Steinitz, Zermelo, and Elkies

by Dan Heisman

In the late 19th century World Champion Wilhelm Steinitz laid down the fundamental principles of positional play. A good summary of his work can be found at the Exeter Chess Club site. Among the several important ideas postulated, one was that a chess game begins in equilibrium and that a player would have to make a mistake in order to lose.

A corollary of Steinitz's work is that a player can't win a drawn game by making brilliant moves – a theoretically drawn game can only be won as a result of an opponent's mistake. It may take a brilliant move to pinpoint that mistake and make it apparent, but the fundamental principle applies: in order for your game to get better, it requires a mistake by your opponent.

Ernst Zermelo

To Steinitz's great credit, the main part of his theory was proven about twenty years later by mathematician Ernst Zermelo, a pioneer in mathematical game theory. If I may paraphrase Zermelo, he stated (among other applications) that for any finite, complete knowledge game (such as chess, go, or checkers), if the game has an initial position that is winning for one player, then the player who is winning will remain winning unless he makes a mistake, and if the game has an initial position that is theoretically drawn it will remain drawn unless one player makes a mistake. This result is basically the same as what Steinitz wrote, except that Steinitz implied that the initial chess position is drawn, while Zermelo extended this to any similar game, including those that begin with a forced win for one player.

In *Applying Steinitz' Laws*, I wrote about some of the ramifications for practical play, including applications from his above principle and others. This time I would like to concentrate on this one issue.

It is fundamental to note that the definition of "better" in mathematical terms means to go from a theoretical loss or draw to a win, or from a loss to a draw, assuming best play on both sides. Conversely, "worse" means to go from a win to a loss or draw, or from a draw to a loss. "Better" in this sense *does not* mean to control more space, obtain a better pawn structure, or even win material unless this changes the expected outcome assuming best play.

To borrow from the earlier article, the proof that one cannot make the position better by making a move is rather straightforward: since evaluation assumes best play, then the best move must leave the evaluation unchanged. In other words, if it is your move, then your position is only as good as your best move, and if you make that best move, you have reached the potential for your move and your position is no better. For example, if you are winning and make a move that checkmates, then your position is no better than before your move; thus, administering the checkmate realizes the evaluation but does not improve it.

As another example, take the initial position of a game. Consider what happens when White plays 1.e4. Indeed, afterwards, White has more central control and greater piece mobility, but in return he has given up the move. If 1.e4 is White's best move, then playing it has reached the potential of the position, nothing more, and White’s position has not improved. And if another move, say 1.d4, is eventually proven best *à la* Hans Berliner’s suggestion, then 1.e4 may, from a practical standpoint, make the position slightly worse.

What is interesting about this argument is that good chess players often disagree, but good mathematicians don’t! My bachelor’s degree is in mathematics, but I am hardly a mathematician, so I called upon someone who is both a strong player and distinguished mathematician, Harvard Professor of Mathematics Noam Elkies. Professor Elkies, who has also achieved a USCF master’s title, has posted on his website a relevant paper “Zermelo and the Early History of Game Theory” by Ulrich Schwalbe and Paul Walker. Professor Elkies was kind enough to reply (and also review this article):

> “As you can see from that paper, the result is indeed usually attributed to Zermelo (also one of the founders of modern mathematical logic) though the attribution may be imprecise. If you use the resulting perfect strategy to define the concepts of "won/drawn/lost position" then it is an immediate corollary that there is no move that can improve your position in the sense of transforming a lost to a drawn or won position, or a drawn position to a won one, and the same is true of what you describe as "Steinitz's conjecture" … This does not contradict the possibility of improving your position in the practical sense of raising the expected value (in the sense of probability) of the game when played between opponents of roughly the same strength as those actually at the board. Between perfect players, that expected value would be constant at 1, 1/2, 0 depending on the won/drawn/lost evaluation of the initial position; between humans the expectation may be quite different, and may change a lot over the course of the game.”

I could not have said it better myself – and that is the truth.

For the purposes of the following discussions, let’s consider the mathematical way of looking at this issue: the “theoretical” way and the “practical” way. Let’s begin by considering a fairly straightforward hypothetical case:

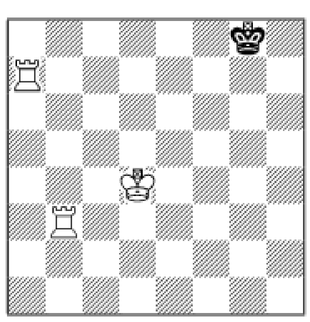
Suppose you have an endgame position where you are “winning” and have thirty different possible legal moves. Further, let’s assume that they break down the following way:

- twelve moves out of the thirty win with further best play on both sides (after the initial move)
- eight of the thirty draw
- ten of the thirty lose

Then, in the theoretical sense, all twelve of the moves that win are “equally” best, although not, as Professor Elkies eloquently points out, in the practical sense. In fact, one or more of these twelve moves might be mate in one!

This leads to a similar example that was recently suggested to me:

White to play



Assuming the game is far from the fifty-move rule draw, White has no moves that lose or draw. So theoretically all moves are equally good but, from a practical standpoint, that is clearly not true. I would certainly not tell my students to play anything other than 1.Rb8#, the only mate in one in the position. But if White does not mate, his position has not gotten “worse” in the sense that he is winning anyway. A win is a win is a win.

I recently heard an anecdote about a game played by former world champion Vladimir Kramnik. Apparently he was running short on time and found an easily winning line. After he won, someone pointed out a much quicker win, but Kramnik correctly noted that it would be impractical for him to spend time looking for an easier win once he had found an easy win. It’s similar to the logic involved with the previous diagram.

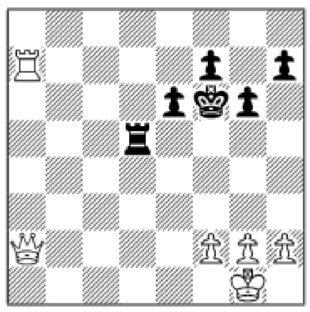
Besides the corollary to Zermelo’s theorem about not being able to win a drawn position with a brilliant move, another corollary is that making a move that is not (equally) best can only deteriorate the position. We can consider this corollary two ways.

In the theoretical sense, playing a move other than 1.Rb8# in the previous diagram is not “inferior” since anything wins. However, in a practical sense delaying the win could result in further mistakes that cost the win or cause a loss on time. For example, if White plays 1.Rbb7, this is not a theoretical blunder in the sense that White can still mate on the next move. White has not made an “inferior” move, so the position did not deteriorate. 1.Rbb7 was theoretically “equal best,” but clearly not as desirable as 1.Rb8# from a practical sense.

Now let’s return to the endgame where there were thirty moves and twelve of those won. If White plays any of those twelve, then, theoretically, the position does not deteriorate (White can still win and that’s the best that can be done) and, from that standpoint, all twelve are considered equally good. Yet if White does play an inferior move, one of the other eighteen, then a win can no longer be forced and the position has deteriorated, both theoretically and practically. So playing a move not (equally) best will, as conjectured, make the position worse.

Finally, here is a “technique” problem I often use:

White to play



From a practical standpoint, I suggest 1.Qxd5 followed by marching the white king to the d-file, stopping the threatened mate and eliminating all counterplay. Clearly 1.Qxd5 is not a theoretical mistake since White is still winning easily. One might argue either way about the benefits of this move: the win has become easy and the chances of losing nonexistent, both of practical benefit, but the mate is further away, which may seem the opposite. Computer engines do not find 1.Qxd5 “best” since it does not result in the highest evaluation nor result in the quickest mate. [This “technique” is often seen in blitz or bullet games – ed.]

Hopefully this article has cleared up some of the concerns about both the theoretical and practical applications of this aspect of Steinitz’s Laws and Zermelo’s work.
