0.062	-38.2	40.8	0.1003	0.6
4	241.13	86158.87	86160.05	0.244	4.7	170.0	0.1173	2.1
5	250.84	86149.16	86148.79	0.342	-17.3	264.2	0.1120	3.1
6	249.02	86150.98	86149.27	0.111	-60.3	229.3	0.0870	1.3
7	243.87	86156.13	86156.19	0.070	74.5	198.0	0.1457	0.5
8	244.85	86155.15	86155.73	0.019	51.2	117.7	0.1492	0.1
9	242.57	86157.43	86158.31	0.171	5.3	311.6	0.1151	1.5
10	247.80	86152.20	86151.98	0.115	-8.4	228.6	0.1133	1.0
11	244.49	86155.51	86155.98	0.070	19.4	61.1	0.1192	0.6
13	240.96	86159.04	86160.30	0.086	-17.3	157.3	0.0998	0.9
14	240.31	86159.69	86160.12	0.278	-44.5	8.2	0.1015	2.7
15	241.69	86158.31	86159.24	0.190	-31.9	343.7	0.1037	1.8
16	243.53	86156.47	86156.91	0.081	-60.9	110.2	0.0921	0.9
17	250.99	86149.01	86147.23	0.129	-32.0	221.7	0.0992	1.3
18	247.63	86152.37	86151.94	0.062	-7.5	322.4	0.1089	0.6
20	241.55	86158.45	86158.81	0.290	-20.3	101.2	0.1118	2.6
21	242.96	86157.04	86157.41	0.164	-46.9	300.0	0.1040	1.6
23	236.53	86163.47	86166.77	0.169	-13.1	6.3	0.0989	1.7
24	251.53	86148.47	86146.34	0.092	-17.6	197.7	0.1020	0.9
25	249.77	86150.23	86149.57	0.207	-87.7	171.4	0.0901	2.3
26	247.10	86152.90	86152.38	0.127	41.0	282.9	0.1368	0.9
27	249.42	86150.58	86150.73	0.299	19.1	129.2	0.1344	2.2
28	245.43	86154.57	86154.76	0.011	61.2	25.9	0.1307	0.1
29	243.74	86156.26	86156.25	0.113	44.4	260.8	0.1423	0.8
30	244.09	86155.91	86156.13	0.077	-43.5	251.2	0.0958	0.8
31	246.29	86153.71	86152.92	0.012	-8.9	42.5	0.1054	0.1

Times for 22 Dec 2003 (day 356), at 19:15 UTC, at 35.881°N, 120.402°W

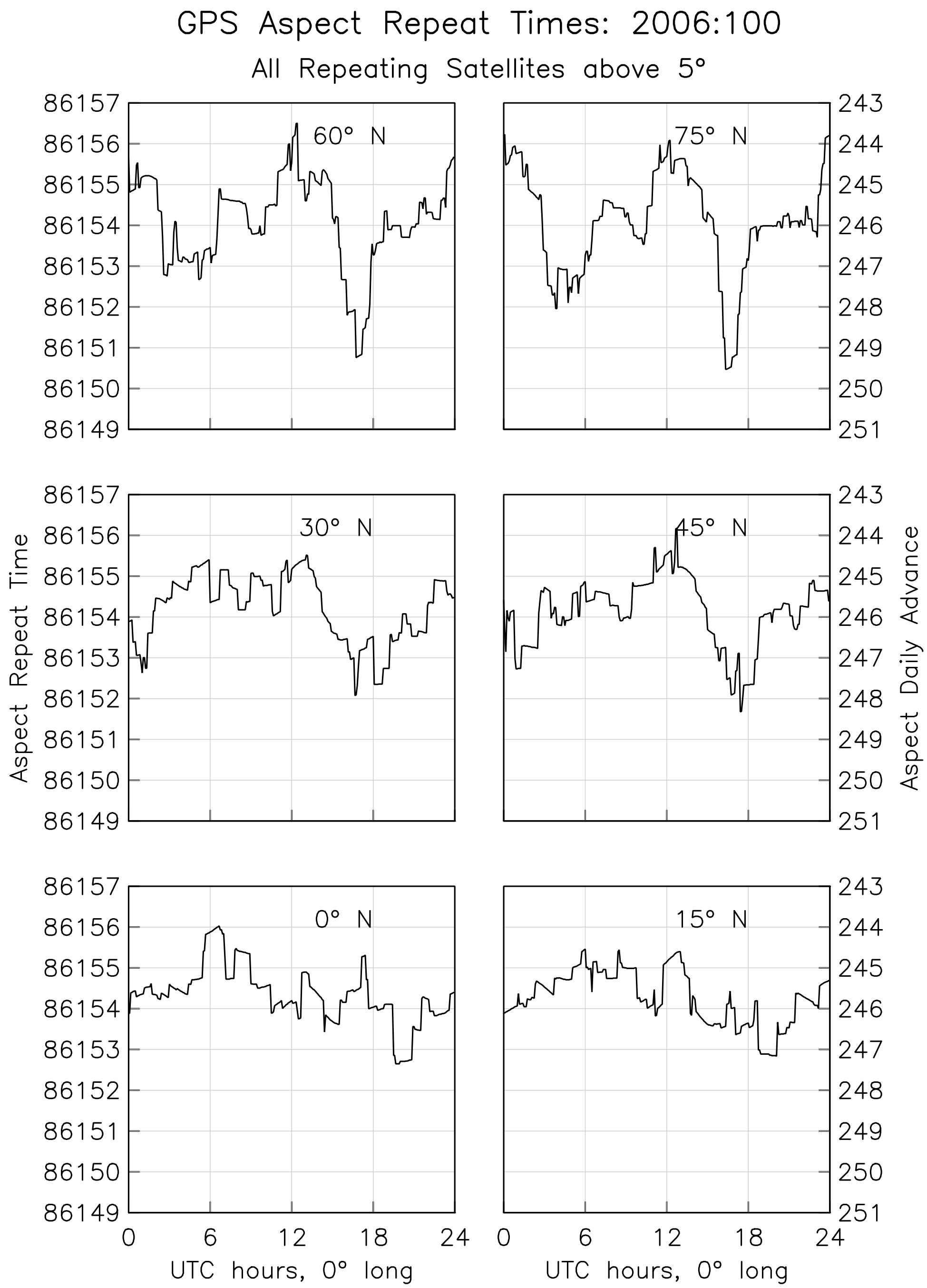

Figure 2: GPS aspect repeat time, (time between the satellite appearing in the same direction from a particular location), averaged over the GPS constellation (visible above 5°) as of 2006:100. Locations are at 0° longitude and latitudes as shown. The average varies with time of day (UTC) and latitude, and averages about 86154 s over the day.

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