CS 3240: Languages and Computation

Problem Set 4 Solutions

Problem 1

Problem 1.40 Part (b) on Page 89 of Sipser.

Solution: Let A be any regular language and let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA that recognizes A. It is easy to see that $\mathsf{NOEXTEND(A)}$ is recognized by the DFA $M' = (Q, \Sigma, \delta, q_0, F')$, where F' consists of every state in F from which there is no path to any state in F. Therefore $\mathsf{NOEXTEND(A)}$ is regular.

□

Problem 2

Problem 2.22 on Page 130 of Sipser.

Solution: Note that a string of the form $x\#y$ is a member of language C if and only if $x \neq y$, and the latter holds if and only if for some position i, $x_i \neq y_i$. A PDA can recognize language C as follows:

1. Skip input symbols and use the stack to remember the number of symbols skipped, until a position in x is nondeterministically guessed.
2. Remember the symbol at this position in its finite control, and skip all the symbols until it seems $\#$.
3. Using the stack, skip an equal number of symbols as in Step 1.
4. Check if the current symbol is different from the symbol remembered in Step 2.

More precisely, the PDA does the following:

1. Read the next input symbol, and push a 1 onto the stack.
2. Nondeterministically jump to either Step 1 or Step 3.
3. Remember the current input symbol in the finite control.
4. Read the next input symbols until $\#$ is read.
5. Read the next symbol, and pop the stack.
6. If the stack is empty, go to Step 7, else go to Step 5.
7. Accept if the current input symbol is different from the symbol remembered in Step 3. Otherwise reject.

□

Problem 3

Problem 2.23 on Page 130 of Sipser.

Solution: We construct a PDA that recognizes language D. A string w is a member of language D if and only if $|w|$ is even and w can be written as $w = xy$ where $|x| = |y|$ and $x_i \neq y_i$ for some position i. The challenge is to guess the right positions in x and y. We make the key observation that if $w = xy$ where $|x| = |y|$, then for each index i, the number of symbols in w from x_{i+1} to y_i, is exactly equal to the number of all other symbols in w. A PDA can recognize language D by guessing a place in x and a place in y, and using the stack to check whether the number of symbols between the two places equals the number of other symbols. More precisely, the PDA does the following:

1. Read the next input symbol and push a 1 onto the stack.
2. Nondeterministically jump to either Step 1 or Step 3.
3. Remember the current input symbol in the finite control.
4. Read the next input symbol and pop the stack. Repeat until the stack is empty.
5. Read the next input symbol and push a 1 onto the stack.
6. Nondeterministically jump to either Step 5 or Step 7.
7. Reject if the current symbol is the same as the symbol remembered in Step 3.
8. Read the next input symbol and pop the stack. Repeat until the stack is empty.
9. Accept if the input is empty.

□