Approximation of Mean Time Between Failure When a System has Periodic Maintenance

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***Abstract*—This paper describes a simple technique for estimating the mean time between failure (MTBF) of a system that has periodic maintenance at regular intervals. This type of maintenance is typically found in high reliability, mission-oriented applications where it is convenient to perform maintenance after the completion of the mission. This approximation technique can greatly simplify the MTBF analysis for large systems. The motivation for this analysis was to understand the nature of the error in the approximation and to develop a means for quantifying that error. This paper provides the derivation of the equations that bound the error that can result when using this approximation method. It shows that, for most applications, the MTBF calculations can be greatly simplified with only a very small sacrifice in accuracy.**

***Index Terms*—Mean time between failure (MTBF), periodic maintenance, reliability modeling.**

ACRONYMS[1]

MTBF mean time between failure

NOTATION

MTBF_A	approximate MTBF
rel.err	relative error
$R_A(t)$	approximate reliability function assuming periodic maintenance every T (hours)
$R_T(t)$	exact reliability function assuming periodic maintenance every T (hours)
t	system operational time
T	periodic maintenance interval (usually in hours).

I. INTRODUCTION

DURING the system-design process, a reliability model is typically developed to describe the system's nominal reliability. It is used to predict reliability performance for comparison against design-goals and system-requirements. A common metric used to assess the reliability of a repairable system is MTBF.

The effect of maintenance on the system MTBF must be considered when a system has redundant components. For example, a system that follows a maintenance policy of repairing failed redundant-subsystem-components prior to a critical system-failure has a higher MTBF than one that waits until a critical failure to repair all failed components. A common maintenance concept is to perform periodic maintenance at regular intervals to repair any failed redundant components before a critical system-failure occurs. This type of maintenance is common for high reliability, mission-oriented, applications where it is convenient to perform maintenance after the completion of the mission.

A mathematical equation that describes the MTBF of a system that has periodic maintenance performed at regular intervals is in reliability handbooks [1], [2]; however, its solution can be cumbersome for highly complex reliability models. Therefore, a much simpler approximation can be used to save computation time. This paper describes this technique and provides an error analysis that shows that, for most applications, the MTBF calculations can be greatly simplified without an appreciable sacrifice in accuracy.

Section II presents some background information about the mathematical theory of computing MTBF. Section III explains the MTBF approximation method. Section IV analyzes the error, and derives the equations that quantify the maximum fractional error.

II. BACKGROUND

The MTBF for a system that has periodic maintenance at a regular interval can be described by [1], [2]:

$$\text{MTBF} = \frac{\int_0^T R_T(t)\,dt}{1 - R_T(t)}. \tag{1}$$

The complexity of solving (1) is found in the complexity of the integration. It can become particularly cumbersome for a system with many components because the reliability function can have 100s or 1000s of factors. While there are software tools that perform integration, quite often these tools are unavailable to the reliability engineer. The approximation method provides an effective way for those without the option of using a mathematical software tool to find a solution, because it can be easily computed using spreadsheet software. For those that find an exact solution to (1), the approximation method can provide a quick check to ensure that the exact method was solved properly.

III. MTBF APPROXIMATION METHOD

The fundamental basis for the approximation method is that the reliability function for a redundant system with periodic

Manuscript received March 29, 1999; revised February 28, 2001.
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Publisher Item Identifier S 0018-9529(02)06091-8.

[1]The singular and plural of an acronym are always spelled the same.

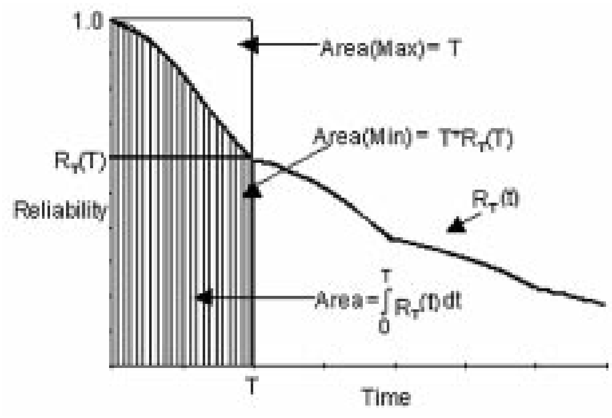

Fig. 1. Analysis of error.

maintenance can be approximated by an exponential time-between-failure distribution with constant failure rate, with reasonable error. Therefore, [3]:

$$R_A(t) = \exp\left(-\frac{t}{\mathrm{MTBF}_A}\right) \tag{2}$$

$$\mathrm{MTBF}_A = -\frac{1}{\log(R_A(t))}. \tag{3}$$

Another fundamental of the approximation method is that the reliability at T equals the approximate reliability at T:

$$R_A(T) = R_T(T). \tag{4}$$

Apply (3) to (4):

$$\mathrm{MTBF}_A = -\frac{T}{\log(R_T(T))}. \tag{5}$$

The value of T is known; thus $R_T(T)$ can be found from the system reliability model using basic reliability equations (e.g., series, parallel, k-out-of-n).

IV. Error Evaluation

Given that this method is only an approximation, it is important to know the rel.err between the exact and approximate MTBF's. The source of error is the difference between areas under $R_T(t)$ and $R_A(t)$, over all time. It is not possible to find the exact error without finding the exact MTBF and, in doing so, defeat the purpose (saving computation time) of using the approximation. However, it is possible to derive equations that will bound the rel.err as a function of $R_T(T)$, which is known.

The derivation begins with the rel.err equation:

$$\text{rel.err} = \left[1 - \frac{\mathrm{MTBF}_A}{\mathrm{MTBF}}\right]. \tag{6}$$

Apply (1) and (5) to (6):

$$\text{rel.err} = \left[1 + \frac{T \cdot (1 - R_T(T))}{\log(R_T(T)) \cdot \int_0^T R_T(t)\,dt}\right]. \tag{7}$$

The integral of $R_T(t)$ is the unknown factor in (7); the striped area under the notional $R_T(t)$ curve in Fig. 1 graphically represents this integral.

TABLE I
Maximum rel.err Using the Approximation Method

$R_T(T)$	un:rel.err	ov:rel.err
0.9999	0.00005	0.00005
0.999	0.00050	0.00050
0.99	0.00500	0.00500
0.9	0.051	0.055
0.8	0.104	0.120
0.7	0.159	0.202
0.6	0.217	0.305

Fig. 1 shows that this striped area has an upper bound defined by Area(Max) and a lower bound defined by Area(Min). Therefore,

$$T \cdot R(T) \le \int_0^T R_T(t)\,dt \le T. \tag{8}$$

Equation (8) can be used to bound the error. There are two types of error: the approximation can: 1) underestimate MTBF or 2) overestimate MTBF. The relative error for each case is found by combining (7) and (8):

$$\text{un: rel.err} \le \left[1 + \frac{1 - R_T(T)}{\log(R_T(T))}\right] \tag{9}$$

$$\text{ov: rel.err} \le \left|\left[1 + \frac{1 - R_T(T)}{R_T \cdot \log(R_T(T))}\right]\right| \tag{10}$$

un: rel.err $\equiv$ underestimating rel.err
ov: rel.err $\equiv$ overestimating rel.err.

These rel.err bounds are a function only of $R_T(T)$. Therefore, one can easily tabulate (Table I) the bounds for rel.error vs. $R_T(T)$.

Table I shows that the maximum error when estimating the MTBF using the approximation method is very small when $R_T(T)$ is close to 1. Fortunately, it is likely that the periodic maintenance schedule will be selected based upon maintaining a high probability (usually greater than 0.9) of completing the task/mission. Therefore, the maximum rel.err for the most typical applications (e.g., high mission reliability) is extremely low. High values of $R_T(T)$ are typical of system architectures using equipment redundancy.

References

[1] W. H. Von Alven, *Reliability Engineering*. Englewood Cliffs, NJ: Prentice-Hall, 1964, pp. 262–264.
[2] *Reliability Toolkit: Commercial Practices Edition*: Rome Laboratory and Reliability Analysis Center, p. 159.
[3] *Electronic Reliability Design Handbook*, U.S. Mil-Hdbk-338B, 1984, pp. 5–7.

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