### 2.1 Functions

A function is a correspondence between 2 sets of numbers, $x$ and $y$, such that each $x$ has exactly one $y$ value.

Three ways to express a function.

1. Equation: $y=2 x-3$
2. Set of points: $\{(1,2),(2,3),(3,4)\}$
3. Graph:


Domain: set of possible $x$ values
Range: set of possible $y$ values
(Domain and range must be real numbers.)
I. Determine whether each relation is a function. Give the domain and range.
a. Equation.

Example: $2 x-y=5$
This equation is a function since any value you want to pick for $\boldsymbol{x}$ will give you exactly one value for $y$.

Domain: all real numbers, since you can use any value for $\boldsymbol{x}$. D: $(-\infty, \infty)$
Range: all real numbers, since you can get any value for $\boldsymbol{y}$ by picking the correct $\boldsymbol{x}$. R: $(-\infty, \infty)$

Example: $y=x^{2}$
This equation is a function since any value you want to pick for $\boldsymbol{x}$ will give you exactly one value for $y$.

Domain: all real numbers, since you can use any value for $\boldsymbol{x}$. D: $(-\infty, \infty)$
Range: all positive real numbers, since $y$ is $x$ squared, you can only get positive numbers for $\boldsymbol{y}$. $\mathbf{R}: \quad[0, \infty)$

Example: $y^{2}=x$
This equation is not a function since some values you pick for $\boldsymbol{x}$ will give you two values for $y$. For example, pick $x=4$.

$$
\begin{aligned}
y^{2} & =4 \\
\sqrt{y^{2}} & = \pm 4 \\
y & =2
\end{aligned}
$$

${ }^{*}$ In equations, $y^{\text {even }}$ is not a function. $y^{\text {odd }}$ is a function.
Example: $x^{2}+2 y=3$ is a function.
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

Example: $y^{4}=16 x$ is not a function.

Example: $y=-\sqrt{x-3}$ is a function.
Domain: Since functions only use real numbers, we cannot take the square root of a negative number.

$$
\begin{aligned}
& \sqrt{ } \geq 0 \\
& x-3 \geq 0 \\
& x \geq 3 \\
& \text { D: }\{x \mid x \geq 3\}
\end{aligned}
$$

Range: $(-\infty, 0]$ since $-\sqrt{ } \leq 0$.

Example: $y^{3}=x$ is a function.
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

Example: $y=\frac{3+x}{x-5}$ is a function.
Domain: Since the denominator cannot be zero,
$x-5 \neq 0$
$x \neq 5$
D: $\{x \mid x \neq 0\}$.
Range: $(-\infty, \infty)$
${ }^{*}$ In equations, the domain is all real numbers, $(-\infty, \infty)$ except when the equation has a radical or a fraction. When there is a radical, what is under the radical must be greater than or equal to zero. When there is a fraction, the denominator cannot be equal to zero.

## Determine whether each relation is a function. Give the domain and range.

b. Set of points.

Example: $\{(4,5),(6,3),(8,5)\}$
Yes, it is a function. (Each $\boldsymbol{x}$ value appears only once.)
D: $\{4,6,8\}$ (the $x$ values)
R: $\{3,5\}$ (the $y$ values.

Example: $\{(\mathbf{2}, \mathbf{3}),(4,5),(4,6)\}$
No, it is not a function. (The $\boldsymbol{x}$ value 4 has two $\boldsymbol{y}$ values.)

Example: $\{(\mathbf{3}, \mathbf{1}),(\mathbf{2}, \mathbf{1}),(5,1)\}$
Yes, it is a function. (Each $\boldsymbol{x}$ value appears only once.)
D: $\{\mathbf{2 , 3}, \mathbf{5}\}$
R: $\{5\}$
*In points, if each $\boldsymbol{x}$ appears only once, it is a function. If any $\boldsymbol{x}$ value appears more than once, it is not a function.

## Determine whether each relation is a function. Give the domain and range.

c. Graphs.

Example

(Assume that arrows are on each end of the graph.)

Yes, it is a function. Each $\boldsymbol{x}$ value has exactly one $\boldsymbol{y}$ value.
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
*In graphs, use a vertical line test to see if the graph is a function. If every vertical line would pass through the graph at only one spot, it is a function.

## Example



No, it is not a function. There is at least one vertical line that would hit the graph at two points.


## Example



Not a function.

## Example



Yes, it is a function.
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

## Example



Yes, it is a function.
Domain: $[2, \infty)$ The $\boldsymbol{x}$ values start a - $\mathbf{2}$ and go to infinity.
Range: $(0, \infty)$ The $\boldsymbol{y}$ values start a 0 and go to infinity.

## Example



Yes, it is a function.
Domain: $[-3,3]$ The $\boldsymbol{x}$ values start a $\mathbf{- 3}$ and go to 3 .
Range: [0, 3] The $y$ values start a 0 and go to 3 .

## II. Evaluate.

a. Equation.

Example. $f(x)=3 x-1$ Is read $\mathbf{f}$ of and is a another way to write functions in an equation form.

Find $f(-1)$. Replace the $\boldsymbol{x}$ in the function with the number $\mathbf{- 1}$ and evaluate.

$$
\begin{aligned}
f(x) & =3 x-1 \\
f(-2) & =3(-2)-1 \\
f(-2) & =-6-1 \\
f(-2) & =-7
\end{aligned}
$$

Example. For $f(x)=x^{2}-10 x-3$, find $\mathbf{f}(-1), \mathbf{f}(-3)$, and $\mathbf{f}(x+2)$.

$$
\begin{array}{ll}
f(-1)=(-1)^{2}-10(-1)-3 & f(-3)=(-3)^{2}-10(-3)-3 \\
f(-1)=1+10-3 & f(-3)=9+30-3 \\
f(-1)=8 & f(-3)=36
\end{array}
$$

$$
\begin{aligned}
& f(x+2)=(x+2)^{2}-10(x+2)-3 \\
& f(x+2)=(x+2)(x+2)-10 x-20-3 \\
& f(x+2)=x^{2}+4 x+4-10 x-23 \\
& f(x+2)=x^{2}-6 x-19
\end{aligned} * \text { Use Foil to multiply }(x+2)(x+2) .
$$

Example. For $g(x)=-x^{2}-2 x+1$, find $g(3), g(-2), g(-x)$, and $g(\mathbf{3 a})$.

$$
\begin{aligned}
& g(3)=-(3)^{2}-2(3)+1 \\
& g(3)=-9-6+1 \quad \text { Notice that the negative is not squared. } \\
& g(3)=-14 \\
& g(-2)=-(-2)^{2}-2(-2)+1 \\
& g(-2)=-4+4+1 \quad \text { Notice that the negative signs. } \\
& g(-2)=1 \\
& g(-x)=-(-x)^{2}-2(-x)+1 \\
& g(-x)=-x^{2}+2 x+1 \\
& g(3 a)=-(3 a)^{2}-2(3 a)+1 \quad \text { Notice that the negative signs. } \\
& g(3 a)=-9 a^{2}-6 a+1 \quad
\end{aligned}
$$

Example. For $f(a)=\sqrt{25-a}-6$, find $\mathbf{f}(16), \mathbf{f}(-24)$, and $\mathbf{f}(\boldsymbol{x})$.

$$
\begin{array}{ll}
f(16)=\sqrt{26-16}-6 & f(-24)=\sqrt{26-(-24)}-6 \\
f(16)=\sqrt{9}-6 & f(-24)=\sqrt{49}-6 \\
f(16)=3-6 & f(-24)=7-6 \\
f(16)=-3 & f(-24)=1 \\
f(x)=\sqrt{26-x}-6 &
\end{array}
$$

Example. For $f(x)=\frac{|x+3|}{x-3}$, find $\mathbf{f}(5), \mathbf{f}(-5)$, and $\mathbf{f}(\mathbf{3})$.

$$
\begin{array}{ll}
f(5)=\frac{|5+3|}{5+3} & f(-5)=\frac{|-5+3|}{-5+3} \\
f(5)=\frac{|8|}{8} & f(-5)=\frac{|-2|}{-2} \\
f(5)=\frac{8}{8} & f(-5)=\frac{2}{-2} \\
f(5)=1 & f(-5)=-1
\end{array}
$$

$$
f(-3)=\frac{|-3+3|}{-3+3}
$$

is undefined. So - $\mathbf{3}$ is not in the domain.
$f(-3)=\frac{|0|}{0}$

## II. Evaluate.

## b. Set of points.

$$
f(x)=\{(3,5),(4,7),(5,9),(6,11)\} \quad \text { Find } f(3) \text { and } f(5)
$$

$f(3)=5$ since for the point $(3,5)$, when $x$ is $3, y$ is 5 .

$$
f(5)=9
$$

## II. Evaluate.

## c. Graph.



Find (1) and f(3).

$$
\begin{aligned}
& \mathrm{f}(1)=0 \quad(\text { when } x=1, y=0) \\
& \mathrm{f}(3)=1(\text { when } x=3, y=1)
\end{aligned}
$$

