

```

> # MAPLE

# KNUDSEN 02.20 (Elastic Collision in 1D)

> restart; #(use SHIFT-ENTER to get a
soft-newline)

> #define the SET of equations
eqns:= { m1*vli+m2*v2i=m1*vlf+m2*v2f,
m1*vli^2/2+m2*v2i^2/2=m1*vlf^2/2+m2*v2f^2/2
};

eqns := { $m_1 v_{li} + m_2 v_{2i} = m_1 v_{lf} + m_2 v_{2f}$ ,  

 $\frac{m_1 v_{li}^2}{2} + \frac{m_2 v_{2i}^2}{2} = \frac{m_1 v_{lf}^2}{2} + \frac{m_2 v_{2f}^2}{2}$ } conservation of momentum conservation of kinetic energy

> KE_transfer_ratio:=-(m1*vlf^2/2-m1*vli^2/2)
/(m1*vli^2/2);
P_transfer_ratio:=-(m1*vlf-m1*vli)/(m1*vli)
;

KE_transfer_ratio := - $\frac{2\left(\frac{1}{2}m_1 v_{lf}^2 - \frac{1}{2}m_1 v_{li}^2\right)}{m_1 v_{li}^2}$  } for later use
P_transfer_ratio := - $\frac{m_1 v_{lf} - m_1 v_{li}}{m_1 v_{li}}$ 

> #use Maple's solve function, and assign the
result to sols
sols:=solve(eqns,{v2f,vlf});

```

the "THEY MISSED" solution

#note the two SETs of solutions

$sols := \{v_{2f} = v_{2i}, v_{lf} = v_{li}\}, \{v_{2f} = \frac{m_2 v_{2i} - v_{2i} m_1 + 2 m_1 v_{li}}{m_2 + m_1}, v_{lf} = \frac{-v_{li} m_2 + m_1 v_{li} + 2 m_2 v_{2i}}{m_2 + m_1}\}$

> ##### use special Maple functions to do symbolic algebra with the solutions

> sol\_interesting:=sols[2];

$sol\_interesting := \{v_{2f} = \frac{m_2 v_{2i} - v_{2i} m_1 + 2 m_1 v_{li}}{m_2 + m_1}, v_{lf} = \frac{-v_{li} m_2 + m_1 v_{li} + 2 m_2 v_{2i}}{m_2 + m_1}\}$

> v2i:=0; #impose the special case when the target was initially at rest

$v_{2i} := 0$

> assign (sol\_interesting);

> vlf;factor(vlf);

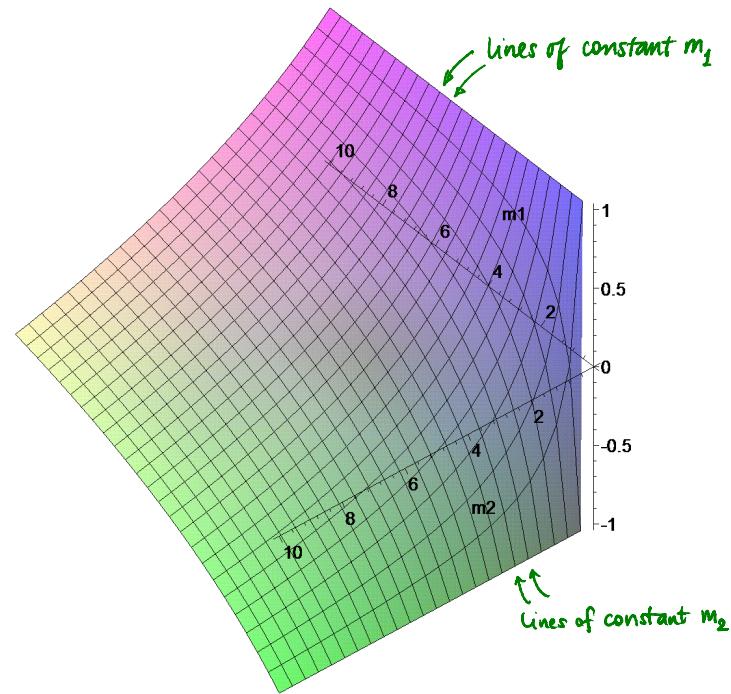
$\frac{-v_{li} m_2 + m_1 v_{li}}{m_2 + m_1}$

$\frac{v_{li} (-m_2 + m_1)}{m_2 + m_1}$

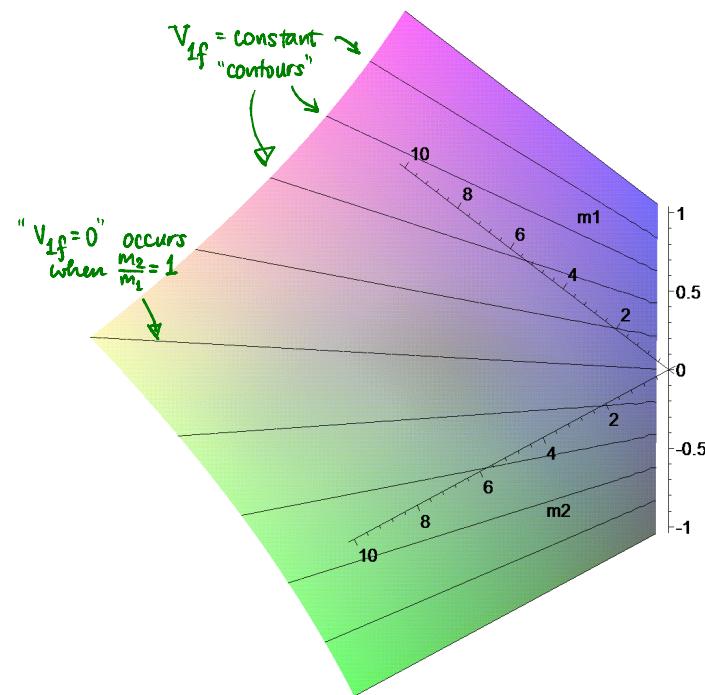
> v2f;

$\frac{2 m_1 v_{li}}{m_2 + m_1}$

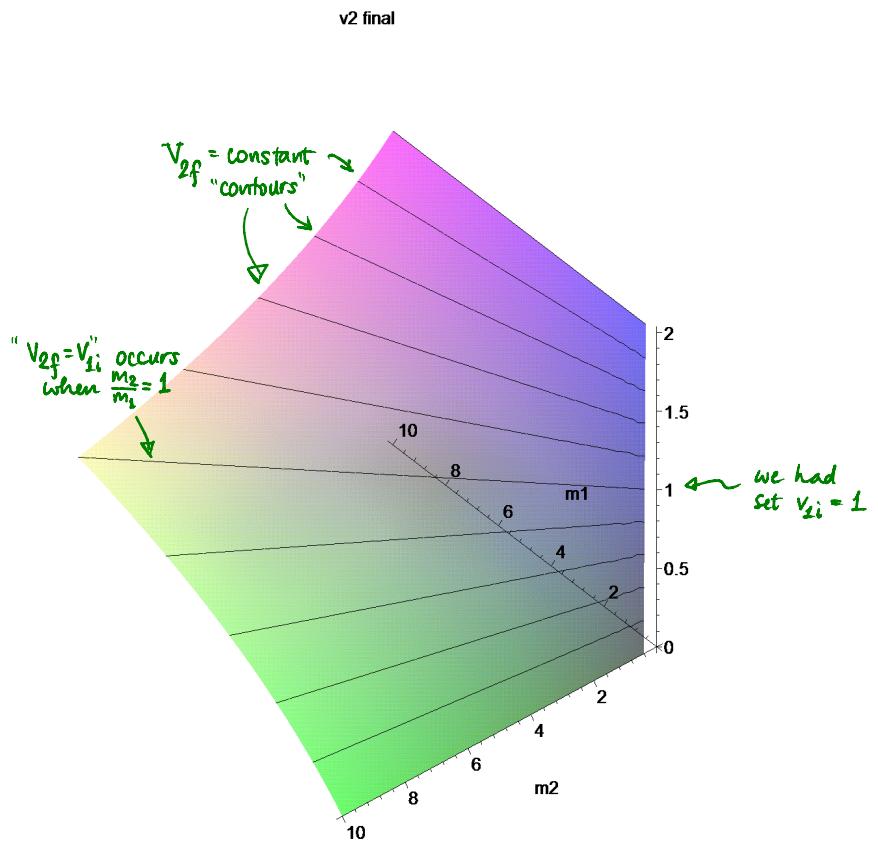
```
> plot3d( subs(v1i=1,v1f), m1=0..10, m2=0..10
,axes=normal,style=patch, title="v1
final",orientation=[140,50]);
v1 final
```



```
,axes=normal,style=patchcontour, title="v2
final",orientation=[140,50]);
v2 final
```



```
> plot3d( subs(v1i=1,v1f), m1=0..10, m2=0..10
,axes=normal,style=patchcontour, title="v1
final",orientation=[140,50]);
plot3d( subs(v1i=1,v2f), m1=0..10, m2=0..10
```



```
P1:=plot3d( subs(v1i=1,v1f), m1=0..10,
m2=0..10 ,axes=normal,style=patchcontour,
title="v1 final"):

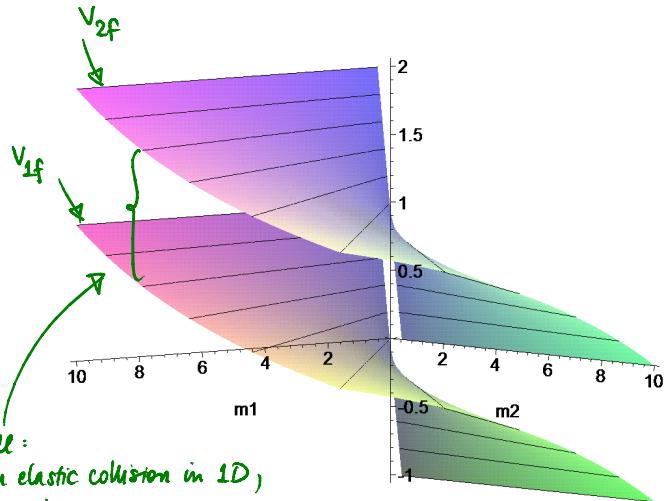
P2:=plot3d( subs(v1i=1,v2f), m1=0..10,
m2=0..10 ,axes=normal,style=patchcontour,
title="v2 final"):

display([P1,P2],title="final
velocities",orientation=[50,85]);
Warning, the name changecoords has been redefined
```

```
> #to use some special Maple plotting
features, load the plots library
with(plots):

#assign names to the plots (for now,
suppress output... note the : instead of ;
)
```

final velocities

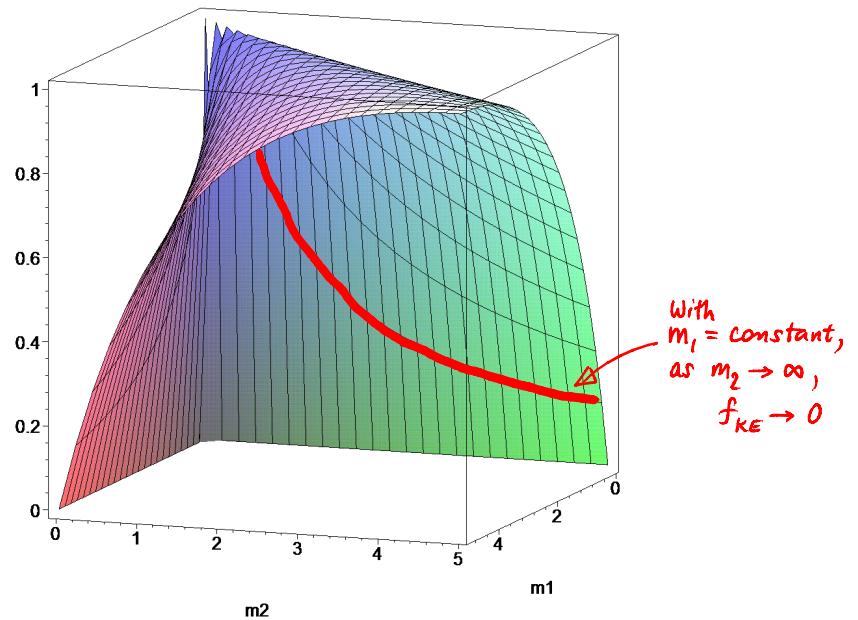


Recall:  
for an elastic collision in 1D,  
 $v_{2f} - v_{1f}$  is independent  
of  $m_1$  and  $m_2$

```
> KE_transfer_ratio:=simplify(%);
plot3d(KE_transfer_ratio,m1=1e-6..5,m2=1e-6
..5,axes=boxed,orientation=[20,80]);
```

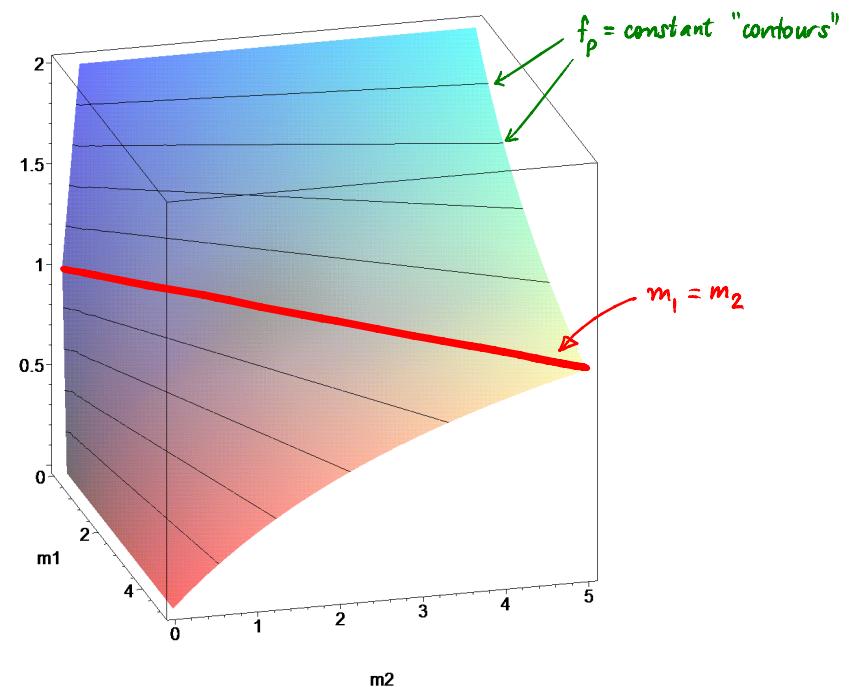
$$-\frac{2 \left( \frac{m1 (-vIi m2 + m1 vIi)^2}{2 (m2 + m1)^2} - \frac{m1 vIi^2}{2} \right)}{m1 vIi^2}$$

$$\frac{4 m1 m2}{(m2 + m1)^2}$$



```
> P_transfer_ratio:=simplify(%);
plot3d(P_transfer_ratio,m1=1e-6..5,m2=1e-6..5,axes=boxed,orientation=[-15,70],style=polygoncontour);
```

$$-\frac{\frac{m1 (-vli m2 + m1 vli)}{m2 + m1} - m1 vli}{m1 vli} - \frac{2 m2}{m2 + m1}$$



[>