AN INTRODUCTION TO STIRLING-CYCLE MACHINES

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Notation:

c_v	specific heat capacity at constant volume (J/kgK)	T	temperature (K)
p	pressure (Pa)	U	internal energy (J)
Q	heat (J)	V	volume (m^3)
R	specific gas constant (J/kgK)	W	work (J)
S	entropy (J/K)	χ	coefficient of performance
		η	efficiency (%)

sign convention: energy into a system is positive, energy out of a system is negative.

1. BACKGROUND

Machines operating on the Stirling Cycle are the most efficient practical heat engines ever built. As an engine they can run on any heat source (including solar heating), and if combustion-heated they produce very low levels of harmful emissions. When operated as a refrigerator or heat-pump Stirling-cycle machines offer the possibility of using safe refrigerants such as air, thus avoiding the environmental damage caused by all refrigerants in current use (NOTE: even the so-called “green” refrigerant R134a is a potent greenhouse gas).

2. THE STIRLING-CYCLE MACHINE

There are five main components in a Stirling-cycle machine, as shown in Figure 2.1.

(a) *Working gas* – the Stirling Cycle is a closed cycle and the various thermodynamic processes are carried out on a working gas that is trapped within the system.

(b) *Heat-exchangers* – two heat exchangers are used to transfer heat across the system boundary. A *heat absorbing heat-exchanger* transfers heat from outside the system into the working gas, and a *heat rejecting heat-exchanger* transfers heat from the working gas to outside the system. For example, on an engine the heat absorbing heat-exchanger might transfer heat from a burner into the working gas, and the heat rejecting heat-exchanger might transfer heat from the working gas to coolant in a water-jacket.

(c) *Displacer mechanism* – this moves (or displaces) the working gas between the hot and cold ends of the machine (via the regenerator).

(d) *Regenerator* – this acts both as a thermal barrier between the hot and cold ends of the machine, and also as a "thermal store" for the cycle. Physically a regenerator usually consists of a mesh material (household pot scrubbers have even been used in some engines), and heat is transferred as the working gas is "blown" through the regenerator mesh. When the working gas is displaced from the hot end of the machine (via the regenerator) to the cold end of the machine, heat is "deposited" in the regenerator, and the temperature of the working gas is lowered. When the reverse displacement occurs, heat is "withdrawn" from the regenerator again, and the temperature of the working gas is raised.

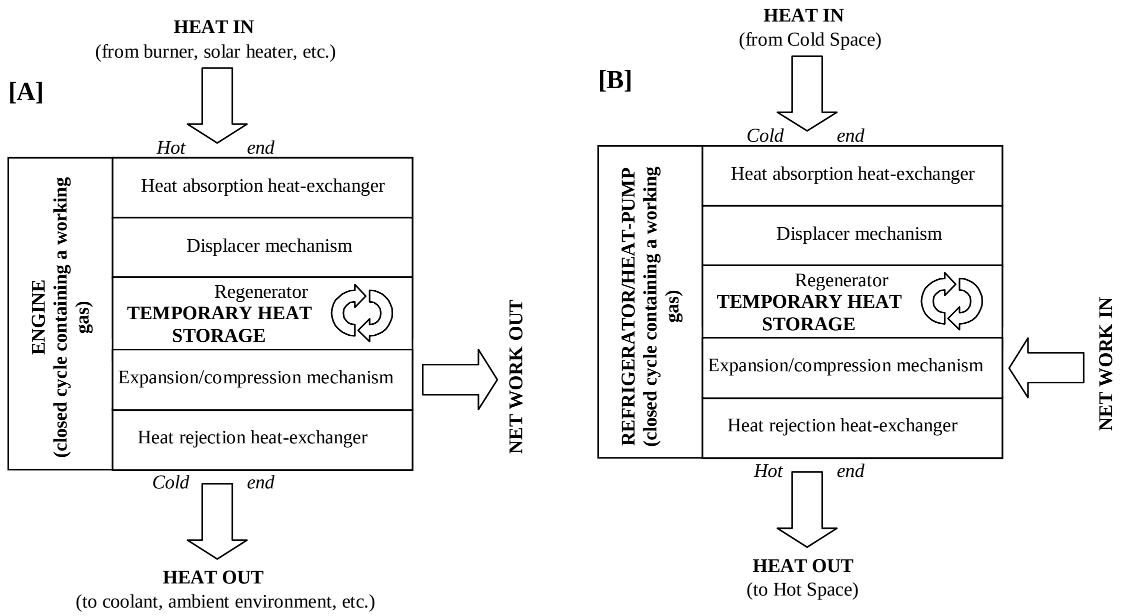

Figure 2.1. Stirling-cycle machine block diagrams: (A) Engine (B) Refrigerator or heat-pump.

(e) *Expansion/compression mechanism* – this expands and/or compresses the working gas. In an engine this mechanism produces a net work output. In a refrigerator or heat-pump a net work input is required to move the heat from a low to a high temperature regime (in accordance with the Second Law of Thermodynamics).

A Stirling-cycle machine can be constructed in a variety of different configurations. For example, the expansion/compression mechanisms can be embodied as turbo-machinery, a piston-cylinder, or even using acoustic waves. Most commonly, Stirling-cycle machines use a piston-cylinder, in either an α or β configuration. An α-configuration machine uses two pistons which displace and expand/compress the gas at the same time. A β-configuration machine has a separate displacer-piston and expansion/compression piston (usually called a power-piston).

3. THE STIRLING-CYCLE AS AN ENGINE

In an ideal Stirling-cycle engine the components of the machine interact to produce four separate thermodynamic processes. These processes are illustrated using a simplified β-configuration machine in Figure 3.2., and are shown on pressure-volume and temperature-entropy diagrams in Figure 3.1. It should be noted that for the ideal Stirling Cycle the heat-exchangers, regenerator, and transfer passages are assumed to have zero volume.

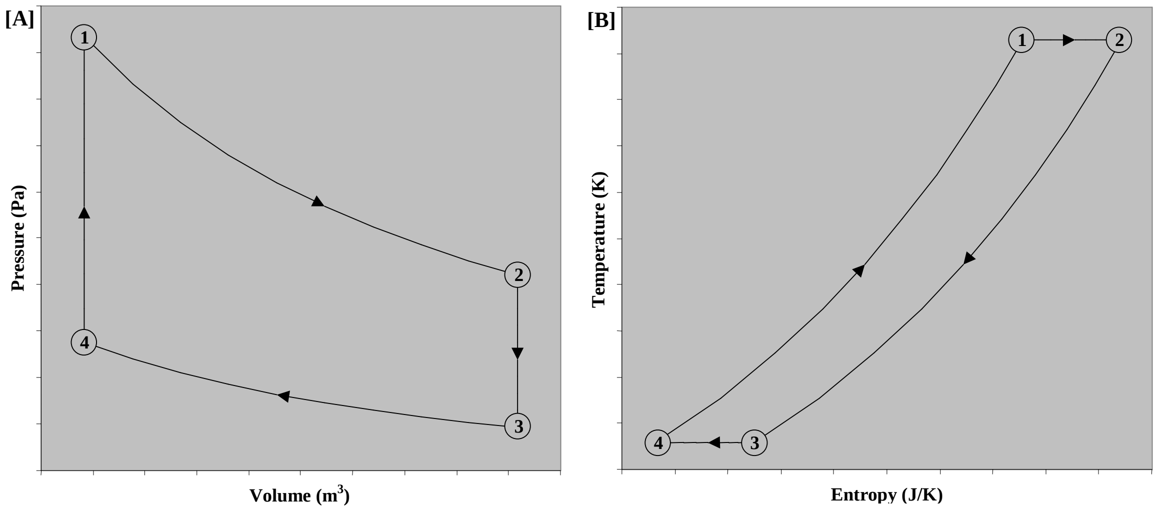

Figure 3.1. Thermodynamic processes in the ideal Stirling-cycle engine. (A) Pressure-volume diagram. (B) Temperature-entropy diagram.

1→2 : Isothermal (constant temperature) expansion – the high-pressure working gas absorbs heat from the hot space (via the heat absorbing heat-exchanger) and expands isothermally, thus doing work on the power-piston.

2→3 : Isochoric (constant volume) displacement – the displacer-piston transfers all the working gas isochorically through the regenerator to the cold end of the machine. Heat is absorbed from the gas as it passes through the regenerator, thus lowering the temperature of the gas to that of the cold space. As the temperature reduces, the gas pressure drops significantly.

3→4 : Isothermal compression – the power-piston does work on the gas and compresses it isothermally at cold end temperature, hence rejecting heat to the cold space (via the heat rejecting heat-exchanger). Because the gas is at low pressure, less work is required for compression than was obtained from the gas during expansion (in 1→2). The cycle therefore has a net work output.

4→1 : Isochoric displacement – the displacer-piston transfers all the working gas isochorically through the regenerator to the hot end of the machine. Heat is delivered to the gas as it passes through the regenerator, thus raising the temperature of the gas to that of the hot space. As the temperature rises, the gas pressure increases significantly, and the system returns to its initial conditions.

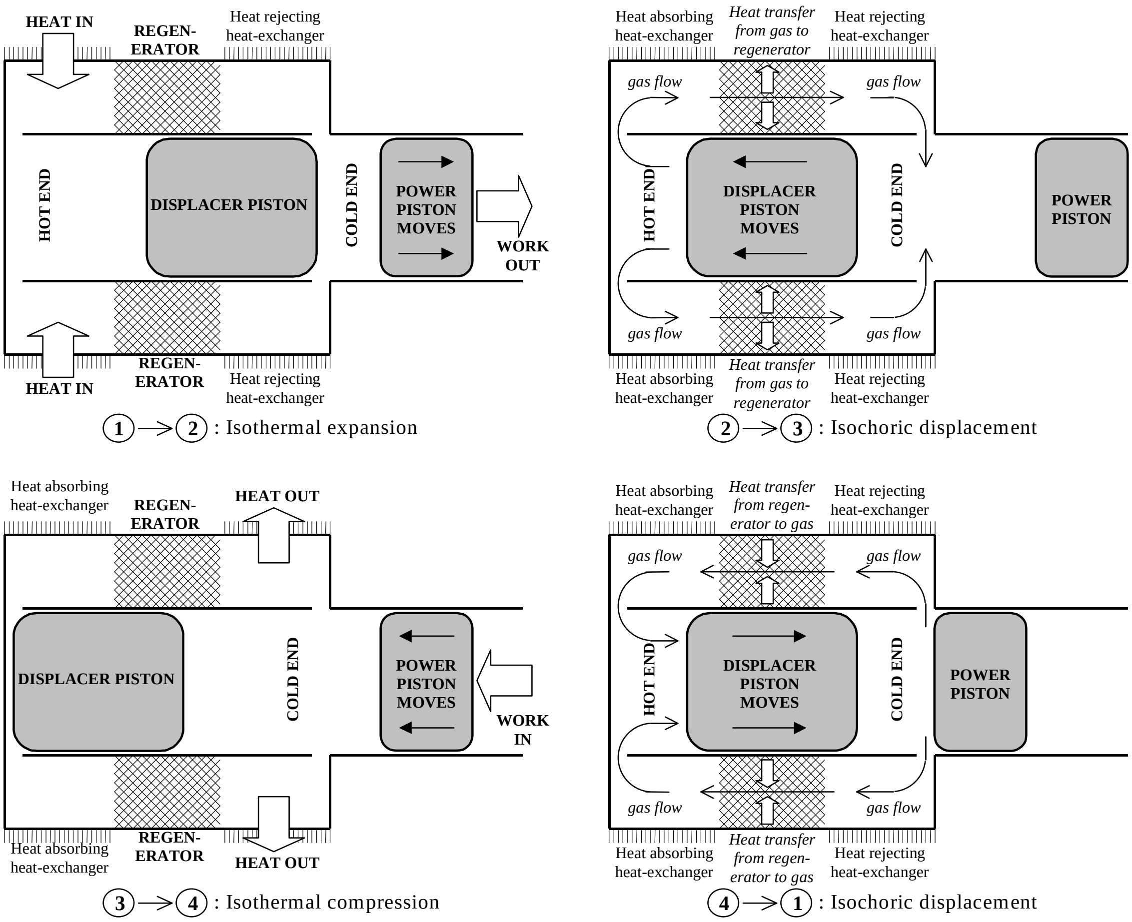

Figure 3.2. Thermodynamic processes in the ideal Stirling-cycle engine as shown on a simplified β-configuration machine.

4. ANALYSIS OF THE STIRLING-CYCLE ENGINE

4.1. Work done by an ideal Stirling-cycle engine

The net work output of a Stirling-cycle engine can be evaluated by considering the cyclic integral of pressure with respect to volume:

$$W = -\oint p\mathrm{d}V$$

This can be easily visualised as the area enclosed by the process curves on the pressure-volume diagram in Figure 3.1.

To evaluate the integral we need only consider the work done during the isothermal expansion and compression processes, since there is no work done during the isochoric processes, i.e.

$$W = -\left[\int_{V_1}^{V_2} p.\mathrm{d}V + \int_{V_3}^{V_4} p.\mathrm{d}V \right] \tag{4.1.}$$

By considering the equation of state:

$$pV = mRT$$

and noting that T is constant for an isothermal process, and m is constant for a closed cycle, then an expression for work done during an isothermal process can be formulated:

$$\int_{V_A}^{V_B} p\mathrm{d}V = \int_{V_A}^{V_B} \frac{mRT}{V}\mathrm{d}V = mRT\left[\ln V\right]_{V_A}^{V_B} = mRT\ln\left(\frac{V_B}{V_A}\right) \tag{4.2.}$$

so that by substitution of Equation 4.2. into Equation 4.1. we can evaluate the work integral:

$$W = -\left[mRT_H \ln\left(\frac{V_2}{V_1}\right) + mRT_L \ln\left(\frac{V_4}{V_3}\right) \right]$$

where the subscripts H and L denote the high and low temperature isotherms respectively.

This equation can then be further simplified by noting that $V_4 = V_1$ and $V_3 = V_2$ so that a final equation for work can be obtained:

$$W = -mR\ln\left(\frac{V_2}{V_1}\right)(T_H - T_L) \tag{4.3.}$$

The work done represents energy out of the system, and so has a negative value according to the sign convention used here.

Inspection of Equation 4.3. therefore shows that the work output for a Stirling-cycle machine can be increased by maximising the temperature difference between hot and cold ends (T_H-T_L), the compression ratio (V_2/V_1), the gas mass (and hence either the total volume of the machine and/or the mean operating pressure), or the specific gas constant.

Material strength/temperature considerations and practicalities such as the overall size of the machine usually limit the amount that the temperature, volume, or pressure can be increased. However, it is interesting to note that the specific work output (i.e. work output per kilogram) can be dramatically enhanced in a Stirling-cycle machine simply by selecting a working gas with a high specific gas constant.

Table 4.1. Specific gas constants for a variety of gases at 300 K. (Source: Van Wylen, et al., 1994)

Gas	Specific gas constant, *R* (J/kgK)
Air	319.3
Ammonia	488.2
Carbon dioxide	188.9
Helium	2077.0
Hydrogen	4124.2
Nitrogen	296.8
Propane	188.6
Steam	461.5

One of the reasons that hydrogen and helium are so often used as the working gas in large Stirling-cycle machines can be deduced by inspection of the values for specific gas constants given in Table 4.1. (another reason is the lower flow losses that occur with smaller molecule gases).

4.2. Heat flow in an ideal Stirling-cycle engine

The heat flowing into and out of a Stirling-cycle engine can be evaluated by considering the integral of temperature with respect to entropy:

$$Q = \int T \mathrm{d}S$$

This can be visualised as the area beneath the process curves on the temperature-entropy diagram in Figure 3.1.

Since the isochoric heat transfers within the regenerator are completely internal to the cycle, i.e. $-Q_{2\rightarrow3} = Q_{4\rightarrow1}$, then to evaluate the heat flows into and out of the system we need only consider the isothermal processes.

For the isothermal expansion process in a closed cycle (where T and m are constant, and where the subscripts H and L denote the high and low temperature isotherms respectively):

$$Q_H = \int_{S_1}^{S_2} T_H \mathrm{d}S$$

this integral can be most easily evaluated by considering the First Law of Thermodynamics in the form:

$$\delta Q = \mathrm{d}U - \delta W$$

and since:

$$\delta Q = T\mathrm{d}S \text{ and } \delta W = -p\mathrm{d}V$$

then it can be said that:

$$T\mathrm{d}S = \mathrm{d}U - \left(-p\mathrm{d}V\right)$$

so that the heat flow during the isothermal expansion process can be expressed in terms of a change in internal energy and volume:

$$Q_H = \int_{U_1}^{U_2} \mathrm{d}U + \int_{V_1}^{V_2} p\mathrm{d}V$$

and by considering the equation of state:

$$pV = mRT$$

then the pressure term can be expressed in terms of volume and temperature, and (noting that there is no change in internal energy during an isothermal process) the integral can be easily solved:

$$Q_H = \int_{U_1}^{U_2} \mathrm{d}U + \int_{V_1}^{V_2} \frac{mRT_H}{V} \mathrm{d}V = 0 + mRT_H \left[\ln V\right]_{V_1}^{V_2}$$

giving:

$$Q_H = mRT_H \ln\left(\frac{V_2}{V_1}\right) \quad (4.4.)$$

which is a somewhat convoluted (but hopefully instructive) method of derivation. The same expression can, of course, be obtained much more easily by simple inspection of Equation 4.3., since the heat and work transfers for an isothermal expansion process are equal but opposite.

The isothermal compression process can also be readily evaluated (noting that $V_4 = V_1$ and $V_3 = V_2$, and where the subscripts $_H$ and $_L$ denote the high and low temperature isotherms respectively), giving:

$$Q_L = -mRT_L \ln\left(\frac{V_2}{V_1}\right) \quad (4.5.)$$

4.3. Efficiency of an ideal Stirling-cycle engine

The efficiency of any heat engine is defined as the ratio of work output to heat input, i.e.

$$\eta = \frac{-W}{Q_H}$$

hence an equation for the efficiency of an ideal Stirling-cycle engine can be developed by considering Equations 4.3. and 4.4., giving:

$$\eta_{STIRLING} = \frac{mR\ln\left(\frac{V_2}{V_1}\right)(T_H - T_L)}{mRT_H \ln\left(\frac{V_2}{V_1}\right)}$$

which simplifies to:

$$\eta_{STIRLING} = \frac{T_H - T_L}{T_H} \quad (4.6.)$$

this demonstrates the interesting fact that the efficiency of an ideal Stirling-cycle engine is dependant only on temperature and no other parameter.

It is worth recalling that the Carnot efficiency for a heat engine is:

$$\eta_{CARNOT} = \frac{T_H - T_L}{T_H}$$

and so it will readily be observed that:

$$\eta_{STIRLING} = \eta_{CARNOT}$$

or, in other words, that the Stirling-cycle engine has the maximum efficiency possible under the Second Law of Thermodynamics. However, it should be noted that unlike the Carnot Cycle, the Stirling-cycle engine is a practical machine that can actually be used to produce useful quantities of work.

5. THE STIRLING-CYCLE AS A REFRIGERATOR OR HEAT-PUMP

The ideal Stirling-cycle refrigerator or heat-pump is, in effect, identical to a Stirling-cycle engine *except that* the heat absorbing end of the machine now becomes the cold region, and the heat rejecting end of the machine becomes the hot region. The thermodynamic processes for a refrigerator/heat-pump are illustrated using a simplified β-configuration machine in Figure 5.2., and are shown on pressure-volume and temperature-entropy diagrams in Figure 5.1. Because refrigerator/heat-pumps tend to have a smaller temperature difference between hot and cold regimes than an engine, the pressure-volume and temperature-entropy diagrams appear somewhat squatter in comparison. It should be noted that for the ideal Stirling Cycle the heat-exchangers, regenerator, and transfer passages are assumed to have zero volume.

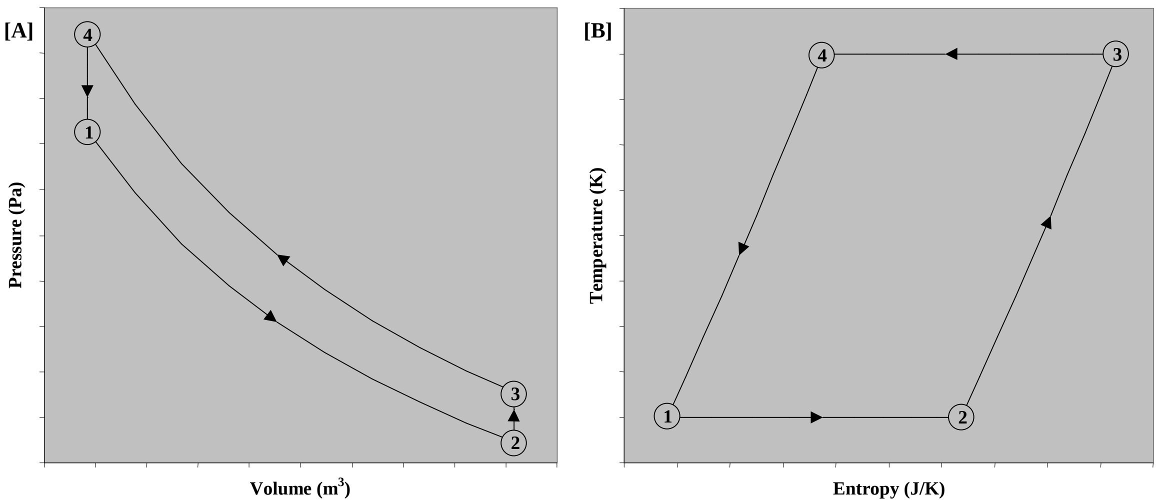

Figure 5.1. Thermodynamic processes in the ideal Stirling-cycle refrigerator or heat-pump. (A) Pressure-volume diagram. (B) Temperature-entropy diagram.

1 → 2 : Isothermal expansion – the low-pressure working gas expands isothermally at cold end temperature, hence absorbing heat from the cold space (via the heat absorbing heat-exchanger) and doing work to the power-piston.

2 → 3 : Isochoric displacement – the displacer-piston transfers all the working gas isochorically through the regenerator to the hot end of the machine. Heat is delivered to the gas as it passes through the regenerator, thus raising the temperature of the gas to that of the hot space. As the temperature rises, the gas pressure increases significantly.

3 → 4 : Isothermal compression – the power-piston does work to the gas and compresses it isothermally at hot end temperature, hence rejecting heat to the hot space (via the heat rejecting heat-exchanger). Because the gas is at high pressure, more work is required for compression than was obtained from the gas during expansion (in $1 \rightarrow 2$). The cycle therefore has a net work input.

4 → 1 : Isochoric displacement – the displacer piston transfers all the working gas isochorically through the regenerator to the cold end of the machine. Heat is

absorbed from the gas as it passes through the regenerator, thus lowering the temperature of the gas to that of the cold space. As the temperature reduces, the gas pressure drops significantly, and the system returns to its initial conditions.

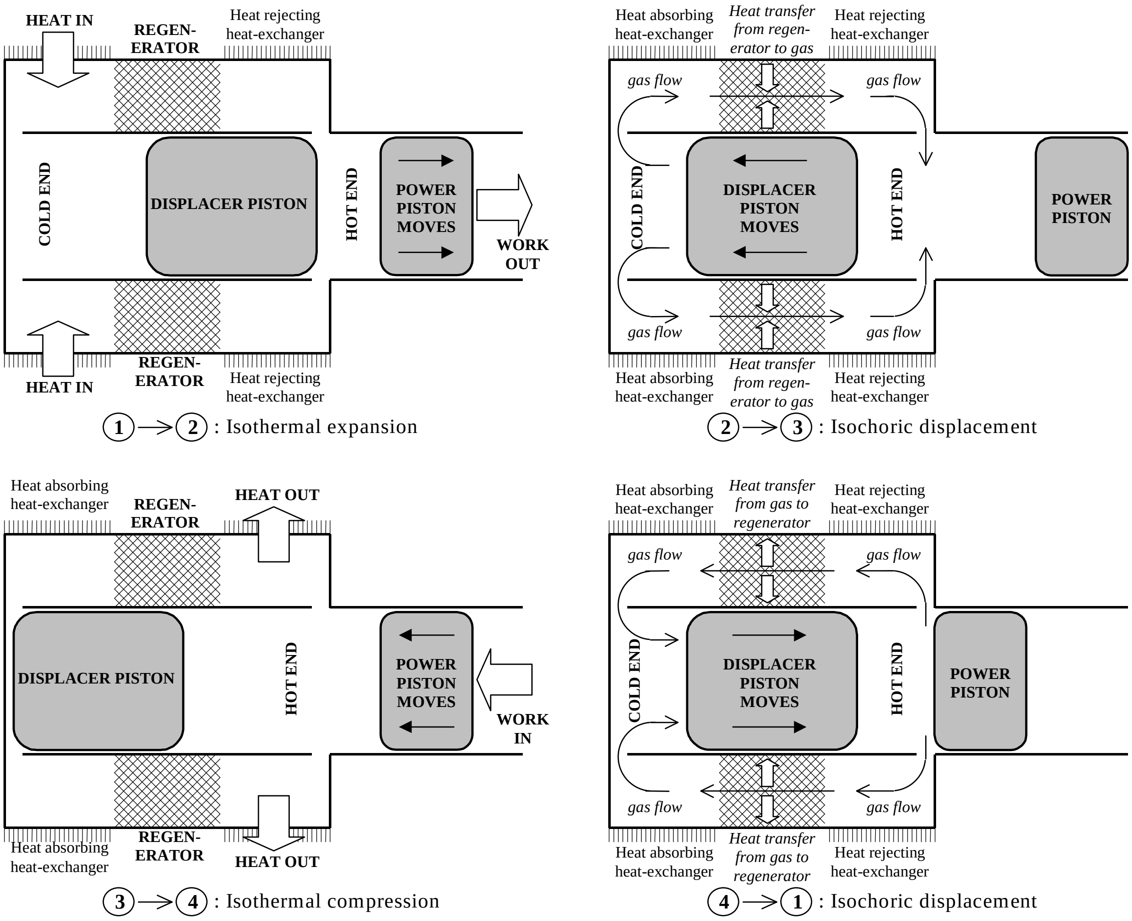

Figure 5.2. Thermodynamic processes in the ideal Stirling-cycle refrigerator/heat-pump as shown on a simplified β-configuration machine.

6. ANALYSIS OF THE STIRLING-CYCLE REFRIGERATOR OR HEAT-PUMP

6.1. Work input to an ideal Stirling-cycle refrigerator or heat-pump

An equation for net work input to an ideal Stirling-cycle refrigerator/heat-pump can be derived in exactly the same way as work output for a Stirling-cycle engine (see Section 4.1.), giving:

$$W = mR\ln\left(\frac{V_2}{V_1}\right)(T_H - T_L) \qquad (6.1.)$$

Note that, unlike the work output from an engine, the refrigerator/heat-pump work has a positive value under the energy sign convention used here, since a net energy input is required to move heat from a low to high temperature regime.

6.2. Heat flow in an ideal Stirling-cycle refrigerator or heat-pump

Equations for the heat flows into and out of an ideal Stirling-cycle refrigerator/heat-pump can be derived in a similar way as heat flows in a Stirling-cycle engine (see Section 4.2.). The main difference is that in a refrigerator/heat-pump the heat flows into the system at a low temperature (T_L) and out of the system at a high temperature (T_H).

For a heat-pump, the heating effect is therefore:

$$Q_H = -mRT_H \ln\left(\frac{V_2}{V_1}\right) \tag{6.2.}$$

And for a refrigerator, the refrigeration effect is:

$$Q_L = mRT_L \ln\left(\frac{V_2}{V_1}\right) \tag{6.3.}$$

6.3. Performance of an ideal Stirling-cycle refrigerator or heat-pump

The coefficient of performance for any refrigerator/heat-pump is defined as the ratio of heating/cooling effect to work input, i.e.

for a heat-pump, the heating coefficient of performance is: $\chi_H = \dfrac{-Q_H}{W}$

for a refrigerator, the refrigeration coefficient of performance is: $\chi_L = \dfrac{Q_L}{W}$

hence equations for coefficient of performance for ideal Stirling-cycle refrigerators and heat-pumps can be developed by considering Equations 6.1., 6.2., and 6.3., giving:

$$\chi_{H\,STIRLING} = \frac{mRT_H \ln\left(\frac{V_2}{V_1}\right)}{mR\ln\left(\frac{V_2}{V_1}\right)(T_H - T_L)} \quad \text{and} \quad \chi_{L\,STIRLING} = \frac{mRT_L \ln\left(\frac{V_2}{V_1}\right)}{mR\ln\left(\frac{V_2}{V_1}\right)(T_H - T_L)}$$

which simplifies to:

$$\chi_{H\,STIRLING} = \frac{T_H}{T_H - T_L} \tag{6.4.}$$

and

$$\chi_{L\,STIRLING} = \frac{T_L}{T_H - T_L} \tag{6.5.}$$

so that (as should be expected from the derivation of efficiency for the Stirling-cycle engine):

$$\chi_{H\,STIRLING} = \chi_{H\,CARNOT} \text{ and } \chi_{L\,STIRLING} = \chi_{L\,CARNOT}$$

7. CONCLUSIONS

It should be noted that the Stirling-cycle machine has only been considered here in its ideal form. Practical Stirling-cycle machines differ from the ideal cycle in several important aspects:

(a) The regenerator and heat-exchangers in practical Stirling-cycle machines have non-zero volume. This means that the working gas is never completely in either the hot or cold end of the machine, and therefore never at a uniform temperature.
(b) The piston motion is usually semi-sinusoidal rather than discontinuous, leading to non-optimal manipulation of the working gas.
(c) The expansion and compression processes in practical Stirling-cycle machines are polytropic rather than isothermal. This causes pressure and temperature fluctuations in the working gas and leads to adiabatic and transient heat transfer losses.
(d) Fluid friction losses occur during gas displacement, particularly due to flow through the regenerator.
(e) Other factors such as heat conduction between the hot and cold ends of the machine, seal leakage and friction, appendix gap effects, and friction in kinematic mechanisms all cause real Stirling-cycle machines to differ from ideal behaviour.

Although Stirling-cycle machines theoretically have Carnot efficiency, the above factors tend to reduce the performance of real machines to significantly less than this value. Further information about the Stirling Cycle and its practical limitations can be found in West [2] and Wurm, et al. [3].

8. REFERENCES

[1] Van Wylen, Sonntag, R.E., Borgnakke, C. *Fundamentals of Classical Thermodynamics*. John Wiley & Sons Inc., New York, 1994.
[2] West, C.D. *Principles and Applications of Stirling Engines*. Van Nostrand Reinhold Company, New York, 1985.
[3] Wurm, J., Kinast, J.A., Roose, T.R., Staats, W.R. *Stirling and Vuilleumier Heat Pumps*. McGraw-Hill Inc., New York, 1991.