

Long term evolution of the spin of Venus - I. Theory.

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Abstract

Due to planetary perturbations, there exists a large chaotic zone for the spin of the terrestrial planets (Laskar and Robutel, 1993). The crossing of this zone in the past, can lead Venus' spin to its present retrograde configuration for most initial conditions, but through two different processes (Correia and Laskar, 2001). Here, we present in full details the dissipative models used for this study of the spin evolution of Venus.

The present state of Venus is an equilibrium between gravitational and thermal atmospheric tidal torques (Gold and Soter, 1969). We present here a revised model for the thermal atmospheric tides which does not suffer the singularity at synchronous states arising in previous studies. This new model should thus provide a more realistic description of the final stages of Venus' evolution. Assuming that the present spin of Venus is in a final state, we describe the resulting constraints on the various dissipative parameters. We show that the capture in the 1:1 spin orbit resonance during Venus' history is unlikely and becomes impossible when the dense atmosphere is present as this resonance becomes unstable. Our study is presented in a very general setting, and should apply to any terrestrial planet with a dense atmosphere.

Key Words: Venus; obliquity; spin dynamics; resonances; atmospheric tides; core mantle friction.

1 Introduction

In 1962, by means of wave radar measurements, the peculiar spin state of Venus was discovered (Smith, 1963, Goldstein, 1964, Carpenter, 1964, 1966, 1970): a slow retrograde rotation, with an obliquity close to 180° and a 243 day period. Since, various hypothesis have been studied for the evolution of Venus' rotation, aiming to answer the question: was Venus born retrograde or not?

The first success was that of Gold and Soter (1969) who proposed that the present spin was near a steady state resulting from a balance between a gravitational tidal dissipation which drives the planet toward synchronous rota-

tion and a thermally driven atmospheric tides which drives it away. However, tidal effects alone could not explain how to prevent Venus' spin axis from rolling over to a prograde orientation (Dobrovolskis, 1978).

Goldreich and Peale (1970) proposed that friction at a core-mantle boundary should drive the spin pole to a fully dampened obliquity state which ends with retrograde rotation. The only requirement for this is that the planet's orientation is already retrograde when the core-mantle friction becomes important. Taking into account the dissipation of tides (both gravitational and thermal) and core-mantle friction, Lago and Cazenave (1979), Dobrovolskis (1980), Shen and Zhang (1989), McCue and Dormand (1993), and Yoder (1995a) proposed different scenarios where the Venusian axis was tilted down during its past evolution, but this still required high values of the initial obliquity.

Laskar and Robutel (1993) discovered that for all terrestrial planets, there is a wide set of possible spin states for which the obliquity undergoes strong chaotic variations with large amplitudes over a few million years. This is due to some resonance overlap between the precession frequency and combinations of secular frequencies of the planetary orbits. The future Earth's passage through the chaotic zone was analyzed as well (Néron de Surgy and Laskar, 1997), and it was shown that our planet's spin has a high probability to reach obliquities as high as 90° within a few billion years. The chaotic zone of Venus (Fig.1) being comparable to the Earth's one, Laskar and Robutel (1993) suggested that planetary perturbations could have played an essential part in the history of the Venusian spin. If Venus was born with an obliquity lower than 90° and initial rotation period faster than 5 days, it surely wandered for a while in the chaotic zone. Indeed, Néron de Surgy (1996) and Yoder (1997) showed that dissipative effects combined with planetary perturbations could tilt the spin axis to 180° starting with any initial obliquity. Finally, Correia and Laskar (2001) confirmed this last result and found that the present spin state of Venus is the most probable for almost any initial condition. They also showed that it is possible to evolve to the present configuration through a different scenario where retrograde ro-

tation is developed while the obliquity goes towards zero (suggested by Kundt, 1977).

Although the outcomes of the dissipative effects are supposed to be well understood, their precise mechanisms are poorly known, as well as the initial spin state of Venus. The present paper is the continuation and presentation with full details of the work by Correia and Laskar (2001). Here, we revisit the theory of dissipative effects, in particular the thermal atmospheric tides. We also discuss the possibilities for the spin evolution and the constraints imposed by the present observed spin state.

In the next section we give the averaged conservative equations in a suitable form for the simulations of the long-term variations of Venus' spin, including the precession motion with planetary perturbations. In section three is defined a model for taking into account the tidal effects and the core-mantle friction. Section four is devoted to dynamical equations' analysis and their implications are discussed in the last section (five). In a companion paper (Correia and Laskar, 2002), we perform massive numerical integrations, where different models and dissipative parameters are tested in order to understand the influence of each unknown quantity on the final evolution.

2 The secular variations of Venus' spin axis

2.1 The conservative Hamiltonian of the system with planetary perturbations

Venus is considered here as an homogeneous rigid body with moments of inertia $A \leq B < C$. As pointed by Yoder (1995a), the axis of rotation differs from the axis of greatest inertia (the C axis) by about 0.5° . However, for long-term integrations we can simplify the equations by averaging the nutation of the axis, which allows us to merge the two axis.

The averaged Hamiltonian of the motion $\overline{\mathcal{H}}$ can be written using canonical Andoyer's action variables (L, X) and their conjugate angles ($\ell, -\psi$) (Andoyer, 1923, Kinoshita, 1977). $L = C\omega$ is the projection of the angular momentum with rotation rate ω on the C axis and X its projection on the normal to the ecliptic; ℓ is the hour angle between the equinox of the date and a fixed point of the equator and ψ the general precession angle. For a slow precessing planet ($\dot{\psi} \ll \omega$), $X \simeq L \cos \varepsilon$, where ε is the obliquity. If we keep the resonant term with argument $(\ell - pM)$ with $p = \pm 1$ and M the mean anomaly, we have (Kinoshita 1977, Laskar 1986, Néron de Surgy and Laskar, 1997):

$$\begin{aligned} \overline{\mathcal{H}} &= \frac{L^2}{2C} - \alpha \frac{X^2}{2L} - \frac{\alpha_r}{2L} (L + pX)^2 \cos 2(\ell - pM) \\ &+ \sqrt{L^2 - X^2} [\mathcal{A}(t) \sin \psi - \mathcal{B}(t) \cos \psi] + 2\mathcal{C}(t)X. \end{aligned} \quad (1)$$

The quantities \mathcal{A} , \mathcal{B} and \mathcal{C} depend on the secular motion of the orbit of Venus undergoing planetary perturbations (Laskar and Robutel, 1993).

$$\alpha = \frac{3Gm_\odot}{2a^3(1-e^2)^{3/2}} \frac{E_d}{\omega} \simeq \frac{3n^2}{2\omega} E_d \quad (2)$$

is called the "precession constant". G , m_\odot , a , n and e are respectively the gravitational constant, the solar mass, Venus' semi major axis, the mean motion and the eccentricity (which varies with time due to the planetary perturbations). E_d is the dynamical ellipticity, which is composed of two different parts:

$$E_d = \frac{C - \frac{1}{2}(A+B)}{C} = \frac{k_f R^5}{3GC} \omega^2 + \delta E_d, \quad (3)$$

where R is the planet's radius and k_f the fluid Love number (pertaining to a perfectly fluid body with the same mass distribution as the actual planet). The first part of this expression corresponds to the flattening in hydrostatic equilibrium (Lambeck, 1980), and δE_d corresponds to the departure from this equilibrium. At present its value is estimated to be 1.3×10^{-5} (Yoder, 1995a), but it may change over time. Finally, the spin-orbit contribution in the synchronous resonance is given by (eg. Goldreich and Peale, 1966):

$$\alpha_r = \frac{3Gm_\odot}{8a^3\omega} \frac{B-A}{C} \simeq \frac{3n^2}{8\omega} \frac{B-A}{C}. \quad (4)$$

When $|\ell - pM| \gg 0$, the mean value of $\cos(\ell - pM)$ is zero which allows us to neglect the contribution of the 1:1 resonance in the averaged Hamiltonian (1). The mean value of the eccentricity of Venus is about 0.036, its maximal value $e \simeq 0.08$ (Laskar, 1994a) and the axially asymmetric distribution of mass $(B-A)/C = (2.16 \pm 0.03) \times 10^{-6}$ (Konopliv *et al.*, 1993). In the present work we will neglect the resonant terms weaker than the product $e \frac{B-A}{C}$ and the only resonances where Venus can be trapped are then the synchronous rotation ones. For more details about the resonances see section 2.4.

2.2 Conservative equations for the precession motion

Since Andoyer variables ($X, -\psi$) are canonical, we have:

$$\frac{dX}{dt} = \frac{\partial \overline{\mathcal{H}}}{\partial \psi}; \quad \frac{d\psi}{dt} = -\frac{\partial \overline{\mathcal{H}}}{\partial X}; \quad (5)$$

that is,

$$\begin{cases} \frac{dX}{dt} = L\sqrt{1 - \frac{X^2}{L^2}} [\mathcal{B}(t) \sin \psi - \mathcal{A}(t) \cos \psi] \\ \quad - p\alpha_r L \left(1 + p\frac{X}{L}\right)^2 \sin 2(\ell - pM), \\ \frac{d\psi}{dt} = \alpha\frac{X}{L} - 2\mathcal{C}(t) + p\alpha_r \left(1 + p\frac{X}{L}\right) \cos 2(\ell - pM) \\ \quad - \frac{X}{L\sqrt{1 - \frac{X^2}{L^2}}} [\mathcal{A}(t) \sin \psi + \mathcal{B}(t) \cos \psi]. \end{cases} \quad (6)$$

The previous system of equations has a singularity for $\sin \varepsilon = 0$, that Venus may encounter during its evolution. Replacing ψ by the complex variable $\Psi = L \sin \varepsilon e^{i\psi}$, we eliminate the singularity and (6) becomes:

$$\begin{cases} \frac{dX}{dt} = \text{Im}(\Psi)\mathcal{B}(t) - \text{Re}(\Psi)\mathcal{A}(t) \\ \quad - p\alpha_r L \left(1 + p\frac{X}{L}\right)^2 \sin 2(\ell - pM), \\ \frac{d\Psi}{dt} = i\left[\alpha\frac{X}{L} - 2\mathcal{C}(t)\right] \Psi + X[\mathcal{A}(t) - i\mathcal{B}(t)] \\ \quad + ip\alpha_r \Psi \left(1 + p\frac{X}{L}\right) \cos 2(\ell - pM). \end{cases} \quad (7)$$

According to (2), the precession rate becomes infinite for $\omega = 0$. This apparent singularity results from the approximation $X \simeq L \cos \varepsilon$, only valid when $\dot{\psi} \ll \omega$. For an almost zero rotation planet, we need to use the complete set of Andoyer variables (Andoyer, 1923). The total angular momentum never being null, then sets an upper limit for the precession rate.

2.3 Global view of the dynamics of the obliquity

The evolution of the precession angle and obliquity is given by a numerical integration of the system of equations (7), in combination with the secular theory of the Solar System (Laskar 1988, 1990). One can then obtain a global view of the rotational dynamics by integrating many initial conditions on α and ε , using frequency map analysis (Laskar 1990, 1999). In the conservative view, that is, over a short time of a few million years, the rotation rate ω of the planet can be considered as a constant, and so will be the precession constant α (Eq.2). But over a very long time interval, of several billion years, the various dissipative effects will change ω and therefore, the precession constant. Nevertheless, a global view of the dynamics of the obliquity can be obtained by constructing the frequency map for a wide range of the α parameter. This was done for all the planets (Laskar and Robutel, 1993), and the results for the planet Venus are given in figure 1.

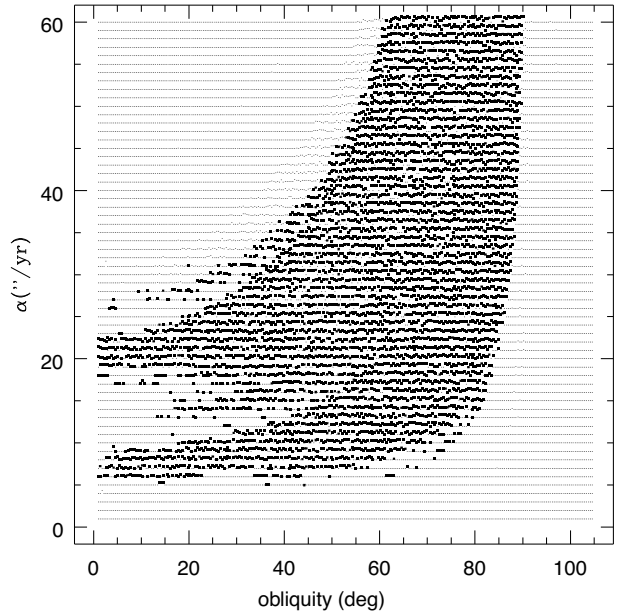


Figure 1: **Venus' chaotic zone (Laskar and Robutel, 1993).** The regions with thin lines correspond to stable trajectories where the obliquity oscillates moderately around a mean constant value. In the region filled with thick points the variations are strongly chaotic due to the overlap of resonances between the precession of the axis and the secular orbital motion. The chaotic diffusion allows the obliquity to wander in a few million years on horizontal lines in the chaotic region.

2.4 Spin-orbit resonances

A spin-orbit resonance occurs when there is a commensurability between the rotation rate ω and the mean motion of the orbit n , with the synchronous rotation of the Moon as the most common example. After the discovery of the 3:2 spin-orbit resonance of Mercury (Colombo, 1965), this effect was studied in great detail (Colombo and Shapiro, 1966, Goldreich and Peale, 1966, Counselman and Shapiro, 1970). Non-synchronous spin-orbit resonances require a large orbital eccentricity, which is not the case of Venus. Thus, here we will only study the particular case of the synchronous resonances, also known as 1:1 resonances. Far from the resonances, the term in α_r appearing in the expression of the averaged Hamiltonian (1) can be neglected, since $(\ell - pM)$ is a fast angle. However, for the 1:1 resonances we obtain

$$\frac{dL}{dt} = -\frac{\partial \bar{\mathcal{H}}}{\partial \ell} = -\alpha_r L \left(1 + p\frac{X}{L}\right)^2 \sin 2(\ell - pM), \quad (8)$$

that is,

$$\frac{d\omega}{dt} = -\frac{3Gm_{\odot}}{8a^3} \frac{B - A}{C} (1 + p \cos \varepsilon)^2 \sin 2(\ell - pM). \quad (9)$$

The width of the corresponding resonance, centered at

$\omega = pn$, is:

$$\Delta\omega = n(1 + p \cos \varepsilon) \sqrt{3 \frac{B-A}{C}}. \quad (10)$$

2.4.1 Capture probabilities

Due to dissipative torques (see section 3) the mean rotation rate of Venus will not remain constant and may cross one of the above resonances where it may be captured. Goldreich and Peale (1966) computed a first estimation of the capture probability P_{cap} , and subsequent more detailed studies proved their expression to be essentially correct (for a review, see (Henrard, 1993)). Since the dissipative torques acting on Venus can be described by means of the torques considered by Goldreich and Peale (1966), we will adopt here the same notations but in a more general formulation. Let

$$\frac{d\omega}{dt} = \langle T \rangle = -W - Z \text{sign}(\dot{\gamma}) - K \frac{\dot{\gamma}}{n}, \quad (11)$$

where $\dot{\gamma} = \omega - pn$ and W a positive and constant torque. K and Z are also constant torques, but not necessarily positive. Thus, with

$$\eta = Z + K \frac{\Delta\omega}{\pi n}, \quad (12)$$

the probability of capture into resonance is given by:

$$P_{\text{cap}} = \begin{cases} 0 & \text{if } \eta \leq 0, \\ \frac{2}{1 + W/\eta} & \text{if } 0 < \eta < W, \\ 1 & \text{if } \eta \geq W. \end{cases} \quad (13)$$

2.4.2 Escape probabilities

The torques K , W and Z , appearing in equation (11), can change with time. It is possible that for a given date η is positive, but later on becomes negative. If the planet was first captured in the resonance (for $\eta > 0$), it will then be forced to abandon it (when $\eta < 0$). However, there are two different possible paths for the rotation rate in that situation: we can leave the resonance with a rotation rate smaller than the resonant rotation ($\omega < pn$) or leave it with a rotation rate larger than the resonant rotation ($\omega > pn$). This distinction is important, because it leads to two distinct evolutions: in the first situation the planet skips the resonance just as if it had never been captured, whereas in the second situation the planet's rotation will increase. To estimate the escape probabilities of each side of the resonance (P^- and P^+ respectively), we use the quantities ΔE and $\Delta E'$ defined by Goldreich and Peale (1966). ΔE can be understood as the energy variation in the semi-cycle of positive $\dot{\gamma}$ inside the resonance and $\Delta E'$ the same quantity corresponding to the

negative semi-cycle. With these notations,

$$P^+ = \frac{\Delta E}{\Delta E + \Delta E'}; \quad P^- = \frac{\Delta E'}{\Delta E + \Delta E'}. \quad (14)$$

Using the generalized torques defined in (11) we have:

$$P^+ = \frac{1}{2} \left(1 + \frac{W}{\eta} \right); \quad P^- = \frac{1}{2} \left(1 - \frac{W}{\eta} \right). \quad (15)$$

For $P^\pm > 1$, we have $P^\pm = 1$, and $P^\pm = 0$ for $P^\pm < 0$. As for $\eta = 0^-$, $P^- = 1$, during the transition from $\eta > 0$ to $\eta < 0$, the planet always leaves the resonance with a rotation rate smaller to the resonant one.

3 Dissipative effects

In the global vision of the stability of Venus' obliquity given in section 2.1, only conservative aspects are considered. Nevertheless, as the dissipative effects have time scales that are in general much longer than the ones involved in the chaotic diffusion, the previous study provides a general framework where all the possible scenarios for the long time evolution of Venus rotation axis will fit. Under billion-year time scales, and specially when considering the past evolution of Venus, the dissipative effects to consider are due to tidal dissipation and core-mantle friction.

3.1 Tidal effects

Tidal effects arise from differential and inelastic deformations of the planet due to a perturbing body. Among these effects we count the gravitational tides and thermal atmospheric tides generated from the solar heating of the atmosphere. The estimations for the contributions to the spin variations are based on a very general formulation of the tidal potential, initiated by George H. Darwin (1880). In both cases, we first write the complete tidal potential expression, U , expressed in the canonical Andoyer's variables. In this formulation, the contributions to the spin are easily obtained as:

$$\frac{dL}{dt} = -m_\odot \frac{\partial U}{\partial l}; \quad \frac{dX}{dt} = m_\odot \frac{\partial U}{\partial \psi}; \quad (16)$$

where m_\odot is the mass of the interacting body which is the Sun in the case of Venus. As we are interested here in the study of the long term evolution of the spin, we will average (16) over the periods of mean anomaly, longitude of node and perigee of the perturbing body. All this work is done with the help of the algebraic manipulator TRIP (Laskar, 1989, 1994b), which expands the potential in Fourier series, as in (Kaula, 1964).

3.1.1 Gravitational tides

As discussed in any text book on tides, the attraction of the Sun at a distance r_\odot from the center of mass of Venus can be expressed as the gradient of a scalar potential which is a sum of Legendre polynomials:

$$V^g = \sum_{l=2}^{\infty} V_l^g = -\frac{Gm_\odot}{r_\odot} \sum_{l=2}^{\infty} \left(\frac{r}{r_\odot}\right)^l P_l(\cos S), \quad (17)$$

where r is the radial distance from Venus' center, and S the angle from the axis from the Sun to the planet center. The distortion of the planet by this potential gives rise to a tidal potential,

$$U^g = \sum_{l=2}^{\infty} U_l^g, \quad (18)$$

where $U_l^g = k_l V_l^g$ at the planet's surface and k_l is the Love number for potential. Since the tidal potential U_l^g is an l th degree harmonic, exterior to the planet it must be proportional to r^{-l-1} (solution of a Dirichlet problem, see eg. Lambeck, 1980). Furthermore, as upon the surface $r = R \ll r_\odot$, we can retain in the expansion only its first term, $l = 2$:

$$U^g \simeq U_2^g = -k_2 \frac{Gm_\odot}{R} \left(\frac{R}{r_\odot}\right)^3 \left(\frac{R}{r}\right)^3 P_2(\cos S). \quad (19)$$

In general, imperfect elasticity will cause the phase angle of U^g to lag behind that of V^g (Kaula, 1964) by an angle $\delta^g(\sigma)$ such that:

$$\delta^g(\sigma) = \frac{\sigma \Delta t^g(\sigma)}{2}, \quad (20)$$

$\Delta t^g(\sigma)$ being the time lag associated to the tidal frequency σ (a linear combination of the inertial rotation rate of Venus, ω , and of the mean orbital motion around the Sun, n). Using (16), we are now able to write the contributions to the spin

$$\begin{cases} \frac{dL}{dt} = -\frac{Gm_\odot^2 R^5}{a^6} \sum_{\sigma} b^g(\sigma) \Lambda_\sigma^g\left(\frac{X}{L}, e\right), \\ \frac{dX}{dt} = -\frac{Gm_\odot^2 R^5}{a^6} \sum_{\sigma} b^g(\sigma) \Xi_\sigma^g\left(\frac{X}{L}, e\right), \end{cases} \quad (21)$$

where e is the eccentricity of the planet's orbit and the series are infinite. However, since the eccentricity is very small, these series can be truncated. The mean and maximal eccentricity of Venus are about 0.036 and 0.08 respectively (Laskar 1994a), so we can neglect the terms in e^2 .

We write then:

$$\begin{aligned} \sum_{\sigma} b^\tau(\sigma) \Lambda_\sigma &= b^\tau(\omega) \frac{3}{4} \frac{X^2}{L^2} \left(1 - \frac{X^2}{L^2}\right) \\ &+ b^\tau(\omega - 2n) \frac{3}{16} \left(1 + \frac{X}{L}\right)^2 \left(1 - \frac{X^2}{L^2}\right) \\ &+ b^\tau(\omega + 2n) \frac{3}{16} \left(1 - \frac{X}{L}\right)^2 \left(1 - \frac{X^2}{L^2}\right) \\ &+ b^\tau(2\omega) \frac{3}{8} \left(1 - \frac{X^2}{L^2}\right)^2 \\ &+ b^\tau(2\omega - 2n) \frac{3}{32} \left(1 + \frac{X}{L}\right)^4 \\ &+ b^\tau(2\omega + 2n) \frac{3}{32} \left(1 - \frac{X}{L}\right)^4 + \mathcal{O}(e^2), \end{aligned} \quad (22)$$

and

$$\begin{aligned} \sum_{\sigma} b^\tau(\sigma) \Xi_\sigma &= b^\tau(\omega - 2n) \frac{3}{8} \left(1 + \frac{X}{L}\right)^2 \left(1 - \frac{X^2}{L^2}\right) \\ &- b^\tau(\omega + 2n) \frac{3}{8} \left(1 - \frac{X}{L}\right)^2 \left(1 - \frac{X^2}{L^2}\right) \\ &- b^\tau(2n) \frac{9}{16} \left(1 - \frac{X^2}{L^2}\right)^2 \\ &+ b^\tau(2\omega - 2n) \frac{3}{32} \left(1 + \frac{X}{L}\right)^4 \\ &- b^\tau(2\omega + 2n) \frac{3}{32} \left(1 - \frac{X}{L}\right)^4 + \mathcal{O}(e^2). \end{aligned} \quad (23)$$

Expression (22) is the same as expression (11) obtained by Dobrovolskis (1980). Dissipation equations must be invariant under the change (ω, ε) by $(-\omega, \pi - \varepsilon)$ which imposes that $b^\tau(\sigma) = -b^\tau(-\sigma)$, that is, $b^\tau(\sigma)$ is an odd function. Although mathematically equivalent, the couples (ω, ε) and $(-\omega, \pi - \varepsilon)$ will correspond to a different physical situation (Fig.5). For gravitational tides, the factor $b^g(\sigma)$ is given by:

$$b^g(\sigma) = k_2 \sin 2\delta^g(\sigma) = k_2 \sin(\sigma \Delta t^g(\sigma)). \quad (24)$$

Dissipation of the mechanical energy of tides in the interior of Venus is responsible for the time delay $\Delta t^g(\sigma)$ between the position of "maximal tide" and the subsolar point. A commonly used dimensionless measure of tidal damping is the quality factor Q (Munk and MacDonald 1960), defined as the inverse of the "specific" dissipation and related to the phase lags by

$$Q_\sigma = \frac{2\pi E}{\Delta E} = \cot 2\delta(\sigma), \quad (25)$$

where E is the total tidal energy stored in the planet, and ΔE the energy dissipated per cycle. We can rewrite (24) as

$$b^g(\sigma) = \text{sign}(\sigma) \frac{k_2}{\sqrt{Q_\sigma^2 + 1}} \simeq \text{sign}(\sigma) \frac{k_2}{Q_\sigma}. \quad (26)$$

As the rheology of terrestrial planets is badly known, the relation between the frequency and the time lag is often subject to some rough approximations. Different models are commonly used to deal with this problem:

The constant Q model. Since Q for the Earth changes by less than an order of magnitude between the Chandler

wobble period of about 440 days and seismic periods of a few seconds, it is common to treat the specific dissipation as independent of frequency. Thus,

$$b^g(\sigma) \simeq \text{sign}(\sigma) \frac{k_2}{Q}, \quad (27)$$

where Q is taken constant with plausible values between 10 and 150 (Goldreich and Soter, 1966).

The viscous model (or “weak friction” model). In this model, we assume that the response time delay to the perturbation is independent of the tidal frequency, i.e., the position of the “maximal tide” is shifted from the subsolar point by a time lag Δt . This model can be made linear (Mignard, 1979, 1980):

$$b^g(\sigma) = k_2 \sin(\sigma \Delta t) \simeq k_2 \sigma \Delta t. \quad (28)$$

This linear approximation is justified because we usually have $\sigma \Delta t \ll 1$. For the Earth $\Delta t \simeq 638$ s (Mignard, 1979, Néron de Surgy and Laskar, 1997) and $\sigma \simeq \omega = 7.29 \times 10^{-5} \text{ rads}^{-1}$, and thus $\sigma \Delta t \simeq 0.05$. For slower rotations (like Venus) the linear approximation is even more accurate.

Interpolated model. The choice of a dissipation model for Venus is not easy: during its evolution Venus is believed to spin rapidly at the beginning which contrasts with the present slow rotation. Most of the previous studies on the rotation of Venus used a constant Q model. However, for near zero tidal frequencies ($\sigma \simeq 0$), which occurs for slow rotation rates ($\omega \sim n$), the dissipation cannot be constant because expression (27) would introduce several discontinuities in dynamical equations (22) and (23). A viscous model seems then more appropriate. Therefore, in our study, we have decided to use an interpolated model which behaves like the viscous model for small tidal frequencies, but which reduces to the constant one for the high tidal frequencies. We choose a natural interpolation function between those two models as:

$$b^g(\sigma) = \text{sign}(\sigma) \frac{k_2}{Q} \left(1 - (1 - Q/Q_n)^{\frac{|e|}{n}} \right), \quad (29)$$

where Q is the quality factor for the fast rotating planet and Q_n the same factor but for $\sigma = n$.

3.1.2 Thermal atmospheric tides

The differential absorption of the Solar heat by the planet’s atmosphere gives rise to local variations of temperature and consequently to pressure gradients. The mass of the atmosphere is then permanently redistributed, adjusting for an equilibrium position. More precisely, the particles of the atmosphere move from the high temperature zone (at the subsolar point) to the low temperature

areas. Indeed, observations on Earth show that the pressure redistribution is essentially a superposition of two pressure waves (see Chapman and Lindzen, 1970): a daily (or diurnal) tide of small amplitude (the pressure is minimal at the subsolar point and maximal at the antipode) and a strong half-daily (semidiurnal) tide (the pressure is minimal at the subsolar point and at the antipode).

The gravitational potential generated by all of the particles in the atmosphere at a generic point of the space \vec{r} is given by:

$$V^a = -G \int_{(\mathcal{M})} \frac{d\mathcal{M}}{|\vec{r} - \vec{r}'|}, \quad (30)$$

where $\vec{r}' = (r', \theta', \varphi')$ is the position of the atmosphere mass element $d\mathcal{M}$ with density $\rho_a(\vec{r}')$ and

$$d\mathcal{M} = \rho_a(\vec{r}') r'^2 \sin \theta' dr' d\theta' d\varphi'. \quad (31)$$

Assuming that the radius of Venus is constant and that the height of the atmosphere can be neglected, we approximate expression (31) as:

$$d\mathcal{M} = \frac{R^2}{g} p_s(\theta', \varphi', t) \sin \theta' d\theta' d\varphi', \quad (32)$$

where g is the mean surface gravity acceleration, and p_s the surface pressure, which depends on the solar insolation. Thus, p_s depends on S , the angle between the direction of the Sun and the normal to the surface:

$$p_s(\theta', \varphi') = p_s(S) = \sum_{l=0}^{+\infty} \tilde{p}_l P_l(\cos S), \quad (33)$$

where P_l are the Legendre polynomials of order l and \tilde{p}_l its coefficients. Developing also $|\vec{r} - \vec{r}'|^{-1}$ in Legendre polynomials we rewrite (30) as:

$$V^a = -\frac{1}{\bar{\rho}} \sum_{l=0}^{+\infty} \frac{3}{2l+1} \tilde{p}_l \left(\frac{R}{r} \right)^{l+1} P_l(\cos S), \quad (34)$$

where $\bar{\rho}$ is the mean density of Venus. Since we are only interested in pressure oscillations, we must subtract the term of constant pressure ($l = 0$) in order to obtain the tidal potential U^a . We also eliminate the diurnal terms ($l = 1$) because they correspond to a displacement of the center of mass of the atmosphere bulge which has no dynamical implications. Thus, since we usually have $r \gg R$, retaining only the semidiurnal terms ($l = 2$), we write:

$$U^a = -\frac{3}{5} \frac{\delta \tilde{p}}{\bar{\rho}} \left(\frac{R}{r} \right)^3 P_2(\cos S), \quad (35)$$

where $\delta \tilde{p} \equiv \tilde{p}_2$. According to expression (16), the dynamical equations are then:

$$\begin{cases} \frac{dL}{dt} = -\frac{3m_\odot R^3}{5\bar{\rho}a^3} \sum_\sigma b^a(\sigma) \Lambda_\sigma^a\left(\frac{X}{L}, e\right), \\ \frac{dX}{dt} = -\frac{3m_\odot R^3}{5\bar{\rho}a^3} \sum_\sigma b^a(\sigma) \Xi_\sigma^a\left(\frac{X}{L}, e\right). \end{cases} \quad (36)$$

Here, too, there is a delay before the response of the atmosphere to the excitation. We name this delay $\Delta t^a(\sigma)$ and the corresponding phase angle $\delta^a(\sigma)$ (Eq.20). The terms Λ_σ^a and Ξ_σ^a are different from their analogs in gravitational tides, Λ_σ^g and Ξ_σ^g . Nevertheless, when neglecting the terms in e^2 , they become equal:

$$\begin{cases} \Lambda_\sigma^a(e=0) = \Lambda_\sigma^g(e=0) = \Lambda_\sigma, \\ \Xi_\sigma^a(e=0) = \Xi_\sigma^g(e=0) = \Xi_\sigma. \end{cases} \quad (37)$$

This allows us to use for thermal atmospheric tides the same expressions (22) and (23) where $b^\tau(\sigma)$ is now:

$$b^a(\sigma) = \delta\tilde{p}(\sigma) \sin 2\delta^a(\sigma) = \delta\tilde{p}(\sigma) \sin(\sigma\Delta t^a(\sigma)). \quad (38)$$

Siebert (1961) and Chapman and Lindzen (1970) showed that when

$$|\delta\tilde{p}(\sigma)| \ll \tilde{p}_0, \quad (39)$$

the amplitudes of the pressure variations on the ground are given by:

$$\delta\tilde{p}(\sigma) = i\frac{\gamma}{\sigma}\tilde{p}_0 \left(\nabla \cdot \vec{v}_\sigma - \frac{\gamma-1}{\gamma} \frac{J_\sigma}{gH_0} \right), \quad (40)$$

where $\gamma = 7/5$ for a perfect gas, \vec{v} is the velocity of tidal winds and J the amount of heat absorbed or emitted by a unit mass of air per unit time. $H_0 = \mathcal{R}\bar{T}_s/\bar{M}g$ is the scale height at the surface, \mathcal{R} the universal gas constant, \bar{M} the mean molecular weight of air and \bar{T}_s the mean temperature at surface level. We can write (40) as,

$$\begin{aligned} \delta\tilde{p}(\sigma) &= \frac{\gamma}{|\sigma|}\tilde{p}_0 \left| \nabla \cdot \vec{v}_\sigma - \frac{\gamma-1}{\gamma} \frac{J_\sigma}{gH_0} \right| e^{\pm i\frac{\pi}{2}} \\ &= |\delta\tilde{p}(\sigma)| e^{\pm i\frac{\pi}{2}}, \end{aligned} \quad (41)$$

where the factor $e^{\pm i\frac{\pi}{2}}$ can be seen as a supplementary phase lag of $\pm\pi/2$:

$$b^a(\sigma) = |\delta\tilde{p}(\sigma)| \sin 2 \left(\delta^a(\sigma) \pm \frac{\pi}{2} \right) = -|\delta\tilde{p}(\sigma)| \sin 2\delta^a(\sigma). \quad (42)$$

The minus sign above causes pressure variations to lead the Sun whenever $\delta^a(\sigma) < \pi/2$ (Chapman and Lindzen 1970, Dobrovolskis and Ingersoll 1980). Unfortunately, our knowledge of the atmosphere response is not as complete as we wished it to be. As for the gravitational tides, models are developed to deal with the unknown part. Dobrovolskis and Ingersoll (1980) use in their paper a so called model, ‘**heating at the ground**’, where they suppose that all the solar flux absorbed by the ground F_s is immediately deposited in a thin layer of atmosphere at the surface. The heating distributing may then be written as a delta-function just above the ground:

$$J(x) = \frac{g}{\tilde{p}_0} F_s \delta(x - 0^+). \quad (43)$$

Neglecting \vec{v} over the thin heated layer, expression (40) simplifies:

$$\delta\tilde{p}(\sigma) = i\frac{5}{16} \frac{\gamma-1}{|\sigma|} \frac{F_s}{H_0} = i\frac{5}{16} \frac{\gamma}{|\sigma|} \frac{gF_s}{c_p\bar{T}_s}, \quad (44)$$

where the factor 5/16 represents the second-degree harmonic component of the insolation contribution (Dobrovolskis and Ingersoll, 1980) and c_p is the specific heat at constant pressure.

Nevertheless, this model has a serious problem: according to (44), if $\sigma = 0$, the amplitude of the pressure variations, $\delta\tilde{p}(\sigma)$, becomes infinite. We know that this cannot be true, as for a tidal frequency equal to zero, a steady distribution is attained, and thus $\delta\tilde{p}(0) = 0$. For $\omega = n$, this effect is known as *solar equilibrium tide* (Chapman and Lindzen, 1970). Indeed, expression (40) is not valid when $\sigma \simeq 0$ because the condition (39) is no longer satisfied. Using the typically accepted values for the Venusian atmosphere, $c_p \simeq 1000 \text{ K kg}^{-1} \text{ s}^{-1}$, $\bar{T}_s \simeq 730 \text{ K}$ and $F_s \simeq 100 \text{ W m}^{-2}$ (Avduvskii *et al.*, 1976) we compute:

$$|\delta\tilde{p}(\sigma)| \simeq 10^{-4} \tilde{p}_0 \frac{n}{|\sigma|}, \quad (45)$$

which means that for $\sigma \sim n$, the ‘heating at the ground’ model of Dobrovolskis and Ingersoll (1980), can still be applied. Since we are only interested in long term behaviors we can set $\delta\tilde{p}(\sigma) = 0$ whenever $|\sigma| < n/100$, and the committed error will be very small. Moreover, for those tidal frequencies the dissipation lag $\sin \sigma\Delta t^a(\sigma) \simeq \sigma\Delta t^a(\sigma)$ also goes to zero. In order to minimize even more this error, in computations, we will use an interpolating function to smooth the discontinuities:

$$|\delta\tilde{p}(\sigma)| = \frac{5}{16} \frac{\gamma}{|\sigma|} \frac{gF_s}{c_p\bar{T}_s} \left(1 - e^{-10^3 \left(\frac{2\sigma}{n} \right)^2} \right). \quad (46)$$

We expect that further studies about the extra solar synchronous planets’ atmospheres, like the one done by Joshi *et al.* (1997), may provide an accurate solution for the case $\sigma \simeq 0$.

In presence of a dense atmosphere, another kind of tides can arise: the atmosphere pressure upon the surface gives rise to a deformation, a pressure bulge, that will also be affected by the solar torque. At the same time, the atmosphere itself exerts a torque over the planet’s bulges (gravitational and pressure bulge). Nevertheless, we do not need to take them into account as their consequences upon the dynamical equations can be neglected (Hinderer *et al.*, 1987, Correia and Laskar, 2002b).

3.2 Core-mantle friction

The last effect to consider is the electromagnetic and viscous friction occurring at the core-mantle boundary. Although without magnetic field, Venus has probably a liquid outer core (Konopliv and Yoder, 1996). This assumption is based on the Earth’s case, which should not be very

different due to the similarities of the density and the size of the two planets. We also make here the assumption that the internal structure of the planet remains constant along its evolution. The core and the mantle do not have the same dynamical ellipticity because of their different shapes and densities. Since the precession torques exerted by the Sun on Venus' core and mantle are proportional to their dynamical ellipticity, the two parts tend to precess differently around an axis perpendicular to the orbital plane (this results from Poincaré's study (1910) on the motion of an inviscid fluid contained in a rotating ellipsoidal shell). This tendency is more or less counteracted by different interactions produced at the interface. The main torques are:

- the torque \vec{N} of non-radial inertial pressure forces of the mantle over the core provoked by the non-spherical shape of their interface.
- the torque of the viscous friction (or turbulent) between the core and the mantle.
- the torque of the electromagnetic friction, caused by the interaction between electrical currents of the core and the bottom of the magnetized mantle.

Rochester (1976) showed that the last two types of friction are collinear. Thus, we can consider these two effects as a single effective friction effect $\vec{\Phi}$, which depends only on one parameter, κ (effective friction coupling constant). At present, Venus does not have a significant magnetic field, maybe because its liquid core has solidified lately (Arkani-Hamed and Toksöz, 1984). In fact, it is believed that Venus' core was certainly liquid before the last great resurfacing event some 0.5 ± 0.3 Gyr ago (Schaber *et al.*, 1992) and hence core friction (viscous and electromagnetic) should have had a major influence on Venus' tidal history. The effect of core viscosity in the Earth's case was treated by Stewartson and Roberts (1963) and Roberts and Stewartson (1965) for low values of viscosity, by linearizing the equations for the viscous boundary layer. Busse (1968) further studied the effect of the non-linear advective term in the equations. As pointed out by Rochester (1976), for the Earth, the results obtained from those studies agree closely with those obtained assuming that the friction torque on the core can be expressed as

$$\vec{\Phi} \simeq -\kappa(\vec{\omega}_c - \vec{\omega}) = -\kappa\vec{\delta}, \quad (47)$$

where $\vec{\omega}_c$ is the core spin vector. Sasao *et al* (1980) showed that the inertial torque can be expressed by:

$$\vec{N} = \vec{\omega}_c \times \vec{L}_c - \vec{P}_c, \quad (48)$$

where \vec{P}_c is the precessional torque on the core, and the subscripts (*c*) and (*m*) refer respectively, to the core and to the mantle. Since the derivative of the angular momentum

is given by the sum of external torques, the contribution of the core-mantle friction (CMF) is the solution of the system:

$$\begin{cases} \frac{d\vec{L}_m}{dt} = \vec{P}_m - \vec{N} - \vec{\Phi} & = \vec{P} - \vec{\omega}_c \times \vec{L}_c + \kappa\vec{\delta}, \\ \frac{d\vec{L}_c}{dt} = \vec{P}_c + \vec{N} + \vec{\Phi} & = \vec{\omega}_c \times \vec{L}_c - \kappa\vec{\delta}. \end{cases} \quad (49)$$

where $\vec{P} = \vec{P}_m + \vec{P}_c$. The contribution of the previous equations to the secular variation of the obliquity is given by (Rochester, 1976, Pais *et al*, 1999):

$$\dot{\varepsilon} \simeq -\kappa \frac{\alpha^2 \cos^3 \varepsilon \sin \varepsilon}{\gamma_{el} C E_c^2 \omega^2}, \quad (50)$$

where E_c is the dynamical ellipticity of the core and $\gamma_{el} \simeq 0.75$ the correcting factor accounting for the elastic deformation of the mantle. In addition, the system of equations (49) also implies that the normal component of the spin momentum $C\omega \cos \varepsilon$ is conserved (neglecting the orbital contribution). Thus,

$$\frac{d(C\omega \cos \varepsilon)}{dt} = \frac{dX}{dt} \simeq 0. \quad (51)$$

We then find the equations (Néron de Surgy and Laskar, 1997) for the secular evolution of the spin of Venus:

$$\begin{cases} \frac{dL}{dt} \simeq -\kappa \frac{9C^3 n^4}{4L^3 \gamma_{el}} \left(\frac{E_d}{E_c}\right)^2 \left(1 - \frac{X}{L}\right)^2 \frac{X^2}{L^2}, \\ \frac{dX}{dt} \simeq 0. \end{cases} \quad (52)$$

For laminar boundary layer or viscous friction, κ is given by (Roberts and Stewartson, 1965, Busse, 1968)

$$\kappa(\text{lam.}) = 2.62 C_c |\omega| \sqrt{\zeta_e}, \quad (53)$$

where

$$\zeta_e = \frac{\nu}{|\omega| R_c^2} \quad (54)$$

is the Ekman number of the core. The kinematic viscosity (ν) is poorly known. Even in the case of the Earth, its uncertainty covers about 13 orders of magnitude (Lumb and Aldridge, 1991). It can be as small as $\nu = 10^{-7} \text{ m}^2 \text{ s}^{-1}$ for the Maxwellian relaxation time and experimental values for liquid metals or as big as $\nu = 10^5 \text{ m}^2 \text{ s}^{-1}$ for the damping of the Chandler wobble or attenuation of shear waves. The best estimate so far of the actual value of this parameter is $\nu \simeq 10^{-6} \text{ m}^2 \text{ s}^{-1}$ (Gans, 1972, Poirier, 1988). However, when we take into account the electromagnetic friction, ν is replaced by an effective friction, which is stronger. Néron de Surgy and Laskar (1997), in a study about the couples $(\Delta t, \nu)$ which give an evolution of the

length of the day similar to the ones given by the observations of the Earth's ground over the last two billion years (Williams, 1989, 1993) give an upper limit for the effective viscosity $\nu \simeq 10^3 \text{m}^2 \text{s}^{-1}$. This value is close to the limit estimated by Toomre (1974): $\nu < 10^2 \text{m}^2 \text{s}^{-1}$.

In addition, unlike Earth's case, friction between the core and the mantle on Venus may become turbulent. Indeed, for slow rotation rates, the Reynolds' number (Re) for precessional flow is so large that turbulence at the core-mantle boundary is almost certain unless the angle between the core and the mantle spin vectors becomes extremely small. Turbulence usually sets in for $\text{Re} \sim 10^5$ to 10^6 and a typical Re is

$$\text{Re} = \frac{\sin^2 \chi}{\zeta_e}, \quad (55)$$

where χ is the angle between $\vec{\omega}_c$ and $\vec{\omega}$ given by:

$$\sin \chi \simeq \frac{\alpha \cos \varepsilon}{\omega E_c} \sin \varepsilon. \quad (56)$$

An estimate of the turbulent stress can be obtained using mixing length theory (Goldstein, 1965, Yoder, 1995a) in which the laminar, viscous boundary layer is replaced by two layers: an interior, laminar viscous sublayer with thickness D which is matched with an exterior turbulent boundary layer. The velocity profile in the laminar sublayer increases linearly with the distance d from the wall ($\vec{u} = \vec{u}_D d/D$) up to a layer thickness D and velocity u_D . The turbulent coupling parameter is then

$$\kappa(\text{turb.}) \simeq \frac{|L|}{2} \left(\frac{u_D}{u_0} \right)^2 \sin \chi, \quad (57)$$

where $\vec{u}_0 = \vec{\delta} \times \vec{R}_c$ is the velocity limiting value in the outer boundary. The value $u_D/u_0 \leq 1/10$ for the expected core parameters. From equations (53), (55) and (57) we have

$$\left(\frac{u_D}{u_0} \right)^2 \simeq \frac{5.2}{\sqrt{\text{Re}}} \times \frac{\kappa(\text{turb.})}{\kappa(\text{lam.})}. \quad (58)$$

In order to ensure the continuity between the κ values when we change from one regime to another, we will use the transition Reynolds' number R_T to evaluate u_D/u_0 . For instance, with $R_T = 10^6$, we have $u_D/u_0 \simeq 1/13.87$.

4 Dynamical evolution

In this section we analyze the dynamical equations obtained previously. The main goal is to describe both evolution and final stages for the spin of Venus in order to understand the results of numerical experiments presented in the companion paper (Correia and Laskar, 2002). This analysis will also allow to determine plausible dissipation coefficients as well as plausible initial conditions for Venus.

4.1 Obliquity calculus

Until now, we have been expressing the variations of the spin in Andoyer's variables. Despite their practical use, these variables do not give a clear view of the obliquity variation. Since $X = L \cos \varepsilon$, one obtains:

$$\frac{d \cos \varepsilon}{dt} = \frac{1}{L} \left(\frac{dX}{dt} - \frac{X}{L} \frac{dL}{dt} \right). \quad (59)$$

For the CMF effect, the variation of ε is easily computed from the previous equation, since $dX/dt \simeq 0$ (Eq.52):

$$\frac{d\varepsilon}{dt} \simeq \frac{1}{L} \frac{dL}{dt} \cot \varepsilon. \quad (60)$$

For tidal effects, we express $d\varepsilon/dt$ using the eccentricity series for dL/dt and dX/dt (Eq.21):

$$\begin{aligned} \frac{d \cos \varepsilon}{dt} &= -\frac{K^\tau}{\omega} \sum_{\sigma} b^\tau(\sigma) \left(\Xi_{\sigma} - \frac{X}{L} \Lambda_{\sigma} \right) \\ &= K^\tau \frac{\sin^2 \varepsilon}{\omega} \sum_{\sigma} b^\tau(\sigma) \Theta_{\sigma}(\cos \varepsilon), \end{aligned} \quad (61)$$

where K^τ is a constant, whose values for each kind of tide ($\tau = g$ or $\tau = a$) are given in table 1, as well as the expression of $b^\tau(\sigma)$. Truncating the series as in (22) and (23) we obtain:

$$\frac{d\varepsilon}{dt} = -K^\tau \frac{\sin \varepsilon}{\omega} \left\{ \begin{array}{l} b^\tau(2n) \frac{9}{16} \sin^2 \varepsilon \\ + b^\tau(\omega) \frac{3}{4} \cos^3 \varepsilon \\ - b^\tau(\omega - 2n) \frac{3}{16} (1 + \cos \varepsilon)^2 (2 - \cos \varepsilon) \\ + b^\tau(\omega + 2n) \frac{3}{16} (1 - \cos \varepsilon)^2 (2 + \cos \varepsilon) \\ + b^\tau(2\omega) \frac{3}{8} \sin^2 \varepsilon \cos \varepsilon \\ - b^\tau(2\omega - 2n) \frac{3}{32} (1 + \cos \varepsilon)^3 \\ + b^\tau(2\omega + 2n) \frac{3}{32} (1 - \cos \varepsilon)^3. \end{array} \right. \quad (62)$$

A similar expression was first established by Dobrovolskis (1980). However, there is a misprint in the sign of the second term $b^\tau(\omega)$ appearing in his expression (12). Shen and Zhang (1988) also reproduce the same error. This is not a very serious mistake, since this term plays a minor role in the evolution. Finally, for the planetary perturbations, we have from the system of equations (6) and from expression (59):

$$\frac{d\varepsilon}{dt} = \mathcal{A}(t) \cos \psi - \mathcal{B}(t) \sin \psi. \quad (63)$$

4.2 Consequences of tidal effects

We have considered two different kinds of tidal effects: gravitational tides (g) and thermal atmospheric tides (a).

Although they have the same nature (a periodic response of the planet to a solar perturbation), their contributions to the dynamical equations are quite different. For example, atmospheric tides can either control the spin evolution or simply be neglected, according to the tidal frequency. The magnitude of each effect depends upon the product of the constant K^τ appearing in expression (62) and the respective dissipation term $b^\tau(\sigma)$. In table 1, we reported their relative strength. The magnitude computation is performed using the same parameters of the *standard model* (Correia and Laskar, 2002) and dividing by the magnitude of the gravitational tides.

Tide (τ)	K^τ	$b^\tau(\sigma)$	Magn.
g	$\frac{Gm_\odot^2 R^5}{Ca^6}$	$k_2 \sin 2\delta^g$	1
a	$\frac{3m_\odot R^3}{5C\bar{\rho}a^3}$	$- \delta\tilde{p} \sin 2\delta^a$	$1.92 \left(\frac{2n}{\sigma}\right)$

Table 1: **Tidal effects comparison.** The tidal effects' contributions to the dynamical equations are the same to first order in e , only the constant K^τ and the dissipation factor $b^\tau(\sigma)$ change for each tidal effect ($\tau = g$ or $\tau = a$).

In the following sections we will analyze each tidal effect separately. We will assume that the rotation rate ω is positive, but this assumption is not restrictive, as results for $\omega < 0$ can be deduced using the symmetry of the dissipation equations (section 3): the couple $(-\omega, \varepsilon)$ behaves identically to the couple $(\omega, \pi - \varepsilon)$.

4.2.1 Gravitational tides

We will first look at the situation $\omega > 2n$, where the planet is believed to spend most of its evolution. Here, the evolution tendency of $d\omega/dt$ is the same for any dissipation model, because all the terms in expression (22) have the same sign. Thus, in this regime, gravitational tides can only brake the rotation rate. On the contrary, when $0 < \omega < 2n$, some terms in expression (22) become negative and we must consider the various dissipation models. Except for the constant Q model, all the dissipation models become linear for these slow rotation rates. In fact, the constant model is not realistic in this situation because when the tidal frequencies becomes null ($\sigma = 0$), a discontinuity occurs in the equations. Using a linearized dissipation model (Eq.28) in the limit of slow rotation rates, one can simplify (22) as,

$$\frac{d\omega}{dt} = -\xi \left[(1 + \cos^2 \varepsilon) \frac{\omega}{2n} - \cos \varepsilon \right] + \mathcal{O}(e^2), \quad (64)$$

with

$$\xi = \frac{3Gm_\odot^2 R^5}{Ca^6} \frac{k_2}{Q_n}. \quad (65)$$

From expression (64) we deduce

$$\frac{d\omega}{dt} \leq 0 \quad \text{if} \quad \omega \geq \left(\frac{2 \cos \varepsilon}{1 + \cos^2 \varepsilon} \right) n. \quad (66)$$

Thus, as long as $\omega > n$, the rotation rate decreases for any value of the obliquity. Conversely, if $\omega < -n$, the rotation rate always increases. For inertial rotation rates values within $-n < \omega < n$, both behaviors are possible and will depend on the obliquity. For each obliquity value ε the transition value is an equilibrium point for the rotation rate:

$$\omega_e = \left(\frac{2 \cos \varepsilon}{1 + \cos^2 \varepsilon} \right) n. \quad (67)$$

When $\omega > \omega_e$ the rotation of the planet decreases, whereas when $\omega < \omega_e$ it increases. Contrary to the rotation rate, in the expression of the obliquity variations for tidal effects (62), different sign terms are present for any rotation rate, so the obliquity variations will always be model dependent. Using as before the linear model, we have:

$$\frac{d\varepsilon}{dt} = -\xi \frac{\sin \varepsilon}{\omega} \left(1 - \frac{\omega}{2n} \cos \varepsilon \right) + \mathcal{O}(e^2), \quad (68)$$

and for a given value of ω , the “equilibrium” obliquity ε_e , obtained as solution of $d\varepsilon/dt = 0$, will be:

$$\begin{cases} \varepsilon_e = \arccos\left(\frac{2n}{\omega}\right) & \text{if } \omega > 2n, \\ \varepsilon_e = 0 & \text{if } 2n \geq \omega > 0. \end{cases} \quad (69)$$

In the case of Venus, the only possible behavior of the gravitational tides alone is thus to lead the planet's spin to the synchronous state ($\omega = n, \varepsilon = 0$), as shown in figure 2. Other resonant states exist but since the eccentricity of Venus and its non-axial deformation are very small, capture in these states is unlikely (see section 2.4). This final evolution can also be visualized in terms of the normal and oblique components of Venus' spin (Dobrovolskis, 1980).

4.2.2 Thermal atmospheric tides

As for gravitational tides, since all terms in expression (22) have the same sign for $\omega > 2n$, the evolution tendency of $d\omega/dt$ in this regime is independent of the dissipation model. However, since this sign (negative for gravitational tides) is positive here, thermal atmospheric tides accelerate the planet's rotation (as long as $\delta^a(\sigma) < \pi/2$). If $0 < \omega < 2n$, the analysis is more complicated. Indeed, due to the surface pressure variations term $\delta\tilde{p}(\sigma)$ (Tab.1), we can no longer establish a simple and general expression as for gravitational tides. Since the function Λ_σ (22) is a polynomial of degree four in $\cos \varepsilon$, all one can say is that there exists for each rotation rate at most four roots of $d\omega/dt$, two corresponding to stable points and the other two to unstable points of the rotation rate. The same difficulty occurs for the study of the obliquity variations, which always depend on $\delta\tilde{p}(\sigma)$. According to expression

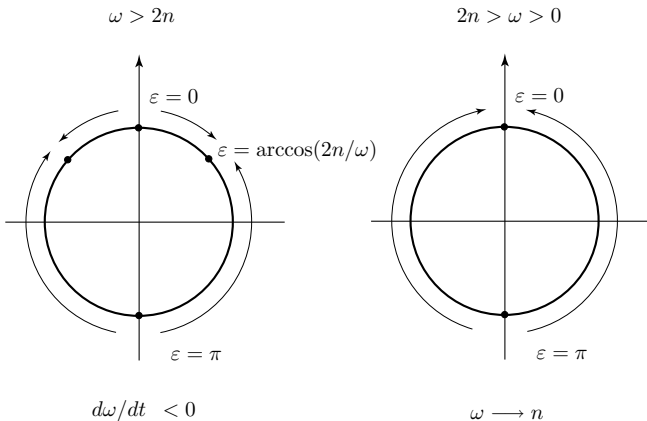


Figure 2: **General consequences of the gravitational tides for ($\omega > 0$).** The planet's spin evolves to the resonant state (1:1). However, during the time that the rotation rate is slowed down, the obliquity tends to a provisional stable point, $\cos \varepsilon = 2n/\omega$.

(40), the surface pressure variations are inversely proportional to σ , so for fast rotation rates ($\omega \gg n$), $d\varepsilon/dt$ is dominated by the first term in (62). There will be two obliquity values where $d\varepsilon/dt$ vanishes, 0° and 180° , the first one corresponding to a unstable position and the last one to a stable equilibrium position for the obliquity.

As we approach slow rotation rates ($\omega \sim n$), all the terms of (62) become important and more complex behaviors are expected. Nevertheless, since the function $\Theta_\sigma(\cos \varepsilon)$ appearing in expressions (61) and (62) is a polynomial of degree three in $\cos \varepsilon$, $d\varepsilon/dt$ has at most three zeros in the interval $[0, \pi]$. These roots can be computed analytically, but we will need to specify the dissipation model. Using (44), we observe that for $\omega > 2n$, the instability point at $\varepsilon = 0^\circ$ will be displaced to an equilibrium obliquity close to ε_e such that

$$\cos \varepsilon_e \simeq -1 + \frac{3\omega}{4n} - \frac{1}{4} \sqrt{9 \frac{\omega^2}{n^2} - 48 \frac{\omega}{n} + 96}, \quad (70)$$

whereas the point $\varepsilon = 0^\circ$ becomes a stable equilibrium point (Fig. 3). When $0 < \omega < 2n$ there are inversions in the signs of the terms $b^a(\sigma)$ and a general solution becomes quite complicated.

All atmospheric tidal terms $b^a(\sigma)$ are proportional to the factor $2n/\sigma$, resulting from the surface pressure variations term $\delta\tilde{p}(\sigma)$ (Table 1). Then, for very low values of σ , their strength become much more important than gravitational tides. On the other hand, for initial rotation rates we have typically $\sigma \sim 100n$ and thus the contribution of thermal atmospheric tides to the rotation rate can be neglected. This result is important as we do not know exactly when a dense atmosphere was formed in the planet's evolution (Walker, 1975, Pepin, 1991, Hunten, 1993). If we assume a fast original rotation, the formation of the

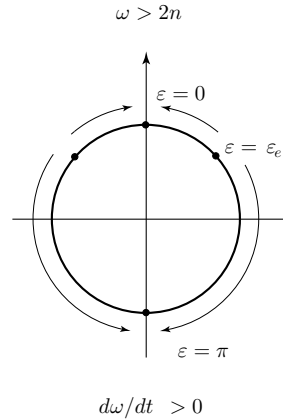


Figure 3: **General consequences of the thermal atmospheric tides for ($\omega > 2n$).** The rotation rate is always accelerated and the obliquity is essentially reversed. However, for slow rotation rates the obliquity decreases when it is inferior to a critical value ε_e (70).

atmosphere in the beginning of the Solar System or only after one billion years will not directly affect the evolution of the rotation rate. However, the same is not true for the obliquity evolution. Due to the presence of the term $b^a(2n) \frac{9}{16} \sin^2 \varepsilon$ (first term in expression 62) there is a constant small contribution from thermal atmospheric tides for obliquities around 90° .

4.3 Consequences of the CMF effect

We now look at the CMF implications. Just as in section 4.2, due to the symmetry between (ω, ε) and $(-\omega, \pi - \varepsilon)$, our analysis is restricted to $(\omega > 0)$. The first equation of system (52) can be rewritten as:

$$\frac{d\omega}{dt} \simeq -\omega K^f(\omega, \kappa) \cos^2 \varepsilon \sin^2 \varepsilon, \quad (71)$$

where

$$K^f(\omega, \kappa) = \frac{\kappa}{\gamma_{el} C} \left(\frac{n}{\omega}\right)^4 \left(\frac{3E_d}{2E_c}\right)^2 \quad (72)$$

is positive. Thus, for any obliquity value, and $\omega \geq 0$,

$$\frac{d\omega}{dt} \leq 0. \quad (73)$$

The contribution of the CMF effect to $d\omega/dt$ is null for $\varepsilon = 0^\circ, 90^\circ$ and 180° . Making use of (60) we obtain from (71) for a given value of ω :

$$\frac{d\varepsilon}{dt} \simeq -K^f(\omega, \kappa) \cos^3 \varepsilon \sin \varepsilon, \quad (74)$$

which implies that, for any rotation rate, the CMF brings the equator to the ecliptic (Fig. 4), while $d\varepsilon/dt$ vanishes for $\varepsilon = 0^\circ$ and $\varepsilon = 180^\circ$ (stable positions) and $\varepsilon = 90^\circ$ (unstable equilibrium). The decrease of the rotation rate and

the obliquity variations are intimately coupled. Indeed, $dX/dt \simeq 0$ (Eq.52) imposes that for an initial rotation rate ω_i and obliquity $\varepsilon_i \neq \pi/2$:

$$\omega = \frac{\omega_i}{\cos \varepsilon_i} \cos \varepsilon . \quad (75)$$

As equation (74) implies that $|\cos \varepsilon| \rightarrow 1$ as time goes on, from the previous expression (75) the equilibrium rotation rate is attained for $\omega_e = \omega_i / |\cos \varepsilon_i|$.

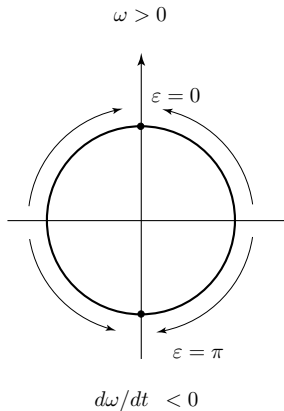


Figure 4: **General consequences of the CMF effect for ($\omega > 0$).** The rotation rate is continuously decreased at the same time that the equator is brought to the ecliptic plane.

4.3.1 Comparison between CMF and tidal effects

Combining expressions (55), (57) and (58) we can write:

$$\kappa(\text{turb.}) \simeq \kappa(\text{lam.}) \sqrt{\text{Re}/\text{R}_T} \geq \kappa(\text{lam.}) , \quad (76)$$

that is, for the same set of parameters, the strength of CMF effect in the turbulent regime is always stronger than its equivalent in the viscous regime. This allows us to extend the conclusions obtained in the viscous regime to the turbulent one. Let β be defined as:

$$\beta = |\omega|^{7/2} K^f(\omega, \kappa) , \quad (77)$$

then, combining expression (53) and (76) we obtain

$$\beta \geq 0.5 \frac{n^4 \sqrt{\nu}}{\gamma_{el} R_c} \left(\frac{E_d}{E_c} \right)^2 . \quad (78)$$

The dynamical ellipticity of the core and the mantle also change slightly with the rotation rate (3). For the Earth values, when the hydrostatic term is dominating (fast rotation rates) we have $E_d/E_c \simeq 4/3$ (Rochester, 1976), while for the non-hydrostatic term (slow rotation rates) $E_d/E_c \simeq 1/4$ (Herring *et al.*, 1986). If we assume that

E_d/E_c is constant, the coefficient β becomes independent of ω and ε . We now rewrite equations (71) and (74) as:

$$\frac{d\omega}{dt} \simeq - \frac{\omega \beta}{|\omega|^{7/2}} \cos^2 \varepsilon \sin^2 \varepsilon , \quad (79)$$

$$\frac{d\varepsilon}{dt} \simeq - \frac{\beta}{|\omega|^{7/2}} \cos^3 \varepsilon \sin \varepsilon . \quad (80)$$

In order to estimate the relative magnitude of CMF and tidal effects we divide $d\omega/dt$ by the magnitude of gravitational tides, ξ (Eq.65), as it was done in Table 1. We have

$$\frac{\beta |\omega|^{-5/2}}{\xi} \geq 7.1 \times 10^3 \sqrt{\nu} \left(\frac{n}{|\omega|} \right)^{5/2} , \quad (81)$$

where we used the non-hydrostatic value as a minimal estimate for E_d/E_c . Yoder (1995a) computes a theoretical value for the non-hydrostatic core ellipticity, $\delta E_c \simeq 29 \delta E_d$ that would set a smaller limit. However, he recognizes (Yoder, 1997) that this value of the non-hydrostatic core ellipticity is probably too large (though not physically unreasonable). Yoder calculated this parameter assuming a constant Q tidal model for slow rotation rates while our model is linear. If we use a constant model, the strength of the thermal atmospheric tides becomes infinite when tidal frequencies are zero which does not seem to be a very realistic situation (see section 3.1.2). Thus, we prefer to assume a Venusian ellipticity closer to the Earth one, instead of using Yoder's estimation. In order to simplify future calculations, we will introduce now the dimensionless parameter ϱ , defined as

$$\varrho = \frac{\beta}{\xi n^{5/2}} . \quad (82)$$

This parameter, which depends on the dissipation models, will be used to quantify the relative strength of the CMF effect to the tidal ones. From (77) and (81), we have:

$$\varrho \geq 7.1 \times 10^3 \sqrt{\nu} . \quad (83)$$

As pointed out by Dobrovolskis (1980) and Yoder (1995a), the CMF effect displays a rapid increase in strength as Venus approaches its present spin state (Eq.81). More, in the limit of slow rotation rates, CMF becomes turbulent, and thus, independent of the effective viscosity ν (see section 3.2). When the regime transition occurs (for $\omega < 4n$), the strength of the turbulent friction becomes, at least, equivalent to the strength of laminar friction with $\nu \simeq 10^{-4} \text{m}^2 \text{s}^{-1}$ (Fig.21, Correia and Laskar, 2002). For lower viscosities, the transition of regime occurs for even faster rotation rates. Then, using that viscosity as inferior limit for ϱ in the turbulent regime, we can deduce

$$\varrho(\text{turb.}) \geq 71 . \quad (84)$$

4.4 Evolution final states

The first measurements of the rotation of Venus (Smith, 1963, Goldstein, 1964, Carpenter, 1964, 1966) led to think that its rotation was in 5:1 resonance with the Earth orbital motion (Goldreich and Peale, 1966). This was unlikely, but the introduction of thermal atmospheric tides seemed to make it possible (Gold and Soter, 1969). However, more accurate measurements found an observed rate ω_o that was not close enough to the resonant rate (243.16 d) to maintain that configuration (Davies *et al.*, 1992, Konopliv *et al.*, 1993). We still do not know if Venus has actually attained its final rotation state, but in this section we will assume that the present measured period $\mathcal{P}_f = 243.0185 \pm 0.0001$ d (Davies *et al.*, 1992) is a final rotation spin state. We will not yet consider the planetary perturbations.

4.4.1 Obliquity final states

For a fast rotation rate ($\omega \gg n$), the surface pressure term (46) remains small and the contribution of atmospheric tides to the rotation rate can be neglected (see section 4.2.2). Since the gravitational tides and the CMF friction both decelerate the planet's rotation, its rate will slow down until it reaches the regime of slow rotation ($\omega \sim n$). Once in this regime, the thermal atmospheric tides can counterbalance the braking effect and give rise to stable positions for both rotation rate and obliquity.

Although many unknowns remain in the physical models and dissipative parameters, it is possible to show (Appendix 5) that in the slow rotation regime, a non-negligible CMF effect controls the general obliquity evolution, and that there are only two stable stable positions for the obliquity, at $\varepsilon = 0^\circ$, $\varepsilon = 180^\circ$. A critical point is also present around $\varepsilon = 90^\circ$, but in the slow regime this critical point is unstable for $\rho > 6$, which is always true in presence of CMF (see the previous section).

4.4.2 Rotation final states

For a planet with a dense atmosphere like Venus, when the final stable positions of the obliquity are $\varepsilon = 0^\circ$ and $\varepsilon = 180^\circ$, there are only four possibilities for its rotation rate (Correia and Laskar, 2001). Indeed, for these two obliquity values, the CMF contribution to $d\omega/dt$ vanishes (71), and the tidal components become very simple, with (at second order in the planetary eccentricity) a single term of tidal frequency $\sigma = 2\omega - 2n$ for $\varepsilon = 0$ and $\sigma = 2\omega + 2n$ for $\varepsilon = \pi$. As $\Lambda_{2\omega-2n}(0) = \Lambda_{2\omega+2n}(\pi) = 3/2$,

$$\left. \frac{d\omega}{dt} \right|_0 = -\frac{3}{2} [K^g b^g (2\omega - 2n) + K^a b^a (2\omega - 2n)] ,$$

$$\left. \frac{d\omega}{dt} \right|_\pi = -\frac{3}{2} [K^g b^g (2\omega + 2n) + K^a b^a (2\omega + 2n)] ,$$
(85)

where K^g and K^a are given in Table 1. Let $f(\sigma)$ be defined as

$$f(\sigma) = \frac{b^a(2\sigma)}{b^g(2\sigma)} .$$
(86)

As $b^\tau(\sigma)$ is an odd function of σ (see section 3.1), $f(\sigma)$ is an even function of σ of the form $f(|\sigma|)$. Thus, at equilibrium, with $d\omega/dt = 0$, we obtain an equilibrium condition

$$f(|\omega - pn|) = -\frac{K^g}{K^a} ,$$
(87)

where $p = +1$ for $\varepsilon = 0$ and $p = -1$ for $\varepsilon = \pi$. Moreover, for all dissipative models (Gold and Soter, 1969, Lago and Cazenave, 1979, Dobrovolskis and Ingersoll, 1980, Shen and Zhang, 1989, McCue and Dormand, 1993, Yoder, 1995a, 1997), f is monotonic and decreasing for slow rotation rates. There are thus only four possible values for the final rotation rate ω_f of Venus, given by

$$|\omega_f - pn| = f^{-1} \left(-\frac{K^g}{K^a} \right) = \omega_s .$$
(88)

Assuming that the present rotation state of Venus corresponds to a stable retrograde rotation, as $\omega_s > 0$, the only possibilities for this final state are $\varepsilon = 0$ and $\omega_o = n - \omega_s$, or $\varepsilon = \pi$ and $\omega_o = \omega_s - n$. In both cases, $\omega_s = n + |\omega_o|$ (ω_s is thus the synodic frequency). With

$$\omega_o = 2\pi/243.0185 \text{ d} ; \quad n = 2\pi/224.701 \text{ d} ,$$
(89)

we have

$$\omega_s = 2\pi/116.751 \text{ d} .$$
(90)

We can then determine all four final states for Venus (table 2). There are two retrograde states (\mathcal{F}_0^- and \mathcal{F}_π^-) and two direct states (\mathcal{F}_0^+ and \mathcal{F}_π^+). The two retrograde states correspond to the observed present state of Venus with period 243.02 days, while the two direct states have a rotation period of 76.83 days. Looking to the present rotation state of the planet, it is impossible to distinguish between the two states with the same angular momentum (Fig.5). These final states do not depend on the precise dissipative model. Indeed, if ω_f^+ is the final rotation rate in a direct state and ω_f^- the final rotation rate in a retrograde state, these two quantities are related by:

$$\omega_f^+ = \omega_f^- + 2pn ,$$
(91)

and when we replace ω_f^- by the observed value ω_o , we have:

$$|\omega_f^+| = |\omega_o| + 2n .$$
(92)

4.4.3 Different observed final rotation rates

To compute the rotation rate in the direct final state, we assumed that the Venusian present rotation rate has reached its final spin value. If that is not the case, for

state	ε	ω	\mathcal{P} (days)	\mathcal{P}_s (days)
\mathcal{F}_0^+	0°	$n + \omega_s$	76.83	116.75
\mathcal{F}_0^-	0°	$n - \omega_s$	-243.02	-116.75
\mathcal{F}_π^+	180°	$-n - \omega_s$	-76.83	116.75
\mathcal{F}_π^-	180°	$-n + \omega_s$	243.02	-116.75

Table 2: **Possible final spin states of Venus.** There are two retrograde states (\mathcal{F}_0^- and \mathcal{F}_π^-) and two direct states (\mathcal{F}_0^+ and \mathcal{F}_π^+). In all cases the synodic period \mathcal{P}_s is the same.

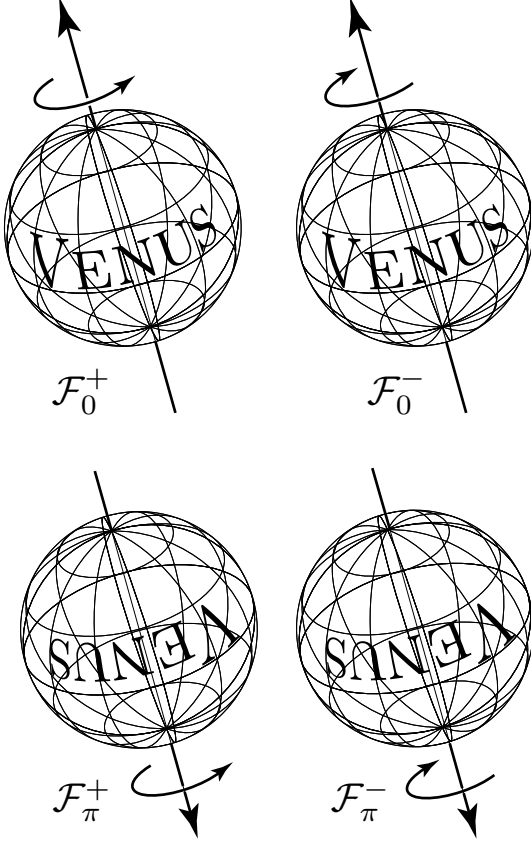


Figure 5: **The four final states of Venus (Correia and Laskar, 2001).** There are two retrograde states (\mathcal{F}_0^- and \mathcal{F}_π^-) and two direct states (\mathcal{F}_0^+ and \mathcal{F}_π^+). The two retrograde states correspond to the observed present state of Venus with period 243.02 days, while the two direct states have a rotation period of 76.83 days. At present it is impossible to distinguish between the two states of each group.

small variations of the present period of rotation \mathcal{P}_o we can deduce from expression (92) a simple formula which gives the variations in the direct final state \mathcal{P}_f^+ :

$$\delta\mathcal{P}_f^+ \simeq \left(\frac{\mathcal{P}_f^+}{\mathcal{P}_o}\right)^2 \delta\mathcal{P}_o \simeq \frac{\delta\mathcal{P}_o}{10}. \quad (93)$$

A variation of 1 day in \mathcal{P}_o gives rise to a variation of only 0.1 day in \mathcal{P}_f^+ . We can then conclude that, in absence of

planetary perturbations, the final rotation period of the direct rotation final state is always close to 76.8 days.

4.4.4 Consequences to the phase lags

Once ω_s is determined, directly from the observations or theoretically, we obtain some constraints on the dissipation phase lags, δ^g and δ^a . Indeed, from equation (88) we have for any final state $\sigma_s = 2\omega_o - p2n = \pm 2\omega_s$:

$$\sin 2\delta^g(2\omega_s) = \frac{|\delta\tilde{p}(2\omega_s)|K^a}{k_2K^g} \sin 2\delta^a(2\omega_s). \quad (94)$$

In addition, since $\sin 2\delta^a \leq 1$, we obtain:

$$Q_{\sigma_s}^{-1} \simeq \sin 2\delta^g(2\omega_s) \leq \frac{K^a}{k_2K^g} |\delta\tilde{p}(2\omega_s)|. \quad (95)$$

Using the ‘heating at the ground’ model (44) for $\delta\tilde{p}(\sigma)$ and the present observed values for k_2 , K^a , K^g and ω_s , we compute:

$$Q_{\sigma_s} \geq 45.3. \quad (96)$$

Inversely, if we are able to measure the Q factor for gravitational tides, we can directly estimate the atmospheric tides phase lags from expression (94):

$$\sin 2\delta^a(2\omega_s) \simeq \frac{k_2K^g}{|\delta\tilde{p}(2\omega_s)|K^aQ_{\sigma_s}}. \quad (97)$$

For instance, with $Q_{\sigma_s} = 50$ we have:

$$\delta^a(\sigma_s) = 33.8^\circ. \quad (98)$$

4.4.5 An atmospheric tides dissipation model

The exact dependency of the dissipation time lags upon the tidal frequency is unknown. However, expression (87) combined with the assumption that Venus is presently in one of its final states (88) provides us some important information about the present ratio between the gravitational Δt^g and the atmosphere Δt^a time lags. Replacing (24) and (42) in expression (87), we obtain

$$\frac{\Delta t^a(2\omega_s)}{\Delta t^g(2\omega_s)} \simeq \frac{\sin(2\omega_s\Delta t^a(2\omega_s))}{\sin(2\omega_s\Delta t^g(2\omega_s))} = \frac{k_2K^g}{|\delta\tilde{p}(2\omega_s)|K^a}. \quad (99)$$

Assuming that this ratio does not change much with the tidal frequency, i.e.,

$$\frac{\Delta t^a(\sigma)}{\Delta t^g(\sigma)} \simeq \frac{\Delta t^a(2\omega_s)}{\Delta t^g(2\omega_s)} \simeq 36.5, \quad (100)$$

we can establish a dissipation model for thermal atmospheric tides. This assumption enables us to simplify the dynamical equations when the obliquity is $\varepsilon = 0^\circ$ or $\varepsilon = 180^\circ$. In fact, equations (85) can be rewritten as

$$\begin{aligned} \frac{d\omega}{dt} &= -\frac{3}{2}K^g b^g(2\omega - p2n) \left(1 + \frac{K^a b^a(2\omega - p2n)}{K^g b^g(2\omega - p2n)}\right) \\ &\simeq -\frac{3}{2}K^g b^g(2\omega - p2n) \left(1 - \frac{|\delta\tilde{p}(2\omega - p2n)|}{|\delta\tilde{p}(2\omega_s)|}\right) \end{aligned} \quad (101)$$

where $p = +1$ for $\varepsilon = 0^\circ$ and $p = -1$ for $\varepsilon = 180^\circ$. The uncertainty remains now only in the gravitational tides dissipation. For small rotation rates, the interpolated model can be linearized, and thus behave like the viscous one (28). Equation (101) simplifies in that case like:

$$\frac{d\omega}{dt} \simeq -\xi \left(\frac{\omega}{n} - p - \frac{\omega_s}{n} \text{sign}(\omega - pn) \right), \quad (102)$$

where ξ is given by expression (65) and $\omega_s/n \simeq 1.92$ (Eq.90). We can now plot the complete evolution of the rotation rate for $\varepsilon = 0^\circ$ or $\varepsilon = 180^\circ$ (Fig.6). These graphics

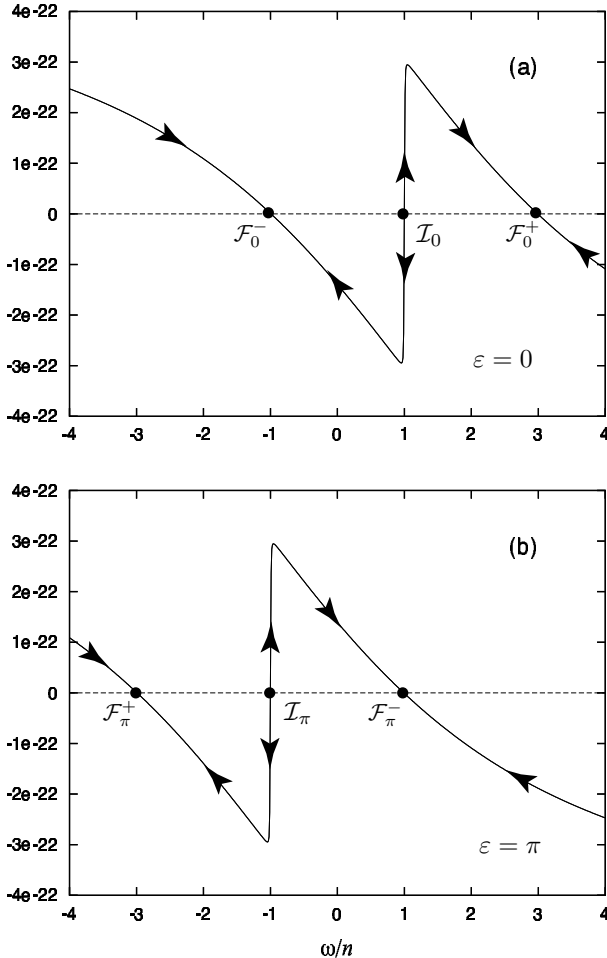


Figure 6: **Variation of $d\omega/dt$ upon ω/n considering all dissipative effects together at $\varepsilon = 0^\circ$ (a) and $\varepsilon = 180^\circ$ (b).** As the central fixed point (\mathcal{I}_0 or \mathcal{I}_π) is unstable, the only possible final evolutions are the four stable points corresponding to \mathcal{F}_0^+ , \mathcal{F}_0^- , \mathcal{F}_π^+ and \mathcal{F}_π^- . Venus rotation rate brakes from fast rotations so we always come from the right hand side of the chart.

are very useful to understand the final evolution of Venus because the final states are equilibrium points ($d\omega/dt = 0$) that correspond to the intersection of the curve with the horizontal axis. As the central fixed point (\mathcal{I}_0 or \mathcal{I}_π) is unstable, the only possible final evolutions are the four stable

points corresponding to \mathcal{F}_0^+ , \mathcal{F}_0^- , \mathcal{F}_π^+ and \mathcal{F}_π^- . Venus rotation rate brakes from fast rotations so we always come from the right hand side of figure 6. If $\varepsilon = 0^\circ$, the only possibility for ω to stabilize is obtained through the \mathcal{F}_0^+ state, since it is the first stability point to be encountered. For the same reason, for $\varepsilon = 180^\circ$ the planet will stabilize its rotation in \mathcal{F}_π^- . However, before reaching 0° or 180° , according to expression (67), the rotation rate can reach any value $\omega \geq -n$. It is then possible that the rotation rate will be comprised between $-n$ and n when the obliquity stabilizes at $\varepsilon = 0^\circ$. In that situation the planet evolves towards \mathcal{F}_0^- as shown in figure 6a.

4.4.6 Relation between the tidal dissipation Q , the initial rotation rate ω_i and the time needed to reach a final state Δt_f .

The initial rotation rate of Venus (ω_i) is not known. Nevertheless, for a given tidal dissipation (quantified by the quality factor Q), the age of the Solar System (~ 4.6 Ga) imposes a constraint on the time (Δt_f) that the planet takes to reach its present configuration. In order to find a relation between these three parameters we need to solve the dynamical equations which is not easy analytically, but we can simplify these equations. For fast rotation rates we can use the constant Q model (see section 3.1) and the contributions from atmospheric tides and CMF to $d\omega/dt$ can be neglected (see sections 4.2 and 4.3). Thus, we have from equations (21) and (22) with $\varepsilon = 0^\circ$:

$$\frac{d\omega}{dt} = -\frac{c}{Q}, \quad (103)$$

where $c = 3k_2 K^g/2$. This equation is valid until the planet reaches the slow rotation regime ($\omega \sim n$) at time $t = t_r$. As we assume that $\omega_i \gg n$, we have $\omega_r - \omega_i \simeq -\omega_i$, and thus

$$t_r - t_i \simeq Q\omega_i/c, \quad (104)$$

where t_i is the initial time. We consider that the planet reaches a final state when $\Delta_\omega = |\omega - \omega_f|/n \ll 1$ (in our computations $\Delta_\omega = 10^{-5}$). In fact, the time $\Delta t_r = t_f - t_r$ spent in the slow regime to reach the final state is much smaller than the total time $\Delta t_f = t_f - t_i$ needed to reach the final state, which we can thus estimate as

$$\Delta t_f \simeq Q\omega_i/c. \quad (105)$$

This expression is very useful as it allows to extrapolate in a easy way the results obtained for a set of initial conditions to another choice of the parameters. We can use a faster initial rotation period for Venus, as long as we increase the dissipation (by reducing Q).

4.5 Effect of the planetary perturbations

In section 4.4.1 we saw that due to the CMF effect, the final obliquity can either be 0° or 180° . However, these final

states correspond to fully damped obliquity states. When planetary perturbations are taken into account, there is always a remaining forced obliquity (Eq.63). In this section, we will only be concerned with small oscillations, that is, with behaviors outside the chaotic zone (Fig.1). Otherwise, the obliquity variations would be very large and we could no longer talk about final states (see section 2.3). Near a final state, we can then write the obliquity as

$$\varepsilon = \varepsilon_e + \delta\varepsilon, \quad (106)$$

where ε_e is the final obliquity in absence of planetary perturbations and $\delta\varepsilon$ the forced obliquity. Usually, $\delta\varepsilon$ never exceed a few degrees (Yoder, 1995a, Correia and Laskar, 2002), so we can assume $\delta\varepsilon \ll 1$ (in radians). The libration amplitude depends on the CMF strength and on the precession constant (Yoder, 1995a, 1997). A strong CMF effect tends to lock the obliquity while a precession constant close to the chaotic zone (Fig.1) increases its oscillations. Taking into account the contribution of this residual obliquity to the rotation rate, we deduce from (79) and (102):

$$\frac{d\omega}{dt} \simeq -\xi \left[\frac{\omega}{n} - p - \frac{\omega_s}{n} \text{sign}(\omega - pn) + \text{sign}(\omega) \varrho \left(\frac{n}{|\omega|} \right)^{\frac{5}{2}} \delta\varepsilon^2 \right] \quad (107)$$

We notice that ω_s does not represent anymore the synodic frequency in a final state. The new rotation rate in a final state is computed from the previous equation when $d\omega/dt = 0$. For the direct states (\mathcal{F}_0^+ or \mathcal{F}_π^+) we have:

$$|\omega_f^+| = \omega_s + n - n\varrho \left(\frac{n}{|\omega_f^+|} \right)^{\frac{5}{2}} \delta\varepsilon_+^2, \quad (108)$$

and for the retrograde states (\mathcal{F}_0^- or \mathcal{F}_π^-):

$$|\omega_f^-| = \omega_s - n - n\varrho \left(\frac{n}{|\omega_f^-|} \right)^{\frac{5}{2}} \delta\varepsilon_-^2. \quad (109)$$

The final rotation rates are no longer fixed and their oscillations $\delta\omega$ follow the forced obliquity variations with

$$|\delta\omega| = n\varrho \left(\frac{n}{|\omega|} \right)^{\frac{5}{2}} \delta\varepsilon^2. \quad (110)$$

Direct rotation final state \mathcal{F}^+ As in section 4.4.2 we can link the spin rate of the direct final state \mathcal{F}^+ to the presently observed value ω_o (92). With $|\omega_f^-| = \omega_o$ in (109), we can solve for ω_s and (108) will give, at first order in $\delta\varepsilon_o^2, \delta\varepsilon_+^2$:

$$|\omega_f^+| = \omega_o + 2n + \delta\omega_f, \quad (111)$$

where

$$\delta\omega_f = n\varrho \left(\frac{n}{\omega_o} \right)^{\frac{5}{2}} \left(\delta\varepsilon_o^2 - \left(\frac{\omega_o}{\omega_o + 2n} \right)^{\frac{5}{2}} \delta\varepsilon_+^2 \right). \quad (112)$$

This result is independent of the atmosphere properties, but depends on the CMF effect and gravitational tides (quantified by ϱ) and on the precession constant (by means of $\delta\varepsilon$). If the obliquity variations $\delta\varepsilon_+$ for \mathcal{F}^+ were identical to the retrograde case, that is, if $\delta\varepsilon_+ \simeq \delta\varepsilon_o$, as $(\omega_o/(\omega_o + 2n))^{5/2} \simeq 0.63$, $\delta\omega_f$ would be positive, and we would expect a period for \mathcal{F}^+ faster than the unperturbed one (76.8 d). However, for the \mathcal{F}^+ states, the planetary perturbations are much larger than in the \mathcal{F}^- states, and the term $\delta\varepsilon_+^2$ becomes dominant. $\delta\omega_f$ is thus negative, and the resulting period becomes larger than in the unperturbed case. Actually, using the dissipation parameters of the *standard model*, we obtained numerically $\mathcal{P}_f^+ \simeq 135 \pm 5$ d (Correia and Laskar, 2002).

4.6 Formation of the atmosphere

The precise evolution of Venus' atmosphere is not known. Nevertheless, it seems that there is an agreement among specialists that the terrestrial planets' atmospheres result from an evolutionary process which takes several hundred million years (Walker, 1975, Hart, 1978, Melton and Giardini, 1982, Zahnle *et al*, 1988, Hunten, 1993, Pepin, 1991, 1994). The present atmosphere of Venus is then a secondary atmosphere that acquired its major properties about 1 Gyr after the formation of the Solar System (eg. Hunten, 1993, Kasting, 1993).

As we have seen in the former sections, the presence of the atmosphere plays a major role in the dynamical evolution of Venus. In its absence, Venus would evolve to a synchronous or near synchronous rotation equilibrium (see section 4.6.1). After the formation of the atmosphere, the planet will inevitably leave the previous configuration and evolve toward one of the four final states. However, the probability of ending in each of these final states will be modified (Correia and Laskar, 2002). The crucial role played by the date of formation of the atmosphere in the Venusian spin dynamics is detailed in the following sections.

4.6.1 Final states in absence of an atmosphere

In absence of atmosphere and planetary perturbations, we saw in section 4.2 that the single action of gravitational tides leads Venus into the synchronous final state. Here, we will show that when CMF effect and planetary perturbations are considered, the synchronous configuration is still possible, though there is another more probable final evolution possibility. The same arguments used in section 4.4.1 to sustain that only two values for the final obliquity are possible, are still valid here, so the final obliquity can either be 0° or 180° . This corresponds to fully damped obliquity states and, as in section 4.5, we must include the contribution of a forced obliquity $\delta\varepsilon$ in the rotation rate equations to take into account the residual

CMF effect provoked by planetary perturbations. From (64) and (79) we write:

$$\begin{aligned}\frac{d\omega}{dt} &\simeq -\xi \frac{\dot{\gamma}}{n} - \frac{\omega\beta}{|\omega|^{7/2}} \delta\varepsilon^2 \\ &= -\xi \frac{\dot{\gamma}}{n} - \frac{p\beta}{|\dot{\gamma} + pn|^{5/2}} \delta\varepsilon^2,\end{aligned}\quad (113)$$

where $\dot{\gamma} = \omega - pn$ and $p = +1$ for $\varepsilon = 0^\circ$ and $p = -1$ for $\varepsilon = 180^\circ$. The equilibrium will then be reached when $d\omega/dt = 0$, that is, when:

$$\frac{\dot{\gamma}}{n} = -\frac{p\beta}{|\dot{\gamma} + pn|^{5/2}} \delta\varepsilon^2. \quad (114)$$

Since we always have $\rho\delta\varepsilon^2 \geq 0$, then $|\dot{\gamma}/n + p| \leq 1$. As a consequence, the equilibrium rotation rate ω_e , solution of the preceding equation (114), must satisfy the following condition (for $p = \pm 1$):

$$|\omega_e| \leq n(1 - \rho\delta\varepsilon^2). \quad (115)$$

The forced obliquity $\delta\varepsilon$ is not constant and its range depends on the CMF strength (ρ): the stronger is the CMF effect, the smaller are the forced obliquity librations. This dependency prevents the equilibrium rotation rate ω_e to become negative (when $\rho \rightarrow +\infty$, $\delta\varepsilon \rightarrow 0$). Roughly, we can approximate ω_e by:

$$\omega_e \simeq n(1 - \rho\delta\varepsilon^2), \quad (116)$$

where ω_e is below the synchronous rotation resonance limits, given by expression (10). Since the planet despins from fast rotation rates, the only possibility to attain this equilibrium is to cross the 1:1 resonance. If the planet is captured, then the final rotation will librate around $\omega = n$, i.e., it will present a synchronous rotation. Otherwise, the rotation rate will librate around an equilibrium ‘‘mean’’ value $\omega = \bar{\omega}_e$, which can be computed using the ‘‘mean’’ value of the forced obliquity in expression (116).

Capture probabilities in the resonance. As we have seen in section 2.4 when the planet crosses a resonance the capture probability is given by expression (13). In the vicinity of the resonance $\dot{\gamma}/n \ll 1$ which allows us to rewrite expression (113) as:

$$\begin{aligned}\frac{d\omega}{dt} &\simeq -\xi \frac{\dot{\gamma}}{n} - \frac{\beta\delta\varepsilon^2}{n^{5/2}} \left(p - \frac{5}{2} \frac{\dot{\gamma}}{n}\right) \\ &= -\xi \left[p\rho\delta\varepsilon^2 + \left(1 - \frac{5}{2}\rho\delta\varepsilon^2\right) \frac{\dot{\gamma}}{n} \right].\end{aligned}\quad (117)$$

Using here the same notations of equation (11) we have:

$$W = p\xi\rho\delta\varepsilon^2, \quad K = \xi \left(1 - \frac{5}{2}\rho\delta\varepsilon^2\right), \quad (118)$$

and since $Z = 0$,

$$\eta = \frac{2K}{\pi} \sqrt{3 \frac{B-A}{C}}. \quad (119)$$

According to (13), the capture probability of the planet into 1:1 resonance will thus be

$$P_{\text{cap}} = \begin{cases} 0 & \text{if } \rho\delta\varepsilon^2 \geq 2/5, \\ \frac{2}{1 + \frac{\pi}{2} \left[\sqrt{3 \frac{B-A}{C}} \left(\frac{1}{\rho\delta\varepsilon^2} - \frac{5}{2} \right) \right]^{-1}} & \text{if } \rho\delta\varepsilon^2 < 2/5. \end{cases} \quad (120)$$

Since the forced obliquity varies continuously, it is possible that for a given instant $\delta\varepsilon$ is small and the planet can be captured into resonance. However, once the forced obliquity increases again, the planet will leave the resonance (see section 2.4). The capture probability should thus be established using the maximal value of the forced obliquity in (120). We should also stress that since $(B-A)/C$ is very small, even when the capture is possible, the capture probability will remain small. In figure 7 we have plotted several examples of the capture probability (Eq.120) as a function of the forced obliquity $\delta\varepsilon$ for different CMF (ν) and gravitational tidal (Q_n) effects. According to Yoder (1995a), the ‘‘mean’’ forced obliquity on Venus is about 2° . For this value, we observe that capture into resonance can only occur for a strong tidal dissipation ($Q_n = 20$) and a weak CMF viscosity ($\nu < 10^{-2} \text{m}^2 \text{s}^{-1}$) with a probability smaller than 5%. Hence we conclude that the capture in the 1:1 resonance is highly improbable.

4.6.2 Evolution of the atmosphere

According to expression (35), the contribution of the thermal atmospheric tides to the dynamical equations in presence of a growing atmosphere only differs from the present contributions by the surface pressure variations factor $\delta\bar{p}(\sigma)$, given by expression (40). The dependency of $\delta\bar{p}(\sigma)$ with the tidal frequency σ is unknown for a primordial atmosphere and the ‘‘heating at the ground’’ model cannot be applied. Indeed, this model works at present because tides in the upper atmosphere are decoupled from the ground by the disparity between their rotation rates, but this may not have been so in the past. However, we can modelize the unknown contribution of the atmosphere by the factor

$$\zeta(t) = \delta\bar{p}(t)/\delta\bar{p}(t_f), \quad (121)$$

which represents the ratio of the surface pressure variations at time (t) over the present one at time (t_f). Assuming that the velocity of tidal winds \vec{v} and that the heating distribution J do not depend much on the tidal frequency, the global dynamical equation for the rotation rate will be the same as in equation (107), but where ω_s is replaced by $\zeta(t)\omega_s$.

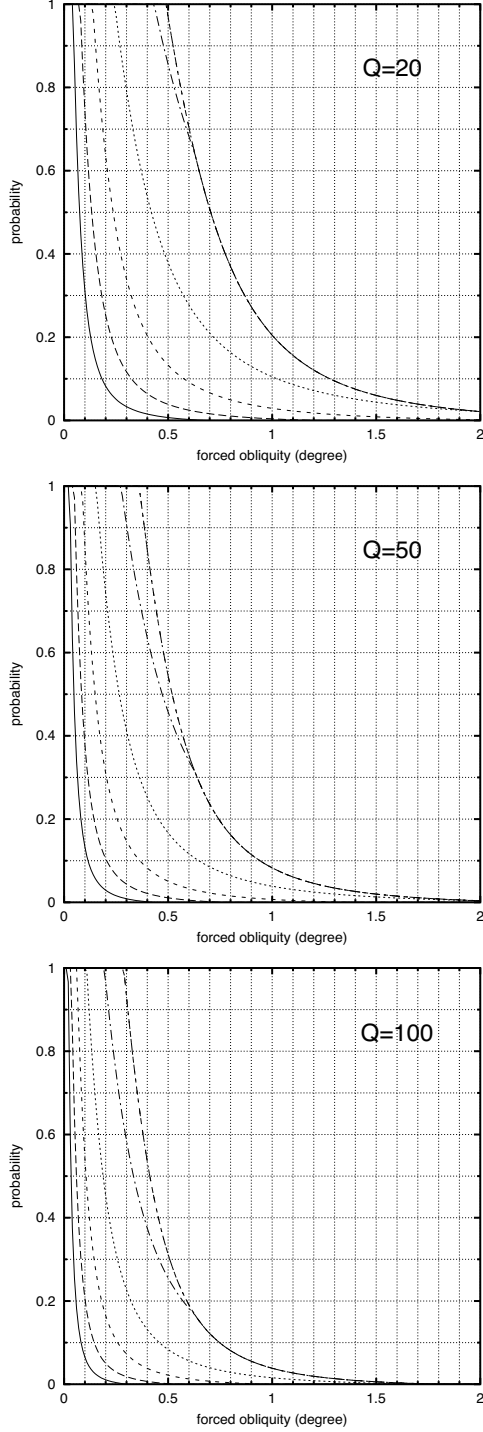


Figure 7: **Capture probabilities for $\delta\varepsilon$ in the 1:1 resonance.** Each curve corresponds to a different viscosity. From left to right we have $\nu = 10^0, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\text{m}^2\text{s}^{-1}$. The last curve on the right also represents the probability for $\nu = 10^{-6}\text{m}^2\text{s}^{-1}$: for these values of ν and $\delta\varepsilon$, we are in the turbulent regime, where CMF does not depend on the viscosity. This is also why the second curve on the right ($\nu = 10^{-4}\text{m}^2\text{s}^{-1}$) merges with the last curve on the right. Each chart corresponds to a different tidal dissipation: $Q_n = 20, Q_n = 50$ and $Q_n = 100$. The lower are the viscosity and Q_n , the higher is the capture probability.

Excluding planetary perturbations. Before taking into account the presence of the atmosphere, the dynamical equations had two possible final states corresponding to the synchronous configuration: ($\omega = n, \varepsilon = 0$) and ($\omega = -n, \varepsilon = \pi$) (see section 4.2). Immediately after the consideration of the faint atmosphere, each synchronous state splits in two, one with a period slightly above the synchronous period and another slightly below. The number of stable equilibrium possibilities increases then from two to four. The stronger is the effect of the atmosphere upon dynamical equations, the larger is the difference between these two rotations rates and the closer they are to the four final states of table 2.

We can understand this bifurcation by looking at figure 8, where we plotted $d\omega/dt$ with $\varepsilon = 0^\circ$ for different stages of the formation of the atmosphere. In absence of the atmosphere ($\zeta = 0$), there is a single equilibrium point where ($d\omega/dt = 0$), for $\omega_e = n$, which corresponds to a stable position for the rotation rate (synchronization). After the introduction of the atmosphere, this equilibrium point becomes unstable, while two other stable equilibrium points appear (one with $\omega_e < n$ and another with $\omega_e > n$). For $\zeta = 0.5n/\omega_s$, they both correspond to direct rotation states. If $\zeta = n/\omega_s$, we have a curious situation, where one of the stable positions corresponds to a planet which does not rotate. As soon as $\zeta > n/\omega_s$, the stable position with $\omega < n$ becomes retrograde (Fig.6a).

Including planetary perturbations. Just as in the case of absence of atmosphere (section 4.6.1), when planetary perturbations are considered, a residual forced obliquity resulting from CMF will prevent the planet from being captured in the synchronous resonance. Indeed, in the vicinity of the resonance we can use the approximation of (Eq. 117), and for $\delta\varepsilon \ll 1$ equations (79) and (102) give

$$\frac{d\omega}{dt} \simeq -\xi \left[\varrho \delta\varepsilon^2 - \frac{\zeta \omega_s}{n} \text{sign}(\dot{\gamma}) + \left(1 - \frac{5\varrho}{2} \delta\varepsilon^2 \right) \frac{\dot{\gamma}}{n} \right]. \quad (122)$$

Keeping the notations of equation (11), the torques W and K are given by expression (118) while the atmospheric torque Z is now:

$$Z = -\xi \zeta \frac{\omega_s}{n} \simeq -2\xi \zeta. \quad (123)$$

According to (13), the capture probability will be zero whenever $\eta = Z + K \frac{\Delta\omega}{\pi n} \leq 0$, that is, from equations (10), (118) and (123), when

$$\varrho \delta\varepsilon^2 \geq \frac{2}{5} \left(1 - \frac{\zeta \pi}{\sqrt{3(B-A)/C}} \right). \quad (124)$$

Since $\varrho \delta\varepsilon^2 \geq 0$, the previous expression is always satisfied when the atmosphere parameter ζ satisfies

$$\zeta \geq \frac{\sqrt{3(B-A)/C}}{\pi}. \quad (125)$$

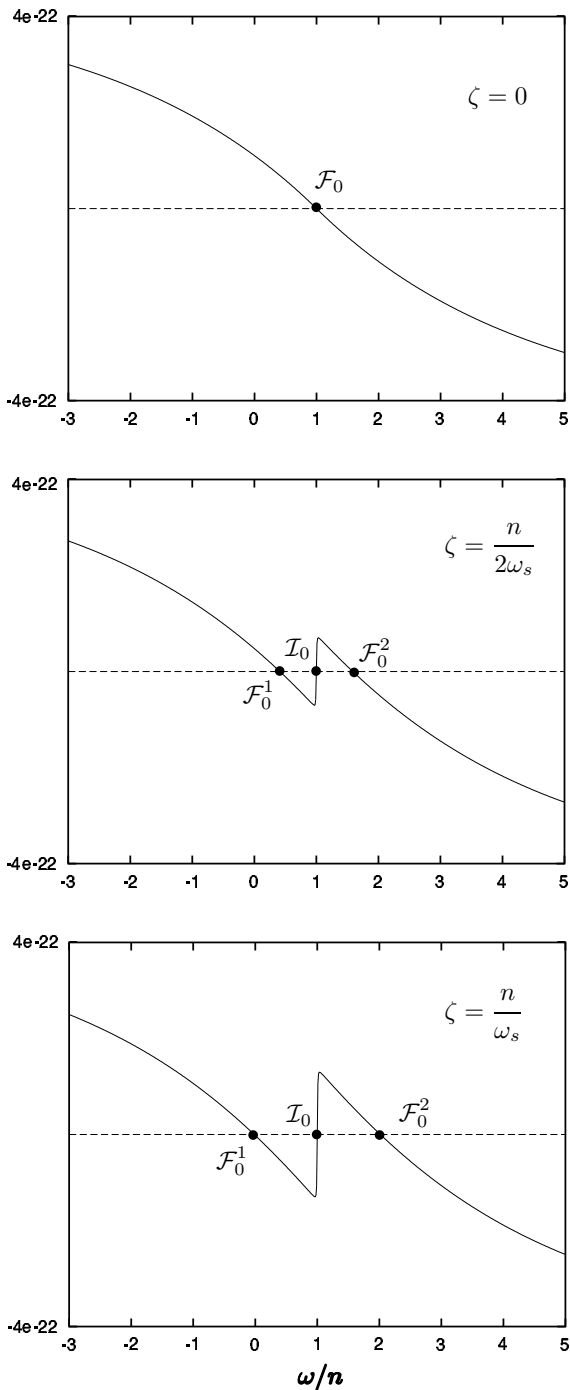


Figure 8: **Bifurcation of the equilibrium states at $\varepsilon = 0^\circ$ for different stages of the atmosphere.** In absence of the atmosphere ($\zeta = 0$), the only stable position corresponds to the synchronization ($\omega = n$). For $\zeta = 0.5n/\omega_s$, we already notice two equilibrium positions, but both corresponding to direct rotation states. If $\zeta = n/\omega_s$, we have a curious situation, where one of the stable positions corresponds to a planet which does not rotate. As soon as $\zeta > n/\omega_s$, one of the equilibrium positions becomes retrograde. As ζ increases, the two equilibria move apart until they stabilize in the final states of figure 6a.

For Venus $(B-A)/C \simeq 2.2 \times 10^{-6}$ (Konopliv *et al.*, 1993), and capture into resonance becomes impossible whenever $\zeta \geq 8 \times 10^{-4}$, that is, in the early stage of the atmosphere formation. Therefore, if the planet spin comes close to the 1:1 resonance, a weak CMF effect (quantified by a small ϱ value) in presence of a faint atmosphere is enough to allow the crossing of the resonance without capture and to let the spin evolve, first to a rotation rate $\omega < n$, and later into the retrograde final state \mathcal{F}_0^- (Fig.6).

Escape probabilities from the resonance. Suppose now, even if it is not very probable (see section 4.6.1), that Venus was captured in the synchronous rotation before the formation of the atmosphere. This requires that the dissipative torque η is positive (12), but as the atmosphere grows, the atmospheric torque Z will no longer be zero (123), and η will decrease. As soon as η becomes negative, Venus will leave the synchronous state, with two different possible paths (see section 2.4), one leading to a direct final state, with probability P^+ , and the other to a retrograde one (Fig.6), with probability $P^- = 1 - P^+$, where P^+ and P^- are given by equation (14). Replacing expressions (118) and (123) in (14) we have:

$$P^- = \frac{1}{2} \left[1 + \frac{\varrho \delta \varepsilon^2}{2\zeta + \frac{2\sqrt{3(B-A)/C}}{\pi} (\frac{5}{2}\varrho \delta \varepsilon^2 - 1)} \right]. \quad (126)$$

If we assume that the planet was captured in the absence of the atmosphere, then $\varrho \delta \varepsilon^2 < 2/5$ (Eq.120) and

$$P^- > \frac{1}{2} \left[1 + \frac{\varrho \delta \varepsilon^2}{2\zeta} \right]. \quad (127)$$

When the thermal atmospheric tides reach their present strength ($\zeta = 1$), we find an inferior limit for P^- (and a superior limit for P^+):

$$P^+ < \frac{1}{2} - \frac{\varrho \delta \varepsilon^2}{4}; \quad P^- > \frac{1}{2} + \frac{\varrho \delta \varepsilon^2}{4}. \quad (128)$$

As a consequence, the probability of ending in a retrograde state is always larger than 50%, independently of the strength of all the involved effects. Moreover, in the early stage of the atmosphere formation, for $\zeta \leq \varrho \delta \varepsilon^2/2$, the escape probability into a retrograde state is always 100% (127). Thus, if we assume a slow transition from $\zeta(t) = 0$ to $\zeta(t) = 1$, the evolution through a retrograde final state is the only possible, i.e., the planet's spin evolves just as if the capture in the resonance never occurred.

5 Conclusion

We have revisited here the theory of thermal atmospheric tides. We confirm that the dense atmosphere of Venus

plays an essential role in the dynamical history of this planet, imposing some constraints upon the spin motion equations which limit its possible evolutions. We show that there are only four possible final states for the spin of Venus, and that gravitational and atmospheric tidal phase lags are correlated and confined within some limits. The existence of the four final states is a general feature for the spin evolution of a planet with a dense atmosphere, that can be applied to future extra solar planets.

We have discussed different dissipation models, their advantages and their applicability to the case of Venus. We show that the constant ‘heating at the ground’ model cannot be applied to a slowly rotation planet, as the amplitude of thermal atmospheric tides becomes infinite for tidal frequencies equal to zero. We have presented a new dissipation model for atmospheric tides that takes into account the constraints imposed by the present observed spin of Venus, and has the particularity of being more realistic near a steady state.

We also analyzed the consequences of a late formation of the atmosphere. Due to a residual CMF effect resulting from the forced obliquity by planetary perturbations, the planet can first evolve toward a near synchronous configuration with $|\omega| < n$ (Eq.115), while capture into resonance is unlikely. As soon as the atmosphere is present, capture into the 1:1 resonance becomes impossible and a previously captured planet will leave the resonance. The crossing of this resonance changes the final evolution of Venus: paths that would normally end in a direct rotation final state, will now evolve into one of the retrograde rotation final states corresponding to the present observed spin of Venus.

The theoretical results established here are illustrated and confirmed by the numerical experiments performed in the companion paper (Correia and Laskar, 2002). There we test several dissipative models for a large set of initial conditions in order to explore all possible evolution scenarios for the planet long term evolution.

Appendix. Obliquity final states

Here we will show that, in the slow rotation regime, a weak CMF effect is sufficient to control the general obliquity evolution, with only two stable positions for the obliquity, at $\varepsilon = 0^\circ$, $\varepsilon = 180^\circ$. Putting together the equations (62) and (80) we rewrite the obliquity variations (for $\omega > 0$) as:

$$\frac{d\varepsilon}{dt} = f_\omega(\varepsilon) = -\frac{\beta}{\omega^{7/2}} \sin \varepsilon \Phi_\omega(\cos \varepsilon) , \quad (129)$$

$$f'_\omega(\varepsilon) = -\frac{\beta}{\omega^{7/2}} [\cos \varepsilon \Phi_\omega(\cos \varepsilon) - \sin^2 \varepsilon \Phi'_\omega(\cos \varepsilon)] , \quad (130)$$

where Φ_ω is given by (Eqs.61, 62):

$$\Phi_\omega(x) = x^3 + \frac{\omega^{5/2}}{\beta} \sum_{\sigma} h(\sigma) \Theta_\sigma(x) , \quad (131)$$

and $h(\sigma) = K^g b^g(\sigma) + K^a b^a(\sigma)$. Since we are in a slow rotation regime, we can use the linear model for gravitational tides (see section 4.2.1) and the approximation (100) for thermal atmospheric tides. Then, just like expression (102), the function $h(\sigma)$ simplifies as:

$$h(\sigma) \simeq K^g b^g(\sigma) \left(1 - \frac{2\omega_s}{|\sigma|}\right) \simeq \frac{\xi\sigma}{3n} \left(1 - \frac{2\omega_s}{|\sigma|}\right) . \quad (132)$$

With these simplifications, (131) becomes:

$$\Phi_\omega(x) = x^3 + \frac{\Gamma_\omega(x)}{\varrho} , \quad (133)$$

where ϱ is given by (82) and

$$\Gamma_\omega(x) = \left(\frac{\omega}{n}\right)^{\frac{5}{2}} \sum_{\sigma} \frac{\sigma}{3n} \left(1 - \frac{2\omega_s}{|\sigma|}\right) \Theta_\sigma(x) \quad (134)$$

is a degree 3 polynomial in x which can be written as

$$\Gamma_\omega(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0 , \quad (135)$$

where the coefficients a_i are functions of ω . We have a stable critical point for the obliquity ε if $f_\omega(\varepsilon) = 0$ and $f'_\omega(\varepsilon) < 0$. An obvious solution for $f_\omega(\varepsilon) = 0$ is obtained whenever $\sin \varepsilon = 0$, that is, for $\varepsilon = 0$ and $\varepsilon = \pi$. The critical point $\varepsilon = 0$ corresponds to a stable position for the obliquity if

$$\Phi_\omega(1) > 0 \Leftrightarrow \varrho > -\Gamma_\omega(1) , \quad (136)$$

and $\varepsilon = \pi$ is stable if

$$\Phi_\omega(-1) < 0 \Leftrightarrow \varrho > \Gamma_\omega(-1) . \quad (137)$$

The two fixed points $\varepsilon = 0$ and $\varepsilon = \pi$ are thus both stable when $\varrho > \max(-\Gamma_\omega(1), \Gamma_\omega(-1))$ (Fig.9). The values needed to stabilize the critical point $\varepsilon = \pi$, are larger than for $\varepsilon = 0$. Indeed, in absence of CMF it is not possible to find stable positions for this obliquity value (Dobrovolskis, 1978). On the other hand, for turbulent friction, $\varrho > 70$ (Eq.84). Thus, in presence of a non-negligible CMF effect the equilibrium points at $\varepsilon = 0$ and $\varepsilon = \pi$ are always stable positions of the spin axis. The other possible critical points are obtained for $\Phi_\omega(\cos \varepsilon) = 0$ (Eq.129). From (136) and (137), using the continuity of Φ_ω , we are certain that there exists at least one additional critical value in $]0, \pi[$. The condition that $x = \cos \varepsilon$ corresponds to a stable critical point can then be expressed as

$$g(x) = (a_3 + \rho)x^3 + a_2 x^2 + a_1 x + a_0 = 0 \quad (138)$$

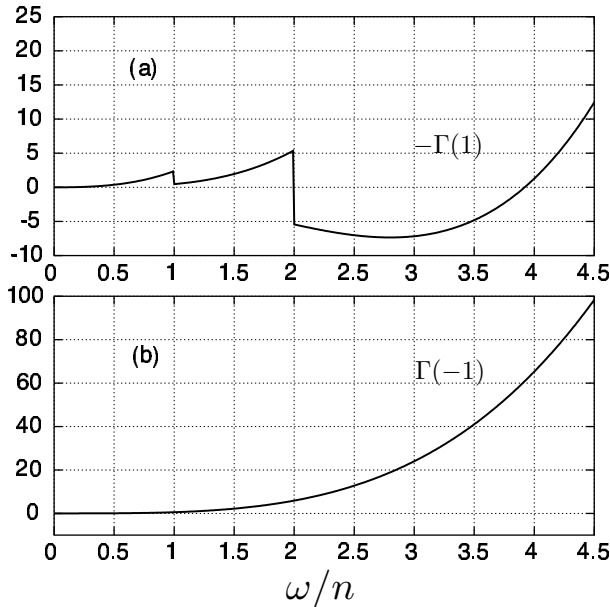


Figure 9: Evolution of $-\Gamma_\omega(1)$ (a) and $\Gamma_\omega(-1)$ (b) with ω/n (Eq.134). $\varepsilon = 0$ (resp. $\varepsilon = \pi$) is a stable critical point when $\rho > -\Gamma_\omega(1)$ (resp. $\rho > \Gamma_\omega(-1)$) (Eqs. 136, 137).

$$g'(x) = 3(a_3 + \rho)x^2 + a_2x + a_1 < 0. \quad (139)$$

When $\rho \rightarrow +\infty$, $g(x) \simeq \rho x^3$ and $g'(x) \simeq 3\rho x^2$, so for large values of ρ , $g(x) = 0$ has a single real root, close to 0, (i.e. $\varepsilon \simeq 90^\circ$), and as $g'(x) > 0$, this solution corresponds to an unstable equilibrium. A necessary condition for this equilibrium to become stable, is that, for a sufficient low value of ρ , we have $g(x) = 0, g'(x) = 0$ (this will also correspond to the bifurcation from one single real root of $g(x) = 0$, to three real roots). We can thus eliminate x in these two relations, and obtain the limit condition equation

$$A_2\rho^2 + A_1\rho + A_0 = 0 \quad (140)$$

with

$$\begin{aligned} A_0 &= +4a_0a_2^3 - 18a_0a_1a_2a_3 - a_1^2a_2^2 + 4a_1^3a_3 + 27a_0^2a_3^2 \\ A_1 &= -18a_0a_1a_2 + 54a_0^2a_3 + 4a_1^3 \\ A_2 &= 27a_0^2 \end{aligned} \quad (141)$$

The discriminant of this second degree equation is

$$\Delta = 4(27a_0^2a_1^2a_2^2 - 9a_0a_1^4a_2 + a_1^6 - 27a_0^3a_2^3). \quad (142)$$

For $2 < \omega/n < 4.5$, as $\Delta < 0$, there are no solutions for the critical equation (141), and thus (138) has a single real root which corresponds to an unstable equilibrium. For $0 < \omega/n < 2$, it becomes possible to have a stable equilibrium, but it requires values of ρ smaller than $\max(r_1, r_2)$, (Fig.10), that are not compatible with the constraint $\rho > 70$, derived in section (4.3.1). We

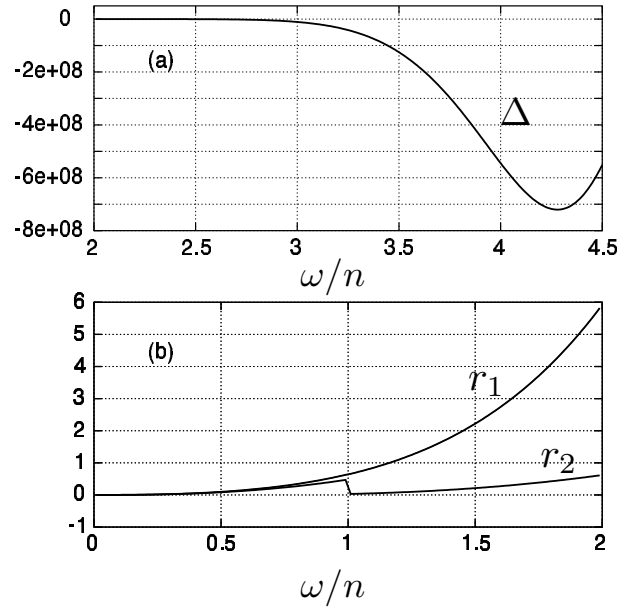


Figure 10: **Solution of the critical equation.** The discriminant Δ of the critical equation (141) is negative for $(2 < \omega/n < 4.5)$ (a). For $0 < \omega/n < 2$, the discriminant is positive, and the two roots r_1 and r_2 of (141) are given in (b).

have thus demonstrated that although there exists an additional equilibrium value for the obliquity around 90° , this value corresponds to an unstable equilibrium, and the only possible stable critical points are thus $\varepsilon = 0^\circ$ and $\varepsilon = 180^\circ$.

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<i>Symbol</i>	<i>Designation</i>	<i>Eq.</i>	<i>Symbol</i>	<i>Designation</i>	<i>Eq.</i>
a	Venus' semi major axis	2	T	general dissipative torque	11
A	minimal moment of inertia	3	\bar{T}_s	mean surface temperature	40
\mathcal{A}	function of the secular orbital motion	1	\vec{v}	velocity of tidal winds	40
$b^a(\sigma)$	atmospheric tides dissipation factor	38	U^a	thermal atmospheric tides potential	35
$b^g(\sigma)$	gravitational tides dissipation factor	24	U^g	gravitational tides potential	18
B	moment of inertia	3	V^a	potential generated by the atmosphere	30
\mathcal{B}	function of the secular orbital motion	1	V^g	gravitational potential	17
c_p	specific heat at constant pressure	44	W	positive and constant torque	11
C	principal moment of inertia	3	X	projection of \vec{L} on the ecliptic's normal	1
\mathcal{C}	function of the secular orbital motion	1	Z	constant torque	11
e	eccentricity of Venus' orbit	2	α	precession constant	2
E_c	core dynamical ellipticity	50	α_r	precession constant for resonances	4
E_d	dynamical ellipticity	3	β	core-mantle friction parameter	77
F_s	solar flux absorbed by the surface	43	γ	perfect gas constant	40
\mathcal{F}^+	direct rotation final state	88	$\dot{\gamma}$	relative rotation rate near resonance	11
\mathcal{F}^-	retrograde rotation final state	88	γ_{el}	correcting factor for elastic deformation	50
g	mean surface gravity	32	$\vec{\delta}$	differential core rotation rate	47
G	gravitational constant	2	δ^a	atmospheric phase lag	38
H_0	scale height at the surface	40	δ^g	gravitational phase lag	20
\bar{H}	averaged Hamiltonian of the motion	1	δE_d	residual dynamical ellipticity	3
J	heating distribution	40	$\delta\tilde{p}(\sigma)$	surface pressure variations	40
k_2	second Love number	19	$\delta\varepsilon$	forced obliquity	106
k_f	fluid Love number	3	Δt^a	atmospheric time lag	38
K	constant torque	11	Δt_f	time needed to reach a final state	105
K^a	atmospheric tides constant	85	Δt^g	gravitational time lag	20
K^f	core-mantle friction function	72	$\Delta\omega$	width of the 1:1 resonance	10
K^g	gravitational tides constant	85	ε	obliquity	9
ℓ	hour angle	1	$\zeta(t)$	ratio of surface pressure variations	121
\vec{L}	angular momentum	1	ζ_e	Ekman number of the core	53
m_\odot	solar mass	2	η	composed dissipative torque	12
M	mean anomaly	1	Θ_σ	tidal coefficient	61
\mathcal{M}	mass of the atmosphere	30	κ	effective friction coupling constant	47
n	mean motion	2	Λ_σ	tidal coefficient	22
\vec{N}	non-radial inertial pressure torque	48	ν	kinematic viscosity	54
\tilde{p}_0	mean surface pressure	33	ξ	viscous model constant	65
p_s	surface pressure	32	Ξ_σ	tidal coefficient	23
\vec{P}	precessional torque	49	$\bar{\rho}$	Venus' mean density	34
P_{cap}	probability of capture into resonance	13	ρ_a	density of the atmosphere	31
P^\pm	escape probability from resonance	14	ϱ	dimensionless parameter	82
P_l	Legendre polynomials	17	σ	tidal frequency	20
\mathcal{P}	rotation period	93	$\vec{\Phi}$	effective friction torque	47
Q	quality factor	25	χ	angle between two spins	56
\vec{r}	radial distance from Venus' center	17	ψ	general precession angle	1
\vec{r}_\odot	Sun distance from Venus' center	17	$\vec{\omega}$	rotation rate	2
R	Venus' radius	3	ω_f	final rotation rate	88
Re	Reynolds' number	55	ω_i	initial rotation rate	104
R_T	transition Reynolds' number	76	ω_o	observed rotation rate	89
S	angle between two directions	17	ω_s	synodic rotation rate	88