POWER FLOW IN TRANSFORMERS VIA THE POYNTING VECTOR

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The fundamental electromagnetic principles associated with the transfer of power from the primary to secondary windings of a transformer are considered. This power flow is by means of the E and H fields and the resulting Poynting vector. The basic transmission line nature of the device that sets up the surface flux, and its diffusion into the core, will only be considered briefly, and we will assume steady state conditions. It is shown that the leakage flux, which is often considered to be of secondary importance, is an essential feature of the ‘ideal’ transformer. The basic concepts are established for the textbook type transformer model, and then applied to various practical transformers.

1. BASIC TRANFORMER OPERATION

Consider the basic ‘text book’ transformer model shown below

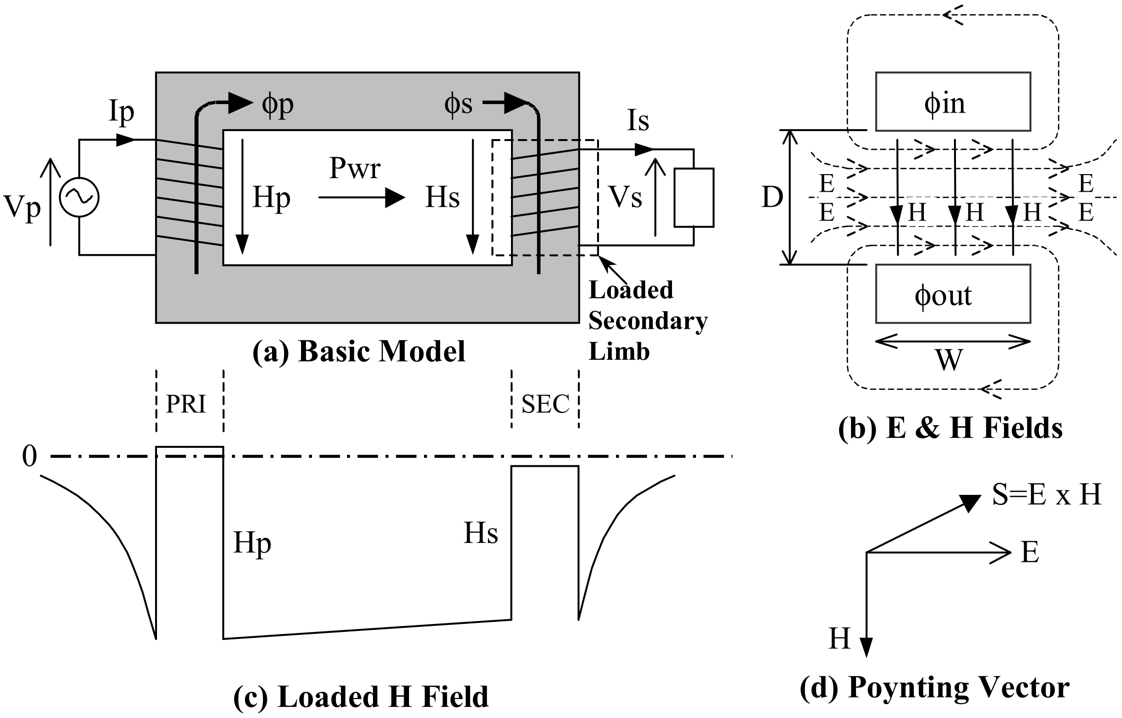


Figure 1- E & H Fields in Basic Text Book Transformer Model

1.1 E Field Around Core

On open circuit the currents are small and the H field is negligible (ideally zero). The primary voltage sets up the magnetic flux, ϕ_p, in the vertical primary limb such that the voltage around the primary limb is:

$$\text{Volts/primary-turn} = \oint E_p . dl = -\frac{d\phi_p}{dt} \approx \frac{V_p}{N_p}$$

Most of this flux is guided by the horizontal limbs to the secondary winding so that the voltage/turn (ie. integral of E) around the secondary limb is only slightly less than that around the primary. The E fields due the changing flux in the upper and lower limbs tend to reinforce each other in the gap between the two limbs and cancel each other in the space beyond. Hence the E field is not uniformly distributed around the limbs, but concentrated into the space between them. Neglecting fringing, the magnitude of the E field between the limbs is

$Eg \approx Vturn/W$, where W is the width of the core.

1.2 Loaded Transformer H Field

When the transformer is loaded its vertical secondary limb plus winding (carrying the load current), can be considered to be represented by an equivalent magnetic limb whose effective parameters (μ_e , σ_e) are such that the electromagnetic fields are unchanged. The losses in this equivalent 'loaded secondary limb' would be equal to the original core losses plus that of the secondary load. As the load on the transformer is increased the effective parameters of the loaded secondary limb (σ_e , μ_e) would be such that the effective ac reluctance ($|H/B|$) of the loaded secondary limb (windings & core) increases, and the primary current increases to produce the greater H field necessary to maintain the core flux.

On crossing a current sheet I, the change in H is given by, $\oint H.dl = \int_S J ds = I$ so the H fields are as indicated in Fig1(c). Most of the H field appears across the horizontal limbs. At the primary end $\Delta Hp = IpNp$ while at the secondary end $\Delta Hs = IsNs$. Hs is slightly less than Hp due to the reluctance of the horizontal magnetic limbs. Thus the average magnetic field strength between the horizontal limbs $Hg \approx (Hp + Hs)/2 \approx Hp \approx Hs$.

1.3 Power Flow

Energy is guided from primary to secondary by means of the horizontal magnetic limbs, in much the same way that copper cables 'transmit' electrical power from a source to a load [1,2]. The magnetic core flux is initially set up on the surface of the core materials (eg. Laminations) by transmission line action, in the same way current is established on the surface of copper conductors, and then diffuses into the core [3]. The power flows in the gap via the Poynting vector $S = E \times H \approx E\,H \sin 90^{O} = E\,H$ VoltAmps/m^2, and is directed towards the secondary winding. The instantaneous power is given by:

$$P(t) = \int_S S.ds = \int_S (Eg(t) \times Hg(t)).ds \text{ VoltAmps.}$$

This Poynting vector takes a small amount of power into the core (reactive and real), due to the small longitudinal component of H required to magnetise the core, while most is directed towards the secondary winding, to energise the load.

The power leaving the primary is,

$$Pp(t) \approx Eg.Hp.W.D = \frac{Vp(t)}{Np.W}.\frac{Ip(t).Np}{D}.W.D = Vp(t).Ip(t)$$

In general, for sinusoidal excitation, the E and H fields will not be in phase and some of this power will be imaginary (ie oscillatory) while some will be real, depending upon nature of the core and the load. The average power flowing from the primary winding will be $Vp.Ip \cos \phi_p$ while the average power arriving at the secondary will be $Vs.Is \cos \phi_s$.

1.4 Leakage Flux

The classical transformer theory the leakage flux is taken to be the component of the total flux that does not link with the secondary winding. While in many instances it is used to give the transformer a particular output reactance, for current limiting and sharing purposes, it is usually associated with some non-ideal characteristic that might be eliminated in an idealised case. This however is not the case for two important reasons.

(a) The gap 'leakage' flux plays a very important part in establishing the main core flux, which creates the E field around the core. In much the same way as the displacement current establishes current on the surface of a conductor, which then diffuses into the interior of the conductor, the leakage flux establishes the main core flux. The flux is initially established and built up on the surface of the core via a transmission line type process, and then diffuses into the interior of the core. Thus at 50Hz the flux only has to move a distance of half the lamination width into the core and this is about the max distance it can diffuse in and out in the 20msecs period of the 50Hz supply. Its phase velocity in the core material is very slow compared with the gap leakage flux that moves at velocity c, and is the real reason for laminating the core. If it was not for this fact the total length of a flux path in the normal laminated core would be limited to less than 1mm, so the gap leakage flux is extremely important in getting the main core flux into the laminations.

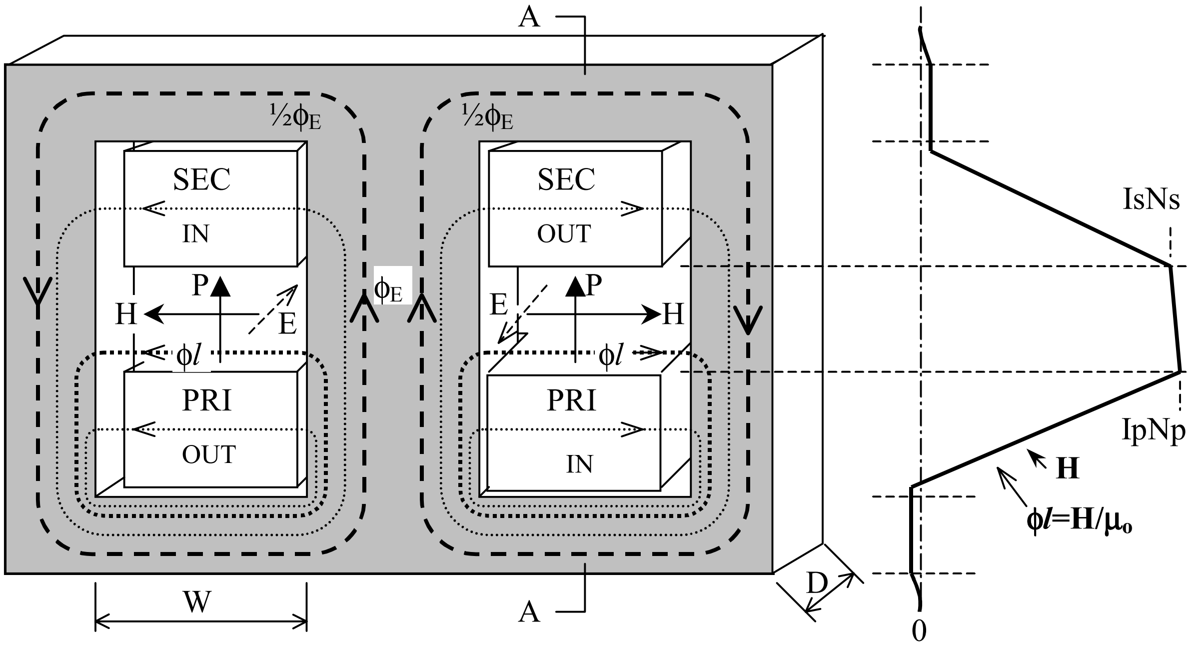
(b)Both the E & H fields are necessary in transferring power from the primary to secondary windings. The H field responsible for power flow to the secondary winding produces the main component of 'leakage' flux ($Bg = Hg/\mu_o$) and is a vital to the transformer operation. This 'leakage' flux mainly embraces the primary winding only and is a consequence of fundamental transformer operation. It would be just as prominent, even if the core were ideal (infinite permeability, no loss etc). As well as this essential leakage flux (ϕl) associated with power flow from primary to secondary, there will of course be a very much smaller component of leakage flux around the conductors that connect the transformer to the electrical system on both the primary and secondary sides. However, even these are ultimately associated with the H fields of the Poynting vector associated with the power flowing in and out of the transformer.

2. SINGLE-PHASE TRANSFORMER WITH AXIAL WINDINGS

Many small single-phase transformers are constructed by winding the primary and secondary windings around each half of the vertical centre limb of the core.

2.1 E & H Fields and Poynting Vector

A cross section of such a the transformer is shown below where the gap between the two windings has been widened to show the E & H fields. The winding pass through the two windows of the core, the width of the windows being W and the depth D as indicated.



(a) Section Showing E & H Fields & Power Flow **(b) H & Leakage Flux Through A-A**

Figure 2 - E & H fields in Transformer With Axial Windings

2.2.1 The E field Around the Core

On open circuit the primary sets up the main flux ϕ_E in the centre limb such that voltage around the limb is

$$\text{Volts / Turn} = Vp/Np = -d\phi E/dt = \oint E\, dl$$

This flux divides equally between the left and right side limbs as shown. Since the flux is directed in opposite directions through the limbs on opposite sides of a window, the E field inside the window is reinforced while outside the window the E fields due to $d\phi_E/dt$ tend to cancel. The net effect is that the E fields are not uniformly distributed around the core but appear mainly along the depth D of the window space. Neglecting fringing the magnitude of the E field in the window space is given by

$$E \approx \frac{\text{Voltage / Turn}}{2D} = \frac{Vp}{2NpD} \ \text{V / m}$$

2.2.2 Electric Field Within the Coil Windings

There is very little electric field within the copper conductors themselves, even along the sections that are inside the window. This is because the longitudinal E field due to $d\phi_E/dt$ tends to be cancelled out by the electrostatic field set up along the conductor due to the terminal voltage. The difference between these two fields is only the relatively low value needed to produce the current ($J_{Cond} = \sigma E_{Cond}$). Thus the longitudinal E field due to $d\phi_E/dt$ in the actual winding appears along the surface of the conductors and in the insulating material inside the window space. As well as this, each turn integrates this longitudinal E field which results in an additional transverse E field in the insulating material and spaces between adjacent turns. Therefore as well as appearing along the insulating material, the E also gets continually concentrated across the insulating material and spaces as inter-turn fields.

2.3 H Field

The low H at the bottom horizontal limb causes the H field to be reflected upwards into the primary winding is shown as shown in Fig 2(b). It can be seen that the field builds up linearly as we move through the primary winding, reaches a maximum in the space between the windings, and drops off as we move through the secondary. In the space between the windings, $H_{max} \approx \frac{IpNp}{W}$ A / m

2.4 The PoyntingVector and Power Flow

The power flows from the primary to the secondary by means of the Poynting vector S = E×H VA/m^2 which is directed upwards from the primary to the secondary in both the left and right hand windows. Although the actual E fields inside the windings themselves are complicated, it will be assumed that their effect on the power flow can be averaged out as equivalent to the longitudinal E existing throughout the winding space. Thus the power flow will be proportional to the magnitude of the H field and builds up as it flows through the primary winding towards the secondary. On reaching the gap between the primary and secondary windings the power in each of the windows is given by

$$P_W = \int_s (E \times H)\, ds = EH\, WD = \frac{Vp}{2NpD} \frac{IpNp}{W} WD$$

$$= \frac{VpIp}{2} \text{ VoltAmps}$$

Total power leaving the primary winding

$$P = 2P_W = VpIp \text{ VoltAmps, as usual.}$$

This power flows in the space between the primary and secondary windings, and apart from some fringing is restricted to the volume inside the windows. It should always be borne in mind that the only power flowing into the conductors themselves is that required for the copper (I^{2}R) losses. The main power required for the secondary load (reactive & real) flows along the insulating spaces between the winding turns and in the space between the primary and secondary windings.

3. TRANSFORMERS WITH CONCENTRIC WINDINGS

Many transformers (single and 3-phase) are constructed by winding the primary and secondary windings concentrically around a limb of the core as illustrated in Fig 3 for the single-phase case

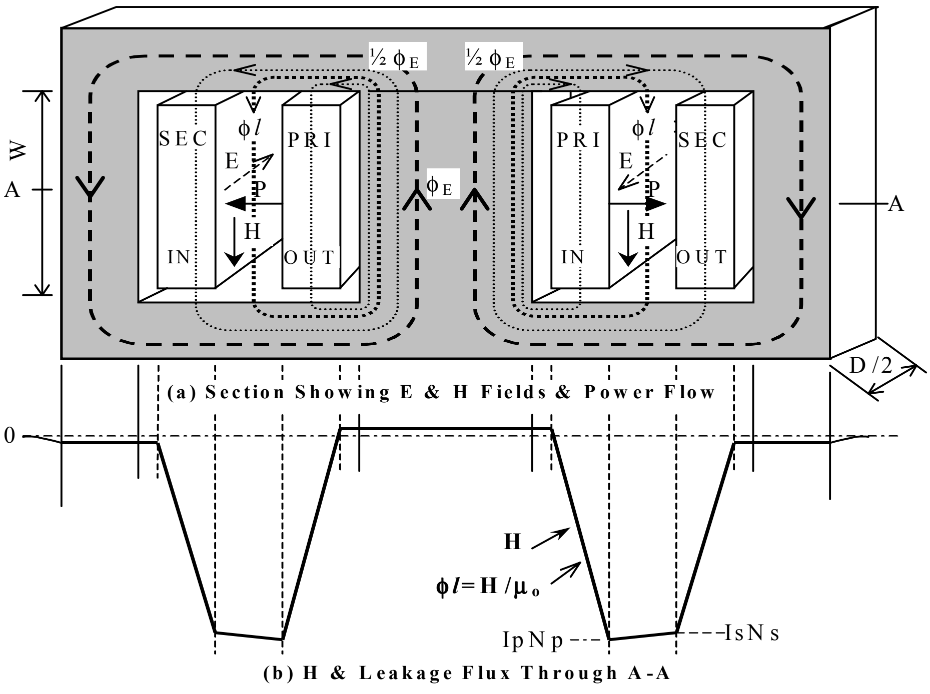


Fig 3 - E & H fields in Transformer With Concentric Windings

The core flux divides equally between the left and right side limbs as shown in Fig 3(a), and produces an E Field due to $d\phi_E/dt$, in the two windows. As previous the E fields are not uniformly distributed around the core but appear mainly along the depth D of the window space. Neglecting fringing the magnitude of the E field in the window space is given by $E \approx \frac{\text{Voltage / Turn}}{2D} = \frac{Vp}{2NpD}$ V / m, as previous

The H field along the length of the transformer is has shown in Fig 3(b). In this case it is the low H in the centre limb that causes the H field and hence the power to be directed towards the secondary windings. Again the field builds up linearly as we move through the primary winding reaches a maximum in the space between the windings, and drops off as we move through the secondary windings. In the gap between the windings, $H_{max} \approx \frac{IpNp}{W}$ A / m , as previous.

The power flowing in the gap between the primary and secondary windings in each of the windows is

$$P_W = \int_s (E \times H)\, ds = \frac{Vp}{2NpD}\frac{IpNp}{W} WD = \frac{VpIp}{2} \text{ VA}$$

as before.

Again this power flows in the space between the primary and secondary windings, and apart from some fringing is restricted to the volume inside the windows.

The leakage flux is an inevitable consequence of the H field in the space between the primary and secondary windings which is necessary for power flow. This leakage flux fundamentally links only with the primary winding, and this would be the case for a single layer secondary winding. In the case of a multi layed secondary winding a relatively small portion of 'leakage' flux may link with some of the secondary turns. In the case of concentrically wound single-phase transformer with the primary inside the secondary, as shown in Fig 3, the leakage flux in the gap between the primary and secondary windings is returned via the horizontal and centre limbs. In the case of a concentrically wound 3-phase transformer with a balanced load the 3 phase leakage fluxes in the spaces between the primary and secondary windings will sum to zero and will not go through the vertical limbs around which the coils are wound. Instead the leakage flux in each of the gaps will be returned via the other two gaps mainly via the horizontal limbs which connect the 3 vertical limbs on which the coils are wound.

4. TOROIDAL TRANSFORMER

A toroidal transformer in which the primary and secondary windings are interwound around a toroidal ring of magnetic material will now be considered.

4.1 Toroidal Transformer On No Load

The cross section of a toroidal step up transformer with a turns ratio of 1:2 is shown in Fig 4, on no load.

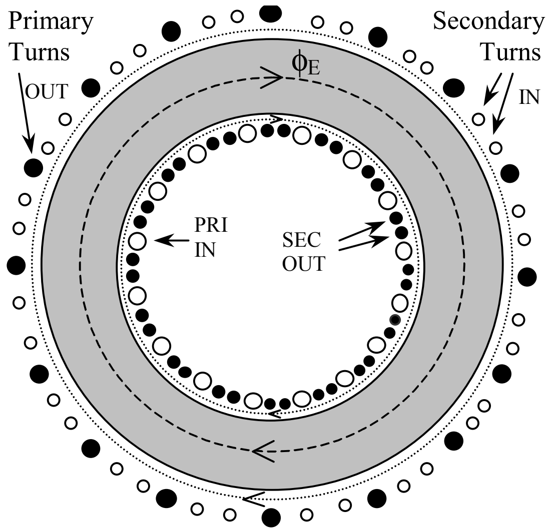


Fig 4 – Toroidal transformer On No Load

The primary sets up the flux ϕ_E such that $d\phi_E/dt = \int E.dl$ around the core = Voltage/Turn. Again this E field is not uniformly distributed around the core, and is greatest around the internal side of the core space, where the effects of the flux in opposite sides of the core are additive. There is a small amount of ϕ_E outside of the core itself due to the finite permeability of the core. In an idealised case the H field around the core would be zero, and the flux ϕ_E restricted to the core itself.

4.2 Loaded Toroidal Transformer

As the secondary is loaded the H field in the radial spaces between the primary and secondary windings increases and power flows from primary to secondary as indicated in Fig 5.

It can be seen that the H field responsible for the power flow surrounds the windings and twists it way around the core. This H field enters the core just below each primary turn and returns to circulate around the outside of the secondary. The resultant leakage flux flows around the outside of the secondary winding and does not link with it.

Most of the power (P) will flow in the inner space of the toroid where the E field is greatest, but a significant amount will also flow around the outer region of the windings particularly if the inner diameter of the toroid is greater than half that of the outer.

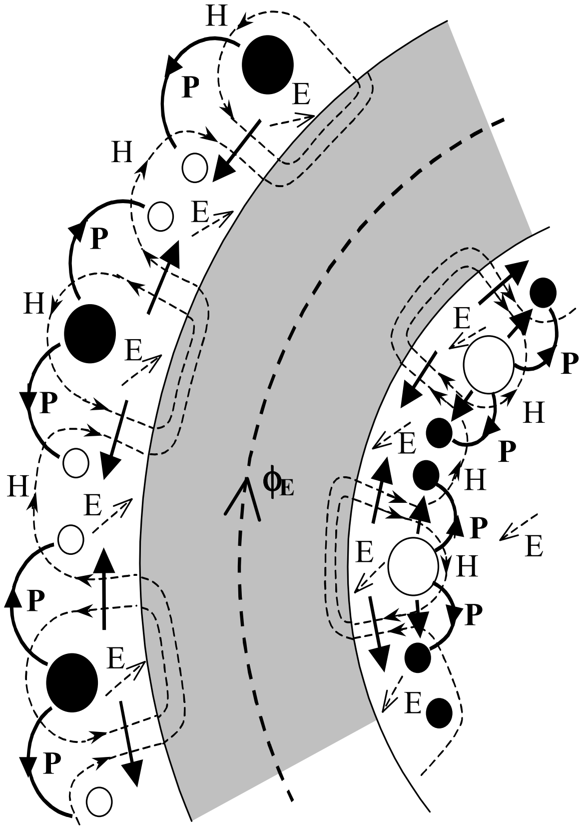


Fig 5 – E & H Fields and Power Flow in Toroidal Transformer

5. CONCLUSIONS

The fundamentals of power flow in transformers have been examined, at least at a conceptual level, and it has been shown that the power flows in the gap between the primary any secondary windings via the E and H fields. The core carries the main flux responsible for the E field, and conditions within the core itself are virtually independent of load.

When the transformer is loaded the core acts as a guide, reflecting the H field and directing the power flow from the primary to the secondary winding.

The so called 'leakage' flux is necessary for setting up the main core flux, and as been shown to be a direct consequence of the H field necessary for power flow, and would exist even in an idealised case.

The results have then been applied to practical transformers, and been seen to produce results which agree with standard circuit theory.

6. REFERENCES

[1] F. Herrmann, G B. Schmid, " The Poynting vector field and the energy flow within a transformer", Amer. J. Physics, 54 , 528-531, June 1986.

[2] W A. Newcomb, "Where is the Poynting vector in an ideal transformer?",Amer. J. Physics, 52, 723-724, August 1984.

[3] J.Edwards, T.K Saha, "Establishment of current in electrical cables by electromagnetic energies and the Poynting vector", AUPEC'98, Vol 2 385-388, Sept 1998.