Journal of Educational and Behavioral Statistics
Winter 2005, Vol. 30, No. 4, pp. 353–368

No Humble Pie: The Origins and Usage of a Statistical Chart

Ian Spence
University of Toronto

William Playfair's pie chart is more than 200 years old and yet its intellectual origins remain obscure. The inspiration likely derived from the logic diagrams of Llull, Bruno, Leibniz, and Euler, which were familiar to William because of the instruction of his mathematician brother John. The pie chart is broadly popular but—despite its common appeal—most experts have not been seduced, and the academy has advised avoidance; nonetheless, the masses have chosen to ignore this advice. This commentary discusses the origins of the pie chart and the appropriate uses of the form.

Keywords: *Bruno, circle chart, Euler, Leibniz, Llull, logic diagrams, pie chart, Playfair*

1. Introduction

The pie chart is more than two centuries old. The diagram first appeared (Playfair, 1801) as an element of two larger graphical displays (see Figure 1 for one instance) in *The Statistical Breviary,* whose charts portrayed the areas, populations, and revenues of European states. William Playfair had previously devised the bar chart and was first to advocate and popularize the use of the line graph to display time series in statistics (Playfair, 1786). The pie chart was his last major graphical invention. Playfair was an accomplished and talented adapter of the ideas of others, and although we may be confident that we know his sources of inspiration in the case of the bar chart and line graph, the intellectual motivations for the pie chart and its parent, the circle chart, remain obscure.

In *The Statistical Breviary,* Playfair sought to display statistical data for European countries around the turn of the 19th century. He did so in graphical form because he believed that "making an appeal to the eye when proportion and magnitude are concerned, is the best and readiest method of conveying a distinct idea" (p. 4). The first chart in the volume (facing p. 13) depicts the countries as they were before the French Revolution of 1789, and the second chart (facing p. 49; and part of our Figure 1) shows how these nations had changed by 1801. Circles of various sizes stand for the total land areas of the countries. Russia, being the largest, is represented by the

This work was partially supported by the Natural Sciences and Engineering Research Council of Canada. The author is grateful for comments on an earlier version of this article (Spence, 2000a) by members of *Les Chevaliers des Albums de Statistique Graphique.*

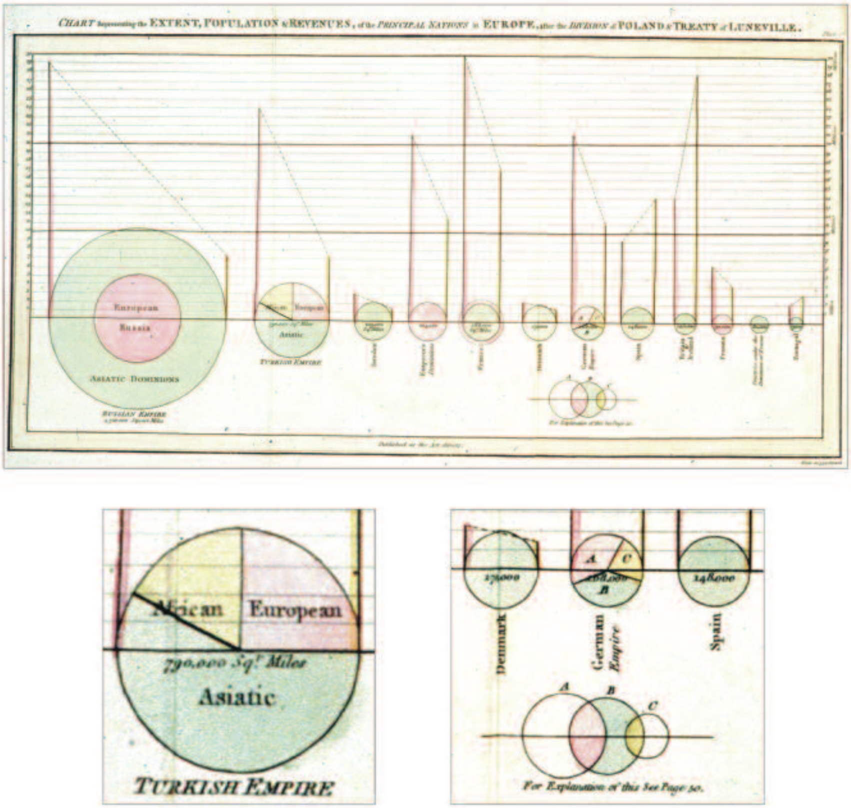

FIGURE 1. *Chart 2, p. 49 in* The Statistical Breviary *displays statistical data for European countries in 1801, the year of the Luneville peace treaty between France and Austria. The area of each circle is proportional to the land area. Two sections of the chart are shown at higher magnification. The lower left panel shows a pie chart for the Turkish Empire and the lower right panel shows a pie chart and a Venn-like diagram for the German Empire.*

circle of greatest diameter, while states such as Portugal are associated with very small circles, indeed. Almost all of the circles also have the numerical values of the land areas in square miles, inscribed just below the horizontal diameter of each circle. In addition to showing the land masses, populations, and revenues of European states, the charts also indicated whether individual countries were maritime powers by coloring the areas green, while the areas of nonmaritime powers were stained red. While the land masses were indicated by the areas of the circles, the sizes of the populations were represented by the vertical red lines on the left of each circle and the tax revenues of each country were shown by the vertical yellow lines on the right. The dotted lines that join the tops of the vertical lines provide an indication of the tax burden on the populations. As Funkhouser (1937) has observed, "the slope of the line is obviously dependent on the diameter of the circle," and so the slope cannot be used as an accurate index of the tax burden. However, based on his comments in the text, it is likely that Playfair merely intended the reader to observe whether the slope was positive or negative. After the original copperplate impressions were made, Playfair himself added the coloring by hand, laboriously, for each copy of the book. Playfair likely also engraved the plates (Spence, 2000b).

To illustrate the political subdivisions in some countries, Playfair used a variety of devices. He divided the Russian empire into European and Asiatic Dominions with the former represented by the inner circle and the latter by the surrounding annulus. This diagram does not indicate inclusion but uses the separate areas to represent the parts of the empire in each continent. The annulus is stained green indicating that the Asiatic dominions were powerful by sea; whereas, the red of the European dominions indicates a land power. The next largest empire, belonging to the Turks, posed more of a problem for Playfair. Here he wished to display the land areas of the Asiatic, European, and African portions of the Turkish Empire. Three concentric circles could have been drawn but a visual comparison of the areas would have been even more difficult than that in the case of the Russian Empire, which is problematic enough. Because Playfair's purpose in creating the circle diagram was to facilitate comparison and to make the data more memorable, the use of concentric circles has severe limitations. Playfair does not discuss these problems in the text, although it is obvious that he must have had to face them.

In the case of the Turkish Empire, he divided the circle into three sectors, each proportional to the Asiatic, European, and African land masses. He tinted the Asiatic green, signifying a maritime power, the European red to denote a land power, and he used yellow to color the African sector. Playfair gave no rationale for his use of these colors. It is not clear whether he was aware that mixing green and red lights produces yellow, and thus used this color to suggest a balance between maritime and land power, or whether the yellow is simply an arbitrary convenience. Whatever his intent with the particular tints, the result is the first pie chart to display empirical proportions and to differentiate the component parts by color (see Tufte, 1983, p. 44, for a color reproduction of Playfair's Chart 1).

The second chart of the *Breviary* (p. 49; our Figure 1) is similar to the first but shows fewer states in 1801, the year of the Luneville peace treaty between France and Austria. The gains made by Napoleon in the course of the European war had forced a number of political realignments and changes. In this chart, Playfair illustrates the subdivision of the German Empire into those parts belonging to Austria and Prussia as well as the remaining territory under the control of the German princes. Two modes of representation are used: the overall area is divided using a pie, as in the case of the Turkish Empire, but, in addition, the joint ownership is indicated by another figure composed of three overlapping circles. Playfair has created what we would now call a Venn diagram. The leftmost circle represents Austrian interests, the rightmost the Prussian dominions, and the central circle—which has the same area as the pie above, represents the German Empire. Thus, the red intersection shows that portion ruled by Austria, the yellow denotes the area under Prussian rule, and the remaining green shows the province of the German princes. The areas in the pie and the intersecting circles diagram do not match very well; Playfair's lack of attention to accuracy was, however, not uncharacteristic.

Thus, in two charts in a single volume, Playfair introduced three new forms of statistical graph: the circle diagram, the pie chart, and the Venn-like diagram, which was used to show joint properties. As in the case of the line and bar charts that he had introduced 15 years previously, his basic designs are sound and have scarcely been improved upon to this day (Biderman, 1990). The areas of the circles are used to represent varying quantities, and while there may be good psychological reasons to question the appropriateness of this particular geometrical form (Cleveland & McGill, 1984; Spence, 1990), the practice of using circles in this fashion persists to the present day, particularly in statistical maps. Playfair used the included angle of the segments to denote proportion, and he used color and labeling to differentiate the segments that make up the whole. The use of Venn-like diagrams to portray statistical quantities is perhaps less common, both today and two centuries ago, but the device is not unfamiliar.

2. Playfair's Inspiration

We have no doubt as to the intellectual origins of Playfair's other graphical inventions, the bar chart and the application of the line graph to statistics. The bar chart was inspired (Playfair, 1801, p. 15) by the chronological diagram (Priestley, 1765). Playfair was a frequent user of timeline charts, which represented the lifespans of individuals by solid black bars staggered in relation to a time scale along the abscissa, and he was also distantly related to one of the principal users of the form (Playfair, J., 1784). In the case of the line graph, he credits his older brother with the idea: John made young William keep daily records of temperature and then chart the data in the fashion of contemporary natural philosophers like himself (Playfair, 1805a, p. xvi). However, the pie remains a mystery; Playfair gives us no indication of his inspiration, and he devotes no discussion to the form. This is in marked contrast to his descriptions of the line graph and bar chart, both of which are accorded full explication in several works. What is the reason for his neglect of the pie, which he did not bother to name? Did he attach no importance to the new design? Today, one might ask whether it is even necessary to look for intellectual precursors: is not the pie chart such a natural and self-evident construction that anyone could have invented it, given the need? By the same token, however, the line, bar, and circle charts might also be said to be self-evident, but the simple fact is that before Playfair nobody else had published charts using lines, bars, circles, and pies to display statistical data. Familiarity has dulled our sense of the importance of Playfair's diagrams and it is easy to underestimate the ingenuity that was required to invent them, even if the sources of inspiration might have been well known.

It is likely that pies, circles, and intersecting circles were such simple and familiar forms to Playfair that he did not think comment necessary. After the death of his father, when William was 12 years old, the eldest son John took on the task of raising and educating his siblings. John was to become one of Scotland's most celebrated mathematicians, as Professor of Mathematics at the University of Edinburgh, and also a world-renowned scientist in geology and physics. Conversations and puzzles involving mathematics are certain to have been a normal part of the Playfair family home life in the small, relatively isolated manse in the country village of Liff, a few miles west of the city of Dundee. Did John pass on mathematical ideas that William used later, even if unconsciously? There are many examples in William Playfair's career as an author and in his trade as an engineer where he adapted the work of others. This copying was often ingenious and creative, as in the case of the bar chart where Playfair extended the original idea from charting lifespans to plotting other numerical data. Perhaps by looking more closely at his brother John, whose influence on William was great, we may be able to guess where William derived the inspiration for the diagrams in his *Statistical Breviary.*

A clue may be found in Playfair's use of intersecting and included circles. Almost a century later, in 1880, Venn was to use very similar diagrams in his work on Boole's system of logic. Contrary to popular myth, logic diagrams of the sort used by Venn were not original with him. Euler had used them for exactly the same purpose in 1768, and Leibniz had employed them in his work on logical propositions in 1666. In turn, both mathematicians had been inspired by Ramón Llull (Raimundus Lullus, 1232–1316), the Catalan mystic, astronomer, and Christian philosopher, and also the ill-fated Dominican friar Giordano Bruno (1548–1600), who was burned at the stake as a heretic in Rome's Campo de'Fiori. Bruno's work in philosophy and astronomy was far-reaching and he rediscovered, revived, and built on many of Llull's ideas. John Playfair, one of the foremost mathematicians of the Enlightenment, was intimately familiar with the work of Euler and Leibniz and would, in consequence, certainly have known of the work of Llull and Bruno. And even if he had not been a mathematician, John was an ordained minister in the Church of Scotland and thus it is inconceivable that he was unaware of the writings of important religious thinkers like Llull and Bruno.

In medieval times, Ramón Llull introduced the beginnings of logic diagrams in his *Ars Magna* (1305–08); he believed that by exploring all possible combinations of certain basic principles, or categories, we could obtain knowledge of all things. He represented concepts by circles and suggested affinities between concepts by overlapping the circles, although he did not develop the modern notion of intersection (Baron, 1969). Llull mechanized part of his scheme by arranging the categories on disks of varying diameters that could be rotated to produce a large number of different combinations of concepts. Llull's combinatorial system had little effect on others until the 15th century when Giordano Bruno attempted to develop his own philosophy by expanding on Llull's ideas. Like Llull, Bruno used circular diagrams where the whole was represented by a circle and the parts by segments—Llull's segments were generally regular but Bruno gives us at least one example where the slices are of different sizes (Figure 2). Bruno also used circular wheels but with a greater number of sectors than Llull. Both philosophers were attempting to create a universal language that would capture the complexity of the human mind, and its

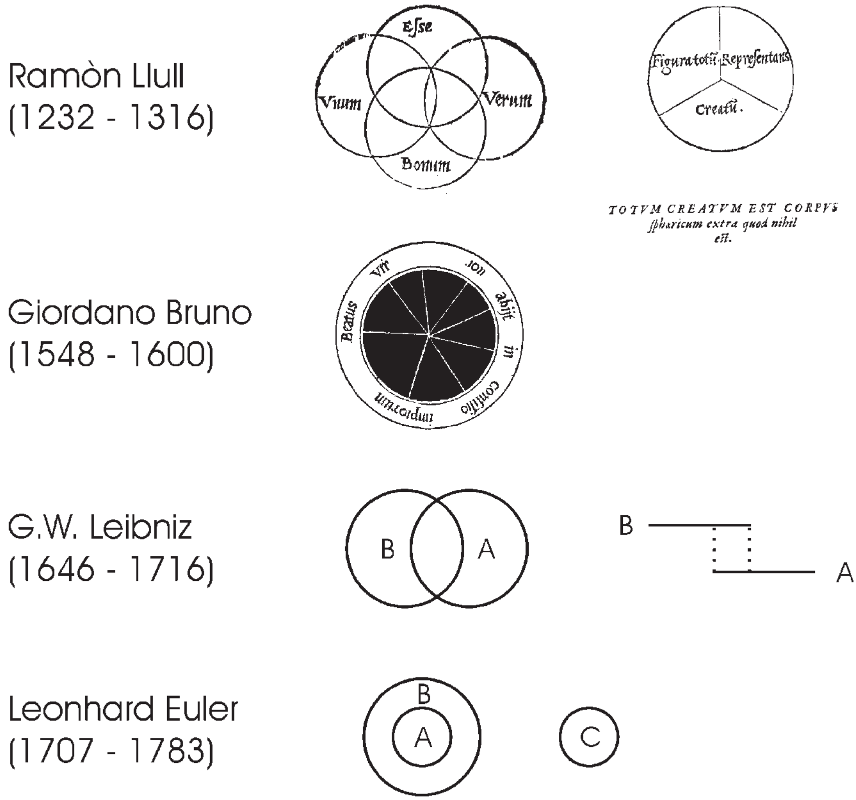

FIGURE 2. *Representative examples of logic diagrams from Llull, Bruno, Leibniz, and Euler. The words* Esse, Unum, Verum*, and* Bonum *in conjunction with the Venn-like diagram of Llull may be interpreted to mean, nothing* Exists *which does not possess* Unity, Truth *and* Goodness. *Note the use of unequal-sized segments in Bruno's diagram and the linear diagram that Leibniz favored over intersecting circles.*

relation to God and the world, by combining a much smaller number of fundamental concepts. It is important to note that none of their diagrams displayed empirical data—they were merely devices for partitioning the universe of discourse. Leibniz was strongly influenced by Bruno and Llull and frequently referred to the latter in his writings. Leibniz was searching for a universal notation and way of developing knowledge inspired by the methods of mathematics; he absorbed Llull and Bruno, adapted their ideas, and proposed the beginnings of an objective logic. In his *Dissertatio de Arte Combinatoria* (1666, p. 168) Leibniz clearly says that his combinatorial art is an elaboration of the ideas of Llull. In a letter to Gabriel Wagner, Leibniz (1696) is even more explicit: he says that it was Llull who brought him to the study of combinations. Inspired by the work of the medievalists, Leibniz was the first mathematician to devote serious attention to the analysis of logical propositions by the use of diagrams. He explored various methods of representing Aristotelian syllogisms by means of geometric figures. These included Venn-like diagrams as well as his own linear versions, which he considered superior. It was, however, Euler who later popularized the use of circle diagrams, although he was clear to point out, in 1768, that the kind of shape was unimportant (Euler, 1859).

John Playfair was intimately acquainted with the work of Leibniz and Euler. One indication of his deep and expert knowledge is his supplement to the fourth edition of the *Encyclopedia Brittanica* entitled, "Progress of mathematical and physical science since the revival of letters in Europe." This dissertation was originally intended as a historical sketch of science for a revision of the *Encyclopedia Brittanica.* John Playfair completed the first of three sections, which treated both Newton and Leibniz, but died as he was about to start the second section, which was to have focused on Euler and d'Alembert. Many think that Playfair's essay is the best short general history of science written in the first half of the 19th century. His appreciation and evaluation of the genius of Leibniz and Newton was universally praised for its insights and clarity of writing. However, as much as he admired Leibniz, John Playfair may have revered Euler even more. Playfair wrote "more is due to Euler than to any other individual" (Playfair, J. G., 1822, p. 267) in regard to Euler's work on higher geometry and he dubbed him "this incomparable man" (Playfair, J. G., 1822, p. 268). Because of his high regard for their ideas, John Playfair could not have failed to make William aware of the work of Leibniz and Euler as he instructed his brother in mathematics, after the early death of their father. The younger brother would have learned much about the diagrams that Leibniz and Euler had used in their efforts to map the structure of logical syllogisms. Indeed, William would have absorbed these ideas so thoroughly that when he came to use circles, included circles, and intersecting circles in his own charts, he would likely have not given any thought to the original inspiration.

3. The Adoption of Circular Charts

By and large Playfair was ignored in Great Britain, and his pies did not catch on. The reasons probably have as much to do with Playfair's dubious personal reputation as anything else. He had been involved in various failed—and sometimes fraudulent—business ventures in London and Paris since the early 1780s. He had run up large debts in London as a young man and had been accused of appropriating the ideas of others in patents that he had obtained (Spence, 2004). He was involved in several financial scandals in revolutionary Paris, and was prosecuted on more than one occasion. His cavalier behavior in financial matters continued on his return to London, where he was embroiled in a dispute with the Bank of England which almost led to his arrest. Curiously, having escaped prosecution in the larger affair, he was subsequently convicted at the Court of Kings Bench for a rather minor fraud. In addition to shady business practices, Playfair was also frequently critical of public figures, often in the form of published manifestos. This behavior invariably attracted official disapproval. Overall, his behavior and reputation (Spence & Wainer, 1997) was the antithesis of that of his brother John, the respected Professor at the University of Edinburgh and minister of the Church of Scotland. William

Playfair's position on the fringes of society cannot have helped acceptance of his charts because reputation was of considerable importance in the late 18th and early 19th centuries, particularly in academic circles. The work of a perceived rogue would have been discounted or would have received little attention.

Playfair's writings fared better in Germany (Alexander von Humboldt, a close friend of brother John, took up his diagrams) and also in France, where his publications attracted particular notice in Paris. Not all the attention was favorable (see Peuchet, 1805), but by midcentury we find Charles-Joseph Minard making significant use of pies in his wonderful *cartes figuratives.* Minard may be the first person to have publicly embraced the pie, which he incorporated into at least two of his statistical maps (see Figure 3 for an example from 1858; and Palsky, 1996, for other examples). The intellectual descendents of Minard in the Ministry of Bridges, Roads, Railways, and Canals (Ponts et Chaussées) also produced several fine examples, as did Bertillon (1891) in his Atlas of the City of Paris (see Figure 4).

Like us, the French employ a gastronomical metaphor when they refer to Playfair's pie chart, but they have preferred instead to invoke the name of the wonderful round soft cheese from Normandy—the camembert. When I spent 4 months in Paris a few years ago, a friend invited my wife and me to lunch with her elderly father who lives in Rouen, Normandy, about an hour North of Paris. Her father inquired—coincidentally during the cheese course—what work I was doing in Paris; I replied that I was researching the activities of a Scot, William Playfair, during the revolutionary period. I told him that Playfair had invented several statistical graphs, including the pie chart, which I referred to, in French, as "le camembert." After a stunned silence of perhaps a couple of seconds, the distinguished elderly gentleman looked me in the eye and exclaimed, "Mon Dieu! Notre camembert?"

4. Empirical Evaluation of the Pie Chart and Other Forms

Belatedly, statistical graphs started to catch on in the United Kingdom. William Stanley Jevons may have been the earliest adopter, basing his own economic atlas on Playfair's example. Indeed, Keynes (1936, 1938) has said that Jevons was making charts in the fashion of Playfair as early as the 1860s. Jevons in turn influenced Karl Pearson who was mainly responsible for making graphics respectable in British statistical circles. However, it was not long before the pie was the subject of criticism. In the first widely circulated textbook on statistical graphs, Brinton (1914) speaks disapprovingly: "the circle with sectors is not a desirable form of presentation." Criticisms like this provoked the first psychological experiments on graphs, where subjects were required to estimate quantities represented in graphic form. Eells (1926) showed that subjects could estimate the size of a proportion more quickly and accurately when the data were in pie chart rather than bar chart form. His advocacy quickly produced critics who conducted their own experiments (Croxton, 1927; Croxton & Stein, 1932; Croxton & Stryker, 1927; von Huhn, 1927), but these early empirical studies were of widely varying quality, were generally inconclusive, and did not convincingly reverse Eells's findings. Nonetheless, by the middle of the 20th century, many statisticians held strong opinions against the pie, although

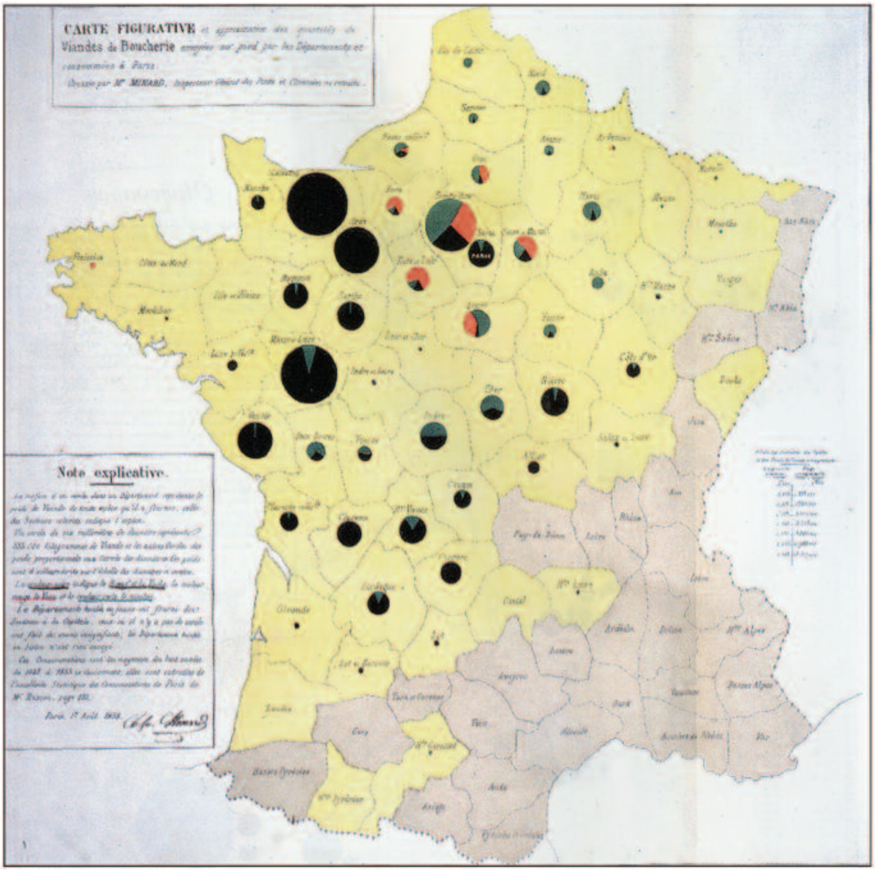

FIGURE 3. Carte figurative *showing the use of pies on a map (Minard, 1858). This chart is to be found along with other fine reproductions in Palsky (1996).*

a number of later studies had demonstrated that the pie was not inferior to the divided bar when users had to estimate or compare simple proportions (Culbertson & Powers, 1959; Peterson & Schramm, 1955). However, in a widely read and influential review, Macdonald-Ross (1977) concluded that the bar chart was superior to the pie chart based on his reading of this problematic literature.

Cleveland and McGill (1984) were the first to develop a theory of graphical perception based on the observation that certain perceptual judgments were made more accurately than others and that, as a consequence, graphs that incorporated these elements would be more-or-less successful depending on which elements they employed. Because judgments of angle are generally less accurate than judgments of length, it would seem to follow that the pie chart cannot be as effective as the bar chart for estimates of proportion or for the comparison of proportions.

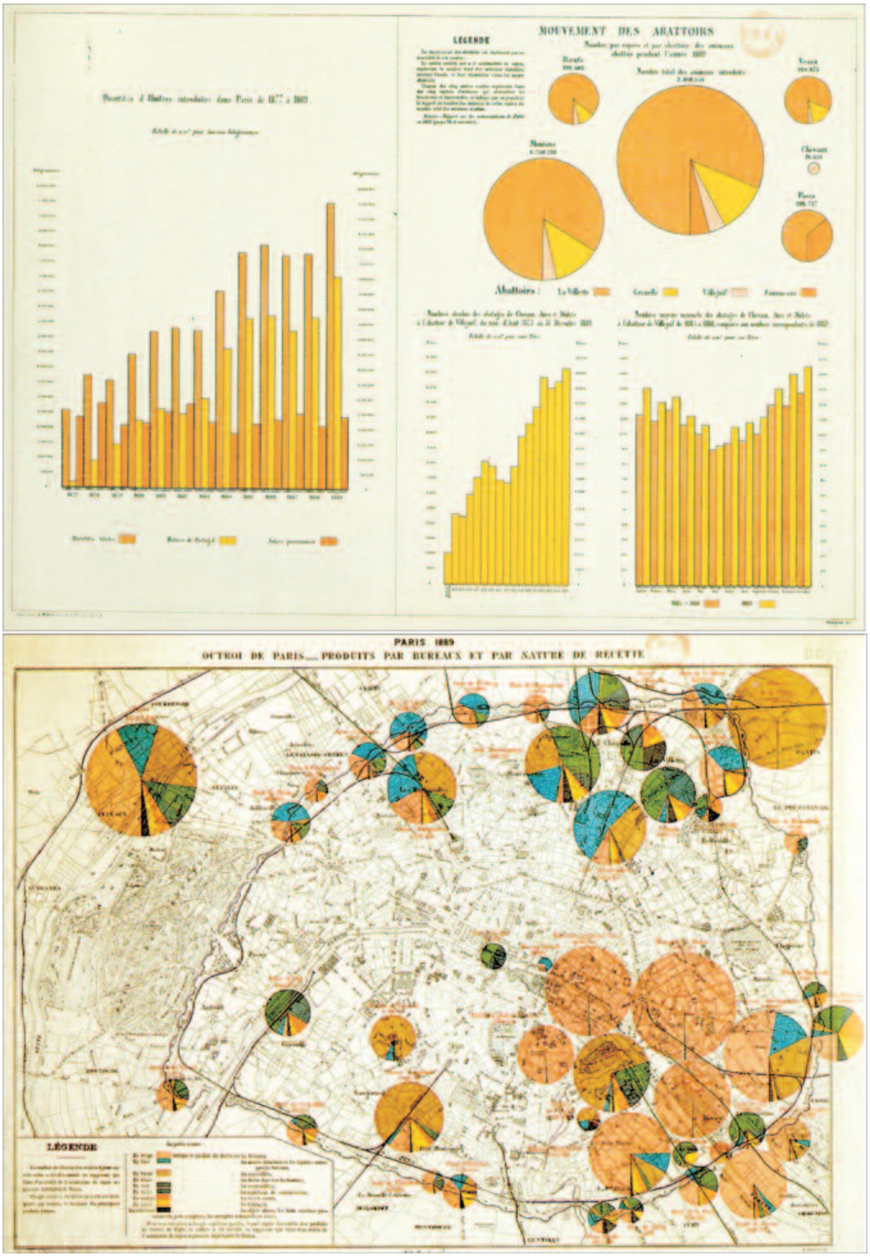

FIGURE 4. *Examples of the use of pies from Bertillon (1891). This reproduction may be found with other charts of Bertillon in Picon and Robert (1999).*

However, Simkin and Hastie (1987) pointed out that the task used by Cleveland and McGill was not a fair comparison of the two forms. In the case of aligned line segments, participants had to judge what proportion one line was of the other, and they also had to perform a similar task with two angles. The former task is consistent with the way people estimate proportion in a bar chart but the latter does not reflect how proportion is estimated in a pie chart where the key judgment is the comparison of the segment angle to the whole 360 degrees. Simkin and Hastie were able to confirm this observation by experiment; they first replicated Cleveland's results and then showed that when the task was to estimate what proportion a bar or a segment was of the whole, the pie chart and bar charts were equivalent.

In a series of psychophysical experiments, Spence (1990) showed that proportions in pies were judged more accurately than with several other basic forms. Spence and Lewandowsky (1991) found that pies and bars performed comparably for simple judgments of proportion but found an advantage for the pie when more complicated judgments were required. They also showed that pies were not inferior to bars when comparisons of proportions were required. Indeed, if the comparison was a complicated one involving compound proportions (for example, A+B vs. C+D) the pie was superior to the bar. Hollands and Spence (1992, 1998, 2001) conducted further experiments and showed that provided the pie chart was used only to display the relative size of a small number of proportions, it fared as well as other commonly used charts.

There seems to be little objective basis for a prejudice against the pie based on considerations of speed or accuracy of estimation—the pie chart does as well, if not better, on simple tasks such as the estimation of a single proportion or the comparison of a small number of proportions. On the other hand, the natural competitors of the pie suffer significant disadvantages. For example, the simple bar chart does not provide an integrated representation of the whole, thus making part–whole estimation more difficult. This drawback may be alleviated by providing a reference bar, but the individual proportions will be at varying distances from the reference bar. The divided bar chart does provide a pictorial representation of the whole but it is less desirable than the pie for exactly the same reason that instruments that have circular dials (speedometers, altimeters, airspeed indicators, clocks, etc.) are generally preferred to those that use linear representations—they take up less space while providing the same or better resolution. Finally, the pie provides at least five natural anchors (0%, 25%, 50%, 75%, 100%) compared to only two, or at most three, for the divided bar (0%, 50%, 100%). A bar chart—without a reference bar—affords no natural anchors to assist in the accurate estimation of proportions (Hollands & Dyre, 2000; Simkin & Hastie, 1987; Spence & Krizel, 1994).

In my opinion, much of the adverse criticism of the pie has come from those who have wished it to do more than it could. The pie chart is a simple information graphic whose principal purpose is to show the relationship of a part to the whole. It is, by and large, the wrong choice as an exploratory device, and it is certainly not the correct choice when the graph maker or graph reader has a complicated purpose in mind, such as displaying small changes in proportion over time, a task that would require several pies. Such a complex task can be made even more difficult if the total area of each pie varies in proportion to the changing quantities that make up the 100% in each pie (Hollands & Spence, 2001). Playfair seems to have been aware of these limitations because he used his new invention only a handful of times in his work. Other than in *The Statistical Breviary* (including in a French edition), he used the pie in only two other publications (Playfair, 1805a; Chart 2—mislabeled Chart 3—facing p.192; Playfair, 1805b) (Figure 5).

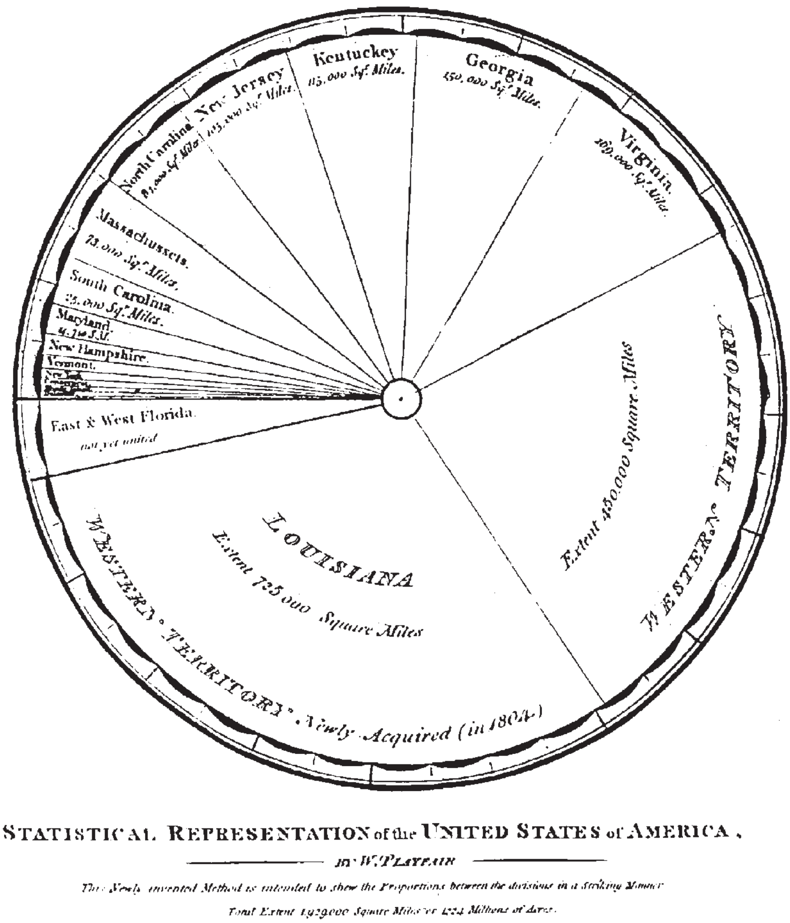

FIGURE 5. *Pie chart from Playfair's translation of Denis Francois Donnant's (1805)* Statistical account of the United States of America. *This chart is unlike other Playfair pies because it contains a much larger number of segments and is decorated around the circumference. There are major tick marks at approximately the 10% positions and also minor marks at the 5% positions. Playfair has labeled the segments and also stated the actual values of the land areas in square miles on each segment.*

5. Contemporary Use

Is the pie chart widely used? To get some idea, I sampled six arbitrarily selected publications that appeared during an arbitrarily selected year, 1998; I included two news magazines (*Time* and *Newsweek*), two business-oriented publications (the Business section of the *Toronto Globe and Mail* newspaper and *The Economist* magazine), and two general science publications (*Science* and *Scientific American*). The relative frequencies of use of all charts, including Playfair's, are shown in six pie diagrams (see Figure 6). It is evident that pie charts are much less used than bar charts or line graphs, and this is true for all types of publication. Indeed,

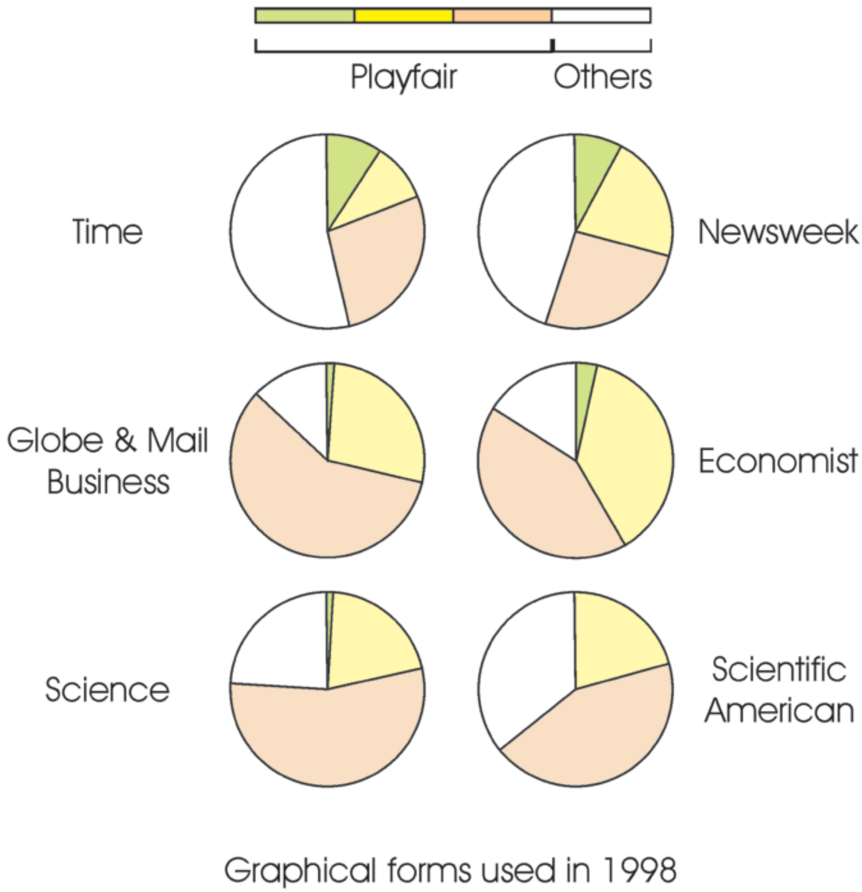

FIGURE 6. *Distribution of statistical graphics used in six publications from 1998. Pies are most frequent in the popular press, accounting for about 10% of usage; they rarely appear in the business and scientific publications. The statistical line graph is the most popular form overall, and all three of Playfair's inventions account from about 50% to 80% of total use.*

the frequency of appearance in business and scientific publications is quite low. The predominant use is in the popular news magazines where approximately one graph in 10 contains a pie chart. The frequency of use of each of the bar and line graphs is about twice that. The statistical line graph is the most popular form overall, accounting for more than half of the charts in some publications. In total, Playfair's graphic creations account for about 50% of all the diagrams used in news magazines and from about 60% to 80% in the business and scientific press. Playfair's legacy is considerable, and while the pie accounts for a relatively small faction of total use, its ability to display a small number of proportions in an appealing fashion has found an ample number of supporters. The pie chart has survived more than two centuries, and it shows no signs of being displaced as an effective and attractive device for the display of a small number of proportions.

References

Baron, M. E. (1969). A note on the historical development of logic diagrams: Leibniz, Euler, and Venn. *Mathematical Gazette, 52* 113–125.

Bertillon, J. (1891). *Atlas de statistique graphique de la ville de Paris, année 1889.* Paris.

Biderman, A. D. (1990). The Playfair enigma: The development of the schematic representation of statistics. *Information Design Journal, 6,* 3–25.

Brinton, W. C. (1914). *Graphic methods for presenting facts.* New York: The Engineering Magazine Company. Reprinted New York, NY: Arno Press, 1980.

Cleveland, W. S., & McGill, R. (1984). Graphical perception: Theory, experimentation, and application to the development of graphical methods. *Journal of the American Statistical Association, 79,* 531–540.

Croxton, F. E. (1927). Further studies in the graphic use of circles and bar II: Some additional data. *Journal of the American Statistical Association, 22,* 36–39.

Croxton, F. E., & Stein, H. (1932). Graphic comparisons by bars, squares, circles, and cubes. *Journal of the American Statistical Association, 27,* 54–60.

Croxton, F. E., & Stryker, R. E. (1927). Bar charts versus circle diagrams. *Journal of the American Statistical Association, 22,* 473–482.

Culbertson, H. M., & Powers, R. D. (1959). A study of graph comprehension difficulties. *AV Communication Review, 7,* 97–110.

Donnant, D. F. (1805). *Statistical account of the United States of America.* (William Playfair, Trans.) London: Greenland and Norris.

Eells, W. C. (1926). The relative merits of circles and bars for representing component parts. *Journal of the American Statistical Association, 21,* 119–132.

Euler, L. (1859). *Lettres d'Euler a une Princesse d'Allemagne sur divers sujets de Physique et de Philosophie accompagnées de l'Éloge d'Euler par Condorcet . . . avec une introduction et des notes par Émile Saisset.* Paris: Charpentier.

Funkhouser, H. G. (1937). Historical development of the graphical representation of statistical data. *Osiris, 3,* 269–404.

Hollands, J. G., & Dyre, B. P. (2000). Bias in proportion judgments: The cyclical power model. *Psychological Review, 107,* 500–524.

Hollands, J. G., & Spence, I. (1992). Judgments of change and proportion in graphical perception. *Human Factors, 34,* 313–334.

Hollands, J. G., & Spence, I. (1998). Judging proportion with graphs: The summation model. *Applied Cognitive Psychology, 12,* 173–190.

Hollands, J. G., & Spence, I. (2001). The discrimination of graphical elements. *Applied Cognitive Psychology, 15,* 413–431.

von Huhn, R. (1927). A discussion of the Eells' experiment. *Journal of the American Statistical Association, 22,* 31–36.

Keynes, J. M. (1936). William Stanley Jevons 1835–1882: A centenary allocation on his life and work as a statistician. *Journal of the Royal Statistical Society, 99,* 516–555.

Keynes, J. M. (1938). Review of The historical development of the graphical representation of statistical data by H. G. Funkhouser. *The Economic Journal, 48,* 281–282.

Leibniz, G. W. (1666). *Dissertatio de arte combinatoria.* Leipzig: Fick und Seubold. In, *Leibniz. samtliche schriften und briefe, Vol. 6.1,* (2nd ed.). Deutsche Akademie der Wissenschaften zu Berlin. Berlin, 1990.

Leibniz, G. W. (1696). Letter to Gabriel Wagner. In *Die philosophischen schriften von Gottfried Wilhelm Leibniz, Vol. 7.* C. I. Gerhardt. Berlin: Weidmannsche Buchhandlung, 1890, 514–527.

Lullus, R. (1517). *Ars magna et ultima.* B. de Lavinheta. Lugduni.

Macdonald-Ross, M. (1977). How numbers are shown. *AV Communications Review, 25,* 359–409.

Minard, C. J. (1858). *Carte figurative et approximative des quantités de viande de bouscherie envoyées sur pied par les départments et consommées à Paris.* Ecole Nationale des Ponts et Chaussées de Paris. Fol. 10975, 10969/c590, p. 44.

Palsky, G. (1996). *Des chiffres et des cartes: Naissance et dévelopement de la cartographie quantitative au XIXe siècle.* Paris: Comité des Travaux Historiques et Scientifiques.

Peterson, L. V., & Schramm, S. (1955). How accurately are different kinds of graphs read? *AV Communication Review, 2,* 178–189.

Peuchet, J. (1805). *Statistique élémentaire de la France.* Paris: Gilbert.

Picon, A., & Robert, J.-P. (1999). *Le dessus des cartes: Un atlas parisien.* Paris: Picard.

Playfair, J. (1784). *A system of chronology.* Edinburgh.

Playfair, J. G. (1822). *The works of John Playfair.* London: Constable.

Playfair, W. (1801). *The statistical breviary.* London: T. Bensley.

Playfair, W. (1805a). *An inquiry into the permanent causes of the decline and fall of powerful and wealthy nations.* London: Greenland & Norris.

Playfair, W. (1805b). *A statistical account of the United States of America by D. F. Donnant.* (William Playfair, Trans.). London: J. Whiting.

Priestley, J. (1765). *A chart of biography.* London: J. Johnson.

Simkin, D., & Hastie, R. (1987). An information processing analysis of graph perception. *Journal of the American Statistical Association, 82,* 454–465.

Spence, I. (1990). Visual psychophysics of simple graphical elements. *Journal of Experimental Pschology: Human Performance and Perception, 16,* 683–692.

Spence, I. (2000a). Le camembert depuis 200 ans: Playfair, Minard, et les autres, Presented at Historical landmarks in statistics and statistical graphics. *Fifth International Conference on Social Science Methodology.* Cologne, Germany, October 5, 2000.

Spence, I. (2000b). The invention and use of statistical charts. *Journal de la Société Francaise de Statistique, 141,* 77–81.

Spence, I. (2004). William Playfair. *Oxford Dictionary of National Biography.* Oxford: Oxford University Press.

Spence, I., & Krizel, P. (1994). Children's perception of proportion in graphs. *Child Development, 65,* 1193–1213.

Spence, I., & Lewandowsky, S. (1991). Displaying proportions and percentages. *Applied Cognitive Psychology, 5,* 61–77.

Spence, I., & Wainer, H. (1997). William Playfair: A daring worthless fellow. *Chance, 10,* 31–34.

Tufte, E. R. (1983). *The visual display of quantitative information.* Cheshire, CT: Graphics Press.

Author

IAN SPENCE is Professor, Department of Psychology, University of Toronto, Toronto, Ontario M5S 3G3, Canada; spence@psych.utoronto.ca. His current research interests are in data visualization and human–computer interaction.

Manuscript received July 20, 2004
Accepted July 28, 2004