Slope, Elasticity, Demand, and Inverse Demand

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1 Slope

Definition 1 *Slope is the ratio of the vertical change to the corresponding horizontal change in a line segment. It is commonly referred as rise over run.*

We calculate the slope of a straight line in the following manner:

$$\begin{aligned} slope &= m = \frac{rise}{run} \\ slope &= m = \frac{\Delta y}{\Delta x} \\ slope &= m = \frac{y_1 - y_0}{x_1 - x_0} \end{aligned}$$

However, remember that there are different units associated to each of the variables in question here. In this case, the y variable represents Price and is measured in monetary units and the x variable represents Quantity which is measured in physical units.

2 Elasticity

Definition 2 *Elasticity is the ratio of the percentage change in quantity demanded to the associated percentage change in price.*[1]

We calculate elasticity in the following manner:

$$elasticity = \varepsilon = \left(\frac{\Delta Q}{\Delta P}\right) \cdot \left(\frac{P}{Q}\right)$$

Let's analyze the different parts that make up the elasticity.

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[1] Baumol and Blinder (2005)

We have two parts $\left(\frac{\Delta Q}{\Delta P}\right)$ and $\left(\frac{P}{Q}\right)$. By looking closely we can identify that the first term is the slope of the demand curve with respect to price. We have the following:

$$\left(\frac{\Delta Q}{\Delta P}\right) = \left(\frac{Q_1 - Q_0}{P_1 - P_0}\right) = m$$

where (Q_0, P_0) can be any arbitrarily chosen point on the demand curve and (Q_1, P_1) can also be any arbitrarily chosen point on the demand curve. However, (Q_1, P_1) must be necessarily different from (Q_0, P_0).

The second term, i.e $\left(\frac{P}{Q}\right)$ is just the base or the factor used to obtain a unitless measure. Therefore, slope and elasticity are not the same. The elasticity involves the slope.

Let's go through an example:

$$Q = 24,000 - 500p \quad (1)$$
$$Q = 6000 + 1000p \quad (2)$$

Assume we have the previous equations. Equation 1 is the demand curve. We know this because of the negative sign associated with the price variable. Therefore, equation 2 is the supply curve. We know this because of the positive sign associated with the price variable.

Let's calculate the price elasticity of demand. First we need to calculate the equilibrium price and quantity. We set Supply equal to Demand.

$$\begin{aligned} 24,000 - 500p &= 6000 + 1000p \\ 18000 &= 1500p \\ \frac{18000}{1500} &= p \\ p &= 12 \\ q &= 18000 \end{aligned}$$

So we are going to calculate the price elasticity of demand at the point $(18000, 12)$.

We know the formula for the price elasticity of demand:

$$\varepsilon = \left(\frac{\Delta Q}{\Delta P}\right) \cdot \left(\frac{P}{Q}\right)$$

Since we are calculating the elasticity at the point[2] $(18000, 12)$ we have the that $\left(\frac{P}{Q}\right) = \left(\frac{12}{18000}\right)$.

[2] In lecture, Professor Gilless calculated elasticity by averaging two points on the demand curve. In this case, I am not following that procedure because I am only interested in the elasticity at a precise point.

We still need to calculate $\left(\frac{\Delta Q}{\Delta P}\right)$. However, we know that $\left(\frac{\Delta Q}{\Delta P}\right) = \left(\frac{Q_1 - Q_0}{P_1 - P_0}\right) = m$ (slope of the demand curve). That number is -500.

Go back to the elasticity formula and plug in the numbers we have calculated:

$$\begin{aligned}
\varepsilon &= \left(\frac{\Delta Q}{\Delta P}\right) \cdot \left(\frac{P}{Q}\right) \\
&= \left(-500 \frac{units}{\$}\right) \cdot \left(\frac{12\ \$}{18000\ units}\right) \\
&= -.3333
\end{aligned}$$

All units cancel out and we have a unitless measure, i.e. a percentage change. The key here is the second factor $\left(\frac{12\ \$}{18000\ units}\right)$ which cancels the units that the slope of the demand curve has.

Now, in the discussion section what I wanted to emphasize was a very subtle distinction. This is very important. When we calculate elasticities we have to make sure we are using the demand curve. We have to make sure that we have solved for quantity before we calculate anything. If we have solved for Price we are using inverse demand and whatever calculations we obtain from this will be the inverse of what we actually want.

Example:

$$Q = 24,000 - 500p$$

The previous equation is a **demand curve**. If we solve for price we obtain **inverse demand**. Which is the following:

$$p = \frac{24000 - Q}{500}$$

What is the slope of Inverse demand? $-\frac{1}{500}$

What is the slope of demand? -500

What happens if by mistake we use the slope of inverse demand to calculate an elasticity? Let's see

We know the formula for the elasticity which is:

$$\begin{aligned}
\varepsilon &= \left(\frac{\Delta Q}{\Delta P}\right) \cdot \left(\frac{P}{Q}\right) \\
&= -\frac{1\ \$}{500\ units} \left(\frac{12\ \$}{18000\ units}\right) \\
&= -0.000001333 \frac{\$^2}{units^2} \\
-0.000001333 &\neq -.3333
\end{aligned}$$

This is not the right answer because $-\frac{1}{500}$ is $\left(\frac{\Delta P}{\Delta Q}\right)$ and not $\left(\frac{\Delta Q}{\Delta P}\right)$ which is the correct slope we need to calculate this elasticity. Additionally, note the units on $-0.000001333\frac{\$^2}{units^2}$ which are dollars square over units square. We do not know what this is. What we learn from this is that we need to pay attention and make sure that we are working with demand and **NOT** inverse demand when we calculate elasticities.

Finally, what do the number we calculate from the price elasticity of demand tell us?

What we see is the following:

Depending on what elasticity value we calculate demand is going to be the following:

$$\begin{bmatrix} & Elastic & Unit\ Elastic & Inelastic \\ elasticity\ values & -\infty < \varepsilon < -1 & \varepsilon = -1 & -1 < \varepsilon < 0 \end{bmatrix}$$

- A good with an elastic demand has plenty of substitutes and it is probably a luxury,good because a slight percentage change in price creates a big percentage change in quantity. An example would be Coke which has plenty of substitutes and a slight change in the price of Coke will create a huge drop in quantity demanded.

- A good with an inelastic demand has only a few substitutes and it is probably a necessity. a big percentage change in price will cause only a slight percentage change in quantity. An example of such a good would be gasoline. People need gasoline in order to drive their cars. The percentage change in the price of gasoline would have to be huge to make people stop using their cars.