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Comment on "Heat capacity, time constant, and sensitivity of Earth's climate system" by S. E. Schwartz

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[1] *Schwartz* [2007] recently evaluated the time constant of Earth's climate system. However, his methodology yields to a significant underestimation of the value of τ and obscures a much more interesting property of the system. Schwartz found τ = 5 ± 1 years. Herein, by using an improved methodology I find that for short time scales from 0 to 2 years τ is of the order of a several months and for larger time scales, at least up to 20 years, τ is at least 70% larger than what Schwartz estimated.

[2] *Schwartz* [2007] hypothesized (his equation (17)) that the climate system behaves as a first-order autoregressive process plus a linear trend. The implicit idea seems that the linear trend represents the effect of the external forcings on climate while the temperature signal, detrended of the above linear component, represents the internal variability of the same. This internal variability is assumed to be described by an AR(1) process whose autocorrelation function, $r(\Delta t)$, decays as an exponential function of the lag time Δt with a given time constant τ: $r(\Delta t) = \exp(-\Delta t/\tau)$.

[3] Although in physics using simple models is useful, the one suggested by *Schwartz* [2007], with a single time constant, is an oversimplification and, as I will prove below, it is inconsistent with the analysis. In fact, it is very well known that climate is the combination, coupling and superposition of several phenomena. Some phenomena respond quickly as the atmosphere, others as the deep ocean respond very slowly. Thus, each climate component responds with its own time constant that might range from a few months to several years or decades.

[4] Given the length limitation of the temperature data herein analyzed (approximately 125 years) the analysis is limited to time scales below 20 years with a monthly resolution and I look for two time constants. The climate model I suggest is

$$R_t = T_t - F_t = X_t + Y_t, \tag{1}$$

where t = 1, 2, ... is a discrete time index and

$$X_t = a_1 X_{t-1} + \zeta_t \tag{2}$$

$$Y_t = a_2 Y_{t-1} + \eta_t. \tag{3}$$

So, we have that T_t represents the global temperature, F_t is the climate effect of the external forcings, R_t the residual signal. The residual signal is made of a slow plus a fast AR(1) processes, X_t and Y_t, respectively, which describe the internal variability of climate. ζ_t and η_t are two independent white noise processes with zero mean and standard deviation σ_1 and σ_2, respectively. The above model has the residual signal R_t characterized by the following autocorrelation function

$$r(\Delta t) = A_1 \exp(-\Delta t/\tau_1) + A_2 \exp(-\Delta t/\tau_2), \tag{4}$$

where $A_1 + A_2 = 1$. By calling $a_1 = \exp(-\Delta/\tau_1)$, $a_2 = \exp(-\Delta/\tau_2)$, where in our case Δ = 1/12 years, we have

$$\frac{\sigma_1^2}{\sigma_2^2} = \frac{A_1\left(1 - a_1^2\right)}{A_2\left(1 - a_2^2\right)}, \tag{5}$$

thus the relative magnitude between A_1 and A_2 is directly proportional to the relative magnitude of the two noise variances.

[5] Figure 1a shows a global surface temperature record from 1880 to 2008 [*Brohan et al.*, 2006] (note that the record includes data from 1850 to 1880 too, but they are excluded herein because *Schwartz* [2007] excluded them). Figure 1b shows the temperature record detrended of the linear component, as Schwartz does. The autocorrelation function of a time series $\{\xi_i\}$ with i = 1, 2, .., N (mean μ and variance σ^2) is

$$r(\Delta) = \frac{1}{(N-\Delta)\sigma^2} \sum_{i=1}^{N-\Delta} (\xi_i - \mu)\left(\xi_{i+\Delta} - \mu\right), \tag{6}$$

where Δ is the lag time. Figure 1c shows $r(\Delta t)$ of the sequence plotted in Figure 1b and its fit with the above autocorrelation function within the interval from 0 to 11 years where the results are more stable. I obtain τ_1 = 0.40 ±

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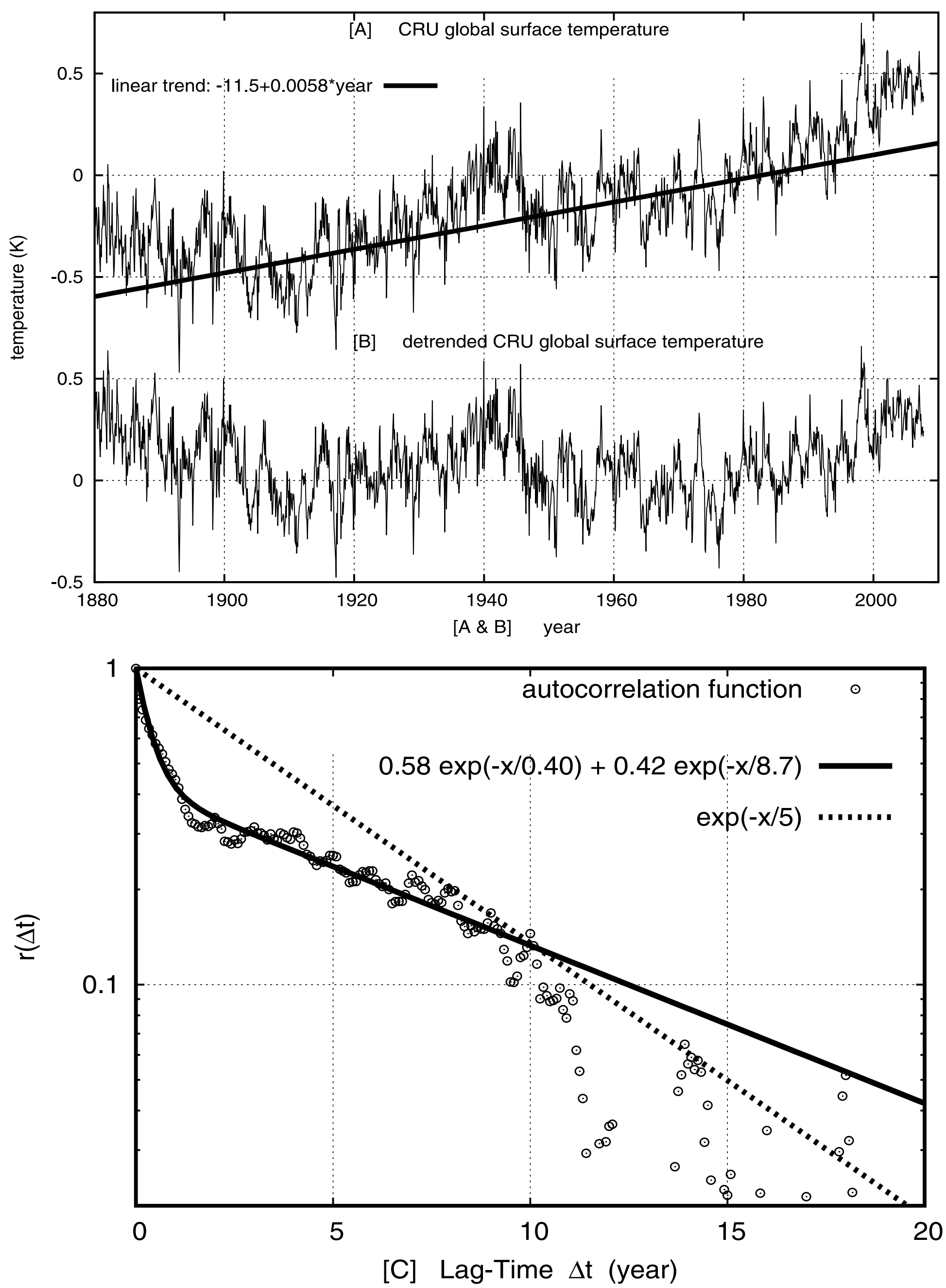

Figure 1. (a) Global monthly average surface temperature [*Brohan et al.*, 2006]. (b) Detrended sequence. (c) Autocorrelation function of the detrended sequence shown in Figure 1b. The *y* axis is in logarithmic scale.

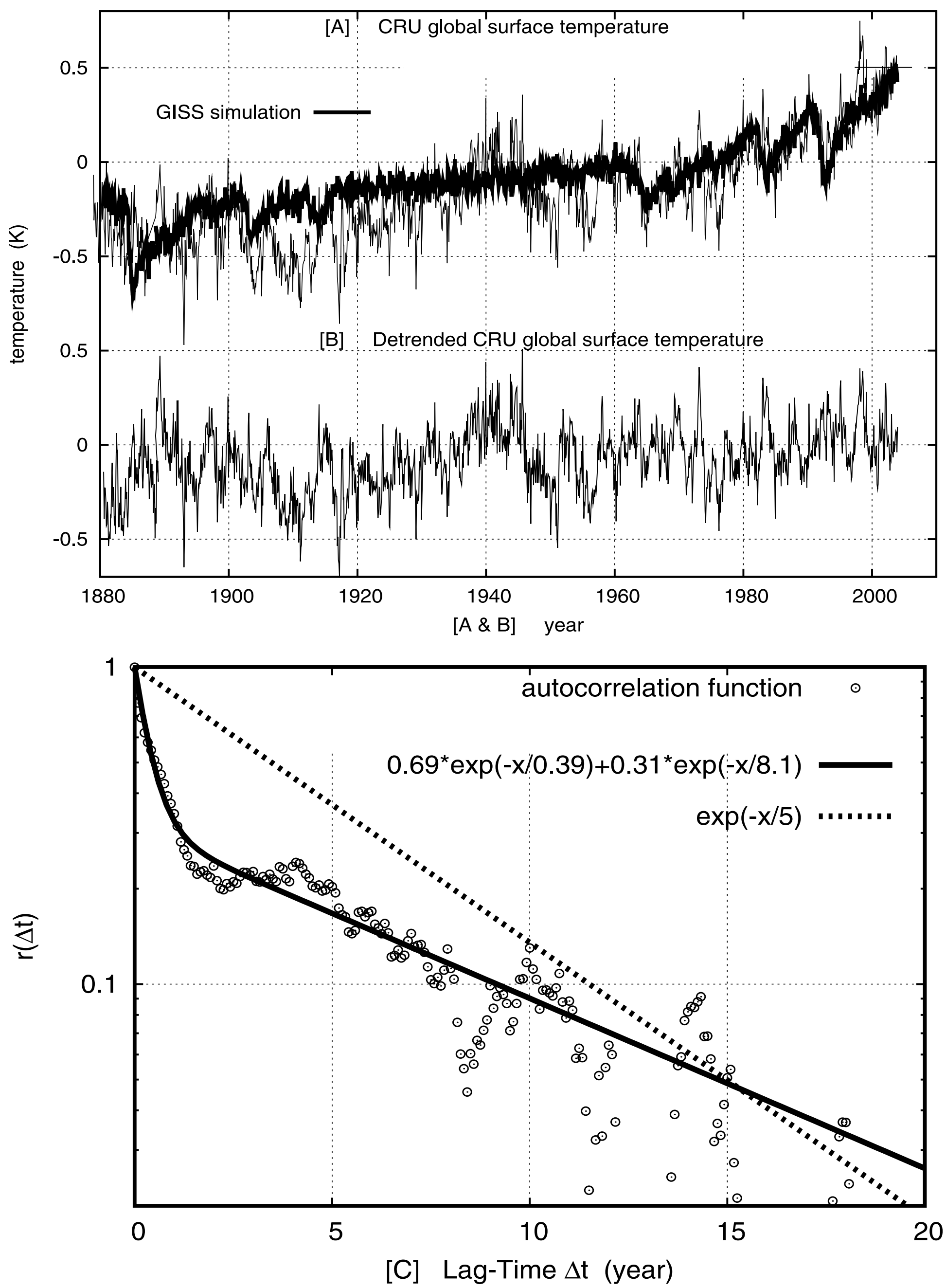

Figure 2. As in Figure 1 but instead of a linear trend I detrend the average global temperature GISS modelE simulation (monthly moving average) obtained by using all forcings [*Hansen et al.*, 2007].

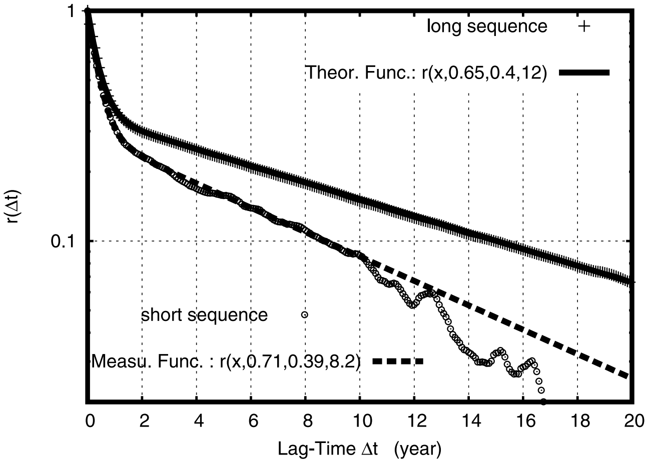

Figure 3. Comparison between autocorrelation functions for computer generated data. A result with a long sequence obtained with A_1 = 0.65, τ_1 = 0.4 years, τ_2 = 12 years is compared with an average result obtained with ten sequences of 125*12 = 1500 data. The curves are fit with the function $r(\Delta t, A_1, \tau_1, \tau_2)$.

0.1 years in the short range and $\tau_2 = 8.7 \pm 2$ years in the long range, respectively. Figure 1c also shows Schwartz's equation with τ = 5 years and it is evident that it does not fit the data. Note that in his Figures 6 and 7, Schwartz claims that $\tau(\Delta t)$ monotonically increases in Δt from 0 to 5 years. But, as Figure 1 shows, this is not correct. Indeed, Schwartz's result is a mathematical artifact of using a single AR(1) process. Also, I find $\sigma_1/\sigma_2 \approx 5$.

[6] However, the above conclusion follows by assuming, as *Schwartz* [2007] did, that the internal variability of climate can be deduced by simply removing a linear trend from the temperature data. This choice is evidently questionable. I repeat the calculation by removing from the temperature data the average global temperature GISS simulation obtained by using all forcings [*Hansen et al.*, 2007]. The rationale is that the residual shown in Figure 2b represents the internal variability of the climate system as obtained by the GISS model. Figure 2c shows the result. I obtain $\tau_1 = 0.39 \pm 0.1$ years in the short range and τ_2 = 8.1 ± 2 years in the long range, respectively. These values do not differ significantly from the previous ones obtained with a simpler linear detrending, and suggest that the above results might be quite robust, unless the GISS model is found to be extremely poor. Also, I find $\sigma_1/\sigma_2 \approx 6$.

[7] However, the length of the time series herein analyzed is quite short, and the time constants might be underestimated. To estimate the magnitude of the statistical bias I compare the autocorrelation function of very long computer generated time series according to the above model with sequences of 125*12 = 1500 data, as the sequences herein analyzed. The results is shown in Figure 3. It seems that τ_1 is not significantly changed, the real τ_2 might be 50% larger than the measured one, and the real A_1 might be 10% smaller than the measured one.

[8] Thus, Figures 1c, 2c, and 3 show that within a time scale of 1–2 years the climate is characterized by a fast time response of about 5 months while for time scales larger than 1–2 years up to 20 years the climate system is characterized by a slower response with a measured time constant of about 8 ± 2 years, which may correspond to 12 ± 3 years by taking into account the statistical bias. These estimates are significantly larger than what *Schwartz* [2007] calculated, but well agree with what *Scafetta and West* [2007] found with an alternative model by adopting the latest solar and temperature proxy sequences since 1600: $\tau = 9 \pm 3.25$ years.

[9] By trusting *Schwartz*'s [2007] equations about the equilibrium climate sensitivity, $\lambda_s^{-1} = \tau/C$, and assuming that $\tau = \tau_2$, I obtain a value that ranges from a measured λ_s^{-1} = 0.5 K/Wm^{-2} to a hypothetical λ_s^{-1} = 0.7 K/Wm^{-2}. These values are below but compatible with the estimates summarized in the Fourth Assessment Report of the IPCC [*Intergovernmental Panel on Climate Change*, 2007]: $\lambda_s^{-1} = 0.8^{+0.4}_{-0.3}$ K/Wm^{-2}. The above values correspond to an equilibrium temperature increase for doubled CO_2, ΔT_{2X}, from a measured best estimate of about 1.7 K to a hypothetical one of about 2.6 K: the IPCC best estimate is about 3 K, from a minimum of 1.5 K to a maximum of 4.5 K. However, the value of τ required in the above equilibrium climate sensitivity might be larger than τ_2 because it might refer to a secular or millenarian time scale, while τ_2 refers to a decadal time scale.

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