Static vs. Dynamic Investment Policy: Matching Asset Management to Investor Risk Preferences

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Abstract

Many investment policies have guideline asset allocations that target a constant proportional weighting between stocks and bonds during both bull and bear markets. A "Constant Mix" asset management approach is defensible under a variety of commonly held assumptions. Generally, a recommendation to "stay the course" during volatile market conditions implicitly assumes that investors, will benefit from a Constant Mix approach. Investors, however, are not a homogeneous group; and, for some, changes in wealth may cause changes in risk tolerance. Wealth management, therefore, may require dynamic changes in both asset allocation as well as other aspects of investment policy such as rebalancing, distribution and monitoring & surveillance policies.

This article outlines several asset management approaches and discusses the importance of aligning Investment Policy with investor risk preferences. If the investment policy ignores this critical "calibration," there is little hope that the investor will be able to maintain the policy during recessionary periods. It presents a follow on simulation study to illustrate the range of possible portfolio evolutions under various asset management approaches. Finally, it focuses on how investment advisors can transition from static ("blueprint" oriented) to dynamic ("systems engineering" oriented) investment policy by using advanced simulation tools and by transforming set-in-stone asset management guidelines into a sequence of periodic decisions regarding the exercise of asset management options.

Static vs. Dynamic Investment Policy: Matching Asset Management to Investor Risk Preferences

Among financial economists there is a general consensus that strategic asset allocation is an important factor in determining a portfolio's long-term expected risk and return.[1] Conversely, short-term traders lack a strategic asset allocation because their investment policy is "to make money" though a rapid series of round-trip transactions in only a few securities (or, baskets of securities).[2] If price prognostications are correct, the trader is successful; if not, the concentrated security positions may prove disastrous. The world of an aggressive *trader* is one of feast or famine.

By contrast, economists assume that most *investors* wish to use capital markets to solve longer-term "intertemporal cash flow problems."[3] An endowment fund wishes to have sufficient money to build a new hospital facility in eight years; a worker wishes to accumulate funds to support consumption during retirement; a young couple wishes to accelerate capital into the present by borrowing to finance a home purchase. Capital markets act like time machines sending money from the present to the future (investing), or from the future to the present (borrowing). In these circumstances, a feast or famine approach is generally inappropriate unless the investor enters the marketplace with entertainment or speculative motives. Although capital markets accommodate a variety of participants ranging from hedgers (farmers selling a futures contract against their crop) to arbitrageurs (buyers and sellers who act when they perceive a violation of the "law of one price"—securities with identical payoffs in all future economic states must sell for the same price), this essay focus primarily on individual investors wishing to accumulate funds for important future goals such as retirement and bequest objectives.

Wealth, Risk, and Required Return

With apologies to Nobel prize economist Kenneth Arrow (who proved that rational, risk-averse investors will always commit at least a small percentage of wealth to a risky investment with an expected return in excess of the risk free rate),[4] we begin by stating that if an investor's current wealth is sufficient to fund critical future objectives by investing only in risk-free investments (U.S. T-Bills), then he need not take additional market-related risk.[5] Bill Gates, for example, need not worry about stock returns to ensure to his personal financial security. That is to say, he does not have an "intertemporal cash flow problem" because it is highly unlikely that he will run out of money in the future. Bill Gates does not need a strategic asset allocation that includes risky asset positions.[6]

Investors with a smaller stock of money, however, must solve this problem, at least in part, by determining an appropriate set of long-term exposures to the risks and returns of capital markets ("systematic risk exposures").[7] If I want to retire comfortably in ten years, what percentage of current wealth should I expose to real estate, foreign small capitalization stock, the S&P 500 stock index, and so forth? If my objectives are ambitious and my current wealth is small, then my allocation must incur substantial systematic risk so that there is the expectation of earning a commensurate reward. If my objectives are modest relative to my current wealth, then the opposite holds true.

But determining hypothetical "optimal" asset allocations based on systematic risk exposures is merely an intellectual exercise. On paper, many investors want to maximize expected return, and many investors have the courage of lions. If all investors share the same capital market efficiency assumptions (markets are the optimal mechanism for allocating societal wealth); if all investors owned the same amount of wealth (identical initial endowments); and, if all investors have the same personal preferences and goals, then the most appropriate wealth accumulation strategy would simply be to buy the capitalization-weighted world capital market and hold it throughout the applicable time horizon [in fact, this is the optimal strategy for certain investors].[8] However, investors have different preferences and endowments. Consequently, the wealth accumulation strategy that is appropriate for one investor may be far from optimal for another.[9] These differences make solving for an optimal wealth accumulation approach more difficult. Investors must resolve at least two complex problems: (1) what is an appropriate strategic asset allocation; and (2) what is the best way to manage the portfolio so that, as it evolves, its risk does not exceed the investor's ability (and willingness) to withstand possible losses? The investment problem becomes at least three-dimensional for portfolios making current cash distributions; but this essay does not explore the complexity of "decumulation" strategies.

A properly drafted Investment Policy Statement [IPS] can help address these problems. The first problem is relatively straightforward. For example, actuarial calculations may suggest appropriate longevity and return assumptions over the applicable planning horizon to determine the "required return" for a portfolio owned by a worker (and, possibly, a spouse) who is twelve years from retirement. If the required return is in excess of what is feasible to earn in the capital markets, the portfolio owner must either decide to delay retirement or reduce planned annual withdrawals during retirement.[10] Addressing the second problem is more difficult. Although more money is always better than less, the pursuit of more money also increases the risk of a future shortfall.[11] A portfolio that generates higher expected ending wealth is not necessarily preferable to a portfolio with both lower expected terminal wealth and lower downside risk.[12] Hence, expected ending wealth should not be the only standard for comparing alternate portfolios. Rather, the investor should select the portfolio that maximizes both return requirements and risk preferences. In the words of economists, the investor seeks to maximize "utility" where utility is defined as satisfaction with the portfolio in terms of its likelihood of meeting both future economic goals and interim risk preferences and investment constraints. The Appendix at the end of the article provides additional insight into the interrelated concepts of "utility," "risk aversion," and "risk tolerance."

The properly drafted IPS should memorialize the investor's specific risk/return preferences, asset allocation, planning horizon, tax and legal issues, liquidity demands, and other constraints.[13] The IPS should reflect each investor's current levels of wealth, personal preferences for future consumption (the future "cash flow" problem), the ability to tolerate downside portfolio risk, the planning horizon, and other critical factors. At the heart of most individual investors' IPS is a strategic asset allocation which defines appropriate systematic risk and return exposures. The correct asset allocation enhances the investor's ability to achieve economic success at an appropriate level of volatility.[14] This essay focuses on how investor risk tolerance affects IPS design; and, specifically, how the investor calibrates risk tolerance and wealth accumulation objectives under conditions of investment uncertainty—i.e., when investing in risky assets.[15]

Constant Absolute Risk Aversion

Consider the following example which assumes only a two asset market—the risk free asset (Treasury-Bill) and a risky asset (Stocks). The risk free T-Bill's return for the forthcoming year is 3%; and the expected return on stocks is 9% with volatility (risk) of 15%. The investor selects a portfolio with a strategic asset allocation of 30% T-Bills and 70% stocks. The expected payoff of this portfolio equals:

$$(30\%)(3\%) + (70\%)(9\%) = 0.9\% + 6.3\% = 7.2\%.$$

If the investor has an initial wealth of $1 million, the portfolio's expected gain at year end equals $72,000. What is the portfolio's downside risk? The risk-free asset has no volatility. Consequently, the portfolio's risk as measured by the annual standard deviation of the risky asset is 10.5%.[16] Given the portfolio's million dollar beginning value, a variance of:

- One standard deviation represents a change in wealth of ± $105,000;
- Two standard deviations represents a change in wealth of ± $210,000; and,
- Three standard deviations represents a change in wealth of ± $315,000.

Assuming the portfolio follows a normal return distribution, the investor runs the risk of the following downside results:

- A 34% probability of one-year portfolio value between $1,072,000 and $967,000;
- A 13.5% probability of one-year portfolio value between $967,000 and $862,000;
- A 2% probability of a one-year portfolio value between $862,000 and $757,000; and,
- A 0.5% probability of a one-year portfolio value below $757,000. [17]

At the end of the year, the investor decides to add additional funds to the portfolio to bring the total portfolio value up to $2 million. However, he does not wish the strategic asset allocation to incur any additional risk to his dollar wealth. In order words, the portfolio should not incur more than a one-standard deviation risk of $105,000. If the expected volatility and returns for the risk-free and risky asset remain the same, the portfolio's asset allocation changes to a 65% T-Bill (risk free) / 35% stocks (risky) asset weighting. The investor will not put any additional funds into stocks because he wishes to keep his dollar-denominated risk to a $105,000 standard deviation. The investor selects an initial strategic asset allocation but, as portfolio wealth changes, he does not "stay the course" in the sense that he chooses not to maintain the initial target allocation throughout all future periods.

The IPS implications are clear—the portfolio should not maintain a constant percentage exposure to the systematic risk of stocks. However, many IPS documents presume that constant risk exposures will be maintained.[18] Traditionally, there have been two primary justifications for the design and implementation of investment portfolios that are periodically rebalanced to their target asset allocations:

1. A mathematical approach based on the concept of maximizing investor utility. Under this approach, deviations from the optimal risk exposures produce disutility; and, therefore, should be corrected provided that the present value cost of rebalancing does not exceed the present value of "utility loss." [19]
2. A statistical approach based on the assumption that a sufficiently long planning horizon produces results that converge to long-term expected values under the "law of large numbers." This means that holding to a constant asset allocation target throughout both up and down market cycles is the best guarantee for long term success.[20]

The issue of the extent to which predictability (forecasting conditional expectations) or variation in the risk premium (non constant investment opportunity set) over time may motivate a change from strategic asset allocation towards tactical asset allocation is beyond the scope of this essay.

Other Risk Aversion Functions and Implications for Investment Policy

The hypothetical investor exhibits a risk aversion function characterized by constant absolute risk aversion or CARA. This risk aversion function is interesting, but is not common. More typically, investors view increases in portfolio dollar value as a "cushion" that, all else equal, allows for an increase in portfolio risk. Having a cushion makes the investor more comfortable accepting risk. Conversely, some investors manifest increasing risk aversion--putting fewer dollars at risk as the portfolio's cushion grows. Other investors exhibit constant relative risk aversion or CRRA. The CRRA investor is willing to risk a constant *percentage* of wealth (as opposed to a constant *dollar value* of wealth) within a reasonable range above and below the current level of wealth. The CRRA investor might be willing to risk 10% of wealth when the portfolio value is $1 million, or 10% of wealth when the portfolio is either $800,000 or $1.2 million.[21] The CRRA investor is most likely to approve and sustain a fixed target asset allocation through both up and down markets. For this investor, the IPS serves primarily as an "architectural document" akin to a building blueprint where all aspects of the portfolio's structure must remain "up to code;" rather than as a "systems engineering" protocol where the portfolio owner periodically evaluates different asset management options.[22]

Blueprint-oriented IPS documents may not work for all investors. Expanding the above example, assume that a financial advisor presents the investor with a series of model portfolios each of which has a different macro allocation between safe and risky assets.

The least volatile portfolio consists of a 100% allocation to T-Bills while the most volatile allocation consists of a 100% allocation to stocks. Other model portfolios consist of various positive weightings of the two assets. Each point on the allocation spectrum determines a tradeoff between reward and risk. The challenge to the investor is to select a macro allocation that best suits his required return objectives and his risk tolerance. The investor decides that, given his current level of wealth and his planning horizon, an allocation of 78% to risky assets and 22% to risk free assets provides maximum expected utility (satisfaction with the portfolio).

Sometimes it may be possible to derive an equation that describes the investor's risk aversion function.[23] Intuitively, the slope of the investor's risk aversion equals the rate at which the investor is willing to trade risk for return. If this were not the case, the investor would move either up or down the risk/return spectrum until he found the tradeoff point that maximizes his expected satisfaction. Of course, every investor wants to earn positive returns; but, when making decisions under conditions of uncertainty (i.e., investing part of wealth in a risky asset), this result can never be guaranteed. What is certain, however, is that, at the moment of portfolio choice, the financial advisor knows the "marginal rate of substitution" of a risk averse investor who prefers more money to less, and who selects a specific risk/return tradeoff. The investor's portfolio selection decision identifies both:

1. The marginal rate of substitution (the ability and willingness to trade risk for return) at the selected asset allocation point; and,
2. The rate of return required to achieve the financial objective.

Together, these two factors jointly determine the most appropriate strategic asset allocation for the investor.[24]

Unfortunately, investors do not walk around with their unique risk aversion equations tattooed on their foreheads. Furthermore, although the financial advisor may infer the correct equation because of the investor's strategic asset allocation choice, the advisor only knows the equation in the "neighborhood" of the asset allocation point (78% risky asset / 22% safe asset, in this example). This limited knowledge could be a problem. Let's jump ahead one year and revisit the hypothetical investor.

It has been a bad year. The dollar value of the portfolio has declined significantly and the applicable planning horizon has also changed. Remember, the investor selected the 78/22 allocation based, in part, on his economic circumstances (level of dollar wealth) at the time of the initial decision. Given the ability to make another decision, based on today's wealth, does the investor retain the 78/22 allocation; or, does the investor wish to leave the 78/22 neighborhood because of a change in his risk aversion function. What would motivate the investor to wander outside the "hood?"

There are several factors that can impact the asset allocation decision when wealth declines due to falling asset prices:[25]

<u>The Wealth Effect</u>: A decrease in wealth may cause the investor's risk aversion to increase (especially if the portfolio values are nearing the critical point at which the

investor has no further wealth "cushion"). As the investor approaches a critical minimum wealth level, he may become more sensitive to volatility. If enough investors exhibit increased risk sensitivity, all else equal, asset prices will fall as risky asset are sold in favor of safe assets. Eventually, this effect can trigger an asset price death spiral as risky asset sales generate a feedback loop that motives even further sales.

The Risk Effect: The increase in downside volatility may cause investors to demand a corresponding increase in compensation for investing in risky assets (an increase in the expected "risk premium"). The increase in the expected future risk premium is implemented in the capital markets by a reduction in price for financial assets. Lower prices translate into higher expected future returns, especially if volatility moderates.[26] The risk effect counterbalances the potential death spiral of the wealth effect. It is the condition that brings value and contrarian investors into the market.

The Liquidity Effect: An increase in downside volatility makes it less likely that a potential buyer will want to purchase your asset. This effect is currently visible in the residential housing market. Three years ago, some real estate agents told customers that their greatest risk was not buying a home immediately because prices were skyrocketing. Delaying the purchase would only result in a more costly future transaction. Currently, the reverse seems to be true. Agents find it difficult to find willing buyers because delaying a purchase may raise the likelihood that a cheaper purchase will be possible in the future. Liquidity (the ability to sell an asset at a reasonable price within a reasonable time) can evaporate rapidly in a deflating market. Facing diminishing liquidity, extremely risk averse investors may sell assets even at substantially reduced prices.

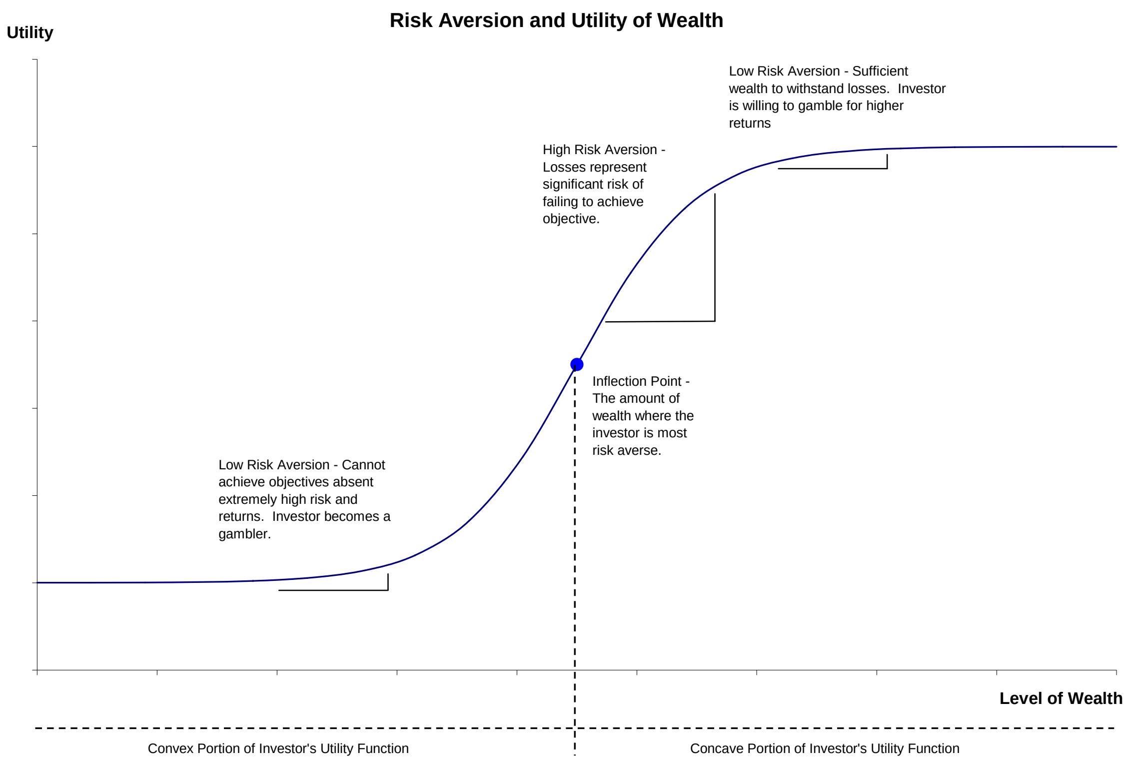
The Diversification Effect: Increased downside volatility can put price pressure on a broad cross section of financial assets. In this scenario, falling prices cause inter-asset correlation to increase (most investments in the portfolio move downward in lockstep). As a result, diversification becomes a less effective portfolio risk management tool.[27] Highly risk averse investors may sell risky investments in an effort to substitute principal guarantees for asset diversification.

Investor Sensitivity to Changes in Wealth

How does the hypothetical investor react to the decline in portfolio dollar value; and, most importantly, how does he wish to position the portfolio for the future? Recall that the portfolio consists of two assets—a risk free position in T-Bills and a risky position in stocks. The initial strategic asset allocation was 78% stocks 22% T-Bills—a ratio of 3.54 [78 ÷ 22]. We know that the asset pricing dynamics caused a decrease in this ratio given the decline in the portfolio's dollar value. The dollar value of the safe asset remains the same while the dollar value of the risky asset falls. If, for example, the $1 million portfolio lost 10% over the year, the ratio is now 3.09 [68 ÷ 22] assuming no net interest on the T-bills. A 20% loss produces a 'risk/safety' ratio of approximately 2.64 with lower ratio values indicating yet a further reduction in portfolio risk due to the deceleration in the rate of future dollar declines.

If the investor's sensitivity increases *more than proportionately* with changes in wealth, then the investor will want to make no changes to the portfolio. Rebalancing to the initial asset allocation target exposures will not seem attractive. To see this, note that the risk/safety ratio improved by 13% as the portfolio dollar value declined by 10% ($1 million to $900,000). The ratio improved by an additional 15% as the portfolio declined by an additional 11% ($900,000 to $800,000). For this hypothetical investor, optimal wealth management, as codified in an IPS, may involve a buy-and-hold portfolio management strategy. Theoretically, if the price of risky assets goes to zero, the portfolio's minimum value is $220,000 (plus interest). If the price of risky assets rises, there is no cap on the portfolio's future dollar value.

If our hypothetical investor is *hypersensitive* to changes in wealth, but still recognizes that failure to use capital markets to solve intertemporal cash flow problems entails too high of an opportunity cost over a long-term horizon,[28] a buy-and-hold asset management strategy may not adjust quickly enough given the investor's steeply sloping utility function. What would cause such hypersensitivity? Consider the following chart that graphs a utility of wealth curve with a critical point. The X axis represents wealth, and the Y axis represents utility. The shape of the curve defines the investor's risk aversion. Where the curve is relatively flat, a large drop in wealth corresponds to just a small reduction in utility (satisfaction). But, where the curve is relatively steep, even a small drop in wealth represents a significant loss of utility.



Conceptually, the critical point on the curve represents the minimum level of wealth necessary to fund an important economic liability. This liability could be a balloon mortgage payment, a retirement nest egg with a minimum value, or so forth. To the right of the critical point the investor has a surplus. To the left of the critical point the investor has a shortfall. As the portfolio's value approaches the critical point from the right (i.e.

moving right to left), the investor becomes more and more sensitive to risk. Moving through to the left of the critical point represents a potential financial disaster.[29] The slope of the investor's risk aversion curve becomes increasingly steeper as the value of the portfolio—change in wealth-- moves closer to the critical point. The slope value measures the investor's sensitivity to changes in wealth—the steeper the slope, the more risk averse is the investor.

Imagine, also, that the hypothetical investor becomes more comfortable as the portfolio's wealth moves above the critical point's dollar value. Whenever there is a cushion, the investor feels like a gambler who plays with "house money." This is an investor that is hypersensitive to losses in the vicinity of a critical point; but, who is willing to take substantial risks in the presence of sufficient investment surplus.[30] Similarly, if the investor's wealth is well below the critical point, he may also be willing to take substantial risks, since the investor's only chance of achieving the critical value is to earn extremely high returns that can only come with extremely high risk. Note, however, that if these extremely high returns become manifest, risk aversion increases rapidly as portfolio values approach the critical point. When the goal is in sight, the pain of unexpected losses becomes much sharper.

The Floor + Equity Multiplier Asset Management Approach

What is an appropriate wealth management strategy for this investor? One critical aspect of an IPS may be to acknowledge a "floor" beyond which the portfolio's dollar value should not decrease. Under the buy-and-hold portfolio described above, the theoretical minimum floor was $220,000. If, in fact, the investor's actual floor value should have been $750,000, then, under the buy-and-hold approach, the initial strategic asset allocation was incorrect. The IPS allocation should have been $750,000 to T-Bills and $250,000 to risky assets. This is clearly a "safer" portfolio; but, unfortunately, it is a portfolio with only limited opportunity for the meaningful long-term growth usually required to solve intertemporal cash flow problems. Instead of a buy-and-hold asset management approach, this investor would better be served by a "floor + equity multiplier" approach.[31] Let's see how this would work in the simple two-asset portfolio example.

The portfolio's initial value is $1 million. Conceptually, its value can be segregated into two pieces: (1) a "floor" of $750,000, and (2) a "cushion" of $250,000. Depending on the curvature (i.e., slope) of the investor's utility of wealth function (as well as on several other factors), a "multiplier" is applied to the cushion. In this case, assume that the multiplier is 2.5. This means that for every $1.00 in the cushion, the investor is willing to place $2.50 into the risky asset. The initial strategic asset allocation is, therefore, the amount of cushion [$250,000] times the equity multiplier [2.5] for an allocation to risky assets (stocks) of $625,000. The formula is:

e = mc; or equity allocation equals multiplier times cushion.

The risk/reward tradeoff (marginal rate of substitution) that best suits hypersensitive investors can be calibrated through the floor and multiplier values. For example, a floor of $700,000 with a multiplier of 3.0 results in an initial strategic asset allocation of 90% risky asset / 10% risk free asset.[32]

The floor + equity multiplier asset management approach is a dynamic wealth management strategy. To see this, assume that the $1 million dollar portfolio with a $750,000 cushion and a 2.5 multiplier is worth only $900,000 at the end of the year. The cushion is now only $150,000; and the required equity allocation must shrink from $650,000 to $375,000 or 42% (375,000 ÷ 900,000). If, at the end of the year, the portfolio value decreased to $800,000, the cushion is a scant $50,000 and the equity allocation must adjust to $50,000 times 2.5; or, $125,000.[33] The new allocation is 16% risky asset / 84% T-Bills. As the portfolio's dollar value approaches the $750,000 minimum floor, the allocation to the risky asset goes to zero. As the value of the portfolio increases above the floor, the investor becomes more comfortable with risk and uses the multiplier to "leverage" upside returns. Risk aversion, and thus strategic asset allocation, may change considerably in areas far away from the "hood."

Note that the floor + equity multiplier strategy is not a market timing strategy. Rather than predicting market movements and changing the asset allocation in anticipation of them, it is a market reaction strategy that increases or decreases risk based on fully-known price moves. Note, also, that a $1 million buy-and-hold portfolio with a floor of $750,000 in T-Bills and a risky asset position of $250,000 is mathematically equivalent to a floor + multiplier portfolio with a $750,000 floor and a multiplier of 1.0. It is clear that the buy-and-hold portfolio management does not entail market timing; and, therefore, neither does the floor + equity multiplier approach.

The Constant Mix Asset Management Approach

If our hypothetical investor exhibits *average sensitivity* to changes in wealth, but still recognizes that failure to use capital markets to solve intertemporal cash flow problems entails too high of an opportunity cost over a long-term horizon, neither a buy-and-hold asset management strategy nor a floor + equity multiplier strategy may be appropriate. What wealth management strategy is suitable for this investor? To answer this question, imagine a gambler in a game with constant positive odds (probability of a win > 50% for any trial) who wishes to develop a strategy that minimizes the probability of ruin (bankruptcy) and that maximizes the chances of long-term success where long-term success is defined in terms of a profit and loss metric per unit of elapsed time in the game. Although this problem seems straightforward, it is not easy to solve; and, in fact, was not solved until J. L. Kelly—a scientist at Bell Labs in New Jersey—published a now famous essay in 1956.[34] If the player wishes to generate quick riches, he bets his entire stake on every trial because, for each trial, the odds are in his favor. However, this strategy will inevitably lead to an exit from the game. At the other extreme, if he wagers a sufficiently miniscule portion of his wealth on each trial, the law of large numbers guarantees that he will eventually win all the money after an infinite time in the game.

Unfortunately, investors with intertemporal cash flow problems may not have creditors with infinite patience.

The problem reduces to solving for an optimal fixed fraction of wealth that should be invested in each trial—under the assumption that the gamble has a positive return expectation (unlike the wagers offered to casino visitors or lotto ticket buyers). Although the mathematics of the solution is complicated, an approximating strategy with good odds for success is:

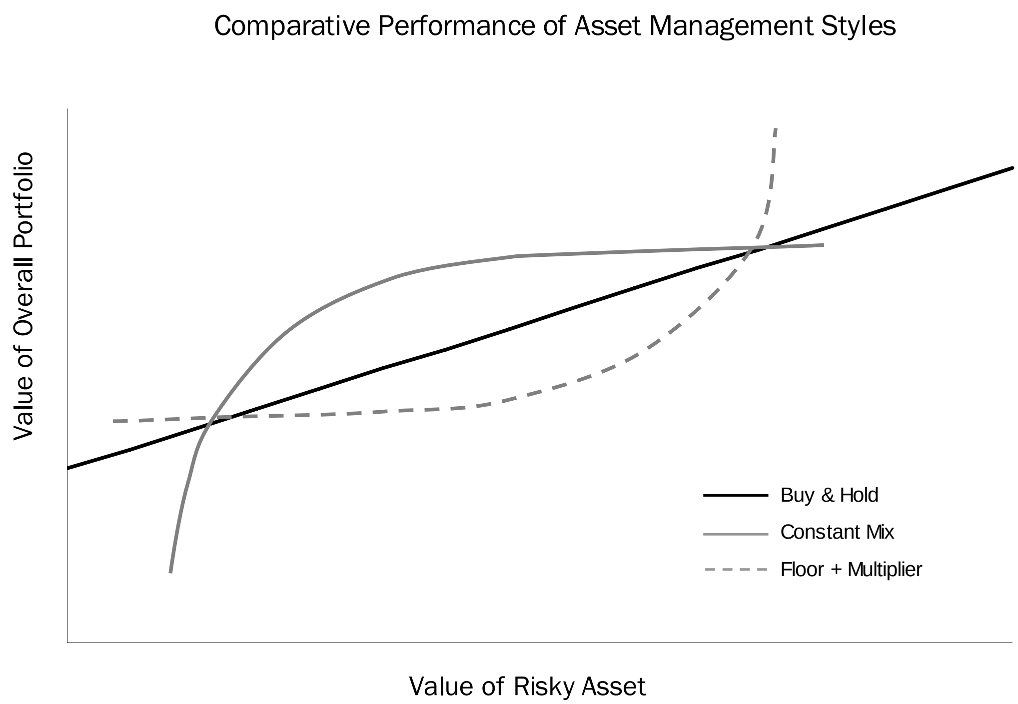
- Investment of a constant fraction of total wealth for each trial; and,
- Determination of the appropriate fraction. This is accomplished by solving for the maximization of "logarithmic utility" of wealth [the logarithmic function is the inverse of the exponential growth function; and, therefore will both optimize the time required to achieve any given level of wealth and minimize the chance of bankruptcy].

A rough investment equivalent of the constant fraction betting strategy is a constant-mix asset allocation. Although the constant fraction strategy has many appealing properties, it is most appropriate for investors with longer-term planning horizons and average risk aversion functions. In terms of our earlier discussion, investors electing to maintain a constant asset allocation throughout all future economies and levels of future wealth exhibit CRRA risk aversion functions (at least within a reasonably large "neighborhood" of their risk/return tradeoff point). Thus, in terms of our hypothetical investor, he will elect to maintain the 78% / 22% allocation throughout all market environments provided his wealth level does not change precipitously.

The constant mix asset management approach is, therefore, also a dynamic strategy that requires periodic portfolio adjustments (rebalancing to the IPS strategic asset allocation target). It should be noted, that the fixed-fraction / log utility strategy will, over time, provide greater odds of success than any other investment strategy; and, as time goes to infinity, its odds of success approach 100% while its odds of bankruptcy approach 0%. However, given the empirical characteristics of risky asset returns, even assuming a normal distribution of those returns, it may take hundreds of years for the constant mix strategy to outperform an all T-Bill portfolio and thousands of years to outperform an all risky asset portfolio (at a 95% confidence level).[35] In any finite time period, the fixed-fraction of wealth (i.e., the solution to maximizing logarithmic utility) strategy may require that the player's wealth shrink to nerve-shatteringly low levels with only the cold comfort of knowing that if the game can be continued infinitely, the player will win. In everyday language, investors must live with *actual* results not *theoretical* results.[36] This is why actual investors wishing to pursue a constant-mix asset allocation may not select the point on the risk/return spectrum that maximizes results over an infinite horizon; but, rather selects the point that keeps a more reasonable downside limit on the portfolio—minimizes the likelihood that it will wander too far away from the "hood." The maximum expected return portfolio gives way to the "safety first" portfolio.[37]

Static vs. Dynamic Wealth Management Approaches

To recap, three asset management approaches are available to investors with multi-period wealth accumulation time horizons: (1) buy-and-hold; (2) floor + equity multiplier; and, (3) constant mix. The approaches require different portfolio management tasks; and provide different expected payoffs to investors. The following graph depicts how payoffs differ under each asset management approach:



Most importantly, investor risk preferences must be calibrated accurately to the selected portfolio management approach to enhance the probability of investment policy success. This means that the IPS must not only specify the initial portfolio asset allocation; but, just as importantly, must specify whether the portfolio's risk will be sustained or will vary according to changes in investor wealth.[38]

The following table summarizes some important conceptual and practical differences among the three strategies:

	Buy & Hold	**Constant Mix**	**Floor + Equity Multiplier**
Operational Costs	Low	Moderate	High
Management Strategy	Static	Dynamic	Dynamic
Risk/Return Payoff	Linear	Concave	Convex
Rebalance Protocol	None	Time/%drift/volatility based	f(Δrisky asset price, floor value, multiplier)
Rebalance goal	Allocation remains unadjusted throughout all future markets	Keep initial allocation proportions constant throughout all future markets	Change initial allocation as a reaction to changes in market value of risky assets
Underlying investment	"efficient market"	Buy Low/ Sell High	Buy High / Sell Low

philosophy		(contrarian)	(momentum)
"Best Market"	Trending	Oscillating	Trending
Utility match in the neighborhood of the initial strategic allocation	Greater than Average Sensitivity to Changes in Wealth	Average Sensitivity to Changes in Wealth	Hypersensitivity to Changes in Wealth

Simulation Analysis

This section presents the results of a simulated multi-asset class portfolio consisting of a safe asset (one-year constant maturity U.S. T-Bills) and several risky assets. The simulation analysis does not incorporate taxes, fees or other investment costs. The initial value of the investment portfolio is $1 million allocated as follows:

Asset Class	Proxy Index	Buy & Hold	Constant Mix	Floor + Multiplier	100% T-Bill
US Large Cap Stock	S&P 500	25%	35%	30%	0%
International Large Cap Stock	MSCI EAFE	15%	25%	10%	0%
Short Term US Gov't Bond	One-Year Constant Maturity T-Bill	0%	10%	60%	100%
Intermediate US Gov't Bond	Lehman Brothers Intermediate Gov't/Credit	0%	15%	0%	0%
Intermediate Global Bond	Citigroup World 1+ Year Gov't Bond Index	0%	15%	0%	0%
Aggregate US Bond	Lehman Brothers Aggregate Bond	60%	0%	0%	0%

The simulation follows the evolution of each portfolio over a 20 year period. For each portfolio, the simulation model generates 5,000 trials and ranks each trial by its dollar value with the lowest values assigned to percentile 1 and the highest to percentile 99 (the median result is the value of the 50^{th} percentile, or the $2{,}500^{th}$ trial). Simulation parameters are based on monthly historical returns beginning in January 1973 (the beginning of the OPEC oil embargo recession) through September 2008. The Lehman Brothers' Aggregate U.S. Bond index begins in January of 1976; and the Citigroup Global Bonds index begins in July of 1989. The Buy and Hold portfolio's initial macro allocation is 40% stock / 60% bond. The Constant Mix portfolio has an initial macro allocation of 60% stock / 40% bond. The Floor + Multiplier portfolio assumes a floor value of $900,000 with an equity multiplier of 4. Given an initial cushion of $100,000, its initial macro allocation is 40% stock / 60% fixed income with all fixed income invested in the "safe" T-Bill asset class. Finally, for comparison purposes, we include a 100% T-Bill portfolio.

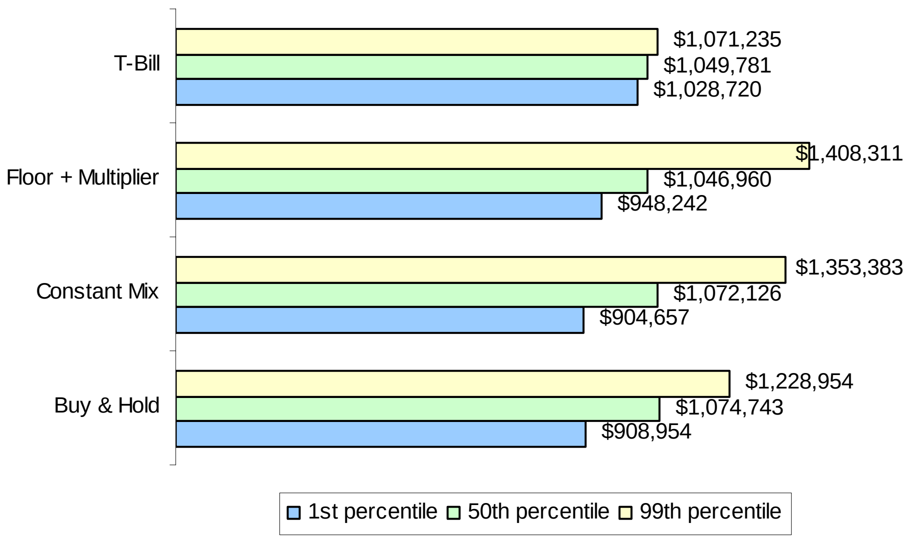
The return generation process is based on an inflation 'state of nature' variable as measured by changes in the CPI.[39] CPI is modeled as an autoregressive variable. The historical variance / covariance matrix is orthogonalized by a Cholesky decomposition which preserves the unconditional correlation structure. The multivariate joint return distribution is assumed to be log normal. This simulation approach is common in the literature; and, although there are a variety of advanced simulation methods to account for conditional moments (expected returns and volatility), dynamic correlation structures, and non-normal distributions, much of these techniques are utilized with high frequency data that exhibit characteristics that are, in effect, "averaged out" over the longer monthly intervals. Given that our main objective is to illustrate the economic differences between asset management strategies, the assumptions underlying the model of the return generating process, albeit simplified, are readily justifiable.

Prior to examining the model's output, it is important to make several observations:

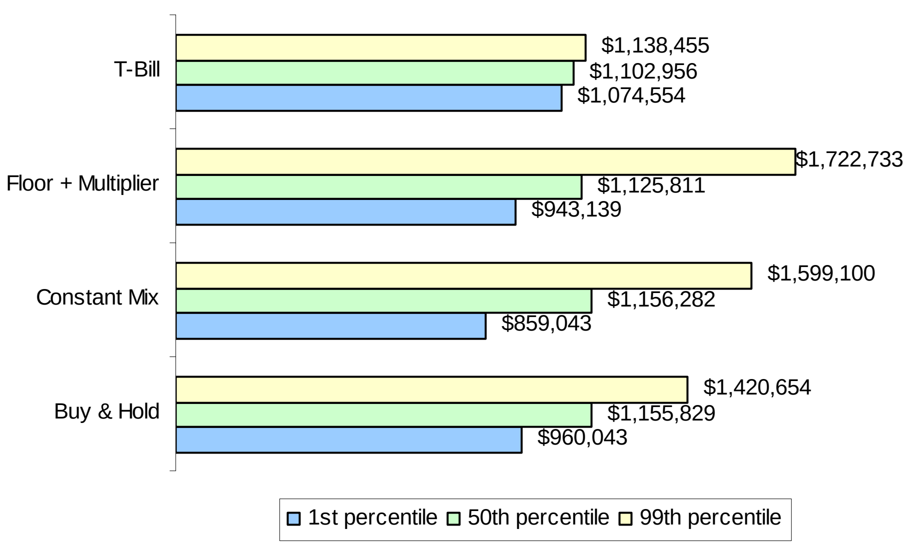
1. The output is mapped to "dollar-wealth" space rather than to "utility of wealth" space. Each investor's risk-aversion curve reflects their unique marginal utility of wealth. Some investors will apply an increasing (convex) utility penalty to decreases in portfolio value; while other investors will attach a more linear penalty under similar conditions. Non-investment factors such as home-ownership/mortgage indebtedness, labor income, closely held business interests also determine the investor's tolerance for losses and satisfaction with gains. Rather than providing an aggregate or an average utility value over the entire return distribution, the model's output is in dollars. Dollar value output requires careful judgment by the investor that the risk/return tradeoffs under one wealth management approach are preferred over the tradeoffs generated by the other approaches.
2. The output is applicable only under the absence of any cash flows into or out of the portfolio. Cash flows create path dependencies that may change investor preferences considerably. Terminal wealth may no longer be the primary economic objective; rather, the ability of wealth to sustain consumption over the entire planning horizon may lead to a "utility of consumption" dimension when selecting the appropriate wealth management approach.
3. The four portfolios are not directly comparable. The differences in their initial macro asset allocation are, in part, designed to account for the phenomenon known as "equity drift." Over time, higher expected equity returns should result in the allocation of the Buy and Hold and Floor + Multiplier portfolios tending towards 100% equity. Given a sufficient amount of time, (perhaps many decades) these portfolios will eventually dominate the 60-40 fixed-mix portfolio. This is one reason why the initial weights to fixed income are greater for the Buy and Hold and Floor + Multiplier portfolios.

The following graphs depict the evolution of the portfolio under various asset management approaches at intervals of 12, 24, 36, and 60 months.

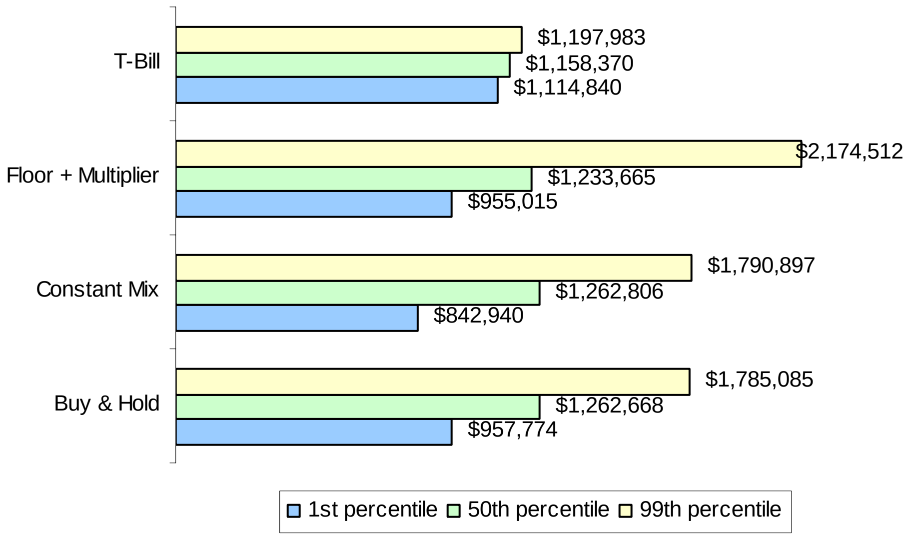
12 month range of returns



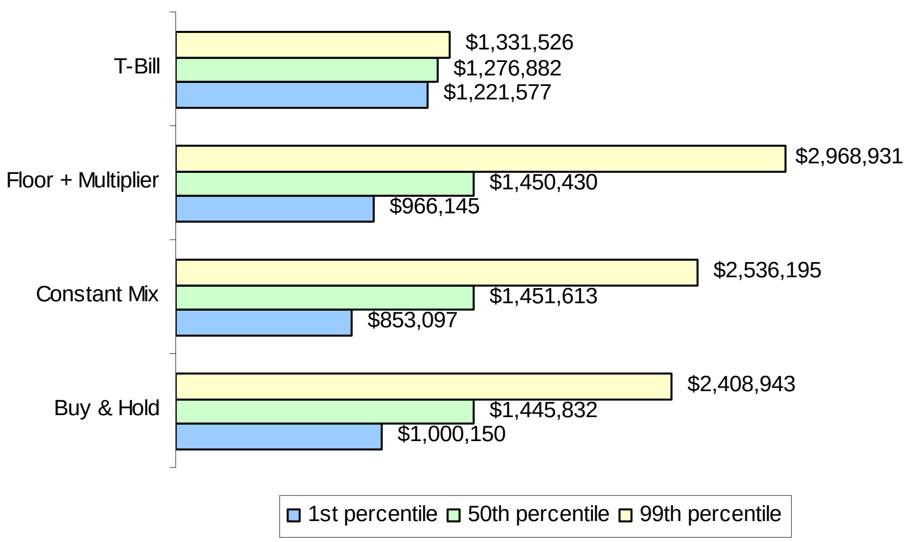
24 month range of returns



36 month range of returns

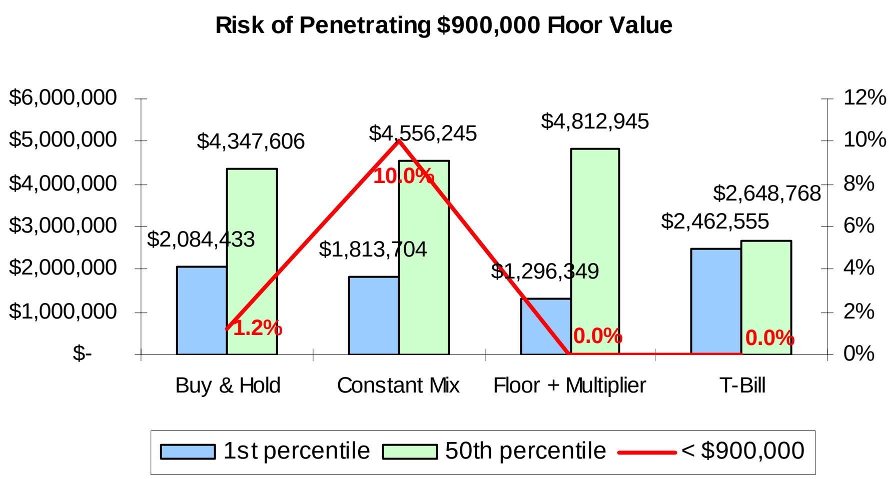


60 month range of returns



The above sequence of charts illustrates the considerable differences in potential outcomes under various asset management approaches.[40] Only investors with economic objectives that require limited growth in assets have the luxury of “managing” risk by “avoiding” risk. These investors may prefer an all T-Bill portfolio. On average, the Constant Mix approach generates favorable results. However, because it requires rebalancing into capital markets that have experienced the greatest relative declines, a prolonged bear market will accelerate losses in the portfolio’s dollar values. At the limit, continued rebalancing into declining capital markets will drive portfolio values close to zero. In the instant model, for example, the graphs indicate that the Constant Mix asset management approach fails to preserve the $900,000 downside floor and would require a more conservative allocation for highly risk averse investors.

The following chart extends the planning horizon to 20 years and compares the probability of penetrating the $900,000 floor value under each asset management approach. Median and "worst case" dollar values are represented by the chart's columns; while the portfolio's downside risk is represented by the superimposed red line.



If the critical dollar value point is $900,000; and, if penetrating this floor value *at any point in the planning horizon* would result in unacceptable disutility, the allocations within both the Buy & Hold and Constant Mix approaches must be changed, or, these approaches must be abandoned.[41]

We also point out that the Floor + multiplier approach requires liquid markets so that leveraged equity positions can be unwound at reasonable costs and within a reasonable time. However, bear markets are characterized by liquidity shortages as investors pile up demand-to-sell pressure to the point where it may overwhelm demand-to-buy. Such market conditions produce price discontinuities that create a positive probability that the minimum floor guarantee cannot be assured.[42] Other than the all T-Bill portfolio (an approach that has a substantial opportunity cost when measured by its long-term expected dollar value), there is considerable downside risk in each asset management approach. For those suggesting that time mitigates investment risk ("time diversification") we refer them to the lower-bound (first percentile) results in the graph which suggest that the all T-Bill portfolio results dominate those generated by the other portfolios.[43]

Investor Utility and the Dynamic IPS

Uncertainty of investment outcomes has profound implications for Investment Policy. A set-in-stone approach to asset allocation takes an "autopilot" approach under all economic circumstances; and may not be appropriate for all investors.[44] Indeed, it is easy to see how such an approach may create risk rather than effectively manage it. There appears to be no bullet-proof approach to asset management that is capable of guaranteeing success under all market conditions. Historically, a blind adherence to a Buy-and-Hold, a Constant Mix or a Floor + Multiplier asset management approach may have jeopardized critical financial objectives, especially in the presence of periodic distributions.

Conceptually, transitioning from an IPS considered as an architectural blueprint to an IPS considered as a systems engineering process, involves a two step process. Financial management consists of (1) portfolio design and implementation issues (the traditional asset allocation function of the IPS); and, (2) the set of future decisions that will make the portfolio evolve in a manner well suited to attain a possibly stochastic set of economic objectives.[45] In some cases, the IPS may reflect investor preferences for relatively static asset management decision making. For example, the investor may initially elect to use low cost indexes to achieve broad diversification; and, over time, may continue to favor this investment strategy. In this example, a feedback loop between changes in capital market *forecasts* may not require a change in investor preference for passive investing. However, there may be feedback loops between changes in *realized returns* and significant changes in personal wealth; and, the investor, under a dynamic IPS model, may have a series of planning options to consider. The Buy-and-Hold investor may wish to recalibrate portfolio risk and reward by readjusting the portfolio's asset allocation following a sustained period of equity drift; the Constant Mix investor may wish to refrain from restoring the full amount of exposure to risky assets lest a continuation of a high volatility regime increase the likelihood of penetrating a minimum wealth level; the Floor + Equity Multiplier investor may wish to raise the floor value to protect equity gains achieved during bull market environments.[46]

Dynamic IPS provisions allow the investor to adjust the portfolio so that it adapts to evolving conditions in way that best enables the finite sum of wealth to discharge the investor's legitimate expectations and objectives. A static IPS may lack the flexibility to employ a reasonable set of decision making tools required to navigate the uncertainty of future wealth, future liabilities, and the complex interrelationship that exists between them. It is particularly helpful to view a Dynamic IPS as a series of options that the investor may (or may not) exercise *after* they know how events unfold.[47] If the recent evolution of the portfolio is favorable, the strategic option may involve a consideration of how future returns (as well as future consumption) can be enhanced, and how present gains can be protected. If the recent evolution of the portfolio is unfavorable, the option decision process may involve a consideration of the adjustments required to preserve the feasibility of attaining critical future objectives. What amount of risk should continue to be borne and what amount of risk can be shed? Initial IPS decisions, rather than acting as irrevocable guidelines, become starting points for making intelligent future decisions that recognize the economic consequences of unfolding events. The indispensible tool for designing a dynamic IPS is advanced simulation capabilities that enable investors to "test drive" the economic consequences of a variety of asset management options prior to their implementation.

The suitability of an investment management approach depends on the investor's risk profile. A static IPS fixes investment decision making at the outset. This type of IPS may be appropriate for investors exhibiting certain attitudes towards risk and reward including Constant Relative Risk Aversion. If the investor assumes that the multivariate distribution of asset returns is approximately log normal, and if the investor is capable of sustaining a "growth optimal" portfolio approach (or, a constant fraction of wealth consumption approach) throughout all levels of portfolio wealth, then a static IPS may fit the bill. If, however, the investor manifests risk/reward preferences that cannot be characterized by a CRRA function, then it is unreasonable to expect the investor to adhere to a static IPS in unfavorable markets. An investor with above average sensitivity to changes in wealth may become too impatient with the rate of wealth growth in bull market regimes; or, too frustrated with the rate of wealth loss in bear market regimes. By contrast, a dynamic IPS has supervision and monitoring protocols that demand investors to pay attention to recent market conditions and current uncertainty; and, by analyzing their current economic circumstances, to make prudent decisions on a go forward basis. The criteria best suited for specifying the Investment Policy Statement's asset management approach is not based on a P&L metric; but, rather on expected utility criteria. No approach is inherently better or worse than another; rather, the informed investor should understand the costs and benefits of each approach, such that the selected approach optimizes the investor's expected utility (satisfaction) from his or her portfolio.*

*The authors wish to thank Jon Chambers and Huy Lam for assistance with graphics and editing.

APPENDIX

UTILITY OF WEALTH: This is a measure of "satisfaction" with any given level of wealth. As wealth changes, the investor's satisfaction also will change with a decrease in wealth generating "disutility" and an increase generating positive "utility." The rate of each investor's unique utility increase or decrease can be represented by a curve graphed in wealth (x-axis) / utility (y-axis) space. For most investors the pain of losing a dollar is greater than the pleasure of gaining a dollar; and the lower the level of wealth, the more the pleasure/pain tradeoff is magnified. As wealth approaches a subsistence level, the pain of further loss may become intolerable.

One important consequence of this observation is that utility is not necessarily "money-equivalent." A dollar is merely a dollar (the unit measurement stays constant), while the amount of pleasure and pain are variables. Investors generally prefer a risk-free payoff to a risky payoff with an equivalent expected value. This is because the risk-free payoff is certain; while the risky payoff may yield an amount greater or lesser than that guaranteed by the risk-free alternative. Given that the pleasure of gain is less than the pain of an equivalent loss, a risky payoff must provide a risk-premium to induce the investor to make the investment. Thus, any payoff must be adjusted to reflect the degree of uncertainty—this adjustment yields a money-equivalent index of satisfaction. For many financial economists, utility measurement is the best way to make such an adjustment. Investor sensitivity to wealth changes is the slope value that determines the investor's utility function curve—the rate at which wealth gains provide increased satisfaction and wealth losses generate disutility.

The existence of fixed commitments alters the asset allocation preferences of many investors. Such commitments may include home mortgages (home foreclosure creates significant disutility), education costs, closely hold business obligations, and, most importantly, a threshold standard of living where the threshold is variously defined as either a subsistence level or preservation of an existing lifestyle. If the return realizations achieved under the investor's asset management election are unfavorable—i.e., increase the likelihood of a failure to fund threshold commitments—an increasingly convex utility penalty may significantly alter the investor's risk tolerance.

RISK AVERSION: This is a mathematical expression of the investor's attitude towards risk. It is measured by the amount of risk premium required by the investor to make the utility of a risky asset position equivalent to that of a risk-free investment. Risk aversion curves are also known as "indifference curves." The curve plots the series of increasingly risky investments that provide equal utility to the investor (hence, the term "indifference"). The more sensitive the investor to a change in wealth, the steeper the slope of the indifference curve—that is to say, the greater the risk premium required to induce the investor away from the risk-free "neighborhood." The steepness of the risk aversion curve is mathematically equivalent to the change ("elasticity") of marginal utility at any given wealth level. Investors exhibiting Absolute Risk Aversion will not risk more than a specific dollar amount on any uncertain venture; investors exhibiting Constant Relative Risk Aversion will not risk more than a specific fraction of their wealth on any uncertain venture. Relative Risk Aversion is, at least theoretically, independent of the actual level of an investor's wealth.

Given that the utility of wealth curves are generally upward slopping--at a decreasing rate of acceleration as wealth grows larger-- it follows mathematically that the upwardly sloping curves have a positive "velocity" and a negative "acceleration." For readers familiar with calculus, the curves have a positive first derivative and a negative second derivative. Although each investor has a unique attitude towards risk [and, therefore, will differ in their preferred risk/reward tradeoffs], it is generally true that the risk aversion function is expressed as follows:

Risk Aversion = -(second derivative of the utility of wealth) ÷ +(first derivative of the utility of wealth).

In other words, an investor's risk aversion function can be derived from the shape of the individual's utility of wealth curve; and the investor's utility of wealth can be recovered from his risk aversion curve.

There is a substantial literature on fitting specific risk aversion functions to investor preferences. Each risk aversion function curve has a unique slope (rate of change) reflecting the individual investor's sensitivity to changes in wealth. It is common to build risk aversion curves that are (1) quadratic (an upside-down bowl shape); (2) exponential (increasing exponential risk aversion where the exponent is a negative fraction); (3) power (where the exponent ranges from a value greater than 0 through to a value of 1); and logarithmic (where the value of the exponent approaches 0). Although many modifications (called "positive affine transformations") are possible for each of the common forms of risk aversion curves, a simple way to gain an intuitive understanding is to compare the "risk premium" required by investors where the probability of loss is equal to the probability of gain (e.g., odds = 50/50). We assume that the hypothetical investor has a current wealth endowment of $1 million.

- The investor with quadratic risk aversion (curve = square root of wealth) requires a payoff of at least $51,282 to risk a loss of $50,000;
- The investor with a power risk aversion exponent of 0.1 requires a payoff of at least $52,356 to risk a loss of $50,000; and,
- The investor with logarithmic risk aversion requires a payoff of at least $52,632 to risk a loss of $50,000.

The payoff (asset allocation risk/reward tradeoff) that will satisfy one investor is unsatisfactory to other investors. If wealth decreases towards a critical level, the slope of the investor's indifference curve may become steeper.

How do we know how much dollar gain must be put on the table to risk the $50,000 loss? For the quadratic risk averse investor, the square root of $1,000,000 equals 1,000. That is to say, $1,000,000 generates 1,000 units of utility. By risking $50,000, the investor may increase wealth to $1,051,282 or decrease wealth to $950,000. But the square root of $1,051,282 is 1,025.32; and the square root of $950,000 is 974.68. Thus, at 50/50 odds, the average of the two risky gamble square roots equals 1,000 units of utility which is the exact value of the risk free $1,000,000. If wealth decreases towards a critical level, the slope of the investor's indifference curve may become steeper. Similar mathematical calculations confirm the values for the power risk aversion (exponent of 0.1) and the log of wealth risk aversion function.

RISK TOLERANCE: This is the reciprocal of the risk aversion function curve [1 ÷ Risk Aversion]. The less risk averse; the more risk tolerant.

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[1] The asset allocation decision's degree of importance to long-term investment results is a subject of some current controversy. An early study suggested that the asset allocation decision is the primary determinant of *return variance* for portfolios with long-term planning horizons [Brinson, Gary P., Singer, Brian D., and Beebower, Gilbert L., "Determinants of Portfolio Performance," Financial Analysts Journal (May-June, 1991)]. Return variance, however, is not the same as realized returns. A more recent study suggests that the importance of asset allocation depends on the investment issue under consideration [Ibbotson, Roger, G., & Kaplan, Paul D., "Does Asset Allocation Policy Explain 40, 90, or 100 Percent of Performance?" Financial Analysts Journal (January/February, 2000), pp. 26-33]. Specifically, the investor might be interested in knowing: (1) What percentage of a portfolio's ups and downs (return variance) is explained, over time, by asset allocation choices; or, (2) how much of the performance difference between two distinct portfolios can, over time, be explained by differences in their asset allocation; or, (3) how much of a specific portfolio's actual returns can, over time, be explained by its asset allocation. These are very different questions; and must be answered by different analytical methods.

[2] Meucci, Attilio, Risk and Asset Allocation (Springer, 2005), p. 240: "For example, traders focus on their daily profit and loss (P & L). Therefore for a trader the investment horizon is one day and the net profits are his objective."

[3] Harris, Larry, Trading & Exchanges, Oxford University Press (2003), pp. 178-180.

[4] Arrow, Kenneth, "The Theory of Risk Aversion," Essays in the Theory of Risk Bearing Markham Press (Chicago, 1971), pp. 90-120.

[5] Even short-term default free instruments, however, are risky from a long-term perspective because their cash flows must be reinvested at uncertain future interest rates. The risk-free instrument for a long-term investor focused on wealth accumulation may be a Treasury Inflation-Protected Security. Inflation indexed annuities, guaranteed by state insurance funds approximate risk-free instruments for investors concerned with the utility of consumption.

[6] Equivalently, if current wealth is sufficient to achieve a terminal wealth objective at a guaranteed rate available in the marketplace, then all wealth may be invested in a default-free, zero-coupon bond maturing at the end of the investor's planning horizon.

[7] Sharpe, William F., Chen, P., Pinto, Jerald E. & McLeavey. Dennis W., "Asset Allocation," Managing Investment Portfolios, (John Wiley & Sons, 2007), pp. 230-320.

[8] See, for example, Bogle, John C., The Little Book of Common Sense Investing," John Wiley & Sons (2007), p. 58: "The winning formula for success in investing is owning the entire stock market through an index fund, and then doing nothing. Just stay the course."

[9] Campbell, John Y. & Viceira, Luis M., Strategic Asset Allocation: Portfolio Choice for Long-Term Investors , Oxford University Press (2002), pp.3-4. See also, Davis, Steven J. & Willen, Paulo, Income Shocks, Asset Returns, and Portfolio Choice," Innovations in Retirement Financing, eds., Olivia Mitchell, Zvi Bodie, P. Brett Hammond, and Stephen Zeldes University of Pennsylvania Press (2002), p. 44: "When labor income and asset returns are correlated, investors are implicitly endowed with certain exposures to

risky financial assets….Because investors typically differ in their endowed exposures, they also differ in their optimal portfolio allocations (levels and shares), even when they have the same tolerance for risk and the same beliefs about asset returns."

[10] Although it is not obvious how much of an adjustment should be made today to enhance the likelihood of achieving a future objective. See, for example, Collins, Patrick J., "Investment Reserves and Precautionary Savings," Wealth Strategies Journal, http://www.wealthstrategiesjournal.com/articles/2008/11/managing-retirement-portfolio.html. If the worker values leisure time (retirement) more than a standard of living goal (consumption in retirement), the asset allocation decision will reflect this tradeoff. Following a bad year, instead of maximizing expected future return by investing heavily in equities, the investor may decide to decrease the retirement income goal so that the target retirement date remains feasible. See, for example, Boscaljon, Brian, "Time, wealth, and human capital as determinants of asset allocation," Financial Services Review (2004), pp. 167-184.

[11] Where shortfall is defined either in absolute terms as an insufficient amount of dollar wealth; or, in relative terms as underperformance versus a comparable benchmark portfolio or versus the compound risk-free rate of return.

[12] High expected returns calculated by averaging portfolio payoffs over all future economic states may not be as important as higher payoffs in a depression or recession but a lower total expected return over all future economic states.

[13] Boone, Norman M. & Lubitz, Linda S., Creating an Investment Policy Statement (FPA Press, 2004), pp. 34-51. See also, Trone, Donald B., Allbright, William R. & Taylor, Philip R., The Management of Investment Decisions, McGraw-Hill (1996), pp. 103-117. The Comptroller of the Currency strongly recommends that national banks design and implement a written Investment Policy Statement as part of their investment management services. For example, the Comptroller's Handbook (August, 2001), p. 110 states: "The creation of an appropriate investment policy document or statement, is the culmination of analyzing the investment assignment, identifying investment objectives, determining asset allocation guidelines, and establishing performance measurement benchmarks. The lack of an investment policy statement, or the existence of a poorly developed one, is a weakness in portfolio management risk control." The Office of Thrift Supervision discusses the role of Investment Policy Statements in the OTS Trust and Asset Management Handbook (July, 2001) in §§800 & 810. The American College of Trust and Estate Counsel, in its "Guide for ACTEC Fellows Serving As Trustees," ACTEC Notes (2001), p. 318 recommends the development of a written "investment plan (which should be in writing)," to facilitate investment of trust assets; and, the Institute of Certified Financial Planners states in the Personal Financial Planning: A CFP Practitioners' Guide (November, 1998), p. 3-11: "Considering the importance of the investment plan to the accomplishment of financial planning objectives, a written investment policy statement appears to be as important as a written financial plan."

[14] See, for example, Restatement (Third) of the Law Trusts: Prudent Investor Rule, The American Law Institute §90 comment g: "…asset allocation decisions are a fundamental aspect of an investment strategy and a starting point in formulating a plan of diversification."

[15] Economists distinguish between 'risk'—a condition to which a probability measure can be assigned; and 'uncertainty'—a condition not conducive to probability measurement. Uncertainty exists when the investor does not know or cannot learn the true parameters of a return generating process.

[16] The risk free asset has no variance. The only term remaining in the calculation of portfolio variance is the weight of the risky asset squared $(.7)^2$ times the variance of the risky asset $(.15)^2$. The standard deviation of the portfolio is the square root of this product, or 10.5%.

[17] These are "back of the envelope" calculations in that the realized distribution of historical returns in many equity markets exhibit leptokurtosis—a greater likelihood of extreme price movements than would be expected in a normal (Gaussian) distribution.

[18] "Glide Path" investment strategies represent a recent variation in asset allocation. This is an age-based strategy in which the amount of equity exposure in the portfolio decreases according to a pre-determined formula. In general, the glide-path formula considers neither investor reactions to market price movements nor the level of the investor's current wealth. For further discussion see, Sharpe, William F., Scott, Jason S. & Watson, John G., "Efficient Retirement Financial Strategies," Recalibrating Retirement Spending and Saving, Oxford University Press (2008), pp. 209-226. There is considerable debate in the qualified retirement plan community regarding the wisdom of focusing exclusively on asset allocation as a function of time until retirement, while ignoring the investor's critical wealth level.

[19] A seminal work in this area is Leland, Hayne, "Optimal Asset Rebalancing In the Presence of Transaction Costs," Working Paper No. RPF-261, Walter A. Haas School of Business Research Program in Finance Working Paper Series (August, 1996). Leland's utility-based argument is an extension of an earlier proof by Merton that, given assumptions of a constant opportunity set (investment returns that exhibit statistical stationarity and 'iid' error terms), investors will wish to maintain a constant asset allocation over all time periods. The proof is reprinted in: Merton, Robert C., Continuous-Time Finance, Blackwell Publishing (1992), pp. 169-173.

[20] For example, "The rate of return obtained in an investment portfolio is a derivative of the level of market risk assumed—or avoided—in the portfolio; the consistency with which that risk level is maintained through market cycles; and the skill with which specific stock risk and stock group risk are eliminated or minimized through portfolio diversification…." Ellis, Charles, D., Investment Policy, Irwin Publishing, Second Edition (1993), p. 50.

[21] Campbell & Viceira, Luis M., *Id.*, pp. 22-30. Strictly speaking, CRRA utility models suggest that portfolio choice is independent of the level of investment wealth. A pauper's lose of a nickel is equivalent to a billionaires loss of $50 million.

[22] For further discussion see, Collins, Patrick J., "A Risk Primer for Investment Fiduciaries," California Trusts and Estates Quarterly (Fall, 2002), pp. 4-24.

[23] Sharpe, William F., ATT Asset Allocation Tools (Second Edition), The Scientific Press (1987), p. 39.

[24] The simple model discussed in this essay ignores intertemporal hedging, precautionary savings, consumption motives and other factors that are potentially important in the asset allocation decision. The required rate of return is akin to the concept of "hurdle rate" common to corporate financial analysis.

[25] The following discussion draws, in part, on Rubinstein, Mark, "Comments on the 1987 Stock Market Crash," Risks in Accumulation Products, Society of Actuaries (2000).

[26] French, Kenneth R., Schwert, G. William & Stambaugh, Robert F., "Expected Stock Returns and Volatility," Journal of Financial Economics (1987), pp. 3-29.

[27] This phenomenon is also known as "correlation clustering." Alexander, Carol, Practical Financial Econometrics, John Wiley & Sons (2008), p. 164.

[28] Ellis, *Id*., p. 27: "Avoidance of market risk does have a real "opportunity cost," and the client should be fully informed of the opportunity cost of each level of market risk *not* taken."

[29] In reality, it is sometimes difficult to fix the liability deterministically. For example, retirement is a feasible objective only if the stochastic present value of assets is equal to or greater than the stochastic present value of liabilities. Usually, there is randomness on both sides of the equation.

[30] When the critical point is defined as a subsistence level, it may be important to incorporate Social Security income into the asset allocation analysis. Social Security represents a forced investment of a portion of total wealth (labor income) into a risk-free, inflation adjusted annuity. Generally, whenever investment wealth must exceed a subsistence level at all times, the investor will decrease allocation to risky assets as their price declines. If the critical minimum wealth level exceeds the present value of Social Security entitlements, the investor allocates 100% of assets to risk-free investments until critical funding is assured. We assume that failure to preserve the critical minimum subsistence wealth level creates infinite disutility. Thereafter, surplus wealth may be allocated, at least in part, to risky assets. A homeowner's ability to acquire a reverse annuity mortgage; or, an executive's entitlement to deferred compensation are further examples of potential factors in the asset allocation decision. The consequences of these observations for investment policy are profound. For example, if one assumes that long-term investors must acquire a *greater* level of risk-free wealth to fund the present value of subsistence over the applicable planning horizon, then older investors having surplus wealth should be *more inclined* to invest in equities—a result that contradicts the glide path assumptions underlying many managed mutual funds. See also, Kyrychenko, Vladyslav, "Optimal asset allocation in the presence of nonfinancial assets," Financial Services Review (Spring, 2008), pp. 69-86; and, Wilcox, Jarrod, Horvitz, Jeffrey E. & diBartolomeo, Dan, Investment Management for Taxable Private Investors, Research Foundation of CFA Institute (2006), p.21. Management of personal investment surplus parallels, in some respects, the theory of liability-relative investing for Defined Benefit Pension Plans.

[31] The floor + equity multiplier" approach is also known as "portfolio insurance" or "constant proportion portfolio insurance." The following sources are helpful for readers wishing further insight into this asset management approach: Black, Fischer & Jones, R., "Simplifying Portfolio Insurance," Journal of Portfolio Management (Fall, 1987), pp. 48-51; Perold, Andre & Sharpe, William, "Dynamic Strategies for Asset

Allocation" Financial Analysts Journal (January/February, 1988), pp 16-26; Cesari, Riccardo & Cremonini, David, "Benchmarking, portfolio insurance and technical analysis: A Monte Carlo comparison of dynamic strategies of asset allocation," Journal of Economic Dynamics & Control (2003), pp. 987-1011; Rubinstein, Mark, "Portfolio Insurance and the Market Crash," Financial Analysts Journal (January/February, 1988), pp. 38-47; Farrell, James L., Portfolio Management: Theory and Application, Irwin McGraw-Hill (1997), pp. 291-297; Sharpe, William F., Investors And Markets, Princeton University Press (2007), pp. 179-181; Grossman, Sanford J. & Zhou, Zhongquan, "Equilibrium Analysis of Portfolio Insurance," Journal of Finance (1996), pp. 1379-1403; and, Boscaljon, Brian & Sun, Licheng, "A Simple Portfolio Insurance Strategy for Retirement Investing," Journal of Financial Service Professionals (September, 2006), pp. 60-65.

[32] A more in-depth discussion of how to calibrate the multiple is found in Hamidi, Benjamin, Maillet, Bertrand & Prigent, Jean-Luc, "A Time-varying Proportion Portfolio Insurance Strategy based on a CAViaR Approach," http://www.finance-innovation.org/risk08/files/7629259.pdf (March 2008). The multiple must be smaller than the inverse of the maximum portfolio drawdown that can occur prior to the rebalancing of the portfolio to its adjusted risk position. Simplistically, if the portfolio is adjusted each year, the multiple cannot be greater than 2.5 assuming a maximum yearly decline of 40% [1 ÷ .40 = 2.5].

[33] At the limit, this strategy would require constant trading to adjust for stochastic price movements. Modeling a floor + multiplier approach under continuous time finance assumptions assumes that the probability of attaining the floor value is zero because of rebalancing during arbitrarily small time intervals. In reality, however, discrete time rebalancing creates a positive probability of penetrating the floor value. If this occurs, it is clear that the investor faces another type of risk that differs from the risk to principal. The portfolio becomes 100% invested in the risk free asset with no opportunity for future growth in excess of the risk-free rate. In practice, the investor in a floor + multiplier approach is unlikely to be able to sustain such a result for the remainder of the planning horizon. Another way to characterize this new risk is "the risk of becoming over insured."

[34] Kelly, J. L., "A New Interpretation of Information Rate," Bell System Technical Journal (July, 1956), pp. 917-926.

[35] Rubinstein, Mark, "Continuously Rebalanced Investment Strategies," Journal of Portfolio Management (Fall, 1991), pp. 78-81.

[36] The "growth optimal portfolio" that maximizes log utility is the preferred portfolio only under a set of extremely restrictive assumptions. These include the assumption that the investor manifests a CRRA utility function over all levels of wealth, and that returns are independent and identically distributed through time. Under these conditions, the probability of loss decreases as the planning horizon grows longer; but, the magnitude of possible loss increases. See, for example, Kritzman, Mark, Puzzles of Finance, John Wiley & Sons (2000), pp. 48-50.

[37] Roy, A.D., "Safety-First and the Holding of Assets," Econometrics (July, 1952), pp. 431-449.

[38] Ellis, *Id*., p. 59. See, also, Wilcox, et al. *id*, p. 11: "… risk aversion may change as a function of substantial changes in wealth. This issue is particularly acute for private investors who experience major personal losses."

[39] All return series from Ibbotson Associates' Portfolio Analytics software application.

[40] It is important to remember that, because of differences in initial asset allocations, investment results are not directly comparable.

[41] We assume that no investor would select an asset management approach that had a positive chance of generating infinite disutility.

[42] Although sound investment policy accommodates investor reactions to changes in asset prices, the price process itself may be a function of the "composition of the behind-the-scenes agents." This is a complicating factor because imbalance among buy & hold, constant mix, and floor + multiplier strategies may exacerbate market volatility. In the extreme, each strategy has a potential to sow the seeds of its own destruction. Jacobs, Bruce I., Levy, Kenneth N. & Markowitz, Harry M., "Financial Market Simulation," The Journal of Portfolio Management (30th Anniversary Issue, 2004), p. 150.

[43] An important caveat with respect to the time diversification argument is to make sure what "risk" metric you are using. The risk of ending up with less than $x at the end of a planning horizon is different from the risk of ever dropping below $x during the planning horizon. These risks, in turn, are very different from the risks of a shortfall relative to a targeted investment rate of return.

[44] Collins, Patrick J., Savage, Sam L. & Stampfli, Josh, "Financial Consequences of Distribution Elections From Total Return Trusts," Real Property, Probate and Trust (Summer, 2000), pp. 243-304.

[45] Amenc, Noel & Le Sourd, Veronique, Portfolio Theory and Performance Analysis, John Wiley & Sons (2003), p. 6.

[46] Boulier, Jean-Francois & Kanniganti, Anu "Expected Performance and Risks of Various Portfolio Insurance Strategies," 5th Annual Actuarial Approach to Financial Risk [AFIR] International Colloquium pp., 1093-1124 discusses various strategies involving dynamic adjustments to the floor and multiplier values.

[47] This discussion draws, in part, from Amram, Martha & Kulatilaka, Nalin, Real Options: Managing Strategic Investment in an Uncertain World, Harvard Business School Press (1999).