Effects and Benefits of Undersampling in High-Speed ADC Applications

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Undersampling or violating the nyquist criterion is a technique often utilised in ADC applications. RF communications and high performance test equipment such as oscilloscopes are to name just a few. This is a 'grey' area and confusion often arises over the necessity to obey Nyquist in order to retrieve the information content of a signal. Examination of both the Nyquist and Shannon theorems will demonstrate that the choice of ADC sampling frequency has a strong correlation to the ratio of maximum input signal frequency to input signal bandwidth.

Nyquist theorem states that to reconstruct an analogue signal waveform without error from sample point taken at equal time intervals, the sampling frequency must be greater than or equal to twice the highest frequency component in the analogue signal.

In practice, analogue signals are not simple sine waves and usually have complex waveforms with components or harmonics at many frequencies. Therefore to reliably reproduce a square wave from time-interleaved samples, the sampling frequency according to Nyquist must be many times higher than the square wave's fundamental frequency.

Sampling an analogue signal fa, at a sampling rate fs, actually produces two aliased components, one at fs+fa and the other at fs-fa. The frequency domain representation of this is shown in figure 1.

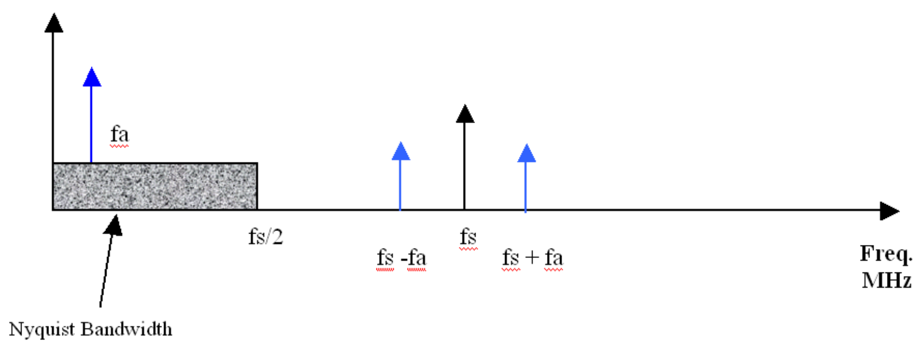

Figure 1. Signal Sampling Produces Aliased Components

Generally, it is only the lower alias component which may cause problems because it can fall within the Nyquist bandwidth and corrupt wanted signals. Based on the aliasing phenomenon of sampling systems the Nyquist criteria requires sampling at a rate of fs > fa in order to avoid overlapping aliased components into the first Nyquist zone. To prevent unwanted interference, any signals which fall outside the bandwidth of interest should be filtered before sampling. This explains the necessity for anti-alias filters in many sampling systems. However, methods exist which use aliasing to one's advantage in signal processing applications.

One can quickly see the limitations implied by the Nyquist theorem. Nyquist assumes that the desired information bandwidth is equal to the Nyquist bandwidth or half the sampling frequency. It should be observed that the required minimum sampling frequency is actually a function of the input signal bandwidth. Shannon's theorem examines this further.

An analogue signal with a bandwidth fb, must be sampled at a rate of fs > 2fb in order to avoid the loss of information. The signal bandwidth may extend from DC to fb (baseband sampling) or from f1 to f2 where fb = f2 - f1 (undersampling).

So Shannon's theorem states that the actual minimum required sampling frequency is a function of the signal bandwidth and not only its maximum frequency component. In general the sampling rate must be at least twice the signal bandwidth and the sampled signal must not cross an integer multiple of fs/2 to prevent overlapping of the aliased components. Note that for large ratios of fMAX (maximum frequency component of the analog signal) to signal bandwidth B, the minimum sampling frequency approaches 2B.

In many applications this greatly reduces the demand placed on the ADC required. Sampling a signal with a maximum signal frequency of 150MHz but with a bandwidth of 10MHz would require perhaps a 22MSPS ADC instead of a >300MSPS ADC as stipulated by nyquist.

Otherwise stated, due to harmonic folding or undersampling, every ADC input frequency component outside the Nyquist bandwidth is always folded back into the first Nyquist zone. Sub-sampling has many uses in practical electronic systems. One such popular application of undersampling is in digital receivers. Lets first explain in a little more detail the process of sub-sampling.

The process of sub-sampling or folding can be thought of as mixing the ADC input signal with the sampling frequency and its harmonics. This means that many frequencies can be mixed down to DC and their original frequency can no longer be determined. Sub-sampling cannot be used if the original input frequency must be determined at the ADC output. This is because the Nyquist criteria is violated. Sub-sampling still proves useful if there is no need to determine the carrier frequency at the ADC output. This is true for many communication systems such as cellular base-station receivers since the receiver only needs to recover the information on the carrier and not the carrier itself.

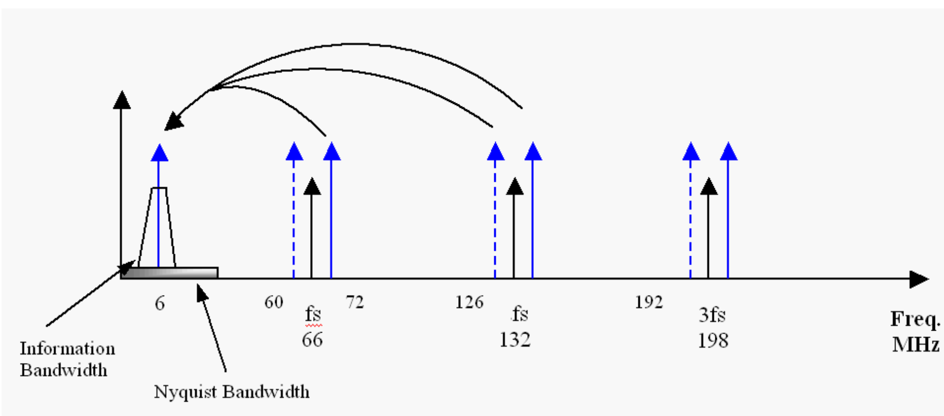

Figure 2. Many Frequencies are mixed down towards DC when Sub-Sampling

Consider a GSM/EDGE base-station with RF carrier frequencies of 900MHz Europe and 1800MHz USA. The receive chain of a mobile base-station is similar to that shown in Figure 3. The high frequency RF carrier signal is first down-converted in the mixer and local oscillator stage to an IF frequency in the range of 150 -190MHz for analogue to digital conversion. Shannon theory shows that the required sampling frequency is a function of the signal bandwidth or 200kHz in a GSM/EDGE system. The dynamic range specifications of a GSM system require a minimum ADC resolution of 10 bits although 12 bits are used in practically all cases. Given the vast choice of high speed ADCs available in the market a digital receiver system designer has to make a choice based upon system dynamic range requirements and also component cost. For these reasons, ADCs with sampling rates of 50 -70MSPS are the most popular choice for GSM receivers. Although the 150 - 190 MHz signal is undersampled at for example 66MSPS (using National's ADC12DL066, dual 12-bit 66MSPS ADC) Nyquist is not violated for the required information bandwidth of 200kHz. This provides more than enough headroom for the 200kHz bandwidth information signal as well as providing over 20dB's of processing gain (explained below). Note that for many reasons, it is not practical to continually increase processing gain by increasing sampling frequency. Higher sampling rate 12-bit ADCs such as the 12-bit 80MSPS ADCs are available (National's ADC12L080) and some >100MSPS 12-bit ADCs exist for specialised applications but the cost differential between sub-100MSPS and >100MSPS sampling rate ADCs is quite significant.

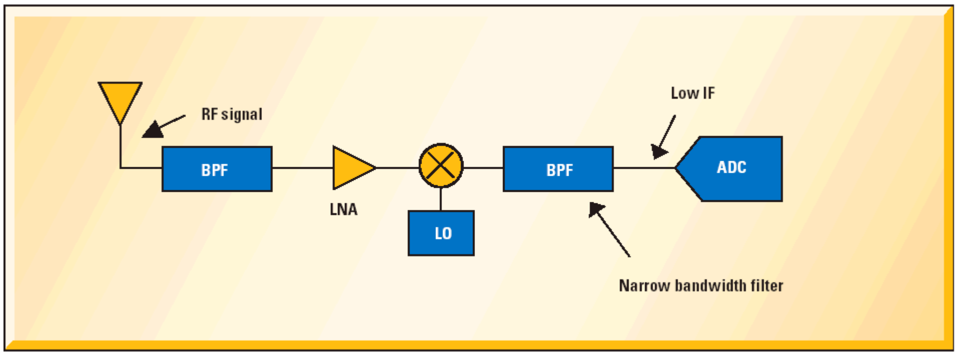

Figure 3. Communications Signal Chain

ADC noise performance is typically limited by thermal noise and when an ADC is specified, the noise bandwidth is normally defined as the nyquist bandwidth. At FS = 66MSPS the integrated noise-floor measurement is expressed in dB's relative to full scale (dBFS) in a 33MHz bandwidth at a particular input signal frequency. For the ADC12L066 the integrated noise-floor is -62dBFS for an input signal frequency of 150MHz. However filtering the ADC output results in a much narrower bandwidth such as that provided by Nationals digital down converter, the CLC5903. The filtering process provides noise processing gain as a function of the bandwidth reduction.

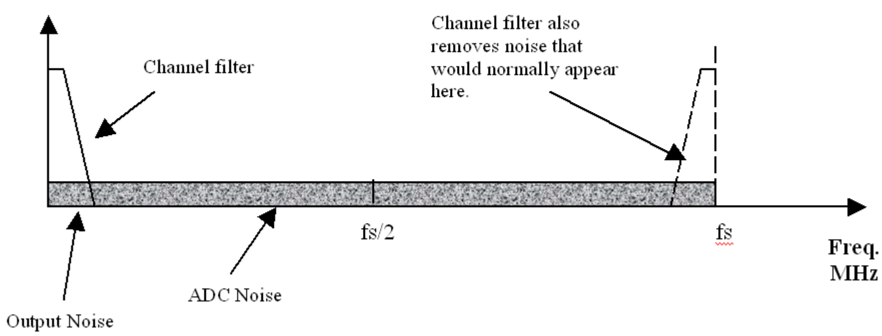

Figure 4. Narrowing the bandwidth by filtering improves noise performance

Choosing the correct sampling rate ADC for an application requires more than just knowledge of the highest analogue frequency for conversion. Shannon's theorem demonstrates that signal bandwidth is of equal importance. We have also discovered that sampling at greater than the Shannon rate brings other benefits such as vast improvements in dynamic range brought about by processing gain. Armed with this knowledge, a system designer can make the right choice for ADC sampling frequency and resolution based upon readily available and affordable standard ADCs.