Greek (Polymaths) Mathematicians

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August 28, 2006

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1 From ancient to moderm times

It is well acknowledged that the ancient Greek mathematicians had a great influence in the establishment of foundations and development of mathematical sciences.

In the past century, influential scientistists like C. Caratheodory and C. Papakyriakopoulos re-established the presense of Greeks in the international mathematical community.

2 Thales (640/610 – 548/545)

Thales of Miletus, **the first man in history to whom specific mathematical discoveries have been attributed.**

Boyer: A History of Mathematics

Five Euclidean theorems have been explicitly attributed to Thales, and the testimony is that Thales successfully applied two theorems to the solution of practical problems.

Being asked what was very difficult, he answered, in a famous apophthegm, **To Know Thyself.** Asked what was very easy, he answered, **To give advice.** To the question, what/who is God?, he anwered, **That which has no beginning or no end. (The infinite!!)**

It is reported that Thales predicted an eclipse of the Sun in 585 BC.

3 Pythagoras of Samos (569BC - 475BC)

Greek philosopher, mathematician, and founder of the Pythagorean brotherhood that, although religious in nature, formulated principles that influenced the thought of Plato and Aristotle and contributed to the development of mathematics and Western rational philosophy.

Pythagorian Theorem and Irrationally.

$a^2 + b^2 = c^2$

Pythagoras is credited with the theory of the functional significance of numbers in the objective world and in music.

4 Heraclitus - the fire priest (540 BC - 480 BC)

This cosmos was not made by gods or men, but always was, and is, and ever shall be ever-living fire.

You could not step twice into the same river; for other waters are ever flowing on to you.

If you do not expect the unexpected, you will not find it, for it is trackless and unexplored.

The road up and the road down is one and the same.

Lambek, J., 1982, **The Influence of Heraclitus on Modern Mathematics,** Scientific Philosophy Today, J. Agassi and R.S. Cohen, eds., Dordrecht, Reidel, 111?"122.

5 Archimedes

What else did he do besides cry **Eureka?**

His research into the equilibriun of floating bodies is the first recorded work in hydrostatics and naval architecture.

π : Defined as the proportion "circumference: diameter".
Archimedes found the first decimal digits of π.

Archimedes used two regular 96-sided polygons, one circumscribing the circle, one inscribing. With the aid of some elegant geometry and arithmetic, he estimated the perimeters of these polygons and therefore the circumference of the circle between them.

6 Anaxagoras

Anaxagoras of Clazomenae (499BC - 428BC)

He was imprisoned for claiming that the Sun was not a god and that the Moon reflected the Sun's light.

There is no smallest among the small and no largest among the large, But always something still smaller and something still larger.

Anaxagoras was the he first mathematician who is on record as having attempted to **square the circle.** Plutarch, in his work On Exile which was written in the first century AD, says:

There is no place that can take away the happiness of a man, nor yet his virtue or wisdom.

Anaxagoras wrote on the squaring of the circle while in prison.

7 Zeno of Elea (490BC - 425BC)

Zeno of Elea was a Greek philosopher famous for posing so-called **paradoxes** which challenged mathematicians' view of the real world for many centuries.

Zeno's book of forty paradoxes was, according to Plato:

... a youthful effort, and it was stolen by someone, so that the author had no opportunity of considering whether to publish it or not. Its object was to defend the system of Parmenides by attacking the common conceptions of things.

Proclus also described the work and confirms that:

... Zeno elaborated forty different paradoxes following from the assumption of plurality and motion, all of them apparently based on the difficulties deriving from an analysis of the continuum.

Aristotle, in his work Physics, gives four of Zeno's arguments, The **Dichotomy, The Achilles, The Arrow, and The Stadium.** For the dichotomy, Aristotle describes Zeno's argument :

There is no motion because that which is moved must arrive at the middle of its course before it arrives at the end.

In order the traverse a line segment (from 0 to 1) it is necessary to reach its midpoint. To do this one must reach the 1/4 point, to do this one must reach the 1/8 point and so on ad infinitum. Hence motion can never begin. The argument here is not answered by the well known infinite sum

$$1/2 + 1/4 + 1/8 + \ldots = 1$$

On the one hand Zeno can argue that the sum $1/2+1/4+1/8+\ldots$ never actually reaches 1, but more perplexing to the human mind is the attempts to sum $1/2 + 1/4 + 1/8 + \ldots$ backwards. This argument makes us realise that we can never get started since we are trying to build up this infinite sum from the "wrong" end. Indeed this is a clever argument which still puzzles the human mind today.

8 Democritus of Abdera (460BC - 370BC)

Democritus is best known for his atomic theory but he was also an excellent geometer.

Quotations by Democritus:

I would rather discover one scientific fact than become King of Persia.

Everything existing in the Universe is the fruit of chance and necessity.

Nothing exists except atoms and empty space; everything else is opinion.

He wrote On numbers, On geometry, On tangencies, On mappings, On irrationals but none of these works survive.

... Democritus was the first to state the important propositions that the volume of a cone is one third of that of a cylinder having the same base and equal height, and that the volume of a pyramid is one third of that of a prism having the same base and equal height; that is to say, Democritus enunciated these propositions some fifty years or more before they were first scientifically proved by Eudoxus.

9 Plato (427BC - 347BC)

Plato is one of the most important Greek philosophers who founded the **Academy in Athens,** an institution devoted to research and instruction in philosophy and the sciences. His works on philosophy, politics and mathematics were very influencial and laid the foundations for Euclid's systematic approach to mathematics.

Plato's Academy flourished until 529 AD when it was closed down by the Christian Emperor Justinian who claimed it was a pagan establishment. **Having survived for 900 years the Plato's Academy of Athens is the longest surviving university known.**

Let no one ignorant of Mathematics enter here.
[Said to have been above the doorway of his Academy.]

Perhaps the best overview of Plato's views can be gained from examining what he thought that a proper course of education should consist. Here is his course of study:

... the exact sciences - **arithmetic, plane and solid geometry, astronomy, and harmonics** - would first be studied for ten years to familiarise the mind with relations that can only be apprehended by thought. Five years would then be given to the still severer study of **dialectic.** Dialectic is the art of conversation, of question and answer; and according to Plato, dialectical skill is the ability to pose and answer questions about the essences of things. **The dialectician replaces hypotheses with secure knowledge, and his aim is to ground all science, all knowledge, on some unhypothetical first principle.**

10 Euclid (325BC - 265BC)

Euclid's **Elements** is a monument of the most profound scientific achievements of the Greek Intellect.

In the Elements, Euclid discussed geometry as well as the theory of numbers and organized the known results of these fields on a systematic axiomatic basis.

Euclid's proof (by contradiction) of the existence of infinitely many prime numbers remains one of the most beautiful proofs in mathematics.

11 Aristotle (384BC - 322BC)

Aristotle was a Greek philosopher and though not primarily a mathematician he made important contributions by systemizing **deductive logic.** He wrote on physical subjects; some parts of his Analytica posteriora show an unusual grasp of the mathematical method. His philosophy had a long-lasting influence on the development of all Western philosophical theories.

The mathematical sciences particularly exhibit order, symmetry, and limitation; and these are the greatest forms of the beautiful.

Metaphysica, 3-1078b.

Those who educate children well are more to be honored than parents, for these only gave life, those the art of living well.

H. G. Apostle, Aristotle's philosophy of mathematics (Chicago, 1952).

12 Eratosthenes of Cyrene (276BC - 194BC)

Eratosthenes was a Greek mathematician who is famous for his work on prime numbers (**Sieve of Eratosthenes**) and for measuring the diameter of the earth.

Eratosthenes also measured the **distance to the sun** as 804,000,000 stadia and the distance to the Moon as 780,000 stadia. He computed these distances using data obtained during lunar eclipses. Ptolemy tells us that Eratosthenes measured the tilt of the Earth's axis with great accuracy obtaining the value of 11/83 of 180degrees, namely 23degrees 51' 15".

Eratosthenes made many other major contributions to the progress of science. He worked out a **calendar that included leap years,** and he laid the foundations of a systematic chronography of the world when he tried to give the dates of literary and political events from the time of the siege of Troy. He is also said to have compiled a **star catalogue** containing 675 stars.

13 Apollonius of Perga (262BC - 190BC)

Apollonius was known as "The Great Geometer". His works had a very great influence on the development of mathematics and his famous 8 books **Conics** introduced the terms **parabola, ellipse and hyperbola.**

A conic or conic section is one of the three curves: a parabola, hyperbola or ellipse which one can obtain by intersecting a plane with a (double sided) cone.

Other books by Apollonius:
Cutting of a ratio (in two books)
Cutting an area (in two books)
On determinate section (in two books)
Tangencies (in two books)
Plane loci (in two books)
Quick Delivery
On verging constructions (in two books).

For example, in Tangencies, Apollonius shows how to **construct the circle which is tangent to three given circles.** More generally he shows how to construct the circle which is tangent to any three objects, where the objects are points or lines or circles.

In the book Quick Delivery, Apollonius obtained an **approximation** for π better than the

$223/71 < \pi < 22/7$

known to Archimedes.

Problem: Prove that the locus of points whose distance from a afixed point A is a multiple of its distance from another fixed point B is a circle (one of Apollonius circles).

14 Diophantus

Diophantine Equations

About all we know of Diophantus's life is his epitaph:

Diophantus spent one-sixth of his life in childhood, one-twelfth in youth, and another one-seventh in batchelorhood. A son was born five years after his marriage and died four years before his father at half his father's final age.

It is easy to compute that Diophantus lived to be 84 years old by solvingthe first order equation:

$$\frac{x}{6} + \frac{x}{12} + \frac{x}{7} + 5 + \frac{x}{2} + 4 = x$$

What about the integral rational solutions of equations like $Ey^2 = Ax^3 + Bx^2 + Cx + D$? In 1923 Mordel proved that such an equation has only a finite number of integral rational solutions.

15 From Epimenides to Gödel

Epimenides' Paradox:
"All Cretans are liars"

Gödel's Incompleteness Theorem:
"All consistent axiomatic formulations of number theory include undecidable propositions"

16 Heron of Alexandria (10 - 75)

Heron or Hero of Alexandria was an important geometer and worker in mechanics who invented many machines ncluding a **steam turbine.**

His best known mathematical work is the **formula for the area of a triangle** in terms of the lengths of its sides:

If A is the area of a triangle with sides a, b and c and $s = (a + b + c)/2$ then $A^2 = s(s - a)(s - b)(s - c)$.

17 Hypatia of Alexandria (370 - 415)

Hypatia was the first woman to make a substantial contribution to the development of mathematics. She was killed by a fanatical Christian sect.

This event marked the beginning of the decline of Alexandria as a major center of ancient learning.

M Dzielska, Hypatia of Alexandria (Harvard, 1995).

18 Kallinikos (7th-century)

In order to maintain all the lands that its precessor once controlled, Byzantium had to put all its forces in one direction : to **defend.** Roman army's only goal was to **conquer.** On the contrary, Byzantine army was in a decisive position. Barbarians from all over saw a great opportunity in the Byzantium, due to its richness and fertility.

Greek, or liquid fire: The Byzantines' most important weapon, which played a decisive role in all naval operations. It consisted of a developed recipe of older chemical compounds attributed to Kallinikos, a Greek engineer from Helioupolis in Syria. Liquid fire contained sulphur, nitrogen, and naphtha (petrol) and other substances and put fear into the enemy fleet, since it burnt even on water.

In 678 The Byzantines utterly destroyed a Muslim fleet (it is believed over 30,000 men were lost) and also in 717-718, when Caliph Suleiman attacked Constantinople. Most of the Muslim fleet was once again destroyed by Greek Fire, and the Caliph was ultimately forced to flee.

19 Constantin Caratheodory (1873 - 1950)

Caratheodory made important contributions to the theory of real functions, to the calculus of variations, and to the theory of point-set measure. His published works include
1918: "Treatise on Real Functions",
1932: "Conformal Representation",
1937: "Geometrical Optics",
1939: "Real Functions", and
1950: "Function Theory" (2 volumes).

Caratheodory contributed important findings to the theory of the functions of several variables and simplified the proof of the main theorem of conformal representation of simply connected regions on the unit-radius circle. His investigations of the geometrical-set theoretic properties of boundaries resulted in his theory of boundary correspondence. He also contributed to thermodynamics and helped to develop Einstein's special theory of relativity.

International Conference on Advances in Convex Analysis and Global Optimization
(Honoring the memory of C. Caratheodory)
June 5-9, 2000, Pythagorion, Samos, Greece

20 Christos Papakyriakopoulos (1914-1976)

The perfidious lemma of Dehn,
Drove many a man insane,
But Christos Popakiriakopolous
solved it without any pain.

The **Oswald Veblen Prize in Geometry** was first awarded in 1964 to C. D. Papakyriakopoulos for his papers,
On Solid Tori, Annals of Mathematics, Series 2, volume 66 (1957), pp. 1-26, and
On Dehn's lemma and the asphericity of knots, Proceedings of the National Academy of Sciences, volume 43 (1957), pp. 169-172.

International Conference on Mathematical Analysis and its Applications
(Honoring the memory of C. Papakyriakopoulos)
August 24-27, 2000 National Technical University of Athens, Greece

“Seekers after gold dig up much earth and find little”

“The lord whose oracle is at Delphi neither speaks nor conceals, but gives signs”

- HERACLITUS