THE BUTTERFLY EFFECT

It is both an honor and pleasure to be receiving the "Premio Felice Pietro Chisesi e Caterina Tomassoni." I want to extend my thanks first to the unknown person who nominated me for this award, next to the committee that devoted much time and energy to deciding that I should be the recipient, and finally to those whose generosity and vision established the award. In its statement of motivation for choosing me, the committee gave prominence to the now familiar expression "butterfly effect," and I have chosen this expression as the title of my lecture.

A word as to the origin and implication of the expression is in order. It was apparently coined by James Gleick, who used it as the title of the leading chapter in his best-selling book "Chaos: Making a New Science." He derived his expression from the title of a lecture—one that I had delivered at a meeting much earlier—entitled "Predictability: Does the Flap of a Butterfly's Wings in Brazil stir up a Tornado in Texas?" In the lecture I did not answer the title question, and even today I am unsure of the proper answer, but I did consider the possibility that the answer might be "yes." The butterfly effect is the process through which, if the answer is "yes," an almost imperceptible flap of a butterfly's wings can lead in due time to the appearance of a tornado that would otherwise not have developed, or, more generally, through which a very small disturbance can lead eventually to a major event that would not have occurred if the disturbance had not been introduced. Systems in which such processes take place have now acquired the generally accepted name of "chaos."

Before I define precisely what systems are or are not chaotic, let me present two examples that will help to clarify certain details. The first is a bit of old English verse, often quoted as indicating an early recognition of possible chaotic behavior. It goes

> "For want of a nail, the shoe was lost.
> For want of a shoe, the horse was lost.
> For want of a horse, the rider was lost.
> For want of a rider, the battle was lost.
> For want of a battle, the kingdom was lost.
> And all for the want of a horseshoe nail."

Let me say right now that I do not feel that this verse is describing true chaos, but better illustrates the simpler phenomenon of instability. The verse assumes the loss of a nail to be detrimental, but, if this were true chaos, losing a nail might equally well lead to the loss or the winning of the kingdom. Moreover, if this were chaos, subsequent small events like the loss of more nails could also lead to the loss or gain of a kingdom. Yet in the verse there is something rather final about the statement "The kingdom was lost." The implication is that subsequent small events will not reverse the outcome.

My second example, which I feel better describes a chaotic process, is a rather trivial anecdote. In it, I am driving along a city street where there are frequent intersections, with traffic lights. As I approach the next intersection, the light is green, and I expect to proceed. However, a jaywalker—a pedestrian intent on crossing the street between intersections—suddenly steps out. I stop to avoid hitting him, and he gets out of the way, but by then the light has turned red. I wait for the next green light, and proceed, and as I approach the next intersection the light there is green. I continue, but as I cross the intersection a small truck on the side street runs through its red light and hits me.

Did the jaywalker cause the accident? Certainly it would not have occurred if he had not stepped out. Yet I find it hard to assign similar blame to him and the truck driver. There was an equal likelihood that the truck would have arrived at the second intersection a minute sooner. In that case, if the jaywalker had not appeared, I would also have arrived a minute sooner, and would have been hit. Because the jaywalker did delay me, I would have reached the second intersection after the truck, and no accident would have occurred. Would the jaywalker then have prevented the accident?

Not only was the jaywalker equally likely to have led to an accident or the avoidance of one, but the actions of all other jaywalkers, at other times or on other streets, also bear similar likelihoods of being associated with accidents. Here I am intentionally disregarding the possibility of another type of accident, in which the jaywalker is the victim. To return to the butterfly, if a butterfly can cause or prevent a tornado, it can do so only in the sense that the jaywalker causes or prevents an accident. Moreover, just as important as the specified flap of a butterfly's wings are all subsequent flaps, and each flap of the wings of innumerable other butterflies, not to mention the actions of members of all other species, some far more powerful.

Having highlighted two special properties of chaos, let me turn to general considerations. Chaos is a property of some but not all dynamical systems. These are systems whose states change as time advances, according to specific laws. In order not to eliminate from the category of dynamical system almost everything that is physically real, we often include systems subject to small random influences, provided that we have reason to believe that the system would behave in the same general manner if the randomness could somehow be suppressed. Even in laboratory experiments, as opposed to naturally occurring systems, where such factors as the intensity of the forcing can be carefully controlled, a small amount of randomness often remains. A dynamical system may be as simple as a pendulum in a clock or as complicated as the global weather pattern. A system is chaotic if, when the laws are applied separately to the system as it stands and as it would stand with the state slightly disturbed, they lead in due time to considerably different states, that is, if the butterfly effect prevails. I should add that, over very long time intervals, the collection of states that will be encountered with or without the disturbance will be about the same, but they will occur in different chronological orders.

Chaos is often identified with unpredictability. In principle, if we knew the governing laws and the present state of a system exactly, and if we knew that no disturbances were forthcoming, we could determine the exact state at any future time. This is all well and good if the system is a mathematical abstraction, and if the laws include such details as the computational round-off procedure, but, with real physical systems, the necessary knowledge is lacking. What we observe to be the present state is actually an unknown true present state plus an unknown small perturbation. It follows that the true state is the observed state plus a small perturbation. When the laws are applied separately to the system with various small perturbations added to the observed state, a collection of rather different states will appear at some later time, if the system is chaotic. Any one of these states might be the one that will actually occur.

A related property of chaos is absence of true periodicity. For many systems, because the number of possible states, each differing more than a given amount from any of the others, is limited, we can be sure that if we wait long enough we shall encounter a state close to one that we have seen before, and we may think of the later state as the earlier one plus a small perturbation. If the system is not chaotic, so that the perturbation does not amplify, the behavior following the second occurrence will nearly repeat that following the first, until, after the passage of a similar time, the state will be nearly repeated again, and periodicity will become established. If, then, the behavior is observed not to be periodic, it must be chaotic.

This conclusion comes with a few caveats. For example, a system supposedly lacking periodicity may actually be periodic, with a period longer than the record of observations.

The term "chaos" has not always enjoyed universal acceptance. Yoshisuke Ueda, possibly the first investigator to have detected apparent chaos in a real system, has noted that over short enough time intervals chaos appears quite regular, and looks random only over longer intervals, and he prefers a term that takes this feature into account.

How can we determine whether a given system will behave chaotically? If it is a mathematical abstraction, defined by equations, there may be no problem. We can determine two or preferably several solutions numerically, and see whether or not they diverge as time advances. For a real physical system we might construct a mathematical model, or attempt to simulate it in the laboratory, but, particularly if the system is rather complicated, there is no assurance that the model and the system will ultimately behave in the same manner. Instead, our opinion as to whether a system is chaotic is likely to be based on its observed behavior.

If we are lucky enough to observe an analogue—a state that closely resembles another observed state—we can examine the variations following each occurrence, and see whether they are similar or widely different. Again however, if the system is complicated, analogues are likely to be far apart, and may never be found.

More commonly, we see whether the system appears to be varying periodically, or whether the variations have a random appearance. Absence of periodicity is, for example, one reason for believing that the weather is chaotic. To be sure, there are periodic components, notably the diurnal and annual cycles, and there is almost no limit to the periodicities that one or another investigator has claimed to have discovered, but, when all known or suspected periods are subtracted out, a strong irregular signal remains.

In short, there are relatively few real systems where one can turn directly to theory and deduce that chaos is present. In fact, in some laboratory experiments, where the value of a constant may be altered, it often happens that one value will produce chaos, while another will produce periodicity. Any theory that correctly predicts the presence or absence of chaos would have to involve the values of the various constants.

Among the real systems where theory may be sufficient are some variants of one where a ball bounces on an irregular or corrugated surface. A slight difference in the path of the ball to the surface can produce a large difference in the angle at which the ball rebounds, and the place where it finally comes to rest will be difficult to predict.

Typical mathematically defined systems also require numerical solution rather than theory to confirm chaos. The most familiar exception is a special case of the so-called logistic equation, which produces a sequence of numbers. In one variant, you simply choose a rational fraction between -2 and 2, say 1.5, as a starting number, and to obtain the following number you simply square the present number and subtract 2. The equation has an explicit solution in terms of cosines, from which chaos can be verified. In a few spare moments, even without a computer, you can calculate a few leading numbers, with two slightly different starting numbers, and observe the divergence.

If you prefer to use a computer, you may want to change the rules and subtract a chosen number somewhat less than 2, say 1.79, after squaring the present number, and you may then observe another common but not universal property of chaos—its selectivity.

Depending upon your choice of the number, you may encounter periodicity or chaos. In the event of chaos, if you calculate a long sequence of numbers, you may find that the numbers in certain ranges occur frequently, while those in adjacent ranges occur far less often or not at all. Of course, chaos is less selective than periodicity.

I want to conclude by considering a system with which I have been intimately concerned—the global weather pattern. Back in my student days the weather system was ordinarily regarded as synonymous with the atmosphere. It was recognized, of course, that the surface temperature of the underlying ocean had an important effect, but this was commonly treated as an external influence. Today our concept of the system includes the whole ocean, or at least its upper layers, and such features as sea ice, or, over continents, snow cover and soil moisture.

What is the evidence for chaos? I have already noted the absence of complete periodicity, but our confidence in the weather's chaotic nature stems largely from the study of mathematical models. These range in complexity from some that represent the entire weather pattern—crudely, of course—by three numbers, to the operational forecasting models at various national and international centers that may use as many as 30 million. A similar number of equations, representing the physical laws, governs the manner in which these numbers change with time. These equations are solved by stepwise numerical procedures. Almost without exception the models produce chaos.

Thirty million numbers may seem redundant and wasteful. Recall, then, that we need a three-dimensional distribution of the values of at least four physical quantities—the temperature, the wind direction and speed, commonly represented by the eastward and northward wind components, and some measure of the humidity. The familiar pressure need be specified at just one level; under the reasonably well satisfied condition of hydrostatic equilibrium, where the vertical forces are in balance, the pressures at other levels can be computed when the temperature and humidity are known. The all-important upward or downward motion need not enter as a separate variable. The horizontal motions are always acting to upset hydrostatic equilibrium, and it is implicitly assumed that the vertical motion field is just the one needed to counteract the horizontal motion and maintain the equilibrium.

The grid of points at which we specify the values of the four variables may occupy 30 elevations. In that case, there will be a quarter million points at each elevation. This may seem extravagant, but simple arithmetic shows that each point must then somehow represent an area of 2000 square kilometers. The weather within such an area can possess many details that will not be resolved, including the structure and sometimes even the presence of a thunderstorm. It appears that as computers become ever more powerful, there will always be some incentive for enlarging the models.

There is overwhelming evidence that weather forecasts are better now than when the models were smaller. This is not entirely because of model improvement. The quality and abundance of weather observations that determine an initial state, and the manner in which we assimilate these observations to update our estimate of the present state, are as important. Currently we begin the assimilation process with a first guess; typically it is a forecast for the present time made a few hours earlier, and is assumed to be good enough for us to regard any subsequent guess that does not resemble it with suspicion. For any possible new guess, we establish a "cost function," in which we are penalized for differences between the new guess and the observations, and differences between the new and the first guess. We then use variational methods to seek, as our final guess, the state that minimizes the cost function. Perhaps surprisingly, the assimilation process tends to require far more computation time than the subsequent numerical integration that produces the forecast.

Unquestionably the recent improvement in forecasting is due partly to improved assimilation. For example, some 30 years ago, when satellite observations were abundant, there was a feeling among many forecasters that they were ineffective in improving the forecast. Today there is little doubt as to their effectiveness. The gain is apparently due not to better satellite observations, although they may be better, but to better procedures for assimilating them.

How does the recognition of chaos affect our approach to the weather? From the research point of view, it opens up new questions to be answered. One of these concerns the rate at which small perturbations will amplify. Recent work indicates that the uncertainty in features such as the locations and structures of major storms will double in two days or a little less. This in turn suggests that reasonably good day-to-day forecasts should be possible a week in advance, but not a month or two in advance. Studies of chaos may thus indicate where we should concentrate our future efforts, if we are to maximize our likelihood of positive results.

The recognition of chaos has also led to a key change in the operational forecasting routine. In 1964 I suggested that, since we do not know the present weather pattern precisely, we might, instead of making a single weather forecast, make a large number, originating from slightly different initial states, any one of which might happen to be, but more likely would not be, the correct initial state. I suspect that the suggestion had been made earlier, although I am unable to find documentation. At ranges where the separate forecasts have begun to show important disagreements, they may still share certain features, suggesting that these features are likely to occur. When the forecasts disagree completely, we may still estimate the probability of occurrence of a specific feature by counting the number of forecasts in which it appears.

It has been a source of pleasure to me to see that this technique, known as "ensemble forecasting," has within recent decades become standard procedure at some operational weather centers. A typical ensemble of initial states may contain 100 members. To avoid a 100-fold increase in computational effort, the ensemble of forecasts is commonly produced with a similar model but with reduced horizontal resolution. In many instances a "consensus" forecast proves to outperform the individual ensemble members.

Chaos has been detected in many fields of endeavor, and I have mentioned but a few. I hope, however, that my remarks give some indication of what chaos is about. I thank you.