Gravity and unification

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Contents

1 Introduction

The subject of the paper is "Gravity and unification", but before we can begin examining gravity, we have to define it. Even though everybody knows what it is, defining it strictly is a hard task, mainly because nobody understands the nature of it. For now we will define gravity in simple terms:

Gravity is the tendency of massess to move towards each other.

Note, that we did not use the word *force* in our definition. Now that we have defined the term, we shall see how scientists understand it. Later on we shall take a look at various attempts to unify gravity and other known forces to form a so called "*Theory of everything*".

2 The history of gravity

For a very long time people have believed that things stay on earth, just because they have to, it was their belief that the earth is flat and everything is placed on it. The first ones to enhance this view were the ancient Greek philosophers. They claimed that everything had a place of rest, for example, flames leapt upwards, because their place of rest was the sky, a stone thrown in the air would fall to it's place - the ground etc. However the Greeks could not explain why are the stars and other objects in the sky and how they move.

The first breakthrough in studying this effect were Galileos observations while throwing cannon balls from the Pisa tower. He noticed that all bodies fall with the same acceleration, independent of their mass.

3 Newtonian gravity

In 1687 Sir. Isaac Newton, an English scientist published his famous book about motion, which is considered to be the beginning of what we call "classical physics". The book contained the famous laws of motion and the first scientific explanation of gravity.

3.1 Definitions

Before stating anything, Newton defined some terms:

- *Mass* is a measure of inertia.
- *Momentum* is a product of mass and velocity
- A *force* is an action that causes a body to change it's motion.

3.2 Newton's laws of motion

Later in the book Newton postulated his famous laws of motion. There are three of them:

1. A body will remain at rest or continue moving in a straight line until it is acted upon by some force.
2. Force is the rate of change of momentum.
3. Every action has a reaction.

Up to now, the term *mass* was defined as the measure of inertia, which is usually denoted as $m^{(i)}$.

3.3 The law of universal gravitation

Last, but not least, Newton declared the famous law of universal gravitation. Gravity was defined as a force, i.e. something that causes bodies to change their steady motion or being at rest. It seemed obvious, that a body thrown in the air falls down with acceleration (recall Galileo's observations). Next, Newton figured out what influences the strenght of the force. He put up an empyrical formula:

$$F_g \sim \frac{m_1^{(g)} m_2^{(g)}}{R^2}.$$

The force with which point (1) attracts point (2) is directly proportional to the product of their "gravitation charges" and inversly proportional to the square of the distance between them.

The "gravitational charge" is some measure of a body's ability to attract other bodies, just like electrical charges. Newton called this property "*the gravitational mass*", thus the subscript "g".

But the most important thing, that Newton discovered, was that gravitational and inertial masses are proportional, thus if some constant is used in the empyrical equation, it becomes precise. Newton called this constant G - the gravitational constant, thus the equation becomes:

$$F_g = G\frac{m_1 m_2}{R^2}.$$

This is the force for two points, to calculate it for two bodies, we have to integrate the equation. It was shown, that for spheres it is the same as if their masses were concentrated in their centers.

The inertial and gravitational mass relation was a very important discovery, which seems trivial to us now, but in Newton's theories the two masses have nothing in common and for some reason they are equal. To this day, no material has been found, which would have differing inertial and gravitational masses.

By using the second and the third laws of motion, Newton was able to calculate everything related to gravity - from the fall of an apple, to the motion of the stars - he had unified the earth and the heavens. What is more, the equation is so precise that scientists used it while calculating spaceship trajectories. This was a huge triumph in science.

3.4 Problems with Newtonian gravitation

For about two hundred years, Newton's equations worked flawlessly, but later they faced some problems.

First of all, Newton had predicted, that gravity is a force, that acts over distance instantaneously. This was bothering Newton himself, because he refused to believe in interaction over distance. In 1905, a young scientist Albert Einstein published his theory of special relativity, in which he showed, that nothing can travel faster than light. Thus came the first problem - gravity just can't travel faster than light, as Newton had shown.

The second problem arose while studying the movement of Mercury. The observations differed from the theoretical predictions, which followed from Newtons equations. Although it was a very small inconsistency, but it was enough to make scientists worried.

What is more, Newtons theory didn't explain why gravitational and inertial masses are equal.

These and other reasons required a new approach on gravity. And so, after more than two hundred years came Einstein.

4 Einstein's aproach

Albert Einstein began his career as a scientist by studying light. Weird as it sounds, this study finaly led to enhancing Newtons view of gravity. It all began in 1905, when Einstein published even 4 papers on various topics, one of them was about special relativity.

4.1 Special relativity

It all began while trying to explain the behaviour of light. Einstein postulated, that the speed of light is a constant in all inertial frames and that all inertial frames are equivalent.

Many intriguing results followed from special relativity, but the most important to our subject are the following:

1. Space and time are inseperable and form a four dimensional space called the spacetime.
2. No signal can be transmited faster than light.

The second outcome conflicted with Newtons calculations, that gravity acts instantaneously. After nearly ten years of racking his brain, Einstein came up with an answer.

4.2 General relativity

In 1915 Einstein published another paper on relativity, this time he generalised his previous theory, which from then was refered to as the theory of special relativity, the new one being the theory of general relativity. The main difference was that this time, the theory adressed reference frames moving with acceleration, including systems acted upon by gravity. According to this theory, gravity is not a force.

In order to explain gravity, Einstein modified Newton's first law of motion, so that it could be applied to a 4 dimensional space. The only difference made was the replacement of the word *line* to a more general term *geodesic*. In simple terms, a *geodesic* in any space or any surface is the shortest curve between two points. Newton though of time and space as two separate things, he also considered space to be a perfect 3 dimensional Euclidian space and time to be running smoothly everywhere. And indeed, up until Einstein's theory nobody doubted that, all measurements showed space to be uniform and smooth with huge precision. Recall that earlier Einstein had unified space and time to a singe entity. This time he stated that this 4 dimensional spacetime can be warped and curved. That's why he needed to modify Newton's first law of motion - a geodesic in curved spacetime is not a line! If we consider space to be perfectly straight, then a geodesic becomes a line and Newton's first law of motion becomes correct. This generalisation of Newton's first law of motion is known as the *Law of geodesic motion*: every body will remain at rest or move steadily along a geodesic unless acted upon by some force.

Next, Einstein showed, that in presence of heavy or very fastly moving objects space is warped. Since time and space are inseparable, this means that time is warped too! Einstein studied these warps in a 4 dimensional space using advanced math such as differential geometry and tensor calculus. This is very complex, so we shall split the warps into space warps and time warps. For an object to warp space around it, it has to be very heavy or moving at a speed close to the speed of light. All such high speed objects usually are very tiny and unnoticable, however we have many examples of heavy objects in our galaxy: the sun, earth and other planets. Let's take the sun and earth as an example system. Since the sun is way heavier that the earth, we can consider earth to be a tiny ball. Einstein showed that the space warps around the sun are very tiny, thus can be ignored in our example, but the time warps are noticable. It turns out that time is warped in such a way that a geodesic around the sun is bent, like a circle. Since earth isn't acted upon by no force (recall that there is no such thing as a gravitational force, according to Einstein), it follows the geodesic formed by the sun's presence. Since it is a curve, earth is spinning around the sun! This way, Einstein defined gravity as an outcome of curved space and the law of geodesic motion, not as a force. If the space warps were to be ignored and only the time warps taken into account, Einstein's equations would reduce to Newton's.

This view is hard to image, but it can be explained considering our four dimensional space as a trampoline, and the sun as a heavy ball put in the center. This way the trampoline is curved inwards and if we put a small ball somewhere near it would roll towards our imaginary "sun". This motion is called gravity.

With this view of gravity Einstein got rid of all the problems that bothered Newtonian gravity. First of all, he calculated the speed of gravity. Again, it may be hard to realise what that is, but image that for no reason our imaginary "sun" disapears from the trapoline. We would see something like a wave, which after some time would reach the other body on the trampoline. This is the speed of gravity, which Einstein calculated to be precisely equal to the speed of light. The inconsistencies in studying Mercury's orbit can be explained taking into account the space warps caused by the sun's presence (recall that Mercury is very close to the sun, so these warps influence it's motion noticably, unlike the other planets). The question why inertial and gravitational massess are equal answers itself in general relativity: in this theory there is no gravitational force, gravity is motion like any other motion described by Newton's laws, thus we use inertial massess.

Up to this day, almost no further inconsistencies have been noticed and for neary a hundred years general realtivity has been working flawlessly.

4.3 Black holes

Shortly after publishing the theory of general relativity scientists began exploring certain outcomes. One of them requires special atention. It was shown that if a heavy body would shrink to a point it would warp spacetime in such a way that nothing could escape the point's pull, even light, thus the name - black holes.

These object's were the first sign, that something is missing in general relativity - if you try to apply it on that point, you get nonsensical results, the theory breaks down. These objects remain as one of the biggest secrets in our universe to this day.

5 Meanwhile ...

While Albert Einstein was busy with general relativity, other scientists continued improving other things. In order to see a bigger picture we need to understand some other important breakthroughs in science.

5.1 Electromagnetism

Before Einstein shocked the science community with relativivy, a Scotish scientist James Clark Maxwell introduced his theory of electromagnetism. Maxwell showed, that electric and magnetic fields are inseparable and everything related to them can be described by four simple euations, which are now better know as Maxwell's equations. Maxwell was even able to calculate the speed with which these fields spread through space. It turned out, that that speed is precisely the same as the speed of light.

Maxwell's main accomplishment was that he unified electric and magnetic forces into one - the electromagnetic force. Einstein had expressed huge admiration for Maxwell because of this.

5.2 Quantum mechanics

Just as Newton's classical theories had failed when applied to fastly moving bodies, it failed when applied to very tiny objects, i.e. atoms and other tiny particles. This inconsistency led to the creation of quantum mechanics.

It turns out, that things in a sub-atomic level are very different from our daily experiences. There everything is ruled by chance. Heisenberg's uncertainty principle states that there is a limit on how accurately you can measure the position and the momentum of a particle. The equation is:

$$\mathtt{d}x \cdot \mathtt{d}p \geq \frac{\hbar}{2}$$

This means, that the more precisely you know the coordinate, the less precisely you know the momentum and vice versa. This is a fundamental limitation of accuracy in nature. Since you can't know for sure, everything in quantum mechanics is expressed by chance.

Einstein was deeply dissapointed by this and for a long time refused to accept it, he believed in a precise and predictable universe. Once he had even said "God does not throw dice". Despite this, quantum mechanics eliminated all problems related to small objects just as Einstein's general relativity did with gravity and up to this day it has been working flawlessly.

5.3 Fundamental interactions

When the electromagnetic force was discovered, scientists realised that every interaction in nature can be expressed in so called *fundamental interactions of nature*. The first one being gravity and the second - the electromagnetic force. It may seem weird that the force we use while interacting with various objects is the electromagnetic force. This can be explained by imagining that particles, which make up things, act like springs - if you push an object, particles propogate the push further thus making an object move. This explanation also shows, that gravity is bilions of times weaker that the electromagnetic force - tiny particles of a pavement can produce a stronger push than the pull of the earth, thus we can walk on a pavement without crushing through.

Studies in quantum mechanics have revealed that there are two more fundamental forces: the strong nuclear force, responsible for keeping protons and neutrons together in the nucleus and the weak force, responsible for radioactivity. These forces are also much stronger than gravity.

So now we can summarize the fundamental interactions and compare their strengths to gravity:

Interaction	**Strength**
Gravity	10^0
Electromagnetic force	10^{38}
Strong nuclear force	10^{40}
Weak nuclear force	10^{15}

As you can see, the strongest interaction, the strong nuclear force, is approximately a hundred times stronger than the electromagnetic force and way stronger that gravity.

These interactions are also sometimes called *the fundamental forces of nature*, however, recall, that Einstein did not consider gravity to be a force, thus the term *interaction* is better in this case.

5.4 The Standard Model

After all of the four fundamental interactions were discovered, scientists began trying to unite some of them. One successful attempt was the unification of the electromagnetic and the weak nuclear forces into a single force, called the electroweak. This unification was shown by using quantum mechanics. Since it dealt with particles so well, scientists began treating forces as particles too. This led to the so called *Standard Model* - a theory which deals with all known elementary particles and forces, interpreted as iteractions of particles, except for gravity.

The electroweak force can be explained as interactions of photons and other particles and the strong nuclear force - as interactions of so called *gluons*. The theory which explains the strong force is called *quantum chromodynamics*.

To date, almost all experimental tests of the three forces described by the Standard Model have agreed with its predictions.

6 Unification

The Standard Model was a successful attempt in unifying the fundamental interactions of nature, however it was not complete. Many scientist are still trying to find an ultimate, so called, *theory of everything*.

6.1 Problems

To understand why unification is needed, we will have a look at some problems with the traditional theories.

The main problem is that general relativity and quantum mechanics have their domain of application: relativity works for heavy objects, like stars or planets and quantum mechanics works well for small particles, such as atoms. However, these theories are drastically different, it almost seems as if they are describing two different worlds, what is more, they don't seem to live well together. The best examples of this "disagreement" are black holes, which were mentioned while talking about general relativity. These objects are both very small and very heavy, so which theory should be applied: for the very small or for the very heavy ? In theory, both should do, however, in reality, they both break down. Nor general relativity, nor quantum mechanics can explain the behaviour of black holes, to this day they are something of a mistery.

Another big problem is related to gravity. Our galaxy, the Milky Way, is spinning around it's center, where a black hole is though to be. If we would try to calculate the mass of our galaxy using general relativity and other theories, we would get, that the mass is five to ten times bigger than the acctual mass of the stars and other objects. This missing part is though to be some other kind of matter, called *dark matter*. Unfortunately, nobody has yet found it, nor nobody know's what it is.

Current theories also fail at explaining other phenomena, such as the big bang and others. These are the main reasons, why scientists struggle to find the theory of everything.

6.2 Einstein's dream

Einstein was probably the first scientist, who tried to find a theory of everything. He admired Maxwell for unifying electric and magnetic forces and thought, that unification should be the primary goal of modern physics.

After he had published his theory of general relativity, he began searching for posibilities to unite gravity with the only known at the time fundamental interaction, electromagnetism. But in the middle of his research, quantum mechanics was born. Einstein refused to accept it and it was his mistake. Exclusion of quantum mechanics and the lack of time are said to be the main reasons why Einstein failed. Some even say that Einstein was so detached from quantum mechanics, that he didn't even know about the weak nuclear force.

6.3 Quantum gravity

The next attempt of unification was and still is quantum gravity. Basically it is an attempt to include gravity to the Standard Model, since it is the only peace of the puzzle missing.

Everything in quantum mechanics is described by particles, thus a new, hypothetical particle was introduced - a graviton. It is considered to be a massless particle, just as a photon, which would cause gravity.

This theory is still being improved and supported, however it hasn't been proven yet.

7 String theory

And finally we will have a closer look to a completely different attempt in unifying the fundamental interactions - the string theory.

7.1 The idea

As the story goes, string theory was discovered almost by accident in the early 70's, while studying the strong nuclear force. Some equations in these studies seemed to describe some new kind of particles, which had internal structure. All particles in the standard model were points, that means they couldn't do very much apart from moving. However these objects, which were later called strings, could wiggle. And thus, string thoery was born, claiming that absolutely everything is made out of these extremely tiny one dimensional objects, strings. The way the strings wiggle determines the basic properties of the objects they form, that is mass, charge etc.

However, the theory comes with a price: in order for it to work, extra dimensions of space need to be included, not one, not two, but even six, making a total of nine dimensions of space. The first question, that comes to mind is where are the dimensions, we can see only three of them. String theorists explain this with an example. Imagine a wire which is very far from a viewer. To him, the wire seems to have only one dimension - it's length, but, say, an ant on the wire can see not only it's lenght, but also it's width, which is too small for the viewer to notice. So just as the wire's width, the six extra dimensions, postulated by string theory, are too small for us to notice.

7.2 A higher dimension

As if the six extra dimensions were not enough, string theorists introduced another dimension of space, which, with the dimension of time, makes up a total of 11 dimensions. This new dimension is different from the others - it is a higher dimension of space. Again, this can be explained with an example. Supppose we have a flat movie screen and a movie playing. The actors in the movie can move in all directions they want, however they can't get reach of the movie screen, it's as if they're stuck in a two dimensional space. But for the viewers, space is three dimensional - back and forht is like a higher dimension. String theorists say, that there actually is a higher dimension to our universe, just like to the movie screen in our example. Strings move and wiggle in this higher dimensional space.

Another quite recent discovery in string theory were membranes. It was shown, that strings can spread in space and if ithey have enough energy, even to an infinity. Such infinately streched strings are called membranes, or branes for short. What is more, it apears that there are many branes floating next to each other in the higher dimension, just like slices of bread.

Other strings normally have lose ends and many of them are tied to branes. And here's the fun part: these branes represent universes and the strings attached to them form the fundamental particles that are available in those universes (in the case of our universe, these are the particles described by the Standard Model). It's almost as if our universe is a huge movie screen, floating somewhere in a higher dimension. Also recall, that there are infinately many of such branes, which means that there are also infinately many universes, paralel to ours. These universes may be very different from ours and maybe some of them even contain living beings.

7.3 Explanation of gravity

Earier we said, that all the attached strings to a brane form the particles, which are described by the Standard Model. However, the Standard Model does not describe gravity, so where does it fit in ?

Theories indicate, that not all strings have lose ends, there appear to be strings with close ends, like rubber bands. The particles they form have no mass. For a long time they were disturbing string theorists, but later they realised, that these particles describe gravity, thus they were called gravitons. Since they are not attached to any particular brane, they can travel through the higher dimension, pervading paralel branes. This view explains why gravity is so weak, compared to other fundamental interactions: unlike attached strings, gravitons interact with branes only for a very short time. This also means, that gravity is the only interaction, which can be transmited accross paralel branes. So theoretically, if there was intelectual life on some other brane, the only way for us to communicate, would be gravity.

This new understanding of gravity can help solving the problems which arose while using the clasical theories. For example, the dark matter problem. Recall, that dark matter is a hypothetical substance introduced in order to explain the fact, that galaxies have less mass than they should, according to calculations. Either that, or there is an extra gravitational pull coming from somewhere. A string theoretic explains this with the fact, that gravity can be transmited through paralel universes. So if there is a heavy object near to a galaxy in a paralel universe, it would explain the extra gravitational pull. Thus no dark matter is needed in string theory.

Apart from other things, string theory can even explain the big bang. It is explained as a collision of two paralel branes. This collison would produce a huge amount of energy, which would result in an expansion on the brane.

However, black holes seem to be a bigger problem than even the big bang, because an explanation for them using string theory is still being developed.

8 Conclusions

Even though gravity was the first fundamental interaction to be discovered, it remains the least undestood. Currently it is the main interest of modern physics. Many efforts are being put in order to explain gravity, the most noticable being huge particle smashers, which are being used with hope to one day discover a graviton. Another interesting project is EINSTEIN@home. Any computer user can download a special program and join the project. The program uses the computer's resources to analyse various data collected from outer space, attempting to detect gravitational waves, predicted by Einstein in general relativity.

All in all, much work still has to be done in order to understand gravity and to finally fulfill Einstein's dream - create a unified theory of everything.

9 Bibliography

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