

DEMOGRAPHIC TRANSITION AND INDUSTRIAL REVOLUTION: A MACROECONOMIC INVESTIGATION

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ABSTRACT

All industrialized countries have experienced a transition from high birth rates, land-based production and stagnant standards of living to low birth rates and sustained income growth. To develop a better understanding of these economic and demographic transformations and the link between them, we construct a general equilibrium framework merging the Hansen and Prescott (2002) model of structural change with the Barro and Becker (1989) model of fertility choice. We find that when the historical changes of youth mortality and sector-specific productivity are introduced into the model, parameterized to capture key moments of 17th century England, it does remarkably well at generating the long-run features of economic and demographic development of England.

Keywords: industrialization, structural transformation, unified growth theory, technological progress, demographic transition, young-age mortality

JEL Codes: J10, O11, O41, O47, E00

I. Introduction

All industrialized countries have experienced a transition from stagnant standards of living to sustained growth in per capita income. Over the same period of time, resources were reallocated from rural to non-rural production, the land share in total income declined significantly, while the labor income share increased. In each case, this economic transition was accompanied by a demographic transition from high to low birth and mortality rates. These key changes together constitute one of the major transformations of modern times.

What factors were responsible for these changes and to what extents? Did the economic and demographic changes transpire through common or distinct channels? To study these questions and further understand the link between economic and population dynamics, we construct a dynamic general equilibrium two-sector model with endogenous fertility. This model is a hybrid of the two well-known and widely used models in the literature: Hansen and Prescott (2002) (henceforth “HP”) and Barro and Becker (1989) (henceforth “BB”). The final good, as in HP, can be produced using two different technologies, the Malthusian, which takes capital, labor, and land as inputs, and the Solow, which employs capital and labor only. We associate the Malthusian technology with rural production and the Solow technology with urban production. This two-technology framework allows us to investigate cross-sector resource allocation and movements in factor income shares. Fertility choice is modeled after BB. Parents place value on both the number of surviving children and their children’s well-being,¹ and thus face a quantity-quality trade-off between having many children each with a small inheritance in the form of capital and land and having few children with a larger inheritance. Parental time is needed for raising each child, including those not surviving to adulthood. The time cost of raising a surviving child thus declines as more newborns survive to adulthood. Changes in the time cost can also be interpreted within our framework as a reduced form representation of such changes as the rise in the value of female time or the introduction of child labor laws and compulsory education reforms.

We parameterize our model to match key moments of 17th century England, without forcing the model to match the time series dimension of any variable. We then ask whether the long-run patterns of development in England are closely captured by the parameterized model dynamics that result when young-age mortality and the sector-specific total factor productivity (TFP) are stipulated to vary over time in accordance with historical data. It is found that the model does remarkably well, which suggests its potential usefulness in future quantitative studies. Because the cost of raising children is not observed in the data, its changes cannot be introduced into the model in the same manner as is done for sector-specific productivity or young-age mortality. We do, however, find that a rise in the time cost further improves the model’s overall performance.

We then use the parameterized model to quantitatively assess the separate contributions of changes in sector-specific productivity, young-age mortality and the cost of children to shaping the demographic and economic transformations in England.

It is important to note that we investigate the model behavior resulting from the historical time series of TFP and young-age mortality without attempting to elucidate the underlying causes for these empirical

¹We use two specifications, one introduced in Barro and Becker (1989) and one introduced in Lucas (2002).

phenomena. Thus, we seek to ascertain the combined influence of all factors on the demographic and economic transformations that are communicated through the TFP and young-age mortality “channels.”² A word of caution is needed here to avoid a possible misinterpretation of our results. For example, our finding that technological progress is quantitatively a major channel through which urbanization transpired does not imply that changes in young-age mortality, or any other force, had no causal effect with regard to this phenomenon. In fact, reductions in young-age mortality, by increasing the population density and thus facilitating the process of idea sharing, could contribute to the productivity growth, and consequently influence urbanization. Instead, this finding leads to the conclusion that if young-age mortality contributed to resource reallocation, its influence must have been communicated predominantly through its effect on TFP. In other words, when evaluating the importance of a specific channel in accounting for a particular phenomenon, we hold constant the influences it may exert through the other channels. There are a number of other potential interactions between the channels studied in this paper. The influence exerted through the productivity channel may cause the mortality decline, as a result of its tendency to increase the standard of living. Changes in productivity, if skill-biased, can also induce quality investment in children, thus contributing to the increased cost of raising children.

Related works have provided a number of illuminating dynamical systems that capture potentially very important mechanisms. However, many of these point to drastically different causes behind the economic and demographic transformations. To give just a few examples, the mechanisms proposed in Greenwood and Seshadri (2002), Jones (2001), Kalemli-Ozcan (2002) and Soares (2005) each generates a drop in fertility and a switch to a sustained growth regime through the exogenous technological change, institutional change, a decline in young-age mortality, and a decline in adult mortality, respectively. Thus, the relative importance of each such mechanism for the case of a particular country remains unclear.

There is a pressing need for more quantitative work in this field. By allowing us to disentangle the roles of the most relevant channels in transforming England, the HP-BB hybrid framework helps us obtain a better understanding of the link between the economic and demographic changes and elucidates the relative importance of different mechanisms developed for the purpose of endogenously generating both the economic and demographic transformations. In fact, most of these existing mechanisms act through some combination of the following channels: technological progress, young-age, adult-age, old-age mortality and the cost of raising children. Future work should attempt to incorporate the remaining channels and attempt to perform a decomposition of the economic and demographic changes into changes transpiring through each of them.³

One advantage of using our framework for assessing the relative importance of the young-age mortality and TFP channels on population, output, resource allocation and factor income shares is that it allows a straightforward mapping to the data. We use standard functional forms, and our choice of parameters is guided strictly by the observables. The time series used in the design of our experiments represent their actual historical estimates. Our framework also allows for TFP time series estimation in the rural and

²We choose to focus on the effect of changes in young-age mortality and TFP because empirical evidence and related historical, demographic, and economic literature overwhelmingly link these factors to economic and demographic variables. See Section III.

³Such growth/demographic accounting framework would be similar in spirit to the business cycle accounting framework developed in Chari et al. (2006).

urban sectors using the available data on wages, land and capital rental rates, and the GDP deflator.⁴ Moreover, such framework allows us to study the channels under consideration both jointly and in isolation. This contrasts with the situation regarding models incorporating intricate collections of forces, which cannot be tested in isolation without shutting down the entire mechanism.

The main contributions of this paper are as follows. First, we construct a general equilibrium framework that combines the HP model of structural change with the BB model of fertility choice and contrast its results with those of HP (See Sections IV and VI). While our results do not support the strict form of the HP conclusions, i.e. that stagnation is generated because the Solow sector is not used and that the structural transformation and takeoff in income growth transpire under constant sector-specific productivity growth rates, we show that the parameterized HP-BB framework featuring historical changes in youth mortality and sector-specific productivity can successfully account for the main patterns of the English economic and demographic transformations. It accounts for roughly 3/4 of the increase in the GDP/capita observed in the data during the period 1650-1950, for over 90% of the movement in factor income shares and resources across sectors, and over 60% of the drop in the crude birth rate⁵ (CBR). Increasing the cost of children further improves the model's fit along the demographic dimension, without compromising its success along the economic dimension. These results lead us to believe that the HP-BB framework will prove useful in future applied work in this area.

The second important contribution is our quantitative assessment of the separate roles of youth mortality, productivity and the cost of raising children. We find that the influence exerted through the productivity channel is largely decoupled from those exerted through the youth mortality and the cost of children channels. Specifically, the productivity channel has a negligible effect on birth rates but accounts for nearly all of the increase in per capita output, industrialization, urbanization, and the decline of land share in total income, while the young-age mortality and the cost of children channels account for almost none of the economic transformation but drive much of the demographic change. Our findings suggest that the quantitatively relevant channels through which the demographic and economic transformations transpired were distinct in the case of England. These results can provide guidance to researchers attempting to model the economic and demographic transformations as endogenous phenomena, and they shed light on the relative importance of several previously proposed theoretical mechanisms.

The third significant contribution is our estimation of the sector-specific productivity changes for the case of England, which relies on the assumption of profit maximization and utilizes historical data for factor prices.

Finally, our analysis shows that endogenizing fertility in the two-sector growth model introduces the possibility of balanced growth even in the presence of differential productivity growth, which represents a noteworthy contribution to the literature on the structural transformation.

It is instructive to briefly highlight the implications of changes in young-age mortality and TFP growth rates for birth rates and the level of industrialization.⁶ One effect of the decline in young-age mortality is

⁴If, instead, we chose a different framework for modeling production, for example, assuming the agricultural good production function used in Greenwood and Seshadri (2002), with skilled labor, unskilled labor, and capital as inputs, we would face a great difficulty in attempting to extract productivity changes for such production function from available data.

⁵The crude birth rate (crude death rate (CDR)) is the number of births (deaths) in a given year per 1000 people.

⁶In this paper we refer to the fraction of non-rural output in the total output as “the level of industrialization” and the fraction of labor employed by the non-rural sector in the total labor as “the level of urbanization.”

that fewer births are needed to realize the desired number of surviving children. In addition, declining young-age mortality lowers the time cost per surviving child, thus relaxing the budget constraint and allowing parents to optimally adjust the number and quality of their surviving offspring. The transition to a more rapidly growing TFP and hence income may also alter fertility choices. On one hand, higher income growth induces higher fertility, because children enter parental utility. On the other hand, it raises the opportunity cost of rearing children measured in terms of foregone wage earnings, thus dampening fertility.

Interestingly, when separate experiments are conducted to independently determine the implications of technological progress and the change in young-age mortality, it is found that each generates full resource reallocation towards the urban sector in the long run. As, according to our estimates, the Solow TFP begins to grow more rapidly than the Malthusian TFP, it attracts an increasingly higher proportion of resources. The result that falling young-age mortality causes an increase in the level of industrialization is less intuitive. As the probability of survival increases, the time cost of raising a surviving child declines, which leads to an augmentation of the aggregate labor supply. As a result, resources tend to shift towards the more labor intensive sector. Although both changes generate a transition from Malthus to Solow, changes in young-age mortality lead to a much slower transition, representing an insignificant contribution to industrialization for the period of interest. This point demonstrates that in carrying out the quantitative analysis it is of utmost importance to solve for the transitional dynamics (from one balanced growth path towards another) triggered by the examined exogenous changes. Indeed, if we had focused on the comparative statics alone, we would have erroneously attributed an important role to youth mortality declines in generating resource reallocation towards the Solow sector.

The remainder of the paper is organized as follows. In Section II we summarize the historical data for England. Related literature and the relationship of our work to the literature on structural transformation are discussed in Section III. In Section IV we set up the model, discuss its equilibrium properties and derive the qualitative differences between the results of this model and those of HP. The model’s calibration and estimation of TFP time series are presented in Section V. The main results are reported in Section VI, where we further contrast our quantitative results with those of HP. We investigate the time cost channel in Section VII. In Section VIII, we present discussion on the correct interpretation of our results and describe the possible causal links between them. We also discuss how these results can be used to guide future research and contrast our work with that of Galor and Weil (2000) (henceforth “GW”) - probably the most well-known unified growth theory. We present a sensitivity analysis in Section IX and conclude in Section X.

II. Motivating Facts about England

We chose to focus on England because its data are most complete.⁷ Floud and Johnson (2004) and Chesnais (1992) describe England during this period. Lee (2006) and Galor (2005) provide general accounts of the demographic change and facts concerning development.

Figure 1a displays the natural log of the real GDP per capita index.⁸ After remaining stagnant

⁷All data sources used in this paper are listed in the appendix.

⁸Data sources for this figure are Clark (2001a) for 1560-1860 and Maddison (1995) for 1850-1992.

for centuries, real GDP per capita took off in the beginning of the 19th century. This period is also characterized by a large-scale shift of the population from the rural sector to the urban sector. As depicted in Figures 1b and 1c, the share of the urban GDP rose from around 30% in the 1550s to roughly 98% in the 1990s, while the share of employment in non-rural production increased from around 40% to 98%.⁹ Further, the land income share fell from as much as 30% at the outset of the 17th century to nearly 0% today (Figure 1d).

The dramatic economic transformation described above was accompanied by remarkable demographic changes (Figure 1e).¹⁰ Before the mid 18th century, both birth and death rates remained high, with the average population growth in the first half of the 18th century remaining low (approximately 0.4% per year.) The decrease in the CDR beginning in the second half of the 18th century was due mainly to declining adult mortality. Sustained decline of the mortality rates for the age groups 5-10, 10-15, and 15-25 began in the mid 19th century, while that for the age group 0-5 began three decades later (Wrigley et al. (1997)). Major factors behind the decline in mortality were the sanitary revolution, which reduced fatalities due to water-borne and food-borne disease and advances in medical science, most notably, the discovery of the benefits of pasteurization, hospitalization, and small pox vaccination.

A sustained fall in birth rates, driven by a fall in marital fertility, occurred from 1870 to 1930, after which both birth and death rates stabilized at lower levels. Previous changes in birth rates resulted from changes in the timing and prevalence of marriage (Floud and Johnson (2004) and Wilson and Woods (1991)). Although the fall in birth rates lagged behind the onset of the fall in death rates, it coincided with the fall in young-age mortality (Figure 1e).¹¹ Note that the lag between the drop in CDR and the drop in CBR resulted in a hump-shaped population growth rate. The general fertility rate (GFR),¹² a measure less sensitive to the age structure of the population than CBR, exhibited similar behavior (Figure 1f). Figure 1f reveals that the number of surviving children also declined during 1870-1930.¹³

Figure 2 plots our own sector-specific TFP estimates. We postpone the discussion of the estimation methodology to Section V. The rural TFP exhibited a somewhat higher growth than the non-rural TFP until the second half of the 18th century, when the growth of the urban TFP sharply increased surpassing that of the rural TFP. Around 1800, the growth of the rural TFP caught up slightly. This short-lived trend marks a small-scale agricultural revolution subsequent to the industrial revolution.

III. Related Literature

Galor (2005) and Doepke (2008) provide useful discussions of the literature on long-run development. We limit the present discussion to work emphasizing the channels of child mortality, technological progress

⁹Data for the level of industrialization and urbanization up to 1860 are taken from Clark (2001a, 2002); the time series are continued using Maddison's data (1995).

¹⁰Data for CBR and CDR are taken from Wrigley et al. (1997) up to 1871 and continued using the data in Mitchell (1978).

¹¹The probability of surviving to age 25 is calculated from age-specific mortality rates taken from Wrigley et al. (1997) and the Human Mortality Database.

¹²The general fertility rate is the number of births in a given year per 1000 females of ages 15-44.

¹³To construct this series, we multiply the probability of survival to the age of 25 by the original GFR series. Note that the Net Reproduction Rate, which is commonly mistaken to reflect the measure of surviving children per woman, instead reflects the number of daughters that would be born to a woman if she went through her lifetime conforming to age-specific fertility and mortality rates in a given year.

and changing time cost of raising children. We also discuss the relationship of our work to the more recent literature on structural transformation.

Among the studies allowing for an important role of *mortality* in driving the demographic and/or economic transition are Ehrlich and Lui (1991), Wolpin (1997), Eckstein et al. (1999), Galor and Weil (1999), Kalemli-Ozcan et al. (2000), Kalemli-Ozcan (2002), Boucekkine et al. (2002), Lagerlof (2003a), Cervellati and Sunde (2005), Doepke (2005), Soares (2005), Tamura (2006).¹⁴ These mechanism build on a quantity-quality trade-off. Boldrin and Jones (2002) also explores the role of child mortality in a framework that reverses the direction of altruism to flow from children to parents. Ehrlich and Lui (1991), Kalemli-Ozcan (2002), Lagerlof (2003a), Soares (2005) and Tamura (2006) explicitly model children's human capital accumulation and assume increasing returns to scale to parents' human capital and time spent with children in production of children's human capital.¹⁵ Because the production of the surviving children's human capital requires a quantity of parents' time that is proportional to their fertility, a drop in young-age mortality raises the return to human capital investment. The necessary parametric restrictions ensure a transition to a sustained growth regime through the substitution of quality for quantity. Several of these studies, in particular, Ehrlich and Lui (1991), Kalemli-Ozcan (2002), Lagerlof (2003a), Soares (2005) and Tamura (2006) conclude that a decline in child mortality results in a significant reduction of the number of surviving children and pulls the economy onto a sustained growth path.¹⁶

The view that *technological progress* is at the center of the demographic and economic transformation is also common. (See Becker and Lewis (1973), Becker (1981), Hotz et al. (1997), GW, Fernandez-Villaverde (2001), Greenwood and Seshadri (2002), HP.) In fact, our findings are qualitatively consistent with this view, as they imply that changes in TFP growth trigger convergence to a sustained growth regime characterized by lower fertility. However, we find the quantitative effect of technological progress on birth rates to be small.

With regard to the method of modeling production, our work is closely related to HP. HP assume an exogenously given population growth as a function of living standards. Both sectors are assumed to enjoy constant TFP growth. Stagnation is generated by (1) assuming parameters such that the Solow sector does not operate and (2) the assumption on the population growth function to guarantee that the Malthusian output per capita is fixed. For a large enough Solow TFP, the Solow sector begins operating and pulls resources out of the land-based sector, thus triggering the structural transformation characterized by increasing standards of living and declining importance of land as a factor of production. The HP calibration exercise aims to capture both stagnation and the growth rate of postwar US income. In this paper, we combine the HP production setup with a model of fertility choice and in addition introduce young-age mortality. In Sections IV and VI, we contrast our results with those in HP.

Fernandez-Villaverde (2001) uses a parameterized framework in which unskilled labor and capital are substitutes, while skilled labor and capital are complements. Capital-specific technological change that matches the fall in the relative price of capital equipment during the years of falling birth rates, 1875-

¹⁴Empirical results pointing to mortality as one of the most important determinants of fertility and/or the onset and speed of its decline are reported in Woods (1987), Bos and Bulatao (1990), Shultz (1997) and Mason (1997a), among others.

¹⁵Doepke (2005) also studies a setup with human capital accumulation. However, he assumes that children's human capital is a decreasing returns to scale function of only parents' time spent with children.

¹⁶Kalemli-Ozcan et al. (2000), Boucekkine et al. (2002), Cervellati and Sunde (2005), Soares (2005) explore the effect of changes in *adult* mortality on human capital accumulation.

1920, is introduced into the model and found to be important in accounting for the observed patterns of fertility and per capita income in England. However, the empirical fact that after 1920 the relative price of capital and capital equipment began to increase reaching almost its 1875 level, while fertility remained roughly constant, is difficult to reconcile with this finding.

Greenwood and Seshadri (2002) uses a two-sector model with exogenous technological progress and endogenous fertility to study the case of the U.S. The preference parameters ensure that as incomes increase, the demand for the agricultural good relative to the manufacturing good declines. Because unskilled labor is not used in the production of the manufacturing good, parents substitute quality for quantity. GW presents a theoretical model with explicit human capital accumulation, endogenous technological change and fertility. In that work, children's human capital is a function of the TFP growth rate and parental time investment in raising children. This function is chosen so as to satisfy several assumptions guaranteeing that parents respond to the acceleration of technological progress, initially fueled by increasing population size, by having fewer, higher quality children. The growing time investment then feeds back into higher technological progress.^{17, 18}

There is also a number of mechanisms that give a central role to the rise of the *time cost* of raising children. The time cost may increase due to changes such as a declining contribution of children to family production with the shift away from agriculture, the rise in the value of female time (Galor and Weil, 1996, Lagerlof, 2003), introduction of child labor laws and education reforms (Hazan and Berdugo, 2005, Doepke, 2004, Doepke and Zilibotti, 2005), changes in the public provision of old-age security or development of credit markets (Boldrin et al. 2005), or parents' explicit decision to spend more time educating offspring in response to, for example, a skill-biased technological change (Greenwood and Seshadri, 2002).

Our paper is also related to the literature on the structural transformation. (See Matsuyama (2005) for a brief outline of the key issues involved.) One strand of this literature assumes lower income elasticity for the agricultural good, and thus generates a structural transformation even in the presence of equal productivity growth rates across sectors. Agricultural productivity growth is necessary for this demand-side explanation to work,¹⁹ and it can be interpreted as releasing labor into the modern sector (See Murphy et al. ,1989, Matsuyama, 1992, Laitner, 2000, Caselli and Coleman, 2001, Gollin et al., 2002, Voigtländer and Voth, 2006.). A different strand of this literature advocates the supply-side explanation, which postulates that the structural transformation is a result of differential productivity growth across sectors. According to this explanation, depending on the substitutability of the two goods, the sector experiencing higher productivity growth will either pull or push resources. The pull effect of advances in industrial productivity is featured in Lewis (1954), Hansen and Prescott (2000), Doepke (2004) and the present paper. The push effect is featured in Ngai and Pissarides (2006) who aim to obtain balanced aggregate growth alongside structural reallocation towards low productivity growth sectors. Note that our observation that beginning in the 17th century factors reallocated towards a more rapidly growing

¹⁷Lagerlof (2006) performs a quantitative test of this model.

¹⁸Boucekkine et al. (2007) suggests that population density, through lowering the cost of school setups, can foster productivity growth, thus providing an explanation for the link between population and productivity postulated in Galor and Weil (2000).

¹⁹Growth in income resulting from TFP growth in the modern sector is not sufficient as its effect on relative consumption demand is offset by the rising relative price of the agricultural good.

modern sector stands in contrast to their description of a more recent structural change. In addition, while their goal of generating balanced growth behavior together with the structural change is justified for capturing the 20th century experience of developed countries, it is not justified for our period of interest, characterized by accelerating income growth, changing factor income shares and increasing capital-output ratio. Although Acemoglu and Guerrieri (2008) assume identical productivity growth across sectors and instead focus on factor allocation across sectors differentiated by capital intensity, they also aim to generate approximate balanced growth. Capital deepening results in factor reallocation away from the capital-intensive sector – a push effect induced by capital accumulation. Our work captures both the effects of differential productivity growth and capital deepening interaction with differential capital intensities. In addition, we study the effect of (endogenous) changes in the population size on factor allocation across sectors with different labor intensities. The important contribution of the present paper to this literature is to show that endogenizing fertility introduces the possibility of balanced growth even in the presence of differential productivity growth.

IV. Model

A. Environment

Technology and Firms

As in HP, firms are endowed with one of two possible technologies to be used in production of the consumption good.²⁰ The Malthusian technology that requires capital, labor, and land as inputs is given the subscript “1,” and it is associated with traditional land-based production taking place in the rural sector. The Solow technology that employs capital and labor as inputs is given the subscript “2,” and it is associated with a modern way of production taking place in the cities. Both technologies exhibit constant returns to scale, which allows us to assume two aggregate competitive firms (sectors). Output production of these firms is described by $Y_{1t} = A_{1t}K_{1t}^\phi L_{1t}^\mu \Lambda_t^{1-\phi-\mu}$ and $Y_{2t} = A_{2t}K_{2t}^\theta L_{2t}^{1-\theta}$, where K_j and L_j denote the capital and labor employed by technology $j \in \{1, 2\}$, and Λ_t denotes the land input. Exogenous technological change augments TFP in both technologies, so that $A_{jt} = A_{j0} \prod_{\tau=0}^{t-1} \gamma_{j\tau}$, $j \in \{1, 2\}$. Letting w_t , r_t , and ρ_t denote the real wage, capital rental rate, and land rental price at time t , we can describe profit maximization by

$$\begin{aligned} \max_{K_{1t}, L_{1t}, \Lambda_t} \quad & A_{1t}K_{1t}^\phi L_{1t}^\mu \Lambda_t^{1-\phi-\mu} - w_t L_{1t} - r_t K_{1t} - \rho_t \Lambda_t, \\ \max_{K_{2t}, L_{2t}} \quad & A_{2t}K_{2t}^\theta L_{2t}^{1-\theta} - w_t L_{2t} - r_t K_{2t}. \end{aligned}$$

Preferences, Households and Dynasties

There is a measure 1 of identical dynasties, each populated by N_t households at time t . Households live for two periods, childhood and adulthood. An adult household derives utility from its own consumption (c_t), the number of its surviving children (n_t), and its children’s average utility according to $U_t = \alpha \log c_t + (1 - \alpha) \log n_t + \beta U_{t+1}$, where $\alpha, \beta \in (0, 1)$. This utility function, also used in Lucas (2002),

²⁰An alternative interpretation of this setup is that the two sectors produce different goods which satisfy the same need and hence enter into households’ utility as perfect substitutes (See Zveimüller (2000)).

is increasing and concave in the number of children, like the utility used in Barro and Becker (1989), $U_t = c_t^\sigma + \beta n_t^{1-\varepsilon} U_{t+1}$. The following proposition states that these preferences are equivalent under certain parametric restrictions. We also explore the BB utility in the sensitivity section.

Proposition 1 *The form of the parental utility used in Lucas (2002), $U_t(c_t, n_t, U_{t+1}) = \alpha \log c_t + (1 - \alpha) \log n_t + \beta U_{t+1}$, represents the same preferences as the Barro and Becker utility, $U_t(c_t, n_t, U_{t+1}) = c_t^\sigma + \beta n_t^{1-\varepsilon} U_{t+1}$, if $\sigma \rightarrow 0$ and $\frac{1-\varepsilon-\sigma}{\sigma} = \frac{1-\alpha-\beta}{\alpha\beta}$.*

Proof. See the appendix. ■

A fraction π_t of children born (f_t) survive to adulthood,²¹ and thus $f_t = \frac{n_t}{\pi_t}$ newborns are needed to realize n_t surviving offspring. A household must spend a fraction a of its time with each born child and an additional fraction b with each child who lives to adulthood.²² Allowing two parameters govern the cost of raising children enables us to capture the young-age mortality profile. For example, a high value of b relative to a captures the empirical observation that children not surviving to adulthood tend to die very early in life. We will return to this discussion when calibrating the model. The total time spent raising children is hence given by $a f_t + b n_t = \left(\frac{a}{\pi_t} + b\right) n_t$. We let $q_t \equiv \frac{a}{\pi_t} + b$ denote the net time cost per surviving child. Observe that q_t is a decreasing function of π_t . Intuitively, as more newborn children survive to adulthood, fewer newborns are needed to realize one surviving offspring and hence less time is spent rearing non-survivors.

An adult household rents out its land holdings (λ_t) and capital (k_t), and devotes all time not spent raising children to work ($l_t = 1 - q_t n_t$). Given $\{w_t, r_t, \rho_t, q_t\}_{t=0}^\infty$, households choose consumption, the number of surviving children, the amount of capital (k_{t+1}) to pass on to each surviving child, and divide their land holdings equally among its descendants. The problem faced by an adult household is thus given by

$$\begin{aligned} U_t(k_t, \lambda_t) &= \max_{c_t, n_t, \lambda_{t+1}, k_{t+1} \geq 0} \alpha \log c_t + (1 - \alpha) \log n_t + \beta U_{t+1}(k_{t+1}, \lambda_{t+1}) \\ \text{subject to } c_t + k_{t+1} n_t &= (1 - q_t n_t) w_t + (r_t + 1 - \delta) k_t + \rho_t \lambda_t, \\ \lambda_{t+1} &= \frac{\lambda_t}{n_t}. \end{aligned}$$

It is common to assume that the conjecture about $U_{t+1}(k_{t+1}, \lambda_{t+1})$ formed by a time t adult household must correspond to the actual level of its children's utility resulting from their optimal response to inheriting (k_{t+1}, λ_{t+1}) . In other words, we focus on subgame perfect equilibria of an infinite horizon dynastic game, in which at each time, current adults solve the above problem. As in Golosov, Jones, Tertilt (2006), it can be shown that the subgame perfect equilibrium outcome of such game is unique²³ and coincides with the unique solution to the dynastic problem below²⁴, where the objective function

²¹There is no uncertainty in the survival of newborns' (as in Sah (1991) or Kalemli-Ozcan (2002)) that would give rise to precautionary motives for having children.

²²If the cost of raising children were to be paid in terms of the final good, the results would not change. In that case, for the existence of a balanced growth path along which per capita variables grow at constant rates, we would need to assume that the goods cost grows in proportion to income.

²³The only equilibria considered are those that are limits of equilibria of the finite horizon truncations of this infinite horizon game.

²⁴Bar and Leukhina (2007) prove uniqueness of the solution to DP.

is obtained applying recursive substitution to household utility. Given $\{w_t, r_t, \rho_t, q_t\}_{t=0}^{\infty}$, the dynastic planner (or the original household) solves

$$\begin{aligned} & \max_{\{c_t, n_t, \lambda_{t+1}, k_{t+1}\}_{t \geq 0}} \sum_{t=0}^{\infty} \beta^t (\alpha \log c_t + (1 - \alpha) \log n_t) \\ \text{subject to } & c_t + k_{t+1} n_t = (1 - q_t n_t) w_t + (r_t + 1 - \delta) k_t + \rho_t \lambda_t, \quad \forall t, \\ & \lambda_{t+1} = \frac{\lambda_t}{n_t}, \quad c_t, n_t, k_{t+1} \geq 0, \quad k_0, \lambda_0 \text{ given.} \end{aligned}$$

Note that we avoid heterogeneity in skills and human capital considerations. This greatly reduces the difficulty of mapping observables into our model, thus later enabling us to estimate sector-specific TFP and calibrate the model in a meaningful way. Note that modeling physical capital accumulation is similar to modeling human capital in the sense that it allows parents to affect future utility of their offspring by spending resources today.

Population Dynamics and Market Clearing

The number of adult households evolves according to $N_{t+1} = n_t N_t$. We use upper case letters to denote aggregate quantities: $C_t \equiv c_t N_t$, $K_t \equiv k_t N_t$, $K_{1t} \equiv k_{1t} N_t$, $K_{2t} \equiv k_{2t} N_t$, $L_t = l_t N_t$, $L_{1t} \equiv l_{1t} N_t$, $L_{2t} \equiv l_{2t} N_t$. The market clearing conditions in the final goods, capital, labor, and land markets are given by

$$\begin{aligned} C_t + K_{t+1} &= A_{1t} K_{1t}^{\phi} L_{1t}^{\mu} \Lambda_t^{1-\phi-\mu} + A_{2t} K_{2t}^{\theta} L_{2t}^{1-\theta} + (1 - \delta) K_t, \\ K_{1t} + K_{2t} &= K_t, \quad L_{1t} + L_{2t} = (1 - q_t n_t) N_t, \quad \Lambda_t = \Lambda. \end{aligned}$$

B. Equilibrium

Definition 1 *A competitive equilibrium, for given parameter values, initial conditions (k_0, N_0) and exogenous sequences $\{\gamma_{1t}, \gamma_{2t}, \pi_t\}_{t=0}^{\infty}$, consists of the allocations $\{c_t, n_t, \lambda_t, k_{t+1}, k_{1t}, k_{2t}, l_t, l_{1t}, l_{2t}, N_{t+1}\}_{t=0}^{\infty}$ and prices $\{w_t, r_t, \rho_t\}_{t=0}^{\infty}$ such that firms' and dynastic maximization problems are solved, and all markets clear.*

In Bar and Leukhina (2007), we prove that the first-order and transversality conditions²⁵ characterize the solution to dynastic problem. It is instructive to review the intuition behind the first-order conditions written in dynastic aggregates,

$$\begin{aligned} (1) \quad & \frac{C_{t+1}}{C_t} = \beta (r_{t+1} + 1 - \delta), \\ (2) \quad & \frac{(1 - \alpha - \beta) C_t}{\alpha N_{t+1}} = q_t w_t - \frac{w_{t+1}}{r_{t+1} + 1 - \delta}. \end{aligned}$$

Equation (1) is a standard Euler equation that describes the optimal intertemporal consumption choice. Condition (2) represents the optimal intratemporal choice between consumption and surviving children: The marginal rate of substitution between children and consumption equals their relative price, that is,

²⁵ $\lim_{t \rightarrow \infty} \beta^t \frac{\alpha(r_{t+1} - \delta)}{(N_t - qN_{t+1})w_t + (r_t + 1 - \delta)K_t + \rho_t \Lambda - K_{t+1}} K_t = 0$ and $\lim_{t \rightarrow \infty} \beta^t \frac{\alpha w_t}{(N_t - qN_{t+1})w_t + (r_t + 1 - \delta)K_t + \rho_t \Lambda - K_{t+1}} N_t = 0$ summarize the transversality conditions.

forgone parental wages due to the time cost of raising children less the present value of the child's earnings at $t + 1$.

Due to decreasing returns to scale in capital and labor, the marginal products of the inputs in the Malthusian technology become very large when its capital and labor inputs approach zero. This guarantees that the Malthusian technology is always employed, and factor prices are determined by

$$\begin{aligned} (3) \quad r_t &= \phi A_{1t} K_{1t}^{\phi-1} L_{1t}^{\mu} \Lambda^{1-\phi-\mu}, \\ (4) \quad w_t &= \mu A_{1t} K_{1t}^{\phi} L_{1t}^{\mu-1} \Lambda^{1-\phi-\mu}, \\ (5) \quad \rho_t &= (1 - \phi - \mu) A_{1t} K_{1t}^{\phi} L_{1t}^{\mu} \Lambda^{-\phi-\mu}. \end{aligned}$$

It is profitable to use the Solow technology as long as $1 \geq \frac{1}{A_{2t}} \left(\frac{\phi A_{1t} K_{1t}^{\phi-1} L_{1t}^{\mu} \Lambda^{1-\phi-\mu}}{\theta} \right)^{\theta} \left(\frac{\mu A_{1t} K_{1t}^{\phi} L_{1t}^{\mu-1} \Lambda^{1-\phi-\mu}}{1-\theta} \right)^{1-\theta}$, that is, as long as its unit cost computed when all resources are employed in the Malthusian sector does not exceed 1. With both sectors operating, factor prices equalize across them: $\phi A_1 K_1^{\phi-1} L_1^{\mu} \Lambda^{1-\phi-\mu} = \theta A_2 (K - K_1)^{\theta-1} (L - L_1)^{1-\theta}$ and $\mu A_1 K_1^{\phi} L_1^{\mu-1} \Lambda^{1-\phi-\mu} = (1 - \theta) A_2 (K - K_1)^{\theta} (L - L_1)^{-\theta}$.²⁶

The proposition below is important for understanding the first-order effects of changes in the labor supply and capital levels on optimal resource allocation across sectors. It states that an increase in the level of an input leads to the reallocation of that input towards the sector in which that input has a higher share. This result hinges on our assumption of a single consumption good, or alternatively, of differential goods entering utility as perfect substitutes.^{27,28} Both sectors exhibit diminishing marginal returns to labor, but the sector with higher labor intensity less so. When labor endowment rises, it is optimal to reallocate it in favor of that sector and substitute that sector's output for the output of the less labor intensive sector. The same intuition extends to the case of a rise in capital endowment.

This theoretical result also reemphasizes the significance of quantitative analysis as it reveals that the relative size of factor shares determines important qualitative dynamics.

Proposition 2 *Suppose both sectors operate. The following then holds: (a) if $1 - \theta > \mu$, then $\frac{\partial}{\partial L} \left(\frac{L_1}{L} \right) < 0$; (b) if $\theta > \phi$, then $\frac{\partial}{\partial K} \left(\frac{K_1}{K} \right) < 0$.*

Proof. See the appendix. ■

Limiting Behavior of Equilibrium Time Paths

We can identify three possible types of limiting behavior of equilibrium time paths (i.e. three types of qualitatively distinct balanced growth), characterized by the properties that (i) the ratio of the output in the Solow sector to total output converges to a constant in the interval $(0, 1)$,²⁹ (ii) the level of output in the Solow sector converges to 0, (iii) the ratio of the output in the Malthusian sector to that of the Solow sector converges to 0. We refer to these types of limiting behavior of equilibrium time paths as convergence to the Malthus-Solow balanced growth path (BGP), Malthus BGP, and Solow BGP, respectively.

²⁶ All formal derivations of optimal resource allocation for given K_t and L_t are presented in Bar and Leukhina (2007).

²⁷ Zweimüller (2000) also models physically different goods that satisfy the same want as perfect substitutes.

²⁸ Bar and Leukhina (2007) theoretically work out the case of a simplified environment but with more general, constant elasticity of substitution, preferences, showing that this result holds if the goods are substitutes, while the reverse holds if the two goods are complements. They also discuss the case of preferences with non-constant elasticity.

²⁹ Along such a balanced growth path, the two sectors operate side by side forever, with the relative outputs constant.

The behavior of equilibrium allocations depends on the choice of the parameter values and initial conditions. In Bar and Leukhina (2007), we derive systems of equations summarizing balanced growth properties and obtain comparative statics results for each type of these balanced growth paths. We also discuss in detail how parameter values and initial conditions affect the limiting behavior of equilibrium time paths.

Along a Malthus-Solow BGP, population growth (n) and per capita output growth (γ) are determined by the TFP growth rates in the two sectors:³⁰

$$(6) \quad \gamma^{MS} = \gamma_2^{\frac{1}{1-\theta}}, \quad n^{MS} = \left(\gamma_1 \gamma_2^{-\frac{1-\phi}{1-\theta}} \right)^{\frac{1}{1-\phi-\mu}}.$$

The growth rate of per capita output is an increasing function of the Solow TFP growth rate, while population growth increases in the Malthusian TFP growth rate and decreases in the Solow TFP growth rate ($\frac{\partial \gamma^{MS}}{\partial \gamma_1} = 0$, $\frac{\partial \gamma^{MS}}{\partial \gamma_2} > 0$, $\frac{\partial n^{MS}}{\partial \gamma_1} > 0$, $\frac{\partial n^{MS}}{\partial \gamma_2} < 0$). The time cost of raising children does not enter these two equations ($\frac{\partial \gamma^{MS}}{\partial q} = \frac{\partial n^{MS}}{\partial q} = 0$), and therefore a rise in π results in a proportional reduction of fertility ($n = \pi f$). For the class of simulations involving an increase in π such that the type of limiting behavior of equilibrium paths is unaltered as a result of this increase, we found that during the transition from the original to a new BGP, population growth exhibits a hump.

Although Malthus BGP and Solow BGP properties do not have a closed-form solution, we derive the following comparative statics results. Along both types of balanced growth, an increase in TFP growth dampens population growth and encourages economic growth ($\frac{\partial n^M}{\partial \gamma_1} < 0$, $\frac{\partial \gamma^M}{\partial \gamma_1} > 0$, $\frac{\partial n^S}{\partial \gamma_2} < 0$, $\frac{\partial \gamma^S}{\partial \gamma_2} > 0$), while a decline in young-age mortality leads to a higher population growth ($\frac{\partial n^M}{\partial q} < 0$, $\frac{\partial n^S}{\partial q} < 0$). Along a Malthus BGP, a decline in young-age mortality, through its positive effect on population growth, tends to slow down economic growth ($\frac{\partial \gamma^M}{\partial q} > 0$), while the growth rate of per capita variables along any Solow BGP, $\gamma^S = \gamma_2^{\frac{1}{1-\theta}}$, is independent of q .

Before moving on, we note that the above comparative statics results should be interpreted with caution. Specifically, it must be kept in mind that it is possible for the dynamic system to undergo a bifurcation in response to a change in parameter values; i.e., it is possible for the type of limiting behavior of equilibrium paths to change qualitatively. In such a situation, the comparative statics results given above are meaningless.

The existence of three types of limiting behavior here contrasts with the situation studied in HP. In that work, which posits population growth to be a particularly chosen function of living standards, only Malthus BGP and Solow BGP can emerge.³¹ The existence of a Malthus-Solow BGP in our model of endogenous fertility choice is of particular significance. The data reveal that as early as 1600, a substantial part of output was already produced by the modern sector, for several decades the relative output ratio fluctuating around a constant (Figure 2). Likewise, output growth, factor shares, birth rates and young-age mortality exhibited no trend during this period, compelling us to exploit this balanced behavior in

³⁰This result is due to the constancy of the interest rate along a Malthus-Solow BGP and the equality of the marginal products of capital in the two sectors. Hence, it is robust to the choice of the utility function.

³¹Note this does not mean that in that work the population growth function could not be chosen to imply existence of a Malthus-Solow balanced growth.

the data to identify the model's parameters. Hence, it is important that the model can generate this behavior qualitatively. By contrast, HP obtain preindustrial balanced growth, or rather stagnation, by assuming it unprofitable to employ the Solow technology.

Furthermore, the growth of per capita variables along the Malthus-Solow BGP is given by $\gamma_2^{\frac{1}{1-\theta}}$ and is thus determined by the Solow productivity growth. Because our estimates reveal slow Solow progress during the period around 1600, our model's ability to successfully capture the experience of this period is further validated. Contrastingly, HP obtain stagnation by assuming that population grows at the rate of the Malthusian output. Thus, our results reveal that the HP production setup when combined with the BB model of fertility choice can deliver preindustrial behavior without relying on the two assumptions made in HP.

It is further instructive to note that if when starting on a Malthus-Solow BGP, a change in parameters causes our dynamical system to exhibit a bifurcation to an asymptotic Solow BGP characterized by a higher growth rate of per capita variables, then this change in parameters must involve an increase in γ_2 . This is true because the relation $\gamma = \gamma_2^{\frac{1}{1-\theta}}$ holds along both Malthus-Solow and Solow BGPs. Even though there is no closed-form solution for the growth rate along the Malthus BGP, it can be shown that an increase in γ_2 is also needed to generate an increase in the growth rate if the system transitions from a Malthus BGP to either a Malthus-Solow or Solow BGP. Thus, our results do not support the strict form of the HP result that the takeoff arises in the presence of fixed productivity growth rates.

Because our analysis shows that productivity acceleration is necessary for the HP framework with endogenized fertility choice to generate the economic transition, the present study has reconciled one of the key differences between the HP model and the GW model. The latter is the most well-known unified growth theory, and it emphasizes the role of productivity acceleration in the process of long-run development.

V. Calibration and TFP estimation

One objective is to calibrate the model parameters so as to match certain key data moments characterizing the English economy at the outset of the 17th century. Because per capita output growth, birth rates, factor shares in total income, young-age mortality, levels of urbanization and industrialization exhibited no trend during the period 1580-1650 and because a significant part of output was already produced non-rurally, we mapped the data moments from this period into the parameters of the model assumed to be on a Malthus-Solow BGP.^{32, 33} Another objective is to estimate the time series of TFP in the rural and non-rural sectors. Because there are no data on time series of inputs and outputs for the two sectors, which are necessary for standard growth accounting, we implemented the dual approach of TFP estimation. This approach employs the assumption of profit-maximization and requires time series

³²This assumption, given the calibrated parameter values, pins down the initial conditions, N_0 and k_0 .

³³Note that a Malthus-Solow type balanced growth occurs on a set of parameter values and initial conditions of positive measure. In Bar and Leukhina (2007), we first show that the Malthus-Solow BGP implied by the calibration is locally structurally stable. In other words, for a small enough change in each calibrated parameter, the limiting behavior of the solution is of Malthus-Solow type, i.e. it remains qualitatively unchanged. Second, we show local Lyapunov stability of the Malthus-Solow BGP for each of these new parameter combinations. In other words, for small enough changes in the initial conditions k_0 and N_0 , the limiting behavior remains quantitatively unchanged.

data on wages in the two sectors, land rental prices, capital rental rates, and the GDP deflator. The procedure we used for TFP estimation is intertwined with calibration, and for this reason we describe both of them in this section.

We chose each time period to represent 25 years. To be calibrated are the Malthusian parameters, A_{10}, γ_1, ϕ and μ , the Solow parameters, A_{20}, γ_2 and θ , the preference parameters, α and β , the cost of children parameters, a, b and π , and the remaining parameters, Λ and δ .

Land is a fixed factor whose value we normalized to 1. Since A_{10} and Λ always appear as a product ($A_{10}\Lambda^{1-\phi-\mu}$), we are allowed a second degree of normalization, and we set $A_{10} = 100$. For simplicity, we also set $A_{20} = 100$, as we lack a criterion for making a more meaningful choice.³⁴ Thus, we have 11 parameters left to calibrate. In order to pin them down, we rewrite the balanced growth path equations in terms of moments and parameters only, and then solve for the model parameters using the 11 pieces of information presented in Table 1.³⁵ The numbers in parenthesis in the table and the rest of the paper represent annual rates.

Note that we do not aim to match per capita output growth and population growth in our model because, although stationary, these moments are quite volatile near the beginning of the 17th century. We do, however, compare these moments to their counterparts predicted by the calibrated model. Historical estimates of the annual depreciation rate range from 2.5% (Clark 2002) to over 15% (Allen 1982). We set $\delta = 0.723$ to realize 5% annual depreciation. The probability of surviving to age 25 around 1600 was roughly constant at approximately 67%. (Wrigley et al. (1997)). Hence, π is also pinned down directly by its data counterpart.

Clark (2001a) provides the labor and capital shares of the total output produced in England, as well as the relative levels of employment and output in the two sectors. The interest rate is taken from Clark (2001b). The fraction of time spent raising children (qn) is set to 0.42. There is no obvious way to infer qn from the data, but a simple example may be illustrative. For a person with 100 hours of time endowment per week, of which he works 40 hours, rests 30 hours and spends 30 hours with children, we would infer $qn = \frac{30}{30+40} \cong .429$, because there is no leisure in our model. Recall that a is the fraction of time spent on each newborn child, while b represents the additional time cost incurred when a child lives to become an adult. We set $\frac{a+b}{a} = 4$, using the data on age-specific mortality and the assumption that the instantaneous cost function of raising a child is a decreasing linear function of the child's age.³⁶ The sensitivity of the results to the choice of δ , qn , and $\frac{a+b}{a}$ is addressed in Section VI. Our method for obtaining $\gamma_{1,1600}$ and $\gamma_{2,1600}$ is described below.

Calibrating ϕ, μ, θ

We determine the labor share $\mu = 0.537$ of the Malthusian technology using $\frac{y_1}{y}$, $\frac{l_1}{l}$, $\frac{wl}{y}$ and the equilibrium property that wages equal the marginal product of labor in the Malthusian sector, $w \frac{l}{y} = \left(\frac{\mu y_1}{l_1} \right) \frac{l}{y}$. With μ known, we pin down the capital share of the Solow technology, θ , by using $\frac{y_1}{y}$, $\frac{wl}{y}$, and the equality of the total labor income and the sum of incomes paid to labor in the two sectors, $\mu \frac{y_1}{y} + (1 - \theta) \frac{y_2}{y} = \frac{wl}{y}$.

³⁴The choice of value for A_{20} affects the magnitude of level variables, such as output or population size. Because we study growth rates and fractions of level variables, our results are insensitive to this choice.

³⁵For a more technical description of the calibration process, which consists of solving this system of linear equations, see Bar and Leukhina (2007).

³⁶See the appendix for a more detailed explanation of how we arrive at this quantity.

This yields $\theta = 0.273$. Similarly, the capital share of the Malthusian technology, ϕ , is determined by $\frac{y_1}{y}$, $\frac{rk}{y}$, and the equality of the total income paid to capital and the sum of rental incomes paid to capital in each sector, $\phi\frac{y_1}{y} + \theta\frac{y_1}{y} = \frac{rk}{y}$. This gives $\phi = 0.104$.

Calibrating γ_1 and γ_2 and Estimating TFP Series

We next explain how $\gamma_{1,1600}$ and $\gamma_{2,1600}$ are obtained. We first estimate TFP time series for each sector during 1585-1915.³⁷ Then, for each of these series we fit a trend consisting of two parts, each characterized by a constant growth rate. The growth rates characterizing the first part of the TFP trends in the two sectors are denoted by $\gamma_{1,1600}$ and $\gamma_{2,1600}$. In order to estimate the TFP time series, we use the inferred factor income shares in the two sectors, ϕ, μ and θ .

From profit maximization of the firms, we derive

$$(7) \quad A_{1t} = \left(\frac{r_t}{\phi}\right)^\phi \left(\frac{w_{1t}}{\mu}\right)^\mu \left(\frac{\rho_t}{1-\phi-\mu}\right)^{1-\phi-\mu},$$

$$(8) \quad A_{2t} = \left(\frac{r_t}{\theta}\right)^\theta \left(\frac{w_{2t}}{1-\theta}\right)^{1-\theta},$$

where r_t (%) is the rental rate of capital, w_t is the real wage measured in units of the final good per unit of labor, and ρ_t is the land rental price measured in units of the final good per acre. Using Clark's data on the time series of r_t (%), nominal wages ω_{1t} and ω_{2t} (£), $\tilde{\rho}_t$ (% return on land rent), $P_{\Lambda t}$ (price of land in £/acre), and the GDP deflator P_t , we infer the real wages w_{it} and the real land rental price ρ_t using $w_{it} = \frac{\omega_{it}}{P_t}$ and $\rho_t = \frac{\tilde{\rho}_t P_{\Lambda t}}{P_t}$. Substituting these into the above (7) and (8) yields equations for TFP estimates \hat{A}_{1t} and \hat{A}_{2t} .

Figure 2 displays these time series together with their trends. To see how a constant growth trend with a regime switch is fitted to a given series, let x_t represent the data and y_t its trend, restricted to the form

$$y_t = \begin{cases} y_0 g_1^t & 0 \leq t \leq \tau \\ y_0 g_1^\tau g_2^{t-\tau} & \tau \leq t \leq T \end{cases},$$

where g_1 and g_2 denote the growth rates in the first and second growth regimes, and τ represents the timing of the regime switch. To find the trend, we solve $\min_{y_0, g_1, g_2, \tau} \sum_{t=0}^T (y_t - x_t)^2$. Note that this procedure determines the two growth rates and the timing of the regime switch. Applying this method to both of the TFP time series, we obtain the TFP growth rates characterizing the first part of the trends, $\gamma_{1,1600} = 1.042$ (0.16%) and $\gamma_{2,1600} = 1.006$ (0.025%), as well as the endpoint growth rates, $\gamma_{1,1900} = 1.126$ (0.4%) and $\gamma_{2,1900} = 1.174$ (0.6%).³⁸

Interestingly, $\gamma_{1,1600}$ and $\gamma_{2,1600}$ yield predictions for the growth rate of the population and per capita output around 1600 (Equation 6). These predictions, $n = \left(\gamma_1 \gamma_2^{-\frac{1-\phi}{1-\theta}}\right)^{\frac{1}{1-\phi-\mu}} = 1.097$ (0.37%) and $\gamma = \gamma_2^{\frac{1}{1-\theta}} = 1.0085$ (0.00034%), are consistent with the data, according to which population grew at the annual rate of 0.4%, while output per capita remained roughly constant.

³⁷See the appendix for a complete description that would allow anyone to reproduce our TFP estimates.

³⁸Our estimation results are in line with those in Antras and Voth (2002). In that work, TFP growth in Britain is estimated for the period 1770-1860, and it is found not to exceed 0.6% annual rate.

Calibrating the Remaining Parameters

The value of preference parameter β is determined to be 0.415 from the Euler equation $\gamma = \frac{\beta}{n} [r + 1 - \delta]$, after we substitute for γ, n , and the gross interest rate.

Time spent with children (qn) and the relation $\frac{a+b}{a}$, together yield $a = 0.085$ and $b = 0.256$. Finally, the balanced growth path feasibility equation, $\frac{c}{k} = r \frac{y}{rk} + 1 - \delta - \gamma n$, gives a prediction for the consumption-capital ratio. Using $\frac{c}{k}, n, \gamma, qn$ and $\frac{l}{l}$ along with the data moments, $r, \frac{rk}{y}$ and $\frac{y_1}{y}$, in the remaining balanced growth path equation, $\frac{(1-\alpha-\beta)(1-qn)}{\alpha\mu} \frac{y}{y_1} \frac{1}{r} \frac{rk}{y} \frac{l}{l} \rho = qn - \frac{\gamma n}{(r+1-\delta)}$, we obtain $\alpha = 0.582$.

The calibrated parameter values are listed in Table 2.

VI. Main Results

In order to quantitatively assess the capability of our model to generate the main features of the economic and demographic change in England, we solve for the calibrated model dynamics while varying the probability of surviving to adulthood according to its historical data and the growth rates of TFP in the two sectors - according to our estimates obtained in Section V. This constitutes our main experiment. The experimental values of γ_1, γ_2 and π are plotted in Figure 3a. Because we do not aim at investigating high-frequency behavior, we smoothed the experimental time series.³⁹

To further assess the separate contributions of young-age mortality and sector-specific productivity channels, two additional experiments were conducted within the calibrated framework, where we employed only one of these two exogenous series. In the first experiment, the growth rates of TFP in the two sectors were varied according to our estimates obtained in Section V with the young-age mortality held fixed at its 1600 level. Thus, we held constant all influences exerted through the mortality channel, including that of productivity. In the second experiment, the probability of surviving to adulthood was varied according to its historical data with the growth rates of TFP in both sectors held at their 1600 values, i.e., holding constant all influences, including that of mortality, communicated through the productivity channel.

In our experiments, the economy starts off on a Malthus-Solow BGP. Each period in the model represents a specific 25-year period in the data. With the appropriate exogenous change fed into the model, the model was solved for the equilibrium dynamics under the assumption of perfect foresight.⁴⁰ Although different types of limiting behavior of equilibrium time paths are possible in our model, in all three experiments, the solution converged to a Solow BGP.⁴¹ Figures 3 and 4 depict the results of the experiments. The figures present the time paths of relevant variables in the data and their model counterparts, result-

³⁹The series for π during the period 1612.5 – 1912.5 was replaced by its 7-period MA. The series for γ_1 and γ_2 were modified by fitting a logistic function to the endpoint growth rates to minimize the distance between the estimated TFP time series and the smoothed trend. The smoothing parameter was restricted to be no more than 3.

⁴⁰The solution method is described in Bar and Leukhina (2007). Briefly, the objective is to find time paths that satisfy the first order, feasibility and transversality conditions. Because the original variables exhibit exponential growth, we work with detrended variables. Since our experiments involve parameter changes, a bifurcation of the dynamical system, i.e., a qualitative change in the type of a BGP towards which convergence takes place, is possible. This forces us to use a non-standard detrending method.

⁴¹For each of the three experiments, the asymptotic BGP towards which convergence takes place is locally stable (possessing a single eigenvalue that is less than 1). This can be understood by noting that if the Malthusian technology is removed, N_t is no longer a state variable. In this case, the only state variable is k_t , and the condition that exactly one eigenvalue be less than 1 is necessary and sufficient for local stability of the BGP towards which convergence takes place.

ing from each of the experiments. The results are summarized in Table 3.⁴² To assess the quantitative importance of different channels in facilitating birth rate dynamics, we compare the model’s results with respect to both CBR and GFR. (Recall that GFR is less sensitive to the population structure, as in its definition it considers births among women of reproductive age.)

Main Experiment: Simultaneous Change in Sector-specific TFP and Young-Age Mortality

The results of this experiment lead us to conclude that the calibrated HP-BB hybrid featuring historical changes in productivity and survival probabilities does remarkably well at generating the main patterns of the economic and demographic development. The dynamics generated by the model are represented by the dashed lines in Figures 3a-3d and 4. Transition from early stagnation to modern growth is well captured by the main experiment: Initially stagnating GDP per capita takes off around 1800 and exhibits a sustained growth of nearly 1% per year (Figure 3b). The increase in per capita GDP obtained in this experiment is approximately 70% of the actual increase in the English per capita GDP during the period 1650 – 1950. The model further captures nearly the entire process of industrialization and urbanization⁴³ (Figures 3d and 4a), generating convergence of the Malthusian output to zero in relative terms. As a result of successfully capturing factor reallocation across sectors with different estimates of factor shares, the model also accounts for over 90% of the long-term trends in the observed income shares (Figures 4b and 4c).

Finally, the model generates a fall in GFR comprising 61% of the fall observed in the data during the period 1650 – 1950 (Figure 3c). Although the population growth rate does increase from 0.37% to 0.8%, this increase is small. It is important to note that since we do not model changes in adult mortality, which greatly affect population growth, we deem it more appropriate to compare the model’s predictions to fertility behavior.^{44, 45}

Overall, the model performs remarkably well, especially in generating the economic changes. In Section VII, we show that introducing an increase in the time cost of children alongside historical changes in productivity and mortality further improves the model’s performance along the demographic dimension, without diminishing its success in generating the economic transformation.

It is useful to further compare our model with the original HP results. As described in Section IV, our model allows for the possibility of a new type of balanced growth which successfully captures the historical behavior found in England around 1600 without relying on the two assumptions used in HP.

Furthermore, there is a major difference between the input series for the sector-specific TFP used in the present work and those used in HP. Contrary to the HP assumption of constant TFP growth rates, our estimates reveal productivity acceleration in both sectors, with the Solow productivity experiencing it to a larger extent. Furthermore, HP choose γ_1 to ensure that output growth matches population growth in the preindustrial period. Anticipating that the economy will converge to the Solow BGP, they choose γ_2

⁴²To understand the values in the table, consider, for example, the first line. Real GDP per capita increased by 379.55% during the period 1600-1950. Experiment 1 generates a smaller change, amounting to 68.34% of 379.55%.

⁴³Levels of urbanization and industrialization are imperfect data counterparts to l_2/l and y_2/y in our model. The main reason is that in the data, rural output is not a perfect substitute for non-rural output, while in the model, the Malthusian good is a perfect substitute for the Solow good. It is, nonetheless, instructive to make these comparisons.

⁴⁴Recall that the observed hump in the English population growth rate resulted from the fact that CDR fell before CBR.

⁴⁵The limiting behavior of the equilibrium time paths is characterized by $y_{t+1}/y_t \rightarrow 1.0088$, $N_{t+1}/N_t \rightarrow 1.008$, $r_t \rightarrow 1.05$, and $c_t/k_t \rightarrow 0.45$.

to match the postwar US income growth. (Population growth is then assumed to equal one.) By contrast, we back out productivity estimates using the dual-approach, which relies on the observed factor prices and the assumption of profit-maximizing behavior. Moreover, we do not constrain the model to match any moments beyond the period around 1600. On the contrary, we use the capability of the calibrated model to generate the observed time series as the main criterion for assessing the model’s quantitative performance.

Note that, consistently with the main hypothesis emphasized in HP, the equilibrium paths of our main experiment converge to an asymptotic Solow BGP, the relative output of the Malthusian sector approaching zero in this limit. We showed that three types of limiting behavior can emerge in our model. Hence, convergence to the Solow BGP could not be anticipated ex-ante, but it was nonetheless obtained as a result of the main experiment. In light of the discussion in Section IV, it should then be pointed out that the uncovered Solow productivity acceleration was necessary for the escape from stagnation characterizing the initial balanced growth and emergence of sustained growth of living standards.

Thus, while our results do not support the strict form of the HP conclusions, i.e. that stagnation is generated because the Solow sector is idle and that the structural transformation and takeoff in income growth transpire under constant productivity growth rates, we conclude that the overall HP production setup combined with a BB-type model of fertility choice can successfully account for the main features of economic and demographic transformations. This framework thus appears to be a useful tool for future studies in this area.

Changes in the Growth Rates of Sector-specific TFP

This experiment reveals that the success of the main experiment along the economic dimension is entirely due to changes in the productivity. In other words, changes in productivity represent an important force behind the observed patterns in per capita income, the level of industrialization and urbanization, and patterns of labor, capital, and land income shares. By contrast, changes in productivity are found to be quantitatively unimportant in driving fertility behavior.

Figures 3b, 3d, 4a-4c indeed reveal that the solid line, which represents the model dynamics when TFP is varied, closely follows the dashed line, which captures the dynamics of the main experiment. In fact, changes in TFP alone generate a slightly larger increase in living standards, explaining 73% of the actual increase in the English per capita GDP during the period 1650 – 1950. The implication of the relative acceleration in the Solow TFP for resource reallocation relies on our assumption of a single consumption good, or alternatively, of differential goods entering utility as perfect substitutes.⁴⁶ Resources tend to reallocate towards the faster growing sector, because from the households’ perspective, output of that sector can be substituted for the output of the slower growing sector.

Interestingly, we find that changes in productivity have a very small quantitative impact on fertility behavior (see Figure 3c⁴⁷). Because children are normal goods, higher income growth exerts upward

⁴⁶Bar and Leukhina (2007) work out the case of a simplified environment but with more general, constant elasticity of substitution, preferences, showing that this result holds if the goods are substitutes, while it reverses if the two goods are complements. For the case of Stone-Geary utility with the Malthusian good assumed to be a necessity (which implies the goods are complements), the relative acceleration is not necessary: reallocation towards the Solow sector is implied by the Malthusian TFP acceleration. We will return to this discussion when discussing the sensitivity of our results with respect to more general preferences.

⁴⁷To compare the results of the experiments to the data we use a 3-period MA representation of CBR and GFR.

pressure on fertility. TFP acceleration also causes an increase in the cost of rearing children through both channels: a rising time cost measured in terms of wages and parents choosing to have higher quality children. Indeed, we can interpret k_{t+1} as a measure of quality, and the ratio k_{t+1}/y_t increases from 0.0675 to 0.113. We find that these two effects nearly offset each other. Through their combined influence, fertility rises slightly, and then declines, with the overall change being small. Similarly, this experiment yields a quantitatively insignificant hump in the population growth rate (see Figure 4d). Starting at the calibrated level of a 0.37% annual rate, the population growth rate increases slightly, and then decreases, converging to a 0.36% annual rate in the limit.⁴⁸

Changes in Young-age Mortality

The results of this experiment suggest that changes in young-age mortality were an important driving force behind the demographic transformation in England but had little influence on the economic changes. Interestingly, the results of the main experiment appear to be a simple sum of changes generated by separately applying TFP growth changes and mortality changes.

Because every child requires an investment of time from the parents, declining young-age mortality lowers the time cost of surviving children, thus relaxing the budget constraint and allowing parents to optimally adjust fertility and the quality of surviving children. The resource cost of children evolves endogenously. In fact, we find that parents choose to raise higher quality children as the ratio k_{t+1}/y_t increases from 0.0675 to 0.1021. Finally, fertility is pressured downwards, because with more newborns living to adulthood, fewer births are needed to realize the desired number of surviving children. The downward pressure on birth rates appears to be stronger overall, as changes in young-age mortality account for nearly 60% of GFR during the period 1650 – 1950 (Figure 3c).

Note that additional factors must have played a role in generating birth rate dynamics. In fact, we explore the significance of the time cost of children channel in the next section.

As π increases, the time spent raising surviving children decreases. As a result, there is more time for work, and the aggregate labor supply increases. Because our calibration of factor shares implies that conditions of Proposition 2 are satisfied, this results in the relative expansion of the urban sector, but the reallocation of resources occurs slowly. Even in 2400, as much as 10% of the total output is still produced in the rural sector. For the period 1650 – 1950, the influence of mortality changes on urbanization and industrialization is found to be quantitatively insignificant (Figures 3d and 4a). Note that if, on the contrary, we performed the comparative statics analysis alone, we would erroneously conclude that a drop in young-age mortality was as important in driving the industrialization/urbanization as sector-specific technical change. Changes in the probability of survival are also found to be quantitatively insignificant in accounting for the GDP per capita dynamics (Figure 3b).⁴⁹

⁴⁸The limiting behavior of the equilibrium time paths is characterized by $y_{t+1}/y_t \rightarrow 1.0088$, $N_{t+1}/N_t \rightarrow 1.0036$, $r_t \rightarrow 1.045$, and $c_t/k_t \rightarrow 0.398$, given in annualized rates.

⁴⁹The limiting behavior of the equilibrium time paths is characterized by $y_{t+1}/y_t \rightarrow 1.00034$, $N_{t+1}/N_t \rightarrow 1.008$, $r_t \rightarrow 1.04$, and $c_t/k_t \rightarrow 0.357$.

VII. On the Time Cost of Raising Children

Throughout the analysis above, we kept a and b constant. With the change in young-age mortality (π rising from 0.67 to 0.98) featured in the main experiment, the time cost of raising children $q = a/\pi + b$ declined in the long run, while the resource cost of children (bequests) evolved endogenously. There are a number of theoretical frameworks, however, that rely on the channel of *rising* time cost (see Section III) in order to generate a fall in the number of surviving children. The rising time cost within our framework can be interpreted as a reduced-form representation of such changes as a declining contribution of children to family production with the shift away from agriculture, a rise in the value of female time, the introduction of child labor laws and compulsory education, or parents' explicit decision to spend more time educating offspring in response to, for example, a skill-biased technological change. The time cost channel in our model should be interpreted very generally as a channel capturing all factors generating income reallocation between consumption spending and spending on children, whether or not these factors directly influence the cost of children relative to consumption.⁵⁰

Because the time cost is not observed in the data, the analysis of the time cost channel cannot be carried out in the same manner as was done for the sector-specific productivity or young-age mortality. It is, however, instructive to investigate whether the rise in the time cost, which can substantially improve the model's match of adult population dynamics, contributes to the model's overall performance, and if so, whether it contributes to both the demographic and the economic dimensions. Indeed, movements in the aggregate labor supply, which is closely tied to the adult population dynamics, have a direct impact on resource allocation and therefore factor shares in total income and economic growth (Proposition 2).

We raise b from its original value of 0.256 to 0.43⁵¹ to guarantee that when sector-specific productivity⁵² and young-age mortality are changed according to their historical paths, the model generates a fall in GFR comparable to that observed historically (from 131 to 66). In other words, we obtain the change in b assuming it to be the residual explanation for the fall in GFR. Such an approach is justified by the more general interpretation of the time cost channel in our model as a reduced form of all factors generating income reallocation between consumption and children. Note that the model's prediction for π (GFR), surviving children per woman, also reflects its fall in the data (from 88 to 65). We implement the change in b by raising it linearly during the period 1887-1987, i.e. beginning at the onset of the fertility decline.

Figures 5b-5f display the model dynamics that result when the original changes in sector-specific TFP and young-age mortality are implemented (dashed line),⁵³ when these changes are accompanied by the rising time cost (circle markers), and when the rise in time cost is implemented alone (solid line). First of all, our results reveal that adding the rising time cost to mortality and productivity changes considerably

⁵⁰ A non-homothetic utility is able to generate income reallocation in the absence of the relative price change.

⁵¹ As a result, the time cost increases from 0.383 to 0.517.

⁵² Recall that the time series of TFP growth rates were estimated on the basis of the data up to 1915. For later years, sector-specific TFP were assumed to retain their constant growth trends ($\gamma_{1,1900}$ and $\gamma_{2,1900}$), although $\gamma_{2,1900}$ is slightly below the estimates obtained when focusing on more recent data. Since the goal here is to obtain the change in the time cost as a *residual* explanation for the fall in GFR, instead of using $\gamma_{2,1900}$ from Table 2, we use $\gamma_{2,1900} = 1.28$ (0.98%), which guarantees that the model matches the growth rate of per capita income in the 20th century (1.4%).

⁵³ Because we are using a slightly higher $\gamma_{2,1900}$ than that used in the original main experiment, this experiment does slightly better along the economic dimension. We present this experiment here in order to allow assessment of the *additional* contribution of the time cost channel.

improves the model’s fit along the demographic dimension⁵⁴ (Figures 5a and 5b), without compromising its capability to successfully capture the economic behavior. In fact, the GDP per capita fit further improves (Figure 5d), while the industrialization and factor shares fit worsens, but only slightly. This finding reinforces our original conclusion that our model, combining the main features of the HH and BB models, appears to be a useful tool for analyzing the long-run development.⁵⁵

Examining the model dynamics resulting from changing the time cost alone reveals that while substantially contributing to the fertility decline, the rising time cost causes resources to reallocate away from the Solow sector (Figure 5c), thus discouraging industrialization. Consequently, it generates movements in factor shares in the opposite direction of their movement in the data (Figures 5e and 5f). These predictions result from the fact that the rising cost of children works to reduce the aggregate labor supply, which, in light of Proposition 2 and our calibration of factor shares, tends to hamper industrialization. Despite its effect on resource allocation away from the faster growing sector, the rising time cost generates a slight increase in the growth rate of GDP/capita because of its dampening effect on population growth (Figure 5d). Comparing the dashed lines with the circles reveals that the additional contribution of the time cost channel to fertility behavior is very important, even more so than the contribution of the mortality channel, while the additional contribution to the economic change is negligible. Interestingly, we see that in the presence of the other channels, the detrimental effect of the rising time cost on structural transformation, present in the experiment involving the cost change alone, is significantly reduced. Our results suggest that, similarly to the influence of the mortality channel, the influence of the time cost channel is largely distinct from the influence of the productivity channel.⁵⁶

VIII. Discussion

In addition to our main result, i.e. that the model combining the HP model of structural change with the BB model of fertility choice, does remarkably well at capturing the main features of the economic and demographic transformation, our analysis of the relative roles of three different channels in the development of England also provides insight. Here, we present a short discussion to avoid the possible misinterpretation of our findings. We then also discuss how our results can provide guidance to researchers attempting to model the economic and demographic transformations as endogenous phenomena. Finally, we contrast our results with those of GW.

By quantifying the influence of a particular channel with regard to a particular phenomenon, we

⁵⁴The rise in b faithfully captures the fall in GFR, but it does not capture its rise during ~1750-1850. Certainly, this rise could be easily generated with a decline in b . However, it seems more likely that gains in adult survival rates played an important role during this period by simply raising marriage rates among the population. In fact, the data given in Wilson and Wood (1991) shows that this rise in GFR is almost entirely due to the rise in marriage prevalence (with marital fertility almost unchanged), while its fall during 1870-1930 is almost entirely due to the fall in marital fertility.

⁵⁵Although this joint variation of the three channels delivers somewhat of a delay in urbanization/industrialization, just as in the original experiment, we believe this is a result of the model’s failure to generate a sufficient rise in the labor supply around 1750–1850, which could most likely be attributed to the declining adult mortality. Note that if adult mortality (or, alternatively, the time endowment) were a part of the model, it would also help explain the rise in GFR that preceded its fall.

⁵⁶We also conducted separate mortality and productivity experiments, each accompanied by a change in the time cost. The results reveal that, even in the presence of a rising time cost, which improves the match of the population dynamics for each of these experiments, our previous finding of largely distinct effects of mortality and productivity channels continues to hold.

ascertained the combined influence of all factors that communicate their effect through that channel. Our finding that the influence exerted by the productivity channel is decoupled from those exerted by the mortality and the time cost channels should thus be interpreted with caution: To the extent that there exist causal links between productivity, mortality and the cost of raising children, the actual effects of these variables on the economic and demographic change need not be decoupled. The limitation of our analysis is that we do not model such causal links. Specifically, when evaluating the influences of a particular channel on the economic and demographic transitions, we held constant the influences that it may exert through the other channels. In what follows, we highlight several potential overlaps between the examined channels.

The influence exerted through the productivity channel may have a causal effect on the mortality decline, as a result of its tendency to increase the standard of living, and consequently improve nutrition and living conditions. Thus, productivity acceleration may influence the demographic change, but the described influence would have to be communicated through the mortality channel. Furthermore, if technological progress is skill-biased, it may induce quality spending at the expense of quantity, the productivity channel thus exerting an additional effect on the demographic change, communicated through the cost channel.

Similarly, factors that tend to lower mortality or the cost of children may affect the economic transformation, but this effect would have to be communicated through productivity acceleration. For example, reductions in young-age mortality could contribute to productivity growth, as described by a number of theories tying increased population size or density to aggregate productivity. If the rural and urban goods are not perfect substitutes, changes in mortality and the cost of children through their effect on the population size would influence the relative price, and hence affect the productivity estimates.

How do our results help guide researchers attempting to model the economic and demographic transformations as endogenous phenomena? First, if human capital accumulation is necessarily associated with a simultaneously rising cost of children, then our results suggest that the technical change in England around the turn of the 19th century could not have been fueled by human capital accumulation, and that it had little causal connection to the demographic behavior. Such a take on English history is consistent with the thesis in Mokyr (2005). Our results, however, do not exclude the possibility that beginning later in the 19th century, technological progress could be fueled by time-intensive human capital accumulation and, through the quantity-quality trade-off, have a stronger connection to the demographic behavior. This means that theories that attempt to endogenize TFP throughout the time period investigated here should allow for the influence of factors other than time-intensive human capital production. Such factors include a surge in the knowledge spillover arising due to the increased population density that results from growing population and urbanization, an improvement in the effectiveness of cross-generational knowledge transmission due to the decline in adult mortality (Bar and Leukhina 2008), the scale effect from the working population size (GW), or the rise in the population quality (Galor and Moav, 2002). Further, our results suggest that forces that manifest themselves through the cost of raising children can potentially account for a large part of the fall in fertility (possibly larger than the direct effect of youth mortality).

It is instructive to contrast the features and predictions of our model with those of GW - the influential unified growth theory. The goal of GW is to generate stylized features of the development process, such

as stagnation and then takeoff, a hump-shaped population growth and productivity acceleration, without any exogenous change. They successfully accomplish this goal. Moreover, because, in addition to the influence of schooling, they allow for the influence of population size on productivity growth, their model is capable of generating productivity acceleration that precedes the fertility decline. In light of the discussion above, this is an important accomplishment. However, as we argue below, their model is a less useful tool for future quantitative studies than the HP-BB hybrid explored in the present study.

First, while the structural transformation was a major part of the economic transformation, the GW model is silent with regard to structural change and, in contrast to empirical observations, generates constant land and labor shares over time.

Second, because the mortality rate is fixed in the GW model, fertility and population growth are represented by the same variable, and hence the increase in the population growth is obtained by modeling the increase in fertility choice.⁵⁷ According to the data, however, the rise in the population growth was largely brought about by declining mortality in the presence of roughly unchanging fertility.⁵⁸ Thus, introducing mortality into the model is essential if one wanted to capture differential empirical behavior of fertility and population growth. Note, however, that as argued in Galor (2005), introducing declining mortality into the GW model would only reinforce the underlying mechanism, which hinges on the early population expansion.

Third, two major predictions of the GW model, human capital and productivity series, do not have observable counterparts in the data, and this makes it difficult to assess the model's validity and to employ it in quantitative studies. Human capital refers to the embodied knowledge used to adopt available techniques for output production. As innovation destroys this knowledge, human capital declines along the simulated time paths. (This does not mean that labor becomes less productive, as gains in technology more than offset the decline in human capital.) Because measures of human capital do not exist in the data, this prediction cannot be tested against the data. More importantly, because both TFP and human capital enter the output production function as unobserved components, it is impossible to infer the empirical counterpart of the GW productivity series from the data, and as a result, the empirical relevance of the productivity series endogenously generated by their model cannot be assessed.

Fourth, the main predictions of the GW model result from the assumptions made regarding the human capital formation function. It is chosen so that education (child quality) increases in the growth rate of technology. An additional assumption is then made to ensure that the optimal education is zero for low levels of productivity growth; i.e., the non-negativity constraint binds. Because the measure of human capital is hard to construct based on the data, it is difficult to assess the empirical relevance of the assumptions made regarding the human capital formation function.

IX. Sensitivity Analysis

TFP Estimates

Recall that the time series of TFP growth rates were estimated on the basis of the data up to 1915. For

⁵⁷Precisely, fertility increases as a result of income growth in the face of a binding subsistence consumption constraint.

⁵⁸In a few countries, fertility did rise slightly, as marriage prevalence increased with growing incomes (Dyson and Murphy (1985)).

later years, sector-specific TFP were assumed to retain their constant growth trends ($\gamma_{1,1900}$ and $\gamma_{2,1900}$). Would changes in TFP growth rates be more successful in accounting for the demographic and economic changes if the growth rate of TFP increased further since 1915? In this sensitivity exercise, we repeat Exp. 1 and 3, but this time with the Solow TFP series updated to guarantee that the model generates the *growth rate* of per capita income in the 20th century (1.4%). Because there is convergence to the Solow BGP, we can determine $\gamma_{2,1900}$ using $\gamma = \gamma_{2,1900}^{\frac{1}{1-\theta}} = 1.4156$ (1.4%). This yields $\gamma_{2,1900} = 1.28$ (0.98%), a slightly higher growth rate than 1.174 (0.64%) used in the original experiments. The original result, that changes in the TFP growth rates drive the economic transformation while having a negligible effect on birth rates, is reconfirmed (Table 4).

Barro and Becker Parental Utility

As proved in the appendix, the parental utility used here, $U_t(c_t, n_t, U_{t+1}) = \alpha \log c_t + (1 - \alpha) \log n_t + \beta U_{t+1}$, is a special case of the BB parental utility, $U_t(c_t, n_t, U_{t+1}) = c_t^\sigma + \beta n_t^{1-\varepsilon} U_{t+1}$, realized when $\sigma \rightarrow 0$ and $\frac{1-\varepsilon-\sigma}{\sigma} = \frac{1-\alpha-\beta}{\alpha\beta}$. Note that this implies that $\varepsilon \rightarrow 1$. A natural question is whether our main results would change if we used the BB parental utility form with $\sigma > 0$ and $\varepsilon < 1$.

First we recalibrated the model under the assumption of the BB utility⁵⁹, using the procedure similar to that described in Section IV, with the only difference being that the calibration procedure used here does not fix both ε and σ . Instead, it pins down the ratio $\frac{1-\varepsilon-\sigma}{\sigma} = 0.0129$, thus allowing one free choice. We performed experiments using several values of ε in the admissible range of (0, 1). For $\varepsilon = 0.9$, which implies that $\sigma = 0.0987$, the results are very close to the original results. Here we report the results for a more extreme case, with $\varepsilon = 0.7$ (and implied $\sigma = 0.2962$).

In this case, again we find the demographic transition is driven mainly by changes in young-age mortality, while the economic transformation is driven mainly by technological progress. However, here we observe that the overall effect on birth rates is weakened.

Sensitivity to δ , $(a + b)/a$, and qn

We find that all of the quantitative results obtained here are extremely robust with respect to changes in δ . Since the estimates of δ vary from 2.5% to 15% in the literature, as mentioned above, we investigated δ in this range.

Recall that $(a + b)/a$ is an estimate of the average time cost of surviving children relative to that of non-surviving children. This quantity only affects the calibration of a and b , and it has no bearing on q . In particular, a decreases and b increases in $(a + b)/a$. An increase of $(a + b)/a$ slightly raises the importance of π in driving the fertility behavior. We examined values of $(a + b)/a$ ranging from 1 to 7, and we found that the results were not affected significantly.

Finally, we set the fraction of time spent raising children, qn , to 0.42. Unfortunately, for $qn \leq 0.411$, we have $1 - \alpha - \beta < 0$, or equivalently $1 - \varepsilon - \sigma < 0$ for the Barro-Becker preferences, which implies that the dynastic utility decreases as the population increases. Although this does not imply that the equilibrium population size will equal zero, as households would still be valued as a factor of production, strict concavity of the objective function would not be guaranteed. For this reason, we only analyzed values of qn in the range [0.411 – 0.7]. For this range, we found little quantitative dependence of the main

⁵⁹We describe the solution and calibration of the model under the assumption of the Barro and Becker parental utility in Bar and Leukhina (2007).

results on the choice of qn .

Fixed Relative Price - Implications for TFP Estimates

We assumed that the two sectors produce a single consumption good. Under the alternative interpretation of two distinct goods entering utility as perfect substitutes, there is an implicit relative price, fixed by the weights of the goods in the utility. While the relative price is fixed in the model, it may change over time in the data. Thus, estimating TFP under this alternative interpretation further requires taking a stand on whether to use the relative price series implied by the model or one implied by the data. Adopting the alternative interpretation (with the relative price series taken from the data) is reasonable, because even though there is a significant overlap of products and services produced in the rural and urban sectors, the rural sector predominately specializes in agricultural and forestry products, and we know that the relative price of these products increased over time.⁶⁰ Note that if the relative price were fixed over time, the interpretation would be inconsequential for the estimated productivity *growth rates*, and hence for the model results.

Assuming the Solow good as the numeraire under the alternative interpretation, profit maximization implies

$$(9) \quad A_{1t}^{alt} = \frac{1}{p_1} \left(\frac{r_t}{\phi} \right)^\phi \left(\frac{w_t}{\mu} \right)^\mu \left(\frac{\rho}{1 - \phi - \mu} \right)^{1 - \phi - \mu},$$

$$(10) \quad A_{2t}^{alt} = \left(\frac{r_t}{\theta} \right)^\theta \left(\frac{w_t}{1 - \theta} \right)^{1 - \theta},$$

where p_1 , w and ρ are now in terms of the numeraire good. We infer the relative price, real wages and the real land rental price from $p_1 = \frac{P_1}{P_2}$, $w_i = \frac{\omega_i}{P_2}$ and $\rho = \frac{\tilde{\rho}P_\Lambda}{P_2}$, where P_1 and P_2 are the price indices of the rural and urban goods, ω_1 and ω_2 are the nominal wages (in \mathcal{L}), $\tilde{\rho}$ is the return on land rent (in %) and P_Λ is the price of land (in \mathcal{L}/acre). Substituting these into the above yields equations for the alternative productivity estimates:

$$(11) \quad \hat{A}_1^{alt} = \frac{P_2}{P_1} \left(\frac{r}{\phi} \right)^\phi \left(\frac{\omega_1}{\mu P_2} \right)^\mu \left(\frac{\tilde{\rho}P_\Lambda}{(1 - \phi - \mu) P_2} \right)^{1 - \phi - \mu},$$

$$(12) \quad \hat{A}_2^{alt} = \left(\frac{r}{\theta} \right)^\theta \left(\frac{\omega_2}{(1 - \theta) P_2} \right)^{1 - \theta}.$$

Under the first interpretation of physically identical goods, or under the alternative interpretation but with the relative price taken to be implied by the model, p_1 would be set to 1 in (9) and we would infer the productivity estimates \hat{A}_1 and \hat{A}_2 as given in Section V.

To determine the difference between the original and alternative estimates, we compute $\hat{A}_1/\hat{A}_1^{alt} = \frac{P_1}{P_2} / \left(\frac{P}{P_2} \right)^{1 - \phi}$ and $\hat{A}_2/\hat{A}_2^{alt} = \left(\frac{P_2}{P} \right)^{1 - \theta}$, where P is the overall price index in the economy, employed in the original TFP estimates. Clark's data on relative prices reveal that while $\frac{P_1}{P_2}$ fluctuated around a constant, it began to rise around 1750. $\frac{P}{P_2}$ also rose, but less so. Consequently, $\hat{A}_1/\hat{A}_1^{alt}$ increased, while $\hat{A}_2/\hat{A}_2^{alt}$ decreased. This means that the alternative TFP estimates would exhibit a larger rise in the industrial sector and a smaller rise in the agricultural sector since 1750. In light of the first discussion given in

⁶⁰The GDP deflator and P_1 are taken from Clark (2001a), and industrial prices were provided by Clark upon our request.

this section, we know that such estimates would further improve the fit of the model without changing the overall conclusions. Note that these alternative estimates would be consistent with any other utility choice that allows for differential relative prices of the Malthusian and Solow goods.

Fixed Relative Price - Implications for the Overall Results

As discussed above, while the relative price of the Malthusian good is fixed in the model, it rises in the data. This rise can potentially lead to changes in the demand for children relative to consumption goods thus influencing the demographics, or to changes in the relative demand of the Malthusian and Solow goods thus contributing to the structural transformation. To the extent that changes in productivity and youth mortality have a causal influence on the relative price behavior in the data, the concern is that the HP-BB framework, by failing to capture the relative price behavior, may miss these additional effects of the examined channels on the demographic and the economic transformations.

First, the effect of changes in the relative price on fertility is present in our model through the time cost channel. To understand this, note that if to raise children, one must feed them with the Malthusian good (e.g., Strulik and Weisdorf (2008)), then a change in its relative price directly affects the cost and hence the demand for children. This effect is captured in our model by the time cost channel. Even if a change in the relative price of the Malthusian good is not directly tied to the cost of children, its influence on fertility, if any, must transpire through income reallocation between consumption and children. Given the general interpretation of our time cost channel, such an effect is also captured by the time cost channel. Hence, the contribution to the drop in fertility that we attributed to productivity or the young-age mortality channels excludes their effect on fertility exerted through their influence on the relative price, but the contribution of the cost channel includes it. Our model, however, does not allow us to identify different sources of the rise in the cost of children. Put differently, our finding that the productivity channel exerted insignificant influence on the demographic change does not contradict the hypothesis that it contributed to the fertility decline by increasing the relative price of the Malthusian good.

Second, the effect of changes in the relative price on the structural transformation is missing from our model. We can thus only discuss how we would expect the influence of productivity and youth mortality to change qualitatively under more general preferences.

To obtain some intuition for how these effects may change, we worked with a simple setup⁶¹ employing several different preferences found in the literature on structural transformation: (1) constant elasticity of substitution utility $[\alpha c_1^\rho + (1 - \alpha) c_2^\rho]^{1/\rho}$ with $\rho = 1$ (perfect substitutes), $0 < \rho < 1$ (substitutes), $\rho = 0$ (Cobb-Douglas), $-\infty < \rho < 0$ (complements), $\rho = -\infty$ (perfect complements), (2) Stone-Geary utility $u(c_1, c_2) = \alpha \ln(c_1 - \tilde{c}_1) + (1 - \alpha) \ln(c_2 - \tilde{c}_2)$ (the two goods always acting as complements), (3) quasi-linear utility $u(c_1, c_2) = \gamma \ln(c_1) + c_2$ (the two goods always acting as substitutes).⁶²

If the two goods are complements, regardless of a particular utility formulation, increasing the labor supply leads to resource reallocation towards the less labor intensive Malthusian sector, i.e., the sector experiencing a larger decline in the marginal product as inputs increase, and thus it cannot be responsible for the structural transformation. This is consistent with our conclusion that young-age mortality, which

⁶¹ $\max_{c_1, c_2, L_1, L_2} u(c_1, c_2)$ subject to $c_1 N = A_1 L_1^{\theta_1}$, $c_2 N = A_2 L_2^{\theta_2}$ and $L_1 + L_2 = N$, $0 < \theta_1, \theta_2 \leq 1$.

⁶²See Bar and Leukhina (2007) for derivations.

results in a larger labor supply, was an important driving force of the structural transformation. As long as the two goods are substitutes, an increase in the labor supply resulting from declining mortality generates resource reallocation towards the more labor intensive Solow sector in the long run, which is consistent with the predictions of our model. Because mortality increased prior to its decline, we expect that mortality would not contribute to the structural transformation in a quantitatively significant manner, as is the case in our model.

Now we turn to the robustness of our result that the structural transformation was a consequence of changes in productivity. As discussed above, taking into account the relative price change when estimating productivity changes would only reinforce our finding that the Solow TFP experienced a larger acceleration. Hence, qualitatively, the exogenous input series of the TFP experiment would be unaltered.

We find that, as long as the two goods are substitutes, in the productivity experiment resources shift towards the faster growing Solow sector, thus generating the economic transformation. This behavior is consistent with our results. Moreover, the relative price movement implied by this experiment is consistent with the trend observed in the data, i.e., the rural good becoming relatively more expensive. When the two goods complement each other, increasing TFP of a particular sector tends to push the resources out and into the other, slower growing sector. Nonetheless, the Stone-Geary utility with the Malthusian good assumed to be a necessity can generate a structural transformation,⁶³ and it does so through the Malthusian TFP pushing resources out of that sector, with the Solow TFP having no effect on resource allocation. This is a competing explanation for the structural change.⁶⁴

Although we cannot assert unequivocally that the structural transformation was not driven by the push effect of the Malthusian TFP, we discuss several factors indicating that the implication of the productivity change with regard to the direction of the structural change in the Stone-Geary mechanism is not robust when considered in the context of several generalizations. By itself, the increase in the Malthusian TFP, which drives the push effect, also counterfactually implies a decline in the Malthusian price. If the relative increase in the Solow TFP is large enough, however, this simple environment can potentially generate both a structural transformation and a relative price series consistent with the data. Here, several caveats need to be mentioned. First, if we opened up the model economy and the relative price series were determined by the rest of the world, then even the Stone-Geary mechanism would work through the pull effect. Indeed, because p is exogenous, L_1 must decline in response to A_2 increasing faster than A_1 . In addition, it is reasonable to assume that the Solow good is also a necessity; after all,

⁶³CES utility with $-\infty < \rho < 0$ cannot generate a structural transformation in either experiment.

⁶⁴To understand the mechanics of the push effect, consider a simple framework with technology given by $Y_1 = A_1 L_1^\mu$ and $Y_2 = A_2 L_2$ and households maximizing Stone-Geary utility from above with $\tilde{c}_2 = 0$ subject to $p c_1 + c_2 = w + \pi$, where π denotes profits. Households' maximization yields $c_1 = \bar{c} + \frac{\alpha}{p} (w + \pi - p\bar{c})$ and $c_2 = (1 - \alpha) (w + \pi - p\bar{c})$. After ensuring the subsistence level, the rest of the income is spent according to preference weights and relative prices. The share of income spent on c_1 is $\frac{p c_1}{w + \pi} = (1 - \alpha) \frac{p\bar{c}}{w + \pi} + \alpha$; i.e., when income is low or the Malthusian good is expensive, the share spent on the Malthusian good is close to 1, subsistence consumption crowding out both the Solow good and children. As income grows, or the Malthusian good becomes cheaper, the share declines and converges to α . Profit maximization gives $w = A_2$ and $\pi = (1/\mu - 1) A_2 L_1/N$. Equating the marginal products of labor gives $p = \frac{A_2}{\mu A_1} L_1^{1-\mu}$. After substituting the optimal p , w and π into the market clearing condition for the Malthusian good, we obtain $(1 - \alpha) \bar{c} + \alpha \frac{\mu A_1}{L_1^{1-\mu}} + \alpha \mu \frac{(1/\mu - 1) A_1 L_1^\mu}{N} = \frac{A_1 L_1^\mu}{N}$. An increase in A_1 leads to a decrease in L_1 , because the demand for the Malthusian good has a lower elasticity than the demand for the industrial good. A change in A_2 has no effect on L_1 , as increases in income are offset by increases in the price of the Malthusian good.

people need shelter and clothing. In this case, depending on the exact parameter values, the push effect originating in the Solow sector may be strong enough to reverse the direction of the structural change, even if $\tilde{c}_2 < \tilde{c}_1$.

Disentangling the relative contributions of the push and pull effects seems to be an important research agenda. However, it is a difficult task for several reasons. Most significantly, it is difficult to construct a model containing both the pull and the push effects. Indeed, none of the models discussed here have both; i.e. it is never the case that $\partial(L_1/L)/\partial A_2 < 0$ and $\partial(L_1/L)/\partial A_1 < 0$ for some parameters and endogenous variables. This fact also reflects the dichotomy of push and pull models present in the literature on structural transformation, discussed in Section III.

X. Conclusion

Mokyr (2005) claims that “the exact connection between the demographic changes and the economic changes in the post-1750 period are far from being understood.” He makes this claim despite the existence of numerous theoretical models connecting the economic change to the demographic variables. The problem is that the existing theoretical models disagree on the main forces behind the economic and demographic transformations. This situation implies the need for more quantitative work.

In order to obtain a better understanding of the relation between the demographic and economic transformations, we constructed a general equilibrium framework that combines the HP model of structural change with the BB model of fertility choice. Our framework possesses standard ingredients, and it maps to observables in a straightforward way. This is a key feature of our model, and a point we wish to emphasize, because it enables us to calibrate the model’s parameters using meaningful criteria based on empirical data. It further enables us to estimate sector-specific TFP time series, which are necessary for the quantitative analysis carried out in this work, by utilizing historical data on factor prices. The model also allows the possibility of balanced growth with the relative sector-specific outputs remaining constant in the presence of differential productivity growth rates, which is a significant contribution to the more recent literature on structural transformation.

While our results do not support the HP conclusions in a strict sense – i.e. that stagnation is generated because the Solow sector is idle and that the structural transformation and takeoff in income growth transpire under constant sector-specific productivity growth rates – we show that the parameterized HP-BB hybrid proposed in this paper does remarkably well at generating the main patterns of the English economic and demographic transformations. We find that when historical changes of youth mortality and sector-specific productivity are introduced, the model accounts for nearly all of the increase in per capita output, industrialization, urbanization, and the decline of land share in total income, while capturing over 60% of the demographic change. Increasing the cost of children further improves the model’s fit along the demographic dimension.

This framework also allows us to study the separate contributions of technological progress, changes in young-age mortality, and the cost of raising children channels to the economic and demographic transformations. In fact, many of the proposed mechanisms developed for the purpose of endogenously generating these transformations act through one or more of these channels. By pinning down the important channels through which the change transpired in a particular country, our framework points to

the class of mechanisms most relevant to the case under study. For the case of England, we find that the influence exerted through the productivity channel is largely decoupled from those exerted through the youth mortality and the cost of children channels. Specifically, the productivity channel has a negligible effect on birth rates but accounts for nearly the entire economic transformation, while the young-age mortality and the cost of children channels account for almost none of the economic transformation but drive much of the demographic change. Our findings suggest that the quantitatively relevant channels through which the demographic and economic transformations transpired were distinct in the case of England.

APPENDIX
Data Sources⁶⁵

Fraction of non-rural labor in total labor (L_2/L): [1565-1865] - Clark (2001a), Table 1, p. 8 (England); [1820 - 1992] - Maddison (1995), p. 253 (UK).

Index of Real GDP per capita (y): [1565-1865] - Clark (2001a), Table 7, p. 30, rescaled to equal 100 in 1565 (England and Wales); [1820-1990] - Maddison (1995), p. 194, rescaled to match Clark's index in 1850 (UK).

Labor Share in Total Income (wL/Y): [1585 - 1865] - Clark (2001a), Table 9, p. 46 (England); [1924 - 1973] - Matthews et al. (1982), p. 164 (UK); Average for [1973 - 1982] - Maddison (1987), p. 659 (UK); 1992 - Gollin (2002), p. 470, Table 2, Adjustment 3 (UK).

Land Share in Total Income ($\rho\Lambda/Y$): [1585 - 1865] - Clark (2001a), Table 9, p. 46 (England); [1873 - 1913] - Matthews et al. (1982), p. 643 (UK); [1987 - 1998] - UK National Statistics.

Capital Share in Total Income: Imputed according to the relation $rK/Y = 1 - wL/Y - \rho\Lambda/Y$.

Fraction of non-rural output in total output (Y_2/Y): [1555-1865] Imputed by dividing the nominal net farm output (alternative labor) obtained from Clark (2002), Table 4, p. 14 (England), by the nominal GDP obtained from Clark (2001a), Table 3, p. 19 (England and Wales), but adjusted for population differential between England and Wales, with the resulting fraction indexed to match Mitchell's estimates in 1800; [1788-1991] - Mitchell, 1978 (UK)

Crude Birth and Crude Death Rates: [1541 - 1871] - Wrigley et al. (1997) (England); [1871 - 1986] - Mitchell, 1978 (England and Wales).

General Fertility Rate: Computed using CBR and the fraction of females in the total population, taken from Wrigley et al. (1997) for [1541 - 1841] (England) and Human Mortality Database for [1841 - 1999] (England and Wales).

Population Growth Rate: [1541 - 1836] - Wrigley et al. (1997) (England); [1841 - 1999] - Human Mortality Database (England and Wales).

Age-specific survival probabilities: [1580-1837] - Wrigley et al. (1997) (England); [1841 - 1999] - Human Mortality Database (England and Wales).

Data used in TFP Estimation: See the appendix on TFP estimation.

Proof of Proposition 1 (Barro and Becker vs. Lucas Utility)

Proof. Let $\frac{1-\varepsilon-\sigma}{\sigma} = \frac{1-\alpha-\beta}{\alpha\beta}$. Consider the following transformation of the BB utility, $W_t(c_t, n_t, U_{t+1}) = (1 - \beta) U_t(c_t, n_t, U_{t+1}) :$

$$W_t(c_t, n_t, W_{t+1}) = (1 - \beta) c_t^\sigma + \beta n_t^{1-\varepsilon} W_{t+1}.$$

⁶⁵Due to data limitations for England, we were forced to draw on the data sources available for England and Wales and UK. Although this inconsistency introduces some degree of error, we believe that it is small for the following reasons. (1) We do not consider level variables, such as GDP or population size, but instead growth rates, indices, and fractions of level variables. (2) For the period under consideration, the population of Wales is less than 6% of that of England. (3) Scotland's population size relative to that of England and Wales falls from 17% in 1820 (the earliest date for which we are forced to use UK data sources) to less than 10% today. (4) Appropriate rescaling was made in all cases.

Next, consider the transformation, $V_t(c_t, n_t, W_{t+1}) = W_t(c_t, n_t, W_{t+1})^{\frac{\alpha}{(1-\beta)\sigma}}$, given by

$$V_t(c_t, n_t, V_{t+1}) = \left[(1-\beta)c_t^\sigma + \beta n_t^{1-\varepsilon} V_{t+1}^{\frac{(1-\beta)\sigma}{\alpha}} \right]^{\frac{\alpha}{(1-\beta)\sigma}} = \left(\left[(1-\beta)c_t^\sigma + \beta \left(n_t^{\frac{1-\varepsilon}{\sigma}} V_{t+1}^{\frac{(1-\beta)}{\alpha}} \right)^\sigma \right]^{\frac{1}{\sigma}} \right)^{\frac{\alpha}{(1-\beta)}}.$$

Now, taking the limit $\sigma \rightarrow 0$ while varying ε in such a manner that $\frac{1-\varepsilon-\sigma}{\sigma} = \frac{1-\alpha-\beta}{\alpha\beta}$, we have

$$\lim_{\sigma \rightarrow 0} V_t(c_t, n_t, V_{t+1}) = \left(\lim_{\sigma \rightarrow 0} \left[(1-\beta)c_t^\sigma + \beta \left(n_t^{\frac{1-\varepsilon}{\sigma}} V_{t+1}^{\frac{(1-\beta)}{\alpha}} \right)^\sigma \right]^{\frac{1}{\sigma}} \right)^{\frac{\alpha}{(1-\beta)}} = \left(c_t^{1-\beta} \left(n_t^{\frac{1-\varepsilon}{\sigma}} V_{t+1}^{\frac{(1-\beta)}{\alpha}} \right)^\beta \right)^{\frac{\alpha}{(1-\beta)}}.$$

Note that $n_t^{\frac{1-\varepsilon}{\sigma}}$ and $V_{t+1}^{\frac{(1-\beta)}{\alpha}}$ remain fixed as $\sigma \rightarrow 0$. Consider the final transformation, $U_t(c_t, n_t, V_{t+1}) = \log V_t(c_t, n_t, V_{t+1})$, which takes the form

$$U_t(c_t, n_t, U_{t+1}) = \frac{\alpha}{(1-\beta)} \left[(1-\beta) \log c_t + \frac{1-\varepsilon}{\sigma} \beta \log n_t + \frac{(1-\beta)}{\alpha} \beta U_{t+1} \right].$$

Simplifying and using the assumption that $\frac{1-\varepsilon-\sigma}{\sigma} = \frac{1-\alpha-\beta}{\alpha\beta}$, i.e., $\frac{1-\varepsilon}{\sigma} = \frac{(1-\alpha)(1-\beta)}{\alpha\beta}$, we obtain

$$\begin{aligned} U_t(c_t, n_t, U_{t+1}) &= \alpha \log c_t + \frac{\alpha}{(1-\beta)} \frac{(1-\alpha)(1-\beta)}{\alpha\beta} \beta \log n_t + \beta U_{t+1} \\ &= \alpha \log c_t + (1-\alpha) \log n_t + \beta U_{t+1}. \end{aligned}$$

■

Proof of Proposition 2 (Effect of Changes in the Labor Supply and Capital Stock on Resource Allocation)

Proof. We drop the time subscript throughout the proof. Let $\kappa = K_1/K$ and $\lambda = L_1/L$. The resource allocation problem can be written in terms of κ and λ as follows:

$$\max_{\kappa, \lambda} \left\{ A_1 \kappa^\phi K^\phi \lambda^\mu L^\mu \Lambda^{1-\phi-\mu} + A_2 (1-\kappa)^\theta K^\theta (1-\lambda)^{1-\theta} L^{1-\theta} \right\}.$$

The first order conditions are

$$(13) \quad \phi A_1 \kappa^{\phi-1} K^\phi \lambda^\mu L^\mu \Lambda^{1-\phi-\mu} = \theta A_2 (1-\kappa)^{\theta-1} K^\theta (1-\lambda)^{1-\theta} L^{1-\theta},$$

$$(14) \quad \mu A_1 \kappa^\phi K^\phi \lambda^{\mu-1} L^\mu \Lambda^{1-\phi-\mu} = (1-\theta) A_2 (1-\kappa)^\theta K^\theta (1-\lambda)^{-\theta} L^{1-\theta}.$$

Dividing (2) by (1) gives $\frac{\mu \kappa}{\phi \lambda} = \left(\frac{1-\theta}{\theta} \right) \left(\frac{1-\kappa}{1-\lambda} \right)$. Solving for κ in terms of λ gives $\left(\frac{\kappa}{1-\kappa} \right) = \frac{\phi}{\mu} \left(\frac{1-\theta}{\theta} \right) \left(\frac{\lambda}{1-\lambda} \right)$. The solution to this equation is $\kappa(\lambda)$, which is an increasing function. Substituting this solution into (1) gives

$$\begin{aligned} \phi A_1 K^\phi L^\mu \Lambda^{1-\phi-\mu} \kappa(\lambda)^{\phi-1} \lambda^\mu &= \theta A_2 K^\theta L^{1-\theta} (1-\kappa(\lambda))^{\theta-1} (1-\lambda)^{1-\theta}, \text{ i.e.,} \\ \phi \bar{Y}_1(K, L) \kappa(\lambda)^{\phi-1} \lambda^\mu - \theta \bar{Y}_2(K, L) (1-\kappa(\lambda))^{\theta-1} (1-\lambda)^{1-\theta} &= 0, \end{aligned}$$

where \bar{Y}_i denotes the maximum possible output in sector i , i.e. output under full employment of available

inputs. In other words, \bar{Y}_i are the end points of the PPF. At the optimal point, the slope of the PPF (in absolute value), i.e., the marginal rate of substitution, equals 1 because the two goods are perfect substitutes:

$$MRT = \frac{\theta \bar{Y}_2(K, L) (1 - \kappa(\lambda))^{\theta-1} (1 - \lambda)^{1-\theta}}{\phi \bar{Y}_1(K, L) \kappa(\lambda)^{\phi-1} \lambda^\mu} = 1.$$

Define

$$\Psi(\lambda, K, L) \equiv \frac{\theta \bar{Y}_2(K, L)}{\phi \bar{Y}_1(K, L)} \frac{\kappa(\lambda)^{1-\phi}}{(1 - \kappa(\lambda))^{1-\theta}} \frac{(1 - \lambda)^{1-\theta}}{\lambda^\mu} - 1,$$

which can be used to obtain $\frac{\partial \lambda}{\partial L} = -\frac{\Psi_L}{\Psi_\lambda}$. In particular, a reallocation from Malthus to Solow will occur if $\partial \lambda / \partial L < 0$. Notice that holding the total resources fixed, we must have $\Psi_\lambda > 0$ because the PPF is strictly concave and the slope in absolute value gets steeper as we move downwards along the PPF. The sign of Ψ_L is the same as the sign of $\frac{\partial}{\partial L} \left(\frac{\bar{Y}_2(K, L)}{\bar{Y}_1(K, L)} \right)$, thus we have

$$\Psi_L \propto \frac{\frac{\partial \bar{Y}_2(K, L)}{\partial L} \bar{Y}_1(K, L) - \frac{\partial \bar{Y}_1(K, L)}{\partial L} \bar{Y}_2(K, L)}{\bar{Y}_1(K, L)^2}.$$

Hence, $\Psi_L > 0$ if and only if

$$\begin{aligned} \frac{\partial \bar{Y}_2(K, L)}{\partial L} \bar{Y}_1(K, L) &> \frac{\partial \bar{Y}_1(K, L)}{\partial L} \bar{Y}_2(K, L), \text{ i.e.,} \\ \frac{\partial \bar{Y}_2(K, L)}{\partial L} \frac{L}{\bar{Y}_2(K, L)} &> \frac{\partial \bar{Y}_1(K, L)}{\partial L} \frac{L}{\bar{Y}_1(K, L)}, \text{ i.e.,} \\ (1 - \theta) \frac{\bar{Y}_2}{L} \frac{L}{\bar{Y}_2} &> \mu \frac{\bar{Y}_1}{L} \frac{L}{\bar{Y}_1(K, L)}, \text{ i.e.,} \\ 1 - \theta &> \mu. \end{aligned}$$

The proof of part (b) is similar. ■

Cost of Raising Children, Measuring $(a + b) / a$

In this appendix we explain our method of determining the average time cost of a surviving child relative to that of a non-surviving child, $(a + b) / a = 4$. Denoting the momentary cost of raising a child by $p(t)$, the total cost of raising a child to age τ is given by $c(\tau) = \int_0^\tau p(t) dt$. Under the assumption that the momentary cost is a decreasing linear function of the form $p(t) = \eta - \frac{\eta}{25}t$, we have $c(\tau) = \tau\eta - \frac{\tau^2}{50}\eta$ and the total cost of raising a surviving child becomes $a + b = c(25) = 25\eta - \frac{25^2}{50}\eta = 12.5\eta$.

Consider the age-specific mortality distribution for people who died before reaching age 25 in early 17th century England. For the five groups corresponding to the age ranges 0-1, 1-5, 5-10, 10-15, and 15-25, mortality rates are given by .45,.22,.12,.05,.16. (Below, we refer to the beginning and ending ages of the i th group as A_i^b and A_i^e , respectively.) The first number, for example, indicates that of all the people who died before reaching age 25, 45% died before age 1. The pattern of age-specific mortality, conditional on dying before age 25, persists throughout the years considered in this paper. Then, assigning to every

child belonging to group i the time cost associated with a child that dies at age $\frac{A_i^b + A_i^e}{2}$, we obtain

$$\begin{aligned} a &= 0.45c(0.5) + 0.22c(3) + 0.12c(7.5) + 0.05c(12.5) + 0.16c(20) = 4\eta, \\ b &= 12.5\eta - 4\eta = 8.5\eta. \end{aligned}$$

It follows that $\frac{b}{a} = 2.15$ and $\frac{a+b}{a} = 3.15$. If, instead, we assign to each child in group i the time cost associated with a child that dies at age A_i^b , we find $\frac{b}{a} = 3.45$, and hence $\frac{a+b}{a} = 4.45$. Finally, because it is reasonable to assume that the average age of death for the children belonging to a given group is closer to A_i^b than to A_i^e , we choose the value $\frac{a+b}{a} = 4$; this corresponds to the assumption that all children belonging to each group i die at age $A_i^b + 0.1(A_i^e - A_i^b)$.

Estimation of TFP Time Series

Given the calibrated values of ϕ , μ and θ and using the assumption of profit maximization, we back out the time series for A_{1t} and A_{2t} given by (7) and (8), where r_t is the rental rate of capital (%/100), w_t is the real wage (final goods per unit of labor), and ρ_t is the rental price of land (final goods per acre).

We work with historical data for r_t (%/100), ω_{1t} (nominal rural wages in £), $\tilde{\rho}_t$ (rental rate of land in %/100), $P_{\Lambda t}$ (price of land in £/acre), and the GDP deflator, P_t . These series yield the real wage and the rental price of land through the identities $w_{it} = \frac{\omega_{it}}{P_t}$ and $\rho_t = \frac{\tilde{\rho}_t P_{\Lambda t}}{P_t}$.

The GDP deflator, P_t , is obtained from Table 9 in Clark (2001a), and for the time period 1875-1910, it is imputed under the assumption that it grew at the same rate as the agricultural prices given in Table 1 of Clark (2002).

Table 1 in Clark (2002) contains nominal wages in the rural sector ω_{1t} (pence per day). Dividing these time series by 240 changes the units into pounds. Further, multiplying the resulting time series by 300 gives the annual nominal wage, ω_{1t} , under the assumption that 300 days are worked per year. We infer ω_{2t} using the time series for the wage bill in the rural sector, $\omega_1 L_1$, the total wage bill in the economy, $\omega_1 L_1 + \omega_2 L_2$, the fraction of rural labor in total labor, $\frac{L_1}{L}$, and the identity $\frac{\omega_1 L_1 + \omega_2 L_2}{\omega_2 L_2} = \frac{\omega_1 L_1}{\omega_2 L_2} + 1$, which implies $\omega_2 = \frac{\omega_1}{\frac{\omega_1 L_1 + \omega_2 L_2}{\omega_2 L_2} - 1} \frac{1}{\frac{L_1}{L} - 1}$.

The time series of the wage bill in the rural sector, $\omega_1 L_1$, is given in Table 3 of Clark (2002). The total wage bill in the economy, $\omega_1 L_1 + \omega_2 L_2$, is taken from Table 3 in Clark (2001a), and for the period 1875-1910, it is imputed using the time series of $\omega_1 L_1$ and the assumption that the ratio $\omega_1 L_1 / (\omega_1 L_1 + \omega_2 L_2)$ continued to fall at the same rate as it did between 1865 and 1875. The fraction of the total labor constituted by rural labor, $\frac{L_1}{L}$, is obtained from Table 1 of Clark (2001a), and for the period 1875-1910 from Maddison (1995) (page 253).

Having obtained ω_{1t} and ω_{2t} , we back out real wages according to the relation $w_{it} = \frac{\omega_{it}}{P_t}$.

We obtain $\tilde{\rho}_t$ (rental rate of land in %/100) from Table 2 in Clark (2002). Following Clark (2002) (p. 6), we infer $r_t = \tilde{\rho}_t + 0.04$, allowing 1.5% for risk premium and 2.5% for depreciation.

Table 4 in Clark (2002b) provides us with ‘‘Total Land Rents and Local Taxes,’’ which represents $\tilde{\rho}_t P_{\Lambda t} \Lambda$, where $P_{\Lambda t}$ is the price of land, £/acre. Dividing this time series by $\Lambda = 26.524$ M acres, taken from Clark (2002) (p. 10), and by P_t , we obtain $\rho_t = \frac{\tilde{\rho}_t P_{\Lambda t}}{P_t}$.

We need to estimate the average size of the population in period t . The number of adults is constant at $2N$ over the duration of a period. The number of children changes during each period due to child mortality. In the beginning of each period, $2fN$ children are born. Using age-specific child mortality rates for the age groups 0-1,1-5,5-10,10-15, 15-25 and the simplifying assumption made above that all children belonging to group i die at age $A_i^b + \nu(A_i^e - A_i^b)$, with $\nu = \frac{1}{10}$, we compute the average population size in each period according to

$$P = 2N + [(\nu + (1 - \nu)\pi_0^1) + 4(\nu\pi_0^1 + (1 - \nu)\pi_0^5) + 5(\nu\pi_0^5 + (1 - \nu)\pi_0^{10}) + 5(\nu\pi_0^5 + (1 - \nu)\pi_0^{10}) + 10(\nu\pi_0^5 + (1 - \nu)\pi_0^{25})] \frac{1}{25} 2fN.$$

The model counterpart of CBR is then given by $CBR = 1000 \frac{2fN}{P}$. Further, GFR is computed as $GFR = 1000 \frac{2fN}{N} = 2000f$.

References

- Allen, Robert (1992)**, *Enclosure and the Yeoman*, Oxford: Clarendon Press.
- Antras, Pol and Voth, Hans-Joachim (2003)**, "Factor Prices and Productivity Growth during the British Industrial Revolution" *Explorations in Economic History*, 40, pp. 52-77.
- Bar, Michael and Leukhina, Oksana (2007)**, "Supplemental Notes to Demographic Transition and Industrial Revolution: A Macroeconomic Investigation," On-line supplement available at www.unc.edu/~oksana/Appendix1.pdf.
- **(2008)**, "The Role of Mortality in the Transmission of Knowledge," Working Manuscript, University of North Carolina
- Barro, Robert J. and Becker, Gary S (1989)**, "Fertility Choice in a Model of Economic Growth." *Econometrica*, 57(2), pp. 481-501.
- Becker, Gary S. and Lewis, H. Gregg (1973)**, "On the Interaction between Quantity and Quality of Children." *Journal of Political Economy*, 81(2, part 2), S279-S288.
- Boldrin, Michele and Jones, Larry E. (2002)**, "Mortality, Fertility and Saving in a Malthusian Economy." *Review of Economic Dynamics*, 5(4), pp. 775-814.
- Boldrin, Michele, Jones, Larry E., and De Nardi, Mariacristina (2005)**, "Fertility and Social Security," Staff Report 359, Federal Reserve Bank of Minneapolis.
- Boucekkine, Raouf; De la Croix, David and Licandro, Omar (2002)**, "Vintage Human Capital, Demographic Trends, and Endogenous Growth." *Journal of Economic Theory*, 104, pages 340-375.
- Boucekkine, Raouf; De la Croix, David and Peeters, Dominique (2007)**, "Early Literacy Achievements, Population Density, and the Transition to Modern Growth." *Journal of the European Economic Association*, MIT Press, vol. 5(1), pages 183-226.
- Francesco Caselli and Wilbur John Coleman II (2001)**, "The U.S. Structural Transformation and Regional Convergence: A Reinterpretation," *Journal of Political Economy*, University of Chicago Press, vol. 109(3), pages 584-616.

- Chari, V. V., Kehoe, Patrick J. and McGrattan, Ellen R. (2006)**, “Business Cycle Accounting.” Forthcoming in *Econometrica*.
- Chesnais, Jean-Claude (1992)**, *The Demographic Transition*, Oxford: Clarendon Press.
- Clark, Gregory (2001a)**, “The Secret History of the Industrial Revolution.” Working Manuscript.
- **(2001b)**, “Debt, Deficits, and Crowding out: England, 1727-1840.” *European Review of Economic History*, December 2001b, 5, pp. 403-436.
- **(2002)**, “The Agricultural Revolution and the Industrial Revolution: England, 1500-1912.” Working Manuscript.
- Doepke, Matthias (2004)**, “Accounting for Fertility Decline during the Transition to Growth.” *Journal of Economic Growth*, 9(3), pp. 347-383.
- **(2005)**, “Child Mortality and Fertility Decline: Does the Barro-Becker Model fit the Facts?” *Journal of Population Economics*, 18(2), pp. 337-366.
- **(2008)**, “Growth Takeoffs,” forthcoming in: *The New Palgrave Dictionary of Economics*, 2nd edition, Steven N. Durlauf and Lawrence E. Blume, eds., London: Palgrave MacMillan.
- Doepke, Matthias and Zilibotti, Fabrizio (2005)**, “The Macroeconomics of Child Labor Regulation,” *American Economic Review*, 95(5), pp. 1492-1524.
- Dyson, Tim and Mike Murphy (1985)**, “The Onset of Fertility Transition,” *Population and Development Review*, 11, pp. 399-440.
- Eckstein, Zvi, Pedro Mira and Kenneth I. Wolpin (1999)**, “A Quantitative Analysis of Swedish Fertility Dynamics: 1751-1990,” *Review of Economic Dynamics*, vol. 2(1), pp. 137-165.
- Fernandez-Villaverde, Jesus (2001)**, “Was Malthus Right? Economic Growth and Population Dynamics.” Working Manuscript.
- Floud, Roderick and Johnson, Paul (2004)**, *The Cambridge Economic History of Modern Britain*. Cambridge University Press, volumes 1-3.
- Galor, Oded (2005)**, “From Stagnation to Growth: Unified Growth Theory.” *Handbook of Economic Growth* (Aghion, Philippe and Durlauf, Steven N., eds.). Amsterdam: Elsevier, volume 1B, pp. 1113-1180.
- Galor, Oded and Omer Moav (2002)** “Natural Selection And The Origin Of Economic Growth,” *The Quarterly Journal of Economics*, 117(4), pages, pp. 1133-1191.
- Galor, Oded and Weil, David N. (2000)**, “Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond.” *American Economic Review*, 90(4), pp. 806-828.
- **(1999)**, “From Malthusian Stagnation to Modern Growth.” *American Economic Review (Papers and Proceedings)*, 89(2), pp. 150-154.
- **(1996)**, “The Gender Gap, Fertility, and Growth,” *American Economic Review*, 86(3), pp. 374-87.
- Golosov, Mikhail; Jones, Larry E. and Tertilt, Michele (2006)**, “Efficiency with Endogenous Population Growth.” Forthcoming in *Econometrica*.

- Greenwood, Jeremy and Seshadri, Ananth (2002)**, “The U.S. Demographic Transition.” *American Economic Review (Papers and Proceedings)*, 92(2), pp.153-159.
- Gollin, Douglas (2002)**, “Getting Income Shares Right.” *Journal of Political Economy*, 110(2), pp. 458-474.
- Douglas Gollin, Stephen Parente and Richard Rogerson (2002)**, “The Role of Agriculture in Development,” *American Economic Review*, American Economic Association, vol. 92(2), pages 160-164.
- Hansen, Gary D. and Prescott, Edward C (2002)**, “Malthus to Solow.” *American Economic Review*, 92(4), pp. 1205-1217.
- Hazan, Moshe and Binyamin Berdugo (2002), “Child Labour, Fertility, and Economic Growth.” *Economic Journal*, 112(482), pp. 810-828.
- Hotz, Joseph V.; Klerman, Jacob Alex and Willis, Robert J. (1997)**, “The Economics of Fertility in Developed Countries,” *Handbook of Population and Family Economics* (Rosenzweig, Mark R. and Stark, Oded, eds.). Amsterdam: Elsevier, volume 1A, pp. 275-347.
- Human Mortality Database**, <http://www.mortality.org>.
- Jones, Charles (2001)**, “Was an Industrial Revolution Inevitable? Economic Growth over the Very Long Run.” *Advances in Macroeconomics*, 1(2), pp. 1028-1028.
- Kalemlı-Ozcan, Sebnem (2002)**, “Does the Mortality Decline Promote Economic Growth?” *Journal of Economic Growth*, 7(4), pp. 411-439.
- Kalemlı-Ozcan, Sebnem; Ryder, Harl E. and Weil, David N. (2000)**, “Mortality Decline, Human Capital Investment and Economic Growth.” *Journal of Development Economics*, 62(1), pp. 1-23.
- Lagerlof, Nils-Petter (2003a)**, “From Malthus to Modern Growth: Can Epidemics Explain the Three Regimes?” *International Economic Review*, 44(2), pp. 755-777.
- Lagerlof, Nils-Petter (2003b)**, “Gender Equality and Long-Run Growth,” *Journal of Economic Growth*, 8, pp. 403-426.
- **(2006)**. “The Galor-Weil Model Revisited: A Quantitative Exercise.” *Review of Economic Dynamics*, 9(1), pp. 116-142.
- Laitner, John P (2000)**, “Structural Change and Economic Growth,” *Review of Economic Studies*, 67(3), pp. 545-561.
- Lee, Ronald D (2006)**, “Demographic Transition.” *The New Palgrave Dictionary of Economics*, (Durlauf, S.N. and Blume, L.E., eds.). Palgrave Macmillan, (forthcoming).
- Lewis, Arthur R (1954)**, “Economic Development with Unlimited Supply of Labor,” Manchester School of Economic and Social Studies 22 (1954), pp. 139-91.
- Lucas, Robert E. (2002)**, “The Industrial Revolution: Past and Future.” *Lectures on Economic Growth*, Cambridge: Harvard University Press.
- Maddison, Angus (1987)**, “Growth and Slowdown in Advanced Capitalist Economies: Techniques of Quantitative Assessment.” *Journal of Economic Literature*, 25(2), pp. 649-698.

- (1995), *Monitoring the World Economy: 1820/1992*. Washington: OECD Development Center.
- Mason, Karen (1997)**, “Explaining Fertility Transitions.” *Demography*, 34(4), pp. 443-454.
- Matsuyama, Kiminori (1992)**, “Agricultural productivity, comparative advantage, and economic growth,” *Journal of Economic Theory*, 58(2), pp. 317-334.
- (2008), “Structural Change,” forthcoming in: *The New Palgrave Dictionary of Economics*, 2nd edition, Steven N. Durlauf and Lawrence E. Blume, eds., London: Palgrave MacMillan.
- Murphy, Kevin M, Andrei Shleifer and Robert W Vishny (1989)**, “Income Distribution, Market Size, and Industrialization,” *The Quarterly Journal of Economics*, vol. 104(3), pages 537-64, August.
- Mokyr, Joel (2005)**, “The Economics of Fertility in Developed Countries.” *Handbook of Economic Growth* (Aghion, Philippe and Durlauf, Steven N., eds.). Amsterdam: Elsevier, volume 1B, pp. 1113-1180.
- Matthews, Robert C.O.; Feinstein, C.H. and Odling-Smee, John C (1982)**, *British Economic Growth: 1856-1973*. Stanford: Stanford University Press.
- Mitchell, Brian R (1975)**, *European Historical Statistics 1750-1970*. New York: Columbia University Press.
- Soares, Rodrigo R (2005)**, “Mortality Reductions, Educational Attainment, and Fertility Choice.” *American Economic Review*, 95(3), pp. 580-601.
- Strulik, Holger and Jacob Weisdorf (2008), “Population, food, and knowledge: a simple unified growth theory,” *Journal of Economic Growth*, 13(3), pp. 195-216.
- Cervellati, Matteo and Sunde, Uwe (2005)**, “Human Capital Formation, Life Expectancy, and the Process of Development.” *American Economic Review*, 95(5), pp. 1653-1672.
- Tamura, Robert F (2006)**, “Human Capital and Economic Development.” *Journal of Development Economics*, 79(1), pp. 26-72.
- U.K. National Statistics**, <http://www.statistics.gov.uk/>.
- Voigtländer Nico and Hans-Joachim Voth (2006), “Why England? Demographic factors, structural change and physical capital accumulation during the Industrial Revolution,” *Journal of Economic Growth*, 11(4), pp. 319-361.
- Wilson, Chris and Woods, Robert (1991)**, “Fertility in England: A Long-Term Perspective.” *Population Studies*, 45(3), pp. 399-415.
- Wrigley, E.A.; Davies, R.S.; Oeppen, J.E. and Schofield, R.S (1997)**, *English Population History from Family Reconstitution 1580-1837*. Cambridge: Cambridge University Press.
- Zweimüller (2000), “Schumpeterian Entrepreneurs Meet Engel’s Law: The Impact of Inequality on Innovation-Driven Growth,” *Journal of Economic Growth*, 5(2), pp. 185–206.

Table 1: England Around 1600: Data Moments Used for Calibration

Data Moment	Description
$\delta = 0.723$ (0.05)	Depreciation
$\pi = 0.67$	Probability of survival to 25
$\frac{l_1}{l} = 0.6$	Fraction of rural labor in total labor
$\frac{y_1}{y} = 0.67$	Fraction of rural output in total output
$\frac{rk}{y} = 0.16$	Capital share in total income
$\frac{wl}{y} = 0.6$	Labor share in total income
$r + 1 - \delta = 2.666$ (1.04)	Interest rate
$qn = 0.42$	Fraction of time spent with children (or not working)
$\frac{a+b}{a} = 4$	Time cost of a surviving child / that of a non-surviving child
$\gamma_{1,1600} = 1.042$ (1.0016)	Growth of rural TFP around 1600
$\gamma_{2,1600} = 1.006$ (1.00025)	Growth of non-rural TFP around 1600

Table 2: Calibrated Parameter Values

Malthusian Technology:	$A_{10} = 100, \gamma_{1,1600} = 1.042, \phi = .104, \mu = 0.537, \gamma_{1,1900} = 1.126$
Solow Technology:	$A_{20} = 100, \gamma_{2,1600} = 1.006, \theta = 0.273, \gamma_{2,1900} = 1.174$
Preferences:	$\alpha = 0.582, \beta = 0.415$
Cost of Children:	$\pi = 0.67, a = 0.085, b = 0.256$
Other:	$\delta = 0.723, \Lambda = 1$

Table 3: Main Results

	1600-1950				1650-1950			
	% Δ Data	%Accounted for by Model with Δ in TFP&Mortality			% Δ Data	%Accounted for by Model with Δ in TFP&Mortality		
		TFP	TFP	Mortality		TFP	TFP	Mortality
y	379.55	65.78	68.34	2.23	348.89	69.33	72.77	1.68
CBR	-48.73	45.85	-0.24	44.67	-39.95	60.72	-0.00	59.06
GFR	-46.28	44.23	-0.56	41.35	-36.45	61.10	-0.01	56.91
$\frac{\rho\Delta}{y}$	-95.32	91.90	92.26	-1.97	-95.68	91.68	91.90	-0.89
$\frac{wl}{y}$	16.67	111.51	111.93	-2.39	20.69	90.95	90.05	-0.88
$\frac{y_2}{y}$	187.88	94.70	95.03	-2.03	177.38	103.30	100.35	-1.00
$\frac{l_2}{l}$	137.25	97.89	98.18	-2.54	113.26	122.17	118.61	-1.46

Table 4: Sensitivity to the Endpoint Solow TFP Growth, $\gamma_{2,1900} = 1.28$

	%Accounted for by Model with Δ in			
	1600-1950		1650-1950	
	TFP	TFP&Mortality	TFP	TFP&Mortality
y	81.93	79.03	87.34	83.42
CBR	-0.26	45.88	-0.03	60.75
GFR	-0.61	44.29	-0.07	61.18
$\frac{\rho\Delta}{y}$	95.88	95.59	95.51	95.31
$\frac{wl}{y}$	116.32	115.98	93.58	94.56
$\frac{y_2}{y}$	98.76	98.50	104.30	107.40
$\frac{l_2}{l}$	101.40	101.17	122.51	126.22

Table 5: Sensitivity to the Choice of Parental Utility, Barro and Becker Form

	%Accounted for by Model with Δ in					
	1600-1950			1650-1950		
	TFP	Mortality	TFP&Mortality	TFP	Mortality	TFP&Mortality
y	61.45	1.09	55.80	65.58	0.34	58.49
CBR	-8.20	41.01	36.67	-9.38	54.26	49.62
GFR	-19.77	32.17	20.53	-23.53	44.40	31.22
$\frac{\rho\Delta}{y}$	97.64	-1.46	97.42	97.26	0.22	97.16
$\frac{wl}{y}$	118.45	-1.77	118.23	95.27	0.22	97.07
$\frac{y_2}{y}$	100.57	-1.51	100.45	106.14	0.25	111.45
$\frac{l_2}{l}$	102.96	-1.93	102.87	124.32	0.26	130.59

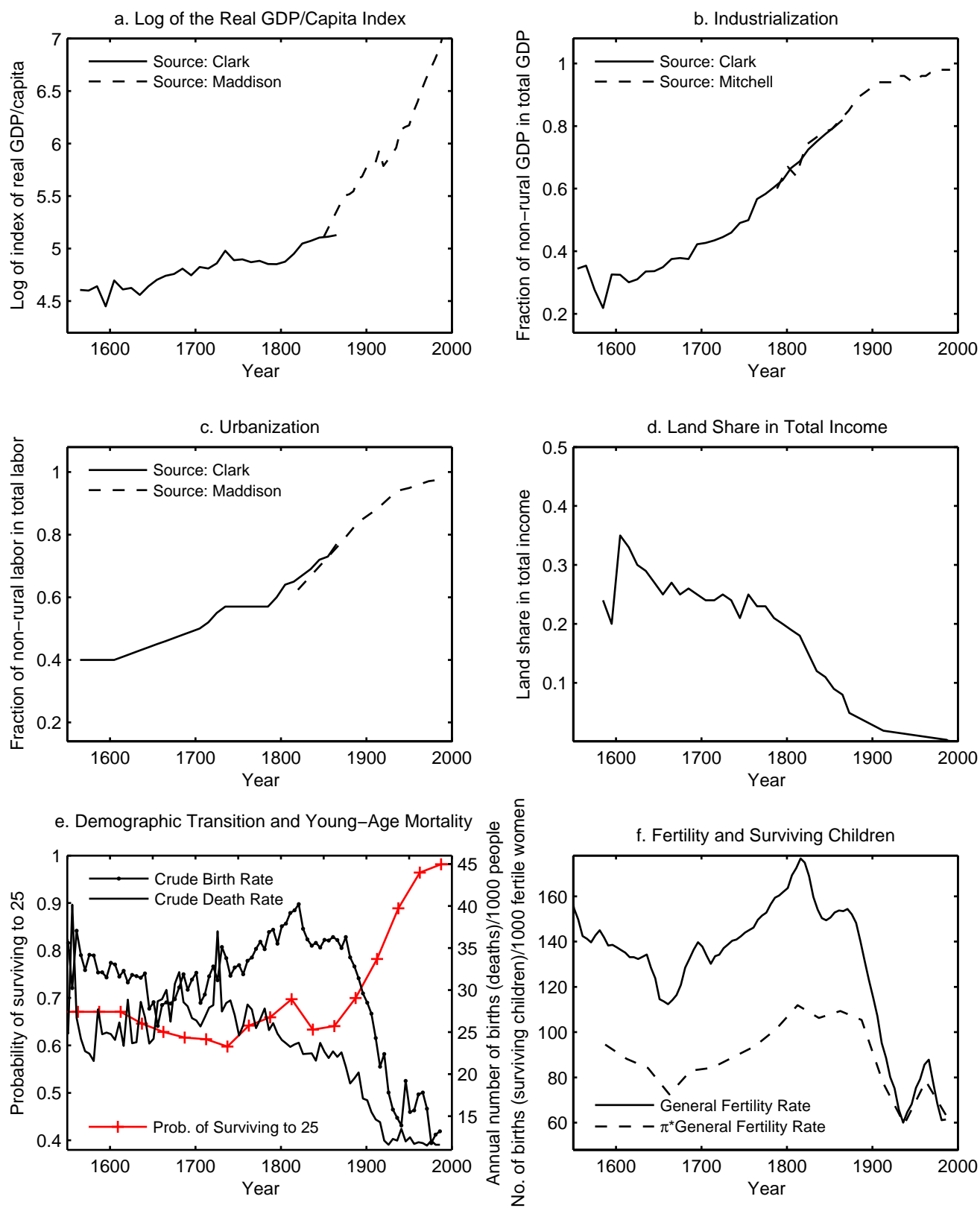


Figure 1: (a) Log of the Real GDP/capita Index, (b) Industrialization, (c) Urbanization, (d) Land Share in Total Income, (e) Demographic Transition, (f) General Fertility Rate and Surviving Children

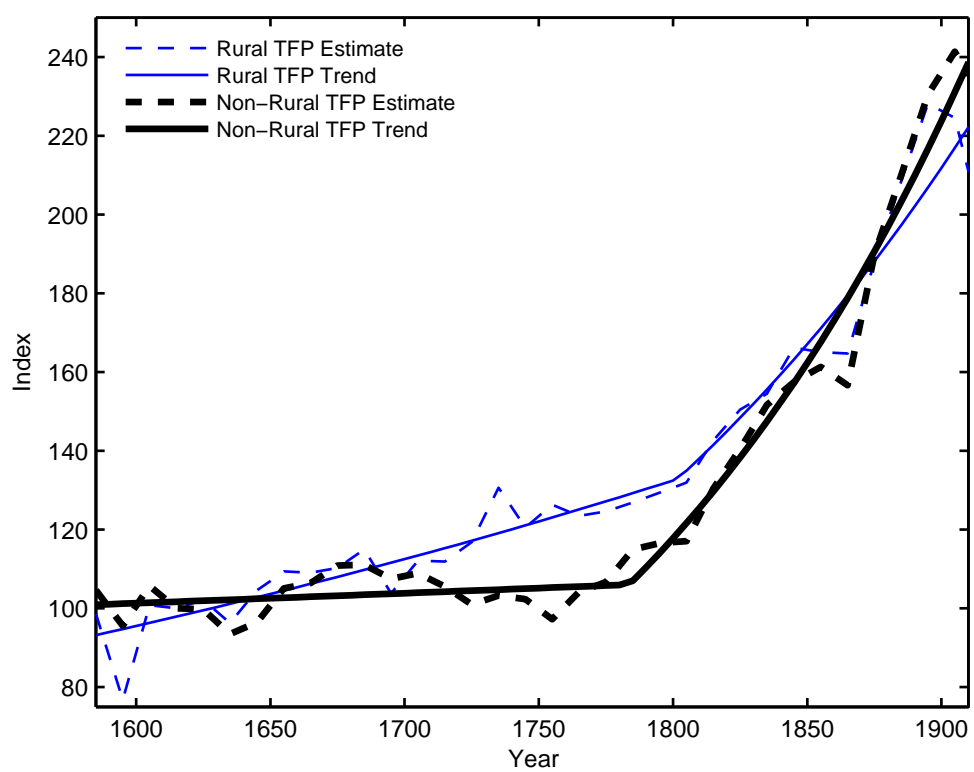


Figure 2: Estimated Sector-Specific Total Factor Productivity

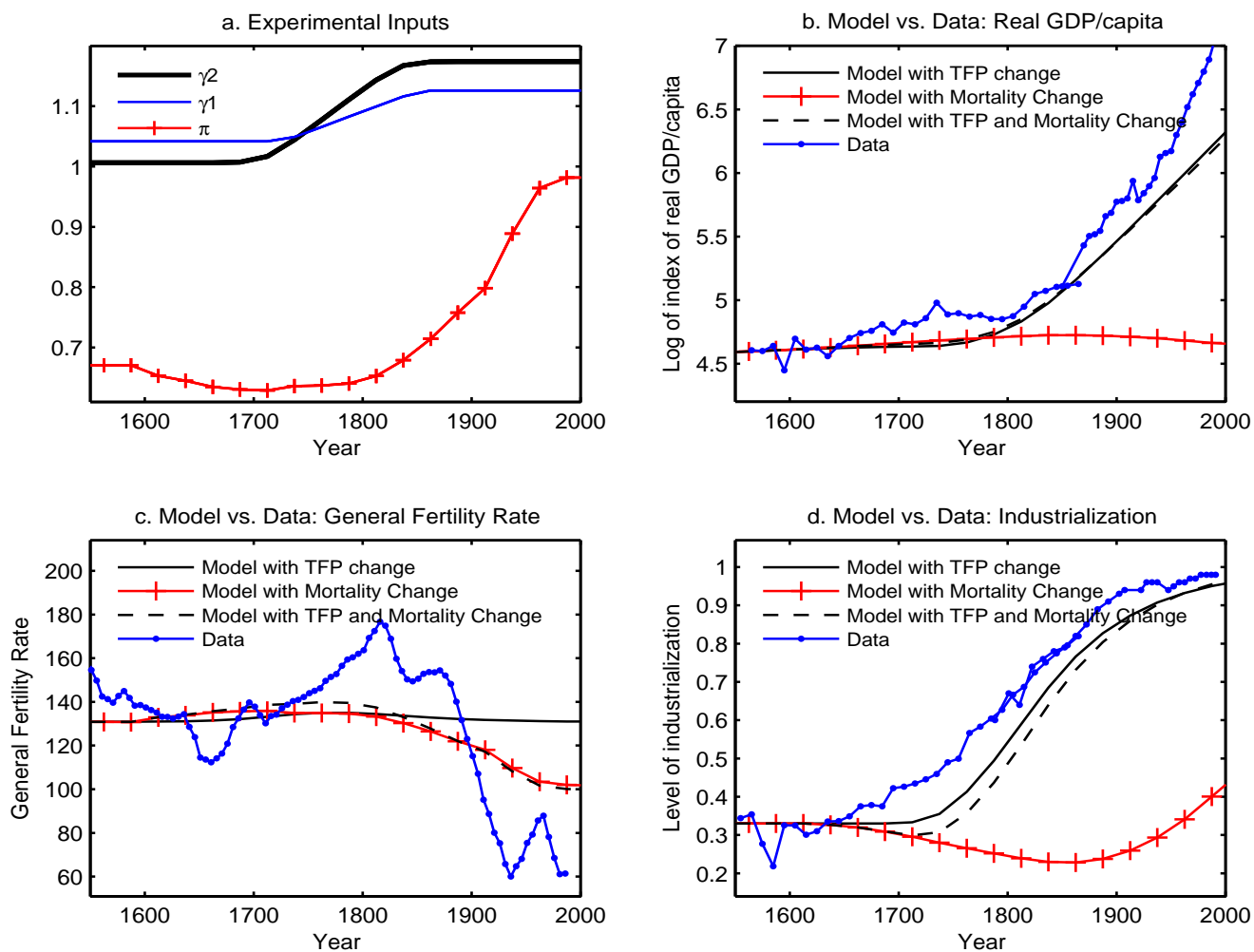


Figure 3: (a) Experimental Inputs, (b) Model vs. Data: Real GDP/capita, (c) Model vs. Data: General Fertility Rate, (d) Model vs. Data: Industrialization

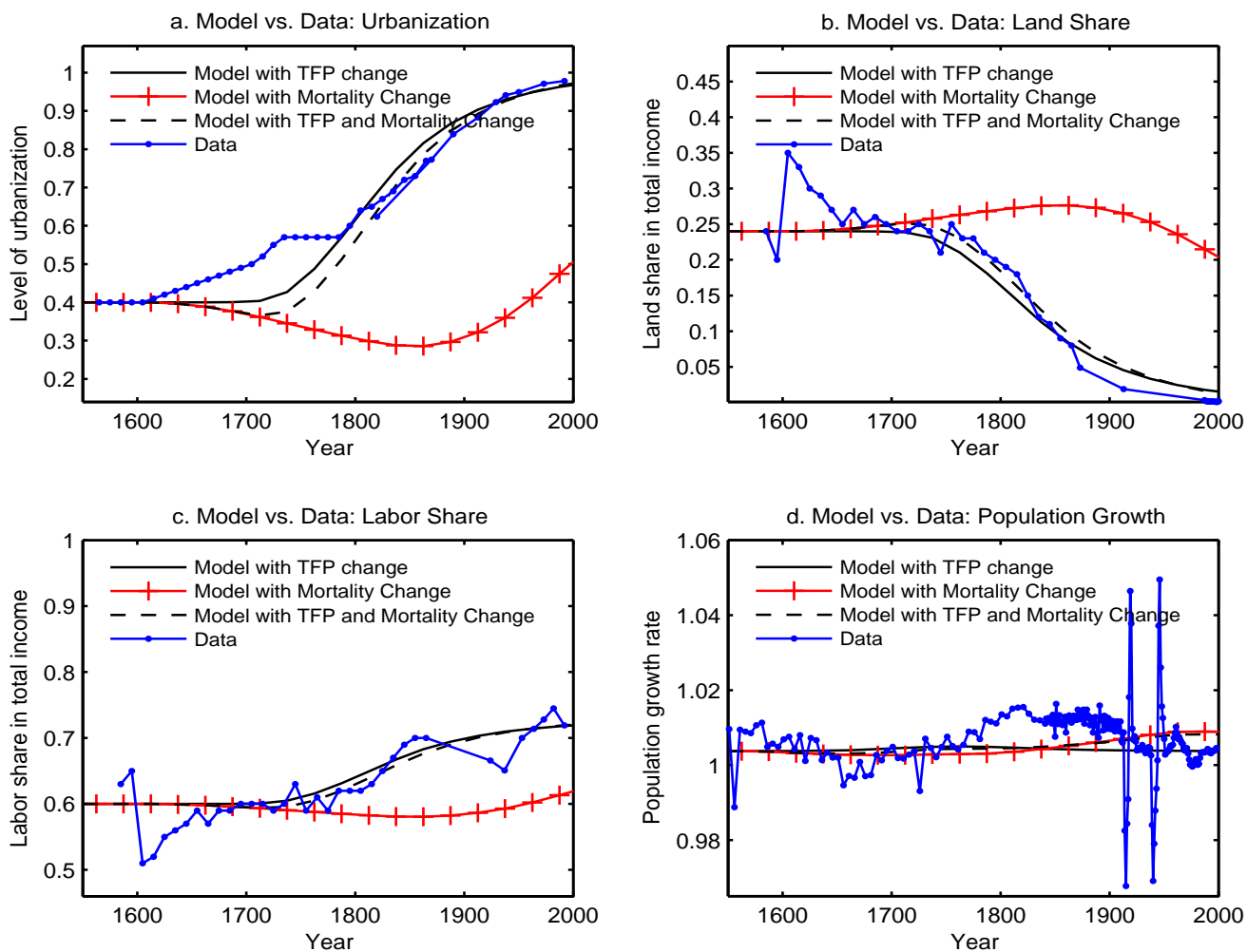


Figure 4: (a) Model vs. Data: Urbanization, (b) Model vs. Data: Land Share in Total Income, (c) Model vs. Data: Labor Share in Total Income, (d) Model vs. Data: Population Growth

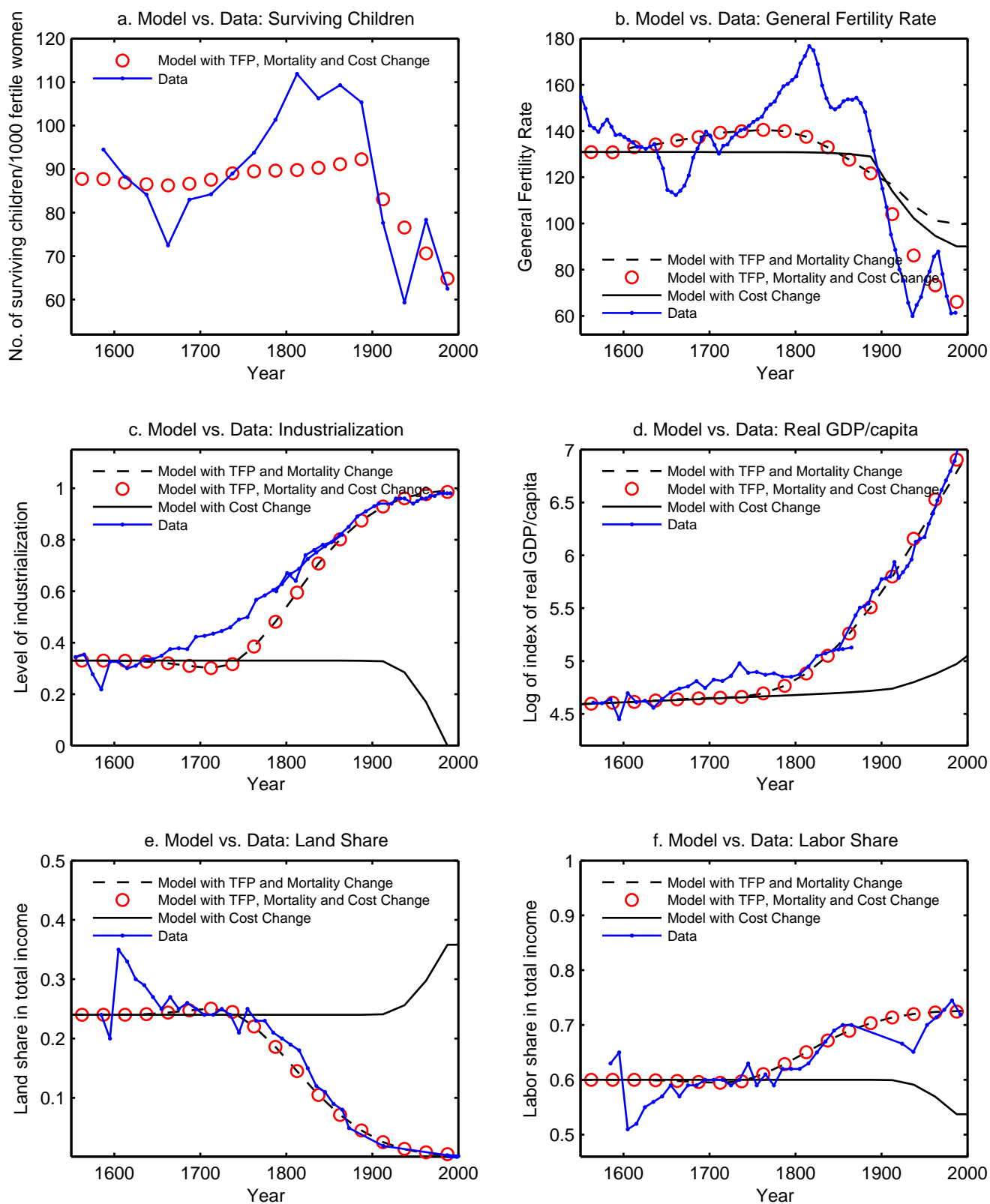


Figure 5: (a) Model vs. Data: Surviving Children, (b) Model vs. Data: General Fertility Rate, (c) Model vs. Data: Industrialization, (d) Model vs. Data: Real GDP/capita, (e) Model vs. Data: Land Share in Total Income, (f) Model vs. Data: Labor Share in Total Income