1. CHARACTERISTICS AND PARAMETERS OF OPERATIONAL AMPLIFIERS

The characteristics of an ideal operational amplifier are described first, and the characteristics and performance limitations of a practical operational amplifier are described next. There is a section on classification of operational amplifiers and some notes on how to select an operational amplifier for an application.

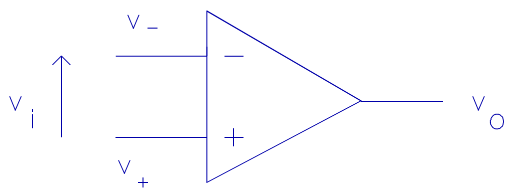
1.1 IDEAL OPERATIONAL AMPLIFIER

1.1.1 Properties of An Ideal Operational Amplifier

The characteristics or the properties of an ideal operational amplifier are:

i. Infinite Open Loop Gain,
ii. Infinite Input Impedance,
iii. Zero Output Impedance,
iv. Infinite Bandwidth,
v. Zero Output Offset, and
vi. Zero Noise Contribution.

The opamp, an abbreviation for the operational amplifier, is the most important linear IC. The circuit symbol of an opamp shown in Fig. 1.1. The three terminals are: the non-inverting input terminal, the inverting input terminal and the output terminal. The details of power supply are not shown in a circuit symbol.



$$v_o = -A_o v_i = -A_o(v_- - v_+)$$

Fig.1.1: Circuit symbol of an opamp

1.1.2 Infinite Open Loop Gain

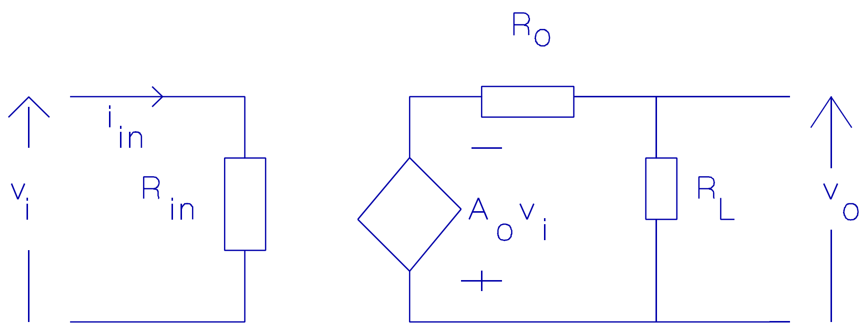
From Fig.1.1, it is found that $v_o = -A_o \times v_i$, where `A_o' is known as the open-loop gain of the opamp. Let v_o be -10 Volts, and A_o be 10^5. Then v_i is 100 μV. Here the input voltage is very small compared to the output voltage. If A_o is very large, v_i is negligibly small for a finite v_o. For the ideal opamp, A_o is taken to be infinite in value. That means, for an ideal opamp $v_i = 0$ for a finite v_o. Typical values of A_o range from 20,000 in low-grade consumer audio-range opamps to more than 2,000,000 in premium grade opamps (typically 200,000 to 300,000).

The first property of an ideal opamp: Open Loop Gain A_o = infinity.

1.1.3 Infinite Input Impedance and Zero Output Impedance

An ideal opamp has an infinite input impedance and zero output impedance. The sketch in Fig. 1.2 is used to illustrate these properties. From Fig. 1.2, it can be seen that i_{in} is zero if R_{in} is equal to infinity.

The second property of an ideal opamp: R_{in} = infinity or i_{in} =0.



Note: R_L is not part of opamp model

Fig.1.2: Model of an Opamp

From Fig. 1.2, we get that

$$v_o = -A_o v_i * \frac{R_L}{R_o + R_L}$$

If the output resistance R_o is very small, there is no drop in output voltage due to the output resistance of an opamp.

The third property of an ideal opamp: R_o = 0.

1.1.4 Infinite Bandwidth

An ideal opamp has an infinite bandwidth. A practical opamp has a limited bandwidth, which falls far short of the ideal value. The variation of gain with frequency has been shown in Fig. 1.3, which is obtained by modelling the opamp with a single dominant pole, whereas the practical opamp may have more than a single pole.

The asymptotic log-magnitude plot in Fig. 1.3 can be expressed by a first-order equation shown below.

$$A(jw) = \frac{A_o}{1 + \frac{jw}{w_H}}.$$

It is seen that two frequencies, w_H and w_T, have been marked in the frequency response plot in Fig. 1.3.. Here w_T is the frequency at which the gain A(jw) is equal to unity. If $A(jw_T)$ is to be equal to unity,

$$A(jw_T) = 1 = \left| \frac{A_o}{1 + \frac{jw_T}{w_H}} \right|.$$

Since A_o is very large, it means that $w_T = A_o * w_H$.

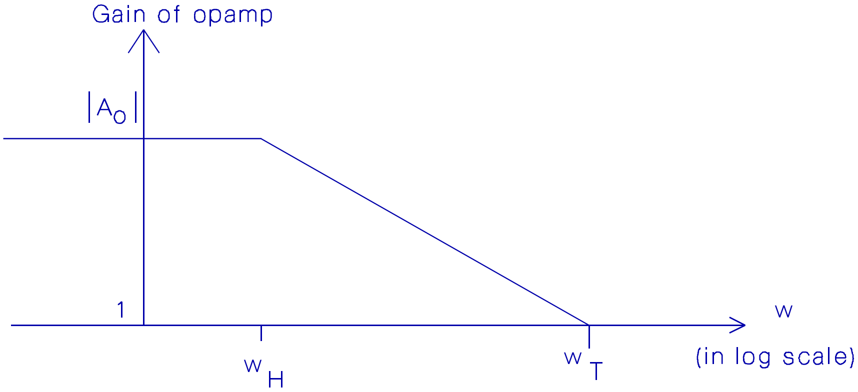


Fig. 1.3: Change in gain of Opamp with Frequency

1.1.5 Zero Noise Contribution and Zero Output Offset

A practical opamp generates noise signals, like any other device, whereas an ideal opamp produces no noise. Premium opamps are available which contribute very low noise to the rest of circuits. These devices are usually called as premium low-noise types.

The output offset voltage of any amplifier is the output voltage that exists when it should be zero. In an ideal opamp, this offset voltage is zero.

1.2 PRACTICAL OPERATIONAL AMPLIFIERS

This section describes the properties of practical opamps and relates these characteristics to design of analog electronic circuits. A practical operational amplifier has limitations to its performance. It is necessary to understand these limitations in order to select the correct opamp for an application and design the circuit properly.

Like any other semiconductor device, a practical opamp also has a code number. For example, let us take the code LM 741CP. The first two letters, LM here, denote the manufacturer. The next three digits, 741 here, is the type number. 741 is a general-purpose opamp. The letter following the type number, C here, indicates the temperature range. The temperature range codes are:

C commercial 0^o C to 70^o C,
I industrial -25^o C to 85^o C and
M military -55^o C to 125^o C.

The last letter indicates the package. Package codes are:

D Plastic dual-in-line for surface mounting on a pc board
J Ceramic dual-in-line
N,P Plastic dual-in-line for insertion into sockets.

1.2.1 Standard Operational Amplifier Parameters

Understanding operational amplifier circuits requires knowledge of the parameters given in specification sheets. The list below represents the most commonly needed parameters. Methods of measuring some of these parameters are described later in this lesson.

Open-Loop Voltage Gain. Voltage gain is defined as the ratio of output voltage to an input signal voltage, as shown in Fig. 1.1. The voltage gain is a dimensionless quantity.

Large Signal Voltage Gain. This is the ratio of the maximum allowable output voltage swing (usually one to several volts less than V- and V++) to the input signal required to produce a swing of ± 10 volts (or some other standard).

Slew rate. The slew rate is the maximum rate at which the output voltage of an opamp can change and is measured in terms of voltage change per unit of time. It varies from 0.5 V/μs to 35 V/μs. Slew rate is usually measured in the unity gain noninverting amplifier configuration.

Common Mode Rejection Ratio. A common mode voltage is one that is presented simultaneously to both inverting and noninverting inputs. In an ideal opamp, the output signal due to the common mode input voltage is zero, but it is nonzero in a practical device. The common mode rejection ratio (CMRR) is the measure of the device's ability to reject common mode signals, and is expressed as the ratio of the differential gain to the common mode gain. The CMRR is usually expressed in decibels, with common devices having ratings between 60 dB to 120 dB. The higher the CMRR is, the better the device is deemed to be.

Input Offset Voltage. The dc voltage that must be applied at the input terminal to force the quiescent dc output voltage to zero or other level, if specified, given that the input signal voltage is zero. The output of an ideal opamp is zero when there is no input signal applied to it.

Power-supply rejection ratio. The power-supply rejection ratio PSRR is the ratio of the change in input offset voltage to the corresponding change in one power-supply, with all remaining power voltages held constant. The PSRR is also called "power supply insensitivity". Typical values are in μV / V or mV/V.

Input Bias Current. The average of the currents into the two input terminals with the output at zero volts.

Input Offset Current. The difference between the currents into the two input terminals with the output held at zero.

Differential Input Impedance. The resistance between the inverting and the noninverting inputs. This value is typically very high: 1 MΩ in low-cost bipolar opamps and over 10^{12} Ohms in premium BiMOS devices.

Common-mode Input Impedance
The impedance between the ground and the input terminals, with the input terminals tied together. This is a large value, of the order of several tens of MΩ or more.

Output Impedance. The output resistance is typically less than 100 Ohms.

Average Temperature Coefficient of Input Offset Current. The ratio of the change in input offset current to the change in free-air or ambient temperature. This is an average value for the specified range.

Average Temperature Coefficient of Input Offset Voltage. The ratio of the change in input offset voltage to the change in free-air or ambient temperature. This is an average value for the specified range.

Output offset voltage. The output offset voltage is the voltage at the output terminal with respect to ground when both the input terminals are grounded.

Output Short-Circuit Current. The current that flows in the output terminal when the output load resistance external to the amplifier is zero ohms (a short to the common terminal).

Channel Separation. This parameter is used on multiple opamp ICs (device in which two or more opamps sharing the same package with common supply terminals). The separation specification describes part of the isolation between the opamps inside the same package. It is measured in decibels. The 747 dual opamp, for example, offers 120 dB of channel separation. From this specification, we may state that a 1 μV change will occur in the output of one of the amplifiers, when the other amplifier output changes by 1 volt.

1.2.2 Minimum and Maximum Parameter Ratings

Operational amplifiers, like all electronic components, are subject to maximum ratings. If these ratings are exceeded, the device failure is the normal consequent result. The ratings described below are commonly used.

Maximum Supply Voltage. This is the maximum voltage that can be applied to the opamp without damaging it. The opamp uses a positive and a negative DC power supply, which are typically ± 18 V.

Maximum Differential Supply Voltage. This is the maximum difference signal that can be applied safely to the opamp power supply terminals. Often this is not the same as the sum of the maximum supply voltage ratings. For example, 741 has ± 18 V as the maximum power supply voltage, whereas the maximum differential supply voltage is only 30 V. It means that if the positive supply is 18 V, the negative supply can be only -12 V.

Power dissipation, P_d. This rating is the maximum power dissipation of the opamp in the normal ambient temperature range. A typical rating is 500 mW.

Maximum Power Consumption. The maximum power dissipation, usually under output short circuit conditions, that the device can survive. This rating includes both internal power dissipation as well as device output power requirements.

Maximum Input Voltage. This is the maximum voltage that can be applied simultaneously to both inputs. Thus, it is also the maximum common-mode voltage. In most bipolar opamps, the maximum input voltage is nearly equal to the power supply voltage. There is also a maximum input voltage that can be applied to either input when the other input is grounded.

Differential Input Voltage. This is the maximum differential-mode voltage that can be applied across the inverting and noninverting inputs.

Maximum Operating Temperature. The maximum temperature is the highest ambient temperature at which the device will operate according to specifications with a specified level of reliability.

Minimum Operating Temperature. The lowest temperature at which the device operates within specification.

Output Short-Circuit Duration. This is the length of time the opamp will safely sustain a short circuit of the output terminal. Many modern opamps can carry short circuit current indefinitely.

Maximum Output Voltage. The maximum output potential of the opamp is related to the DC power supply voltages. Typical for a bipolar opamp with ± 15 V power supply, the maximum output voltage is typically about 13 V and the minimum - 13 V.

Maximum Output Voltage Swing. This is the maximum output swing that can be obtained without significant distortion(at a given load resistance).

Full-power bandwidth. This is the maximum frequency at which a sinusoid whose size is the output voltage range is obtained.

1.2.3 Comparisons and Typical Values

Table 1.1 presents a summary of features of an ideal and a typical practical opamp.

Table 1.1: Comparison of an ideal and a typical practical opamp

Property	Ideal	Practical(Typical)
Open-loop gain	Infinite	Very high (>10000)
Open-loop bandwidth	Infinite	Dominant pole(≅10 Hz)
CMRR	Infinite	High (> 60 dB)
Input Resistance	Infinite	High (>1 MΩ)
Output Resistance	Zero	Low(< 100 Ω)
Input Bias Currents	Zero	Low (< 50 nA)
Offset Voltages	Zero	Low (< 10 mV)
Offset Currents	Zero	Low (< 50 nA)
Slew Rate	Infinite	A few V/μs
Drift	Zero	Low

Table 1.2 shown below presents a summary of the effects of opamp characteristics on a circuit's performance. It is a simplified summary.

1.2.4 Effect of Feedback on Frequency Response

The effect of feedback on the frequency response of a system has already been described. Here the effect of feedback is described using the log-magnitude plot. Given that the transfer function of the forward path is specified as:

TABLE 1.2 EFFECTS OF CHARACTERISTICS ON OPAMP APPLICATIONS

Opamp Characteristic that may affect performance	OPAMP APPLICATION: DC amplifier		AC amplifier	
	Small output	Large output	Large output	Small output
1. Input bias current	Yes	Maybe	No	No
2. Offset current	Yes	Maybe	No	No
3. Input offset voltage	Yes	Maybe	No	No
4. Drift	Yes	Maybe	No	No
5. Frequency Response	No	No	Yes	Yes
6. Slew rate	No	Yes	No	Yes

$$A(s) = \frac{A_o}{1 + sT_1} \quad .(1.1)$$

If the closed-loop transfer function T(s) of the circuit is

$$T(s) = \frac{A(s)}{1 + \beta A(s)} \quad .(1.2)$$

On substituting for A(s) by its expression in equation (1.1), we get that

$$T(s) \simeq \frac{\frac{1}{\beta}}{1 + \frac{sT_1}{A_o\beta}}, \; if \; A_o\beta \gg 1 \quad .(1.3)$$

The plot of frequency response for open loop and closed-loop is shown in Fig. 1.4.

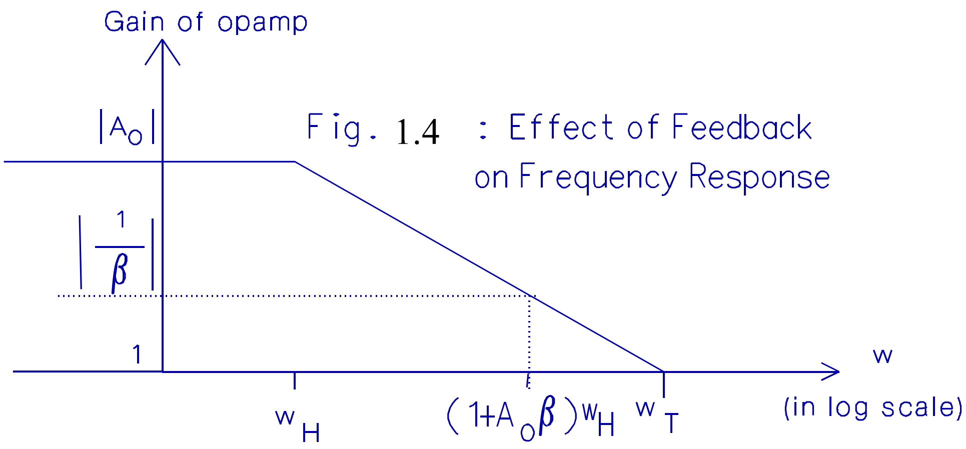


Fig. 1.4 : Effect of Feedback on Frequency Response

1.3 CLASSIFICATION OF OPAMPS

The classification of an opamp can be based on either its function or its family type. The classification based on function is described below.

i. General-purpose amplifier. These general purpose opamps are neither special purpose or premium devices. Most of them are internally compensate, so designers trade off bandwidth for inherent stability. A general purpose opamp is the default choice for an application unless a property of another class brings a unique advantage to this application.

ii. Instrumentation amplifier. Although an instrumentation amplifier is arguably a special purpose device, it is sufficiently universal to warrant a class of its own.

iii. Voltage Comparators. These devices are not true opamps, but are based on opamp circuitry. While all opamps can be used as voltage comparators, the reverse is not true. The special feature of a comparator is the speed at which its output level can change from one level to the other.

iv. Low Input Current. The quiescent current needed for these opamps is low. This class of opamps typical uses MOSFET , JFET or superbeta (Darlington) transistors for the input stage instead of npn/pnp bipolar devices.

v. Low Noise. These devices are usually optimized to reduce internally generated noise.

vi. Low Power. This category of opamp optimizes internal circuitry to reduce power consumption. Many of these devices also operate at very low DC power supply potentials.

vii. Low Drift. All DC amplifiers suffer from drift. Devices in this category are internally compensated to minimize drift due to temperature. These devices are typically used in instrumentation circuits where drift is an important concern, especially when handling low level input signals.

viii. Wide Bandwidth. The devices in this class are also called as video opamps and have a very high gain-bandwidth level, as high as 100 MHz. Note that 741 has a gain-bandwidth product of about 1 MHz .

ix Single DC Supply. These devices are designed to operate from a single DC power supply.

x. High Voltage. The power supply for these devices can be as high as ± 44 V.

xi. Multiple devices. Two or quad arrangement in one IC.

The classification based on family type:
i. Bipolar opamps, ii. BiFET opamps, iii. JET opamps, iv. CMOS opamps etc. The characteristics of opamps change with the internal architecture also. Some opamps have two-stage architecture, whereas some have three-stage architecture.

The purpose of this section is to highlight the facts that it is necessary to select a suitable opamp for the application in hand and that there is a wide choice available. Choosing the right opamp is not simple. Aspects to be considered are: technology,dc performance,ac performance,output drive requirements, supply requirements, quiescent current level, temperature range of operation, nature of input signal, costs etc. Table 1.3 presents a summary of characteristics of a few selected opamps. It is preferable to go through the databooks on linear ICs for selecting the right opamp.

1.3.1 BiFET OPAMP

Although the LM741 and other bipolar opamps are still widely used, they are nearly obsolete. Bipolar technology has been replaced by BiFET technology. The term, "BiFET" stands for bipolar-field effect transistor. It is a combination of two technologies, bipolar and junction field-effect, making use of the advantages of each. Bipolar devices are good for power handling and speed whereas field-effect devices have very high input impedances and low power consumption. Most modern general-purpose opamps are now produced with BiFET technology.

BiFET opamps generally have enhanced characteristics over bipolar opamps. They have a much greater input impedance, a wider bandwidth, a higher slew rate and larger power output than the corresponding ratings of bipolar opamps. A variety of BiFET opamps are now available: the TL060 low-power, TL070 low-noise and TL080 general purpose from Texas Instruments, LF350 and LF440 series from National semiconductor, the MC34000 and MC35000 series from Motorola etc. The performance of most of them are similar and are normally pinout compatible with 741C.

Extremely low bias currents make a BiFET opamp to be more suitable for applications such as an integrator, a sample and hold circuit and filter circuits. But the bias currents double for every 10^o C and at high temperature, a BiFET opamp may have a larger bias current than a bipolar opamp! Both BiFET and CMOS have less noise current, which is an important consideration when dealing with sources of high impedance.

A BiFET opamp has some disadvantages too. It tends to have a far greater offset voltage than its bipolar counterpart. The offset voltages tend to be unstable too. In addition, a BiFET opamp has poorer CMRR, PSRR and open-loop gain specifications. But some of the recent BiFET do not have these drawbacks. It may please be noted that a BiFET opamp needs a dual power supply.

Unlike 741C, TL080 does not have internal compensation and needs an external capacitor of value ranging from 10 pF to 20 pF to be connected between pins 1 and 8. The smaller the capacitor is, the wider the bandwidth is, but the opamp tends to become more underdamped at higher frequencies.

1.3.2 CMOS OPAMPs

Although originally considered to be unstable for linear applications, a CMOS opamp is now a real alternative to many bipolar, BiFET and even dielectrically isolated opamps.

The major advantage of a CMOS opamp is that it operates well with a single supply. The input common-mode range is more or less the same as the power supply range. A CMOS opamp needs very low supply currents, less than 10 μA and can operate with a supply voltage as low as 1.4 V, making it ideally suitable for battery-powered applications. In addition, a CMOS opamp has high input impedance and low bias currents. On the other hand, a CMOS tends to have limited supply voltage range. Its offset voltages tend to be higher than those of a bipolar opamp.

TABLE 1.3 Typical Performance of Selected Opamp Types

	Type 741 2-stage	LM 118 3-stage	LM 108 Super β	AD 611 BiFET	AD 570K Wide-band
Input offset voltage (mV)	<5	<4	<2	<0.5	<5
Bias Current (nA)	<500	<250	<2	<0.025	<15
Offset Current (nA)	<200	<50	<0.4	<0.01	<15
Open-loop gain (dB)	106	100	95	98	100
Input Resistance (MΩ)	2	5	100	10^6	300
Slew Rate (V/μs)	0.5	>50	0.2	13	35
Unity-gain bandwidth (MHz)	1	15	1	2	35
Full-power bandwidth (kHz)	10	1000	4	200	600
Settling time(μs)	1.5	4	1	3	0.9
CMRR(dB)	80	90	95	80	100

2. BASIC OPAMP APPLICATIONS

2.1 NONINVERTING AMPLIFIER

The basic noninverting amplifier can be represented as shown in Fig. 2.1. Note that a circuit diagram normally does not show the power supply connections explicitly.

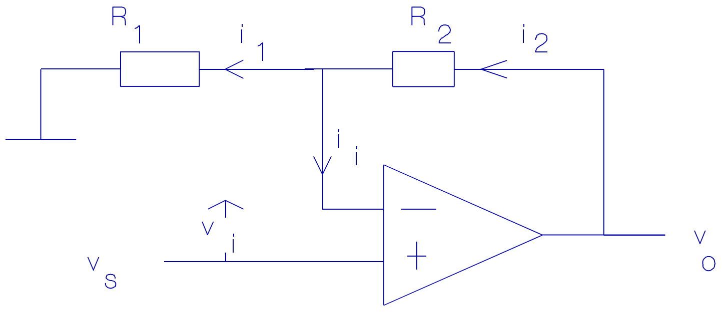


Fig.2.1: NON-INVERTING AMPLIFIER

2.1.1
Analysis For An Ideal Opamp

An ideal opamp has infinite gain. This means that

$$v_i = 0,\ if\ v_o\ is\ finite. \quad (2.1)$$

Thus,

$$v_- = v_+ = V_s\ . \quad (2.2)$$

An ideal opamp has infinite input resistance. That is,

$$I_i = 0. \quad (2.3)$$

$$\therefore I_2 = I_1 = \frac{V_s}{R_1}. \quad (2.4)$$

We obtain the output voltage as:

$$V_o = V_i + I_2R_2 = V_s * \left(1 + \frac{R_2}{R_1}\right). \quad (2.5)$$

The gain of the noninverting amplifier is then:

$$\frac{V_o}{V_s} = \left(1 + \frac{R_2}{R_1}\right). \quad (2.6)$$

2.1.2 Analysis For an Opamp with a finite gain, A_o

Let the opamp have a finite gain. Then the noninverting amplifier can be represented by the equivalent circuit in Fig. 2.2.

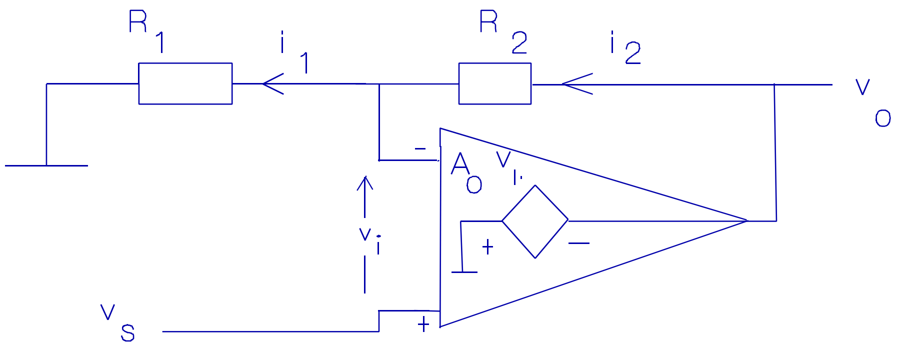


Fig. 2.2: Equivalent Circuit

From Fig 2.2,

$$\frac{(V_s + v_i)}{R_1} = \frac{V_o - (V_s + v_i)}{R_2}, \ and\ v_i = -\frac{V_o}{A_o}.$$

On re-arranging,

$$\left(\frac{V_o}{V_s}\right) = \frac{\left(1 + \frac{R_2}{R_1}\right)}{1 + \frac{1}{A_o} * \left(1 + \frac{R_2}{R_1}\right)}. \quad (2.7)$$

We can represent the circuit in Fig. 2.2 by a block diagram that represents the feedback that is present in the circuit. From Fig. 2.2, we can state that,

$$v_i = \frac{R_1}{R_1 + R_2} * V_o - V_s, \; and \; V_o = -A_o v_i \; .$$

The above equation can be represented by a block diagram as shown in Fig. 2.3. From the block diagram, we get the same expression for the gain of the circuit. It can be seen that if the open loop gain A_o tends to infinity, equation (2.7) reduces to equation (2.6).

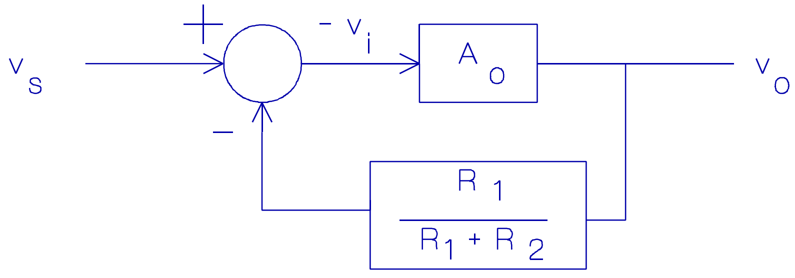


Fig. 2.3: Block Diagram of Noninverting Amplifier

Next we analyse the same circuit when the opamp has a finite input resistance, a finite gain and a nonzero output resistance.

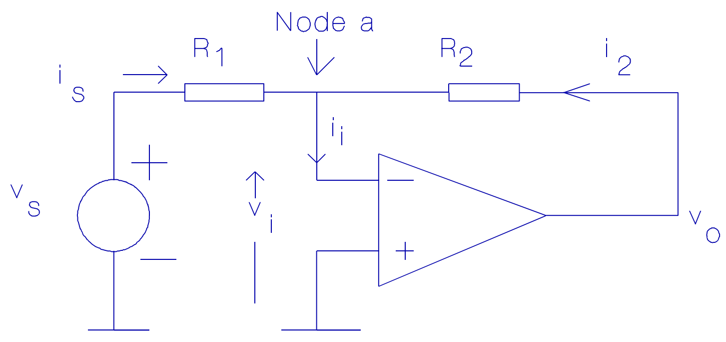


Fig. 2.4 : An Inverting Amplifier

2.2 INVERTING AMPLIFIER

The inverting amplifier is analysed below using both network theory and feedback theory approach.

2.2.1 Analysis Based On Circuit Theory

Analysis is as follows. Apply KCL (Kirchoff's Current Law) at node `a' in Fig. 2.4.
Then

$$i_s + i_2 = i_i .$$

For an ideal opamp, $i_i = 0$. Hence $i_s + i_2 = 0$. Thus the KCL at node 'a' is:

$$\frac{v_s - v_i}{R_1} + \frac{v_o - v_i}{R_2} = 0 .$$

For an ideal opamp, its output resistance is zero. Hence $- A_o v_i = v_o$.

When the gain is infinity, v_i is also zero. Therefore,

$$\frac{v_s}{R_1} + \frac{v_0}{R_2} = 0 .$$

In other words,

$$\frac{v_o}{v_s} = - \frac{R_2}{R_1} .$$

When an opamp is considered to be ideal, v_i and i_i have zero value. If the NI input (noninverting input) is grounded, the inverting input is at zero potential. We can find i_s & i_2 by treating the potential at inverting input terminal to be zero volts. In this condition, the inverting input terminal behaves as if it is grounded and is called as `virtual ground'. When the NI input is not grounded, the inverting input is not at ground potential,it does not behave as if it is grounded and it is no longer called the virtual ground. All that happens is that its potential is the same as that at the NI input.

2.2.2 Analysis Based on Negative Feedback

Now the circuit in Fig. 2.4 is to be represented as a system with feedback. From Fig. 2.4, we get that

$$V_i = \frac{R_2}{R_1 + R_2} V_s + \frac{R_1}{R_1 + R_2} V_o .$$

We can arrive at the result shown above by the use of either superposition theorem or by adding the drop across R_1 to V_s. From Fig. 2.4, we get that

$$V_o = -A_o v_i ,$$

where A_o is the gain of the opamp. Here it is appropriate to call it as the opamp's open loop gain. The above two equations can be represented by a block diagram as shown in Fig. 2.5.

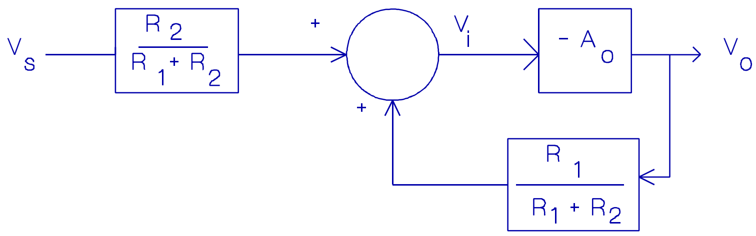


Fig. 2.5 : An Opamp Circuit Shown Connected as a System With Negative Feedback.

For the block diagram in Fig. 2.5, we get the ratio V_o/V_s as

$$\frac{V_o}{V_s} = -\frac{R_2/R_1}{1+\frac{1}{A_o}(1+R_2/R_1)} .$$

Since the open loop gain tends to be infinite, we get the same ratio for V_o/V_s as obtained earlier. In this case, the feedback that is present in the circuit is negative because the opamp has a negative gain. An opamp has a negative gain when v_i is measured as shown in Fig. 2.4.

2.2.3 Applications and Extensions to Inverting Amplifier

An inverter is a basic application using opamp. An opamp's output can be described by the equation

$$v_o = A_{cm} v_{cm} + A_d v_d,$$

where

$$v_{cm} = \frac{(v_+ + v_-)}{2}, v_d = (v_+ - v_-), A_d = A_o.$$

With $v_+ = 0$, and $v_- = 0$, $v_{cm} = 0$. Even though the common-mode gain A_{cm} of opamp may not be zero(common-mode gain is zero for ideal opamp), its contribution to output is almost zero in the case of an inverting amplifier, because $v_{cm} = 0$. The circuit of an inverting amplifier can be extended to more than one input. A circuit with two inputs is shown in Fig. 2.6. For the circuit in Fig. 2.6,

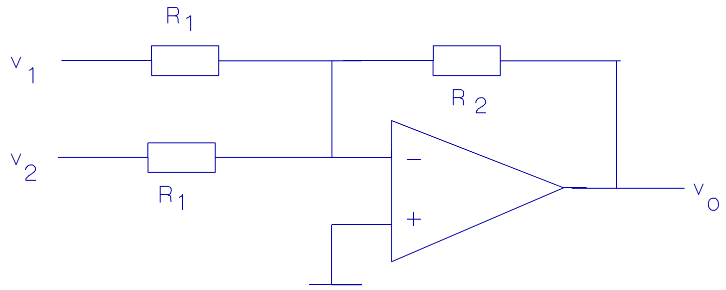


Fig. 2.6 : A Summer

$$v_o = -\frac{R_2}{R_1}(v_1 + v_2).$$

If v_1 and v_2 be of opposite signs, the circuit in Fig. 2.6 can be used as a proportional controller. Even though the inverting amplifier is a reliable and useful circuit,it is not suitable if the source v_s has a large source resistance. Normally the value of source resistance is not known precisely. With a source resistance R_s, the output of circuit in Fig. 2,4 is given by

$$v_o = -\frac{R_2}{R_s + R_1} * v_s.$$

It can be seen that the output may be unreliable in this case. For such applications, we need a circuit with a very high input resistance. A non-inverting amplifier built with an opamp is highly suitable for this purpose.

2.3 DIFFERENCE AMPLIFIER

The circuit of a difference amplifier is shown in Fig. 2.7. Here we find out its output assuming that the opamp is ideal. It is easy to get the output using the superposition theorem. When we apply this theorem, we consider one input at a time. With $v_1 = 0$, we can find output due to v_2.

Due to v_2 only:

Using the result obtained for the non-inverting amplifier, we get

$$v_o = v_+ (1 + \frac{R_2}{R_1}).$$

Given v_2,

$$v_+ = v_2 * \frac{R_4}{R_3 + R_4}.$$

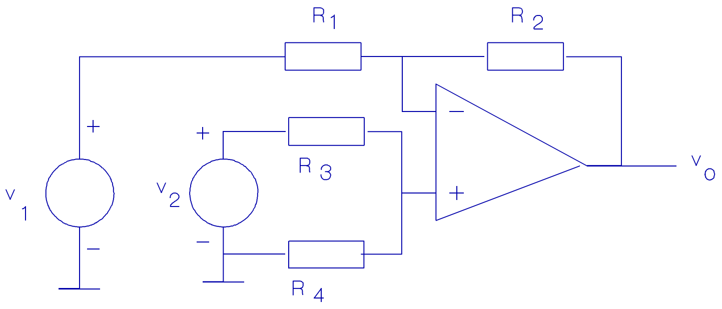


Fig. 2.7 : Difference Amplifier

Then,

$$v_o = v_2 * \frac{R_4}{R_3 + R_4} * \left(1 + \frac{R_2}{R_1}\right).$$

Due to v_1 only:

Here we get the output from the result obtained for the inverting amplifier.

$$v_o = -\frac{R_2}{R_1} v_1.$$

Due to both v_2 and v_1 :

$$v_o = \frac{R_4}{(R_3 + R_4)} * \left(1 + \frac{R_2}{R_1}\right) * v_2 - \frac{R_2}{R_1} * v_1.$$

If $R_1 = R_3$ and $R_2 = R_4$, then

$$v_o = \frac{R_2}{R_1} * (v_2 - v_1).$$

The above equation explains why this circuit is known as the difference amplifier. The difference amplifier circuit is used for measuring the difference between two sources. Such circuits, or improved difference amplifier circuits are used widely in instrumentation. Configuration using three opamps, with two opamps as buffer and the third as the difference amplifier, is used more often.

3 AN INTRODUCTION TO FEEDBACK IN AMPLIFIERS

3.0 OBJECTIVES

(i) To show how to identify negative and positive feedback in a circuit.
(ii) To identify applications suitable for positive or negative feedback.
(iii) To outline the effects of feedback.
(iv) To stress the need for negative feedback in amplifiers by outlining the advantages of using feedback.
(v) To show the block diagram of the four feedback topologies.

3.1 IDENTIFYING THE NATURE OF FEEDBACK IN A CIRCUIT

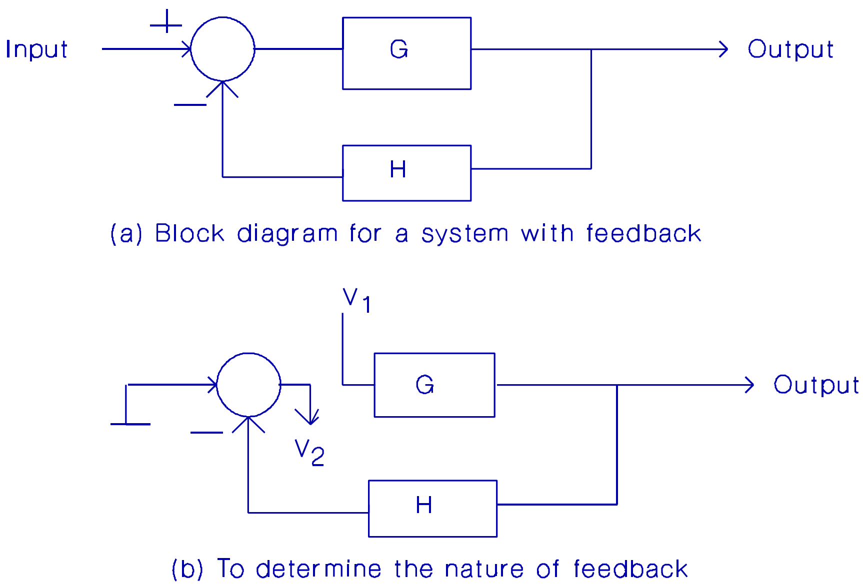


Fig. 3.1: A Block diagram for Illustrating Feedback in a circuit

A system with feedback is usually represented by a block diagram as shown in Fig. 3.1(a). To identify the nature of feedback,

(i) assume that the input is grounded,
(ii) break the loop as shown in Fig. 3.1b, and
(iii) find the ratio of V_2 / V_1 .

This ratio of V_2 / V_1 is known as the loop gain. If the loopgain is negative, then the system has negative feedback and the system has positive feedback if the loop gain is positive.

Quiz Problem 3.1

Identify the nature of feedback for the block diagrams shown in Fig. 3.2.

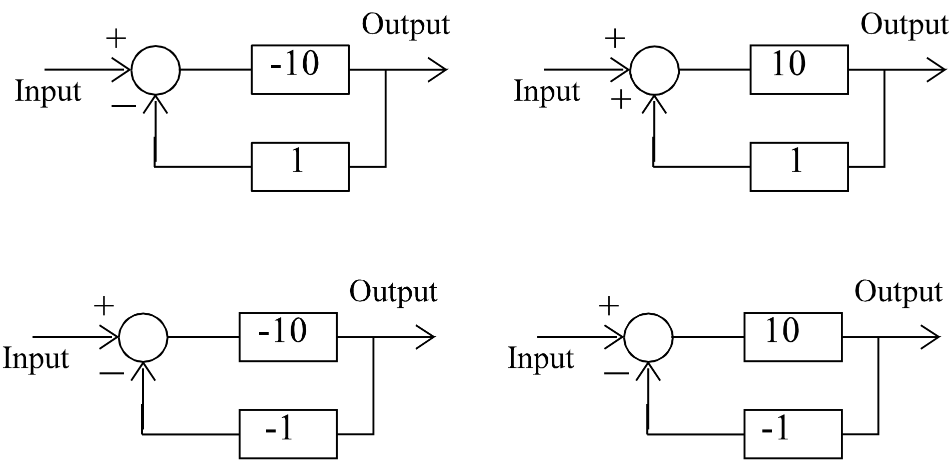


Fig. 3.2: For Quiz Problem 3.1

Answer:

(a) Positive , (b) positive, (c) negative, and (d) positive.

3.2 EFFECT OF FEEDBACK ON PERFORMANCE

Feedback in a system can be either positive or negative. Negative feedback improves the linearity of a system and is hence employed in a linear system such as an amplifier. On the other hand, positive feedback tends to produce a two-state output in a system and is used in circuits such as square-wave oscillators, comparators, Schmitt triggers etc. However, it may be noted that every circuit with positive feedback need not have a two-state output. For example, a sinewave oscillator circuit has positive feedback without having a two-state output. The Laplace transform of

$$v(t) = E \sin(wt).u(t)$$

is

$$V(s) = \frac{w}{s^2 + w^2} E.$$

It can be seen that V(s) has a conjugate pair of poles on the imaginary axis of s-plane. A physical system tends to have poles on the left-hand side of s-plane. Positive feedback tends to shift some of the poles of the system towards the right-side of s-plane. Due to positive feedback, a system can have conjugate poles on the imaginary axis and such a system oscillates. In sinewave oscillators, positive feedback must be closely controlled to maintain oscillations and waveform purity. On the other hand, in a square-wave oscillator or a comparator, the effect of positive feedback at cross-over points is to increase the speed of transition from one level to the other and prevent unwanted spikes at changeover points. In terms of control theory terminology applied to linear systems, an amplifier in general represents a stable system, whereas a sinewave oscillator is a marginally stable system and a square-wave or a Schmitt trigger circuit is an `unstable' system. It is necessary to know what stability means. Where `stability' is desired as in the case of an amplifier, negative feedback is used. Where two-state/digital output is desired, positive feedback is used.

It is worth remembering that the nature of feedback can change with frequency. Feedback may change from negative to positive as the frequency varies and the system may be unstable at high frequencies. Gain of the system also affects stability. Variation in component values and characteristics of devices due to operating conditions such as temperature, voltage, or current, can bring about instability despite negative feedback. Ageing leads to variation in component values and can hence affect stability.

Negative feedback is widely used in an amplifier design because it produces several benefits. These benefits are:

i. Negative feedback stabilizes the gain of an amplifier despite the parameter changes in the active devices due to supply voltage variation, temperature change, or ageing. Negative feedback permits a wider range for parameter variations than what would be feasible without feedback.

ii. Negative feedback allows the designer to modify the input and output impedances of a circuit in any desired fashion.

iii. It reduces distortion in the output of an amplifier. These distortions arise due to nonlinear gain characteristic of devices used. Negative feedback causes the gain of the amplifier to be determined by the feedback network and thus reduces distortion.

iv. Negative feedback can increase the bandwidth.

v. It can reduce the effect of noise.

These benefits are obtained by sacrificing the gain of a system. Another aspect to be borne in mind is that unless the feedback circuit is properly designed, the system tends to be unstable. Mathematical analysis to support the above statements can be found in section 3.4.

In a circuit with negative feedback, the gain of the circuit with feedback depends almost only on the feedback network used if the loop gain is sufficiently large. It is often possible to build the feedback circuit by using only passive components. Since the passive components tend to have a stable value, the circuit performance is then independent of the parameters of the active device used, and the circuit performance becomes more reliable.

The four basic feedback configurations are specified according to the nature of the input signal/input circuit and output signal/feedback arrangement. Since both the input and the output signals can be either voltage or current, there are four configurations as described in section 3.1.

3.3. FEEDBACK TOPOLOGIES

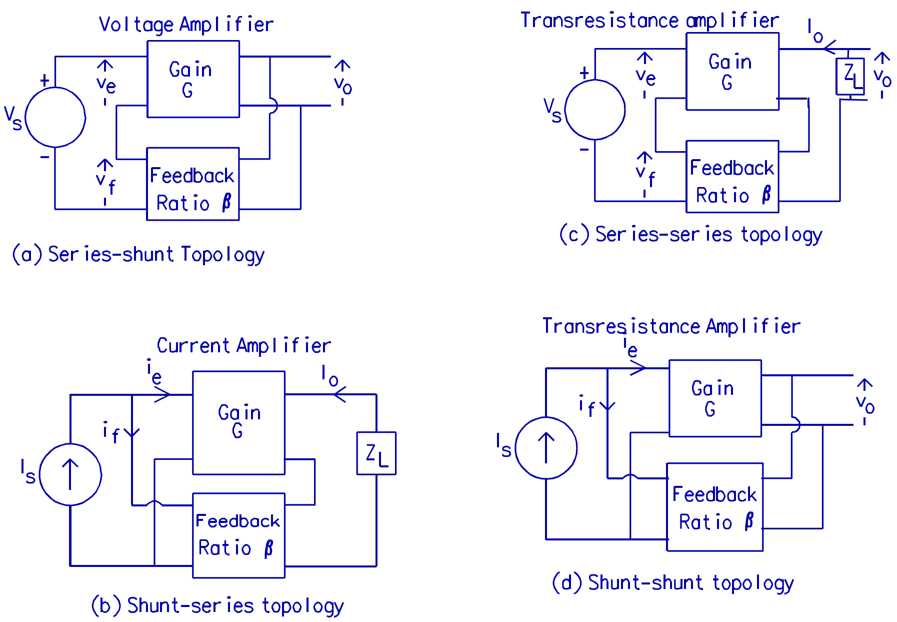


Fig.3.3 : Feedback Topologies

The four feedback topologies are:

(i) Series-shunt topology
(ii) Shunt-series topology,
(iii) Series-series topology and
(iv) Shunt-shunt topology.

The block diagrams for these topologies are shown in Fig. 3.3. It is seen that each topological configuration is described by two terms. The first term refers to the input stage and the second to the output. If the two signals are connected in series, the term 'series' is used. It is logical to connect two voltage signals in series. For series-shunt topology, the source signal v_s and the feedback signal v_f are connected in series at the input and the difference v_e is applied to the voltage amplifier.

On the other hand if two signals are connected in parallel, the term 'shunt' is used. It is logical to connect two current sources in parallel. If the amplifier in the forward path is either a current or a transresistance amplifier, it needs a current signal at its input, which in turn is obtained as the difference of the source signal I_s and the feedback current I_f. In either of these cases, the first term to be used is 'shunt'.

The second term that describes a feedback topology is related to its output. If the output signal is a voltage and a feedback signal is to be derived from it, then the feedback network must be connected across the output terminals. It is appropriate to use the term 'shunt' here. On the other hand, if a feedback signal is to be obtained from the output current, the feedback network must be connected such that the output current flows through it to produce a feedback signal and the feedback network is then connected in 'series'.

At this stage, identifying the feedback topology of a circuit may appear to be simple. Given a circuit with feedback, it turns out that it is not so straightforward to identify its topology. This aspect will be described in greater length after the study of the four topological configurations.

3.4 ANALYSIS

3.4.1 Gain Sensitivity

The block diagram of a system with feedback is shown in Fig. 3.1, where G is the gain of the amplifier in the forward path. In most practical situations, gain G of the amplifier in the forward path is not well defined. For example, if we consider BJT devices with the same type number, the current gain can vary from one device to another, by as much as 50%. In addition, the gain of the forward amplifier is dependent on temperature, and other operating conditions. If an amplifier is used without feedback, the output is more sensitive to the changes in the gain of the amplifier.

For example, let V_o be the output voltage and V_s be the input voltage of an amplifier with gain G. Then

$$\frac{V_o}{V_s} = G \,. \quad (3.1)$$

If the gain of the amplifier changes by ΔG, the output changes by ($\Delta G * V_s$).

The fractional change in output is then (ΔG) / G . Thus for an open-loop amplifier,

$$Fractional\ change\ in\ output = \frac{\Delta G}{G}. \quad (3.2)$$

With a closed-loop system, the output is not as sensitive to changes in gain G. Ideally, variations in G should not affect output at all. From Fig. 2.1, the gain T of the closed-loop system is:

$$T = \frac{G}{1 + GH}. \quad (3.3)$$

Then

$$\frac{\delta T}{\delta G} = \frac{1}{(1 + GH)^2}. \quad (3.4)$$

From equations (3.3) and (3.4), we get that

$$\frac{\delta T}{T} = \frac{\frac{\delta G}{G}}{1 + GH}. \quad (3.5)$$

For example, let G = 100, δG = 5 and H = 0.1. From equation (3.2), the fractional change in the output of the amplifier operating in open-loop is 0.05. On the other hand, the fractional change in the output of the circuit operating in closed-loop is only (0.05/11). It is seen that the output is less sensitive to changes in the gain of the amplifier in the forward path. We define the sensitivity function as follows:

$$Sensitivity\ function, S = \frac{\frac{\delta T}{T}}{\frac{\delta G}{G}} = \frac{1}{1 + GH}. \quad (3.6)$$

3.4.2 The Effect of Feedback on Nonlinear Distortion

Feedback reduces the nonlinear distortion in output. Let an amplifier be operating in open loop, as shown in Fig. 3.4a. Here it is assumed that essentially all the nonlinear distortion is produced by the output stage of the amplifier, which is typically the case. From Fig. 3.4a, we can state that

$$V_o = G * V_s + V_d, \quad (3.7)$$

where V_d represents the nonlinear content in output. As illustrated in Fig. 3.4b, if the same amount of nonlinear distortion is produced at the output stage in an amplifier with feedback, the resultant output is obtained to be

$$V_o = \frac{G}{1 + GH} * V_s + \frac{1}{1 + GH} * V_d . \quad (3.8)$$

For the same amount of distortion V_d introduced, the circuit with feedback has less distortion in its output. From equations (3.7) and (3.8),

$$\frac{\text{Distortion in the circuit with feedback}}{\text{Distortion in the circuit with no feedback}} = \frac{1}{1 + GH} . \quad (3.9)$$

3.4.3 The Effect of feedback on Frequency Response

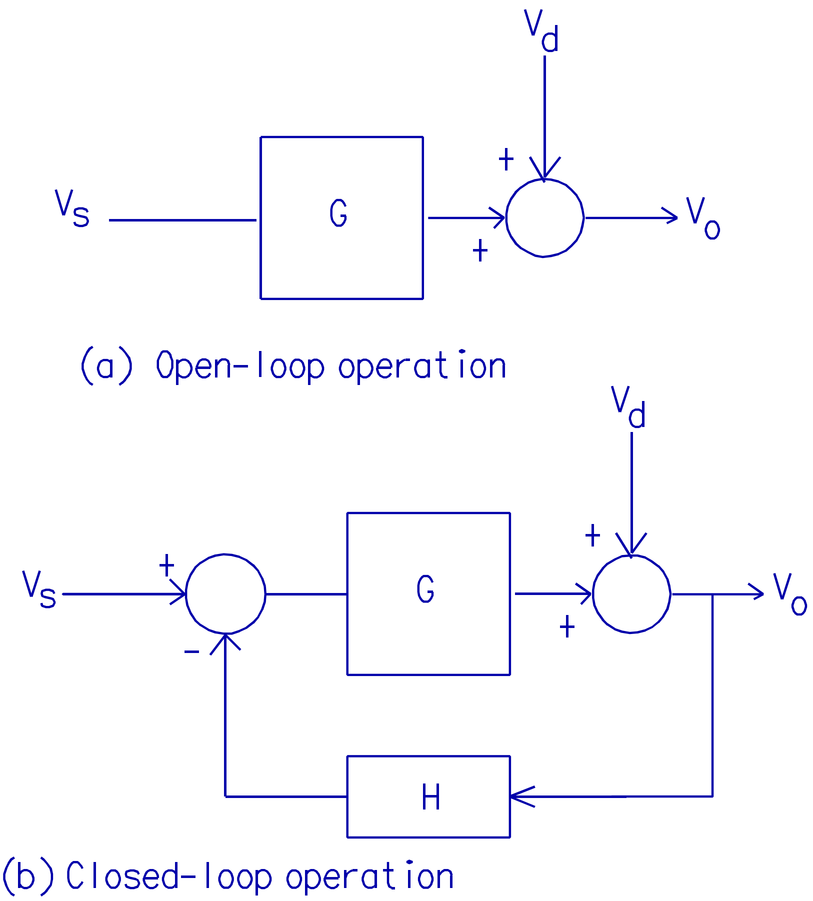


Fig.3.4 : Effect of feedback on distortion

If the gain of the amplifier in the forward path be A and the feedback factor be β, the closed-loop gain T is then A/(1 + Aβ). Gain A, instead of being be a constant. can be a function of frequency, with poles and zeros. Let A be defined as follows.

$$A(s) = \frac{A_o * s}{(s + w_L)\left(1 + \frac{s}{w_H}\right)}, \quad (3.10)$$

where A_o is the mid-frequency gain, w_L is the low-frequency pole and w_H is the high-frequency pole. Then the frequency response is as shown in Fig. 3.5. Since the gain is a function of frequency, the closed-loop gain T also becomes a function of frequency.

At low frequencies, where w << w_H, we can approximate A(s) as

$$A(s) = \frac{A_0 s}{(s + w_L)} \quad (3.11)$$

Since

$$T(s) = \frac{A(s)}{1 + \beta A(s)}, \quad (3.12)$$

we get the closed-loop gain at low frequencies by substituting for A(s) from equation (3.11) .

$$T(s) = \frac{A_o}{1 + A_o\beta} * \frac{s}{s + \frac{w_L}{(1 + A_o\beta)}}. \quad (3.13)$$

We get the expression shown above by assuming that the high frequency pole would not the affect the performance at low frequencies and hence it can be neglected. At high frequencies, we neglect both the zero at origin and the low frequency pole. Then at high frequencies, the closed-loop gain can be stated to be

$$T = \frac{A_o}{1 + A_o\beta} * \frac{1}{1 + \frac{s/w_H}{1 + A_o\beta}}. \quad (3.14)$$

Combining the two equations (3.13) and (3.14) , we get that

$$T(s) = \frac{A_o}{1 + A_o\beta} * \frac{s}{s + \frac{w_L}{(1 + A_o\beta)}} * \frac{1}{1 + \frac{s}{w_H(1 + A_o\beta)}}. \quad (3.15)$$

The frequency response of closed-loop system is also shown in Fig. 3.5. It is seen that the range of frequency response has increased significantly. In Fig. 3.5,

$$w_L = \frac{1}{T_1}, and\ w_H = \frac{1}{T_2}. \quad (3.16)$$

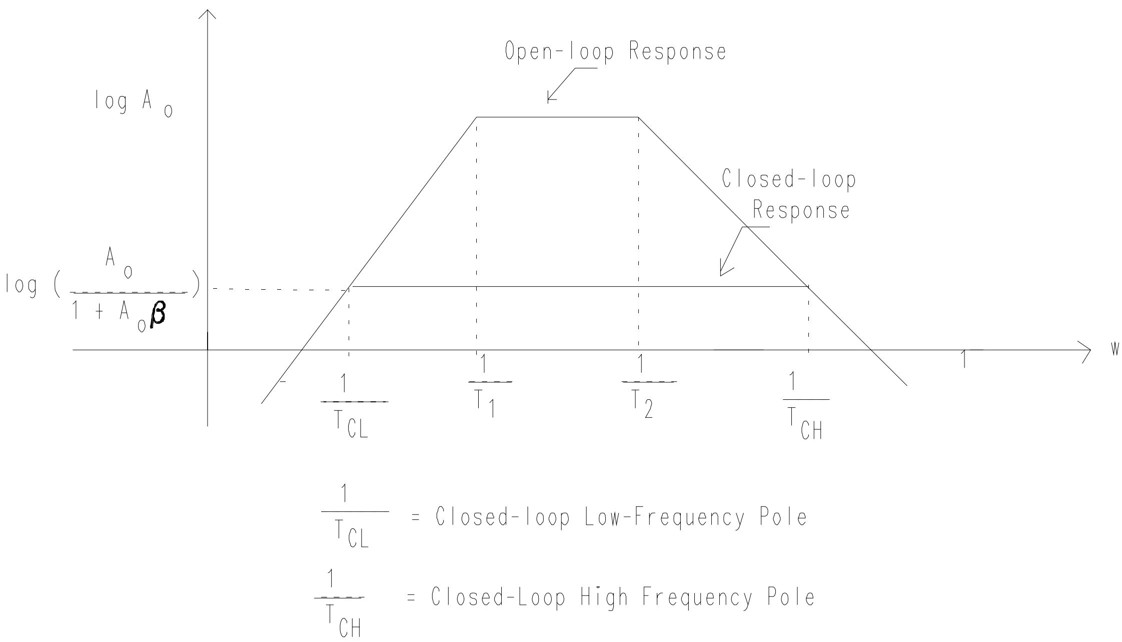


Fig. 3.5 : Frequency Response

3.5 SUMMARY

This chapter has described briefly the four types of feedback topologies and has highlighted the effect of feedback

i. on the sensitivity of amplifier to changes in the gain of the amplifier in the forward path,

ii. on the nonlinear distortion and

iii. on the frequency response.

4 SERIES-SHUNT TOPOLOGY

4.1 IDEAL SERIES-SHUNT NETWORK

The block diagram of a circuit with series-shunt feedback configuration is shown in Fig. 4.1. It is seen that the amplifier in the forward path is an ideal voltage amplifier. An ideal voltage amplifier has an infinite input resistance and a zero output resistance. It can be seen that the feedback network senses the output voltage v_o and produces a feedback voltage signal, v_f. For this feedback network, v_o is the input voltage signal and v_f is its output signal and hence the feedback network is functionally only a voltage amplifier with a low gain, called here as the feedback ratio β. The block representation is thus drawn based on the assumption that the voltage amplifier and the feedback network are ideal. This implies that:

i. the input resistance of voltage amplifier is infinity,
ii. the output resistance of voltage amplifier is zero,
iii. the input resistance of feedback network is infinity and
iv. the output resistance of feedback network is zero.

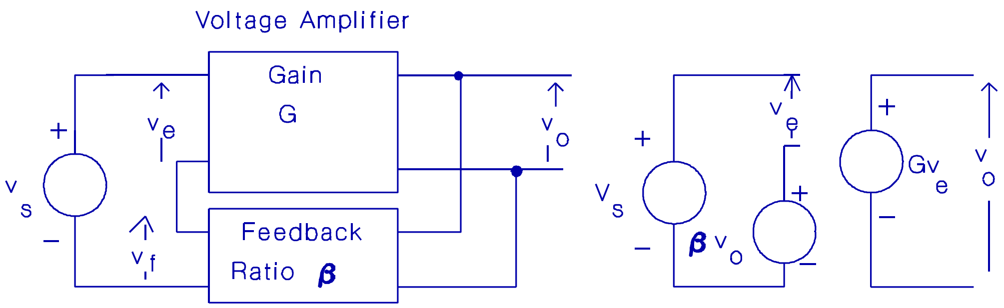


Fig.4 .1 -Series-Shunt Topology
Ideal Network

4.2 NONIDEAL NETWORK

When a voltage amplifier is nonideal, it has a large input resistance and a low output resistance. Since the feedback network functions as a voltage amplifier, its equivalent circuit or model is the same as that of a voltage amplifier and in addition, it should have a large input resistance and a low output resistance, like a voltage amplifier. That the feedback network has a large input resistance means that it draws negligible current from the output port of the voltage amplifier in the forward path and that it does not load the voltage amplifier. That the feedback network has a low output resistance means that the error that can result from its non-zero output resistance is less and often negligible.

Even though the feedback network functions as a voltage amplifier, it must be borne in mind that it is usually a passive network and that we usually associate the word 'amplifier' with a circuit that contains an active device.

The equivalent circuit of the series-shunt topology with these nonideal parameters is shown in Fig. 4.2. It is worth remembering that a practical circuit will have other limitations such as limited frequency response, drift in parameters due to ageing, ambient temperature , but these aspects have been ignored here.

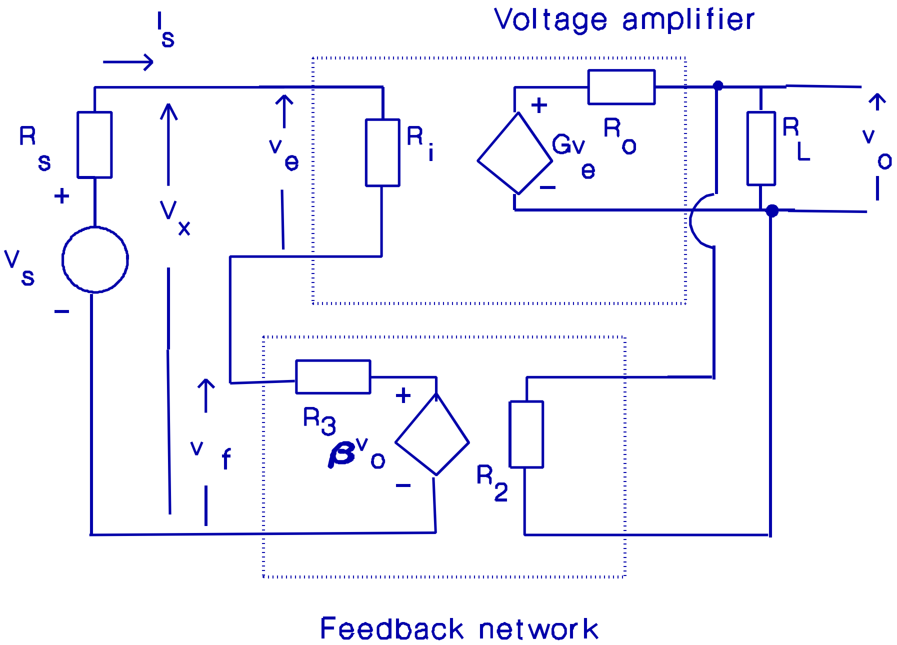


Fig. 4 .2: Series-shunt topology

Representation with nonidealities

4.3 EFFECTS OF FEEDBACK

The principal aim of using feedback is to make the gain of the closed-loop system to be dependent on the feedback ratio β and to free it from its dependence on the gain of the amplifier in the forward path. As outlined earlier, the closed-loop system has larger bandwidth. In addition, feedback has the effects stated below for a series-shunt topology.

(i) The input resistance of the closed-loop system is far greater than the input resistance R_i of the voltage amplifier.

(ii) The output resistance of the closed-loop system is far lower than the output resistance R_o of the voltage amplifier.

4.4 RESULTS OF ANALYSIS OF THE CIRCUIT WITH FEEDBACK

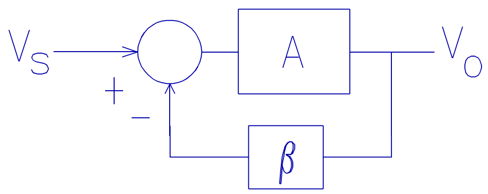


Fig. 4.3: Series-shunt Topology Represented in the Conventional form

The circuit in Fig. 4.2 can be reduced to the block diagram shown in Fig. 4.3. From Fig.4.3, the closed-loop gain T is expressed to be:

$$T = \frac{V_o}{V_s} = \frac{A}{1 + A\beta}, \quad (4.1)$$

where A is the gain of the forward path. It is shown in section 4.5.1 that the gain A of the forward path is:

$$A = \frac{GR_p}{R_o + R_p} * \frac{R_i}{R_i + R_3 + R_s}, \text{ where } R_p = (R_2 || R_L). \quad (4.2)$$

The value of A is not normally much less than G, the gain of the voltage amplifier.

The input resistance R_x of the closed-loop network is obtained to be:

$$R_x = (R_i + R_3 + R_s) * (1 + A\beta) - R_s. \quad (4.3)$$

It is seen that R_x is much larger than R_i, the input resistance of the voltage amplifier since the magnitude of loop gain is much greater than unity. The derivation of expression (4.3) for R_x is shown in section 4.5.2.

The output resistance R_y of the closed-loop network is obtained to be:

$$R_y = \frac{1}{\frac{1}{R_{of}} * (1 + A\beta) - \frac{1}{R_L}}, \text{ where } \frac{1}{R_{of}} = \frac{1}{R_o} + \frac{1}{R_L} + \frac{1}{R_2}. \quad (4.4)$$

It is seen that R_y is much smaller than R_o. The derivation of expression (3.4) for R_y is shown in section 4.5.3.

4.5 DERIVATIONS

4.5.1 Obtaining an expression for Gain of the forward path

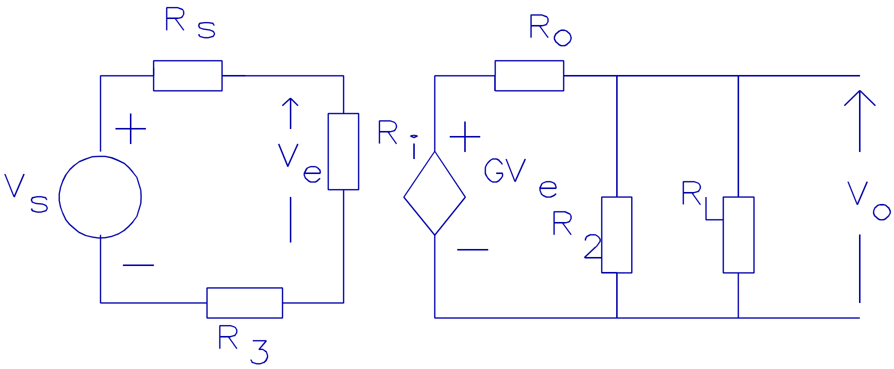


Fig.4.4: For finding forward gain A of series-shunt topology

The circuit for obtaining the gain A of the forward path is shown in Fig. 4.4. This gain is the gain that would exist were there no feedback. This means that the dependent source representing the feedback ratio in Fig. 4.2 is to be bypassed. The bypass arrangement should ensure that there is a path for current I_s shown in Fig. 4.2 , and that the dependent source βV_o does not affect I_s. This condition is met if the dependent source βV_o is replaced by a short-circuit. The circuit that results by replacing βV_o by a short-circuit is shown in Fig. 4.4. We define A as

$$A = \left(\frac{V_o}{V_s}\right)\Bigg|_{(\beta V_o = 0)}$$

From Fig. 4.4, we get that

$$\frac{R_i}{R_i + R_3 + R_s} = \frac{v_e}{v_s}, \frac{GR_p}{R_o + R_p} = \frac{v_o}{v_e}, \frac{GR_p}{R_o + R_p} * \frac{R_i}{R_i + R_3 + R_s} = \frac{V_o}{V_e} * \frac{V_e}{V_s} = \frac{V_o}{V_s} = A. \quad (4.5)$$

While the dependent source βV_o is replaced by a short-circuit, its output resistance R_3 is left in the circuit. (Why ?)

4.5.2 Input Resistance with Feedback

The input resistance is the resistance seen by the source at its terminals. If the source, as shown in Fig. 4.2, has an internal resistance R_s which is normally referred to as the source resistance, then the input resistance R_x is defined to be

$$R_x = \frac{V_x}{I_s}, \quad (4.6)$$

where V_x is the voltage at the terminals of the source and I_s is the current supplied by the source. Please note that the source voltage V_s and the voltage V_x at its terminals are not one and the same. Normally V_x would be slightly less than V_s in magnitude.

The input resistance R_x is obtained to be:

$$R_x = (R_i + R_3 + R_s) * (1 + A\beta) - R_s = R_{if} * (1 + A\beta) - R_s, \quad (4.7)$$

where R_{if} is the input resistance of the circuit used for obtaining the forward gain A. It is fairly easy to remember the above expression. Draw the circuit for obtaining the forward gain, as shown in Fig. 4.4. Then R_{if} is the input resistance of that circuit which includes R_s too. This resistance gets magnified $(1+A\beta)$ times and the value of the source resistance R_s is subtracted from the product $R_{if}(1+A\beta)$ to obtain R_x, the input resistance with feedback. For a circuit with negative feedback, the real part of loop gain $(-A\beta)$ is negative, which means that the product $(A\beta)$ has a positive real part. It means that if A has a positive real part, then β also has a positive real part and that if A has a negative real part, β also has a negative real part.

The derivation of an expression for R_x is as follows. From Fig. 4.2, we can state that

$$V_x = V_e + I_s R_3 + \beta V_o . \quad (4.8)$$

Since R_x is just the ratio (V_x/I_s), we can get an expression for R_x if we can express V_e and V_o in terms of I_s. From Fig. 4.2, we find that

$$V_e = R_i I_s . \quad (4.9)$$

In addition, we have that

$$V_o = \frac{G * R_p}{R_o + R_p} . V_e = \frac{G * R_p}{R_o + R_p} R_i I_s, \text{ where } R_p = \frac{R_L * R_2}{R_2 + R_L} . \quad (4.10)$$

Here the load on the output port is R_p, the parallel value of load resistance

R_L,and the input resistance R_2 of the feedback network.

Substitute for V_e and V_o in equation (4.8) from equations (4.9) and (4.10) and then obtain the ratio of V_x/I_s.

$$R_x = \frac{V_x}{I_s} = R_i\left(1 + G\frac{R_p}{R_o + R_p}\beta\right) + R_3. \quad (4.11).$$

Usually R_p is far greater than R_o. Then

$$\frac{R_p}{R_p + R_o} \simeq 1. \quad (4.12)$$

In addition, the approximation that $R_x \approx R_i(1 + G\beta)$ is usually valid. It may be seen from Fig. 4.2 that R_3 is the output resistance of the feedback network. The feedback network functions as a voltage amplifier, but its output resistance need not be negligible since a passive network is used in the feedback path. On the other hand, the input resistance R_i of the voltage amplifier is large and in reality, it by itself should be much greater than R_3. In addition, the term $(1 + G\beta)$ is also much greater than unity. Hence

$$R_i(1 + G\beta) \gg R_3. \quad (4.13)$$

It is seen that negative feedback has increased the input resistance seen by the source.

Equation (4.11) expresses the input resistance in terms of the voltage gain G of the voltage amplifier. An alternative expression for R_x can be obtained in terms of the forward gain A, the feedback factor β and the other resistors. From equation (4.5), we get that

$$\frac{GR_p}{R_o + R_p} = \frac{R_i + R_3 + R_s}{R_i} * A. \quad (4.14)$$

Therefore, based on equation (4.14), equation (4.11) becomes

$$R_x = R_i\left[1 + A\beta\left(\frac{(R_i + R_s + R_3)}{R_i}\right)\right] + R_3 = (R_i + R_3 + R_s) * (1 + A\beta) - R_s. \quad (4.15)$$

A more compact expression can be obtained as follows.

$$R_x = R_{if} * (1 + A\beta) - R_s, \text{ where } R_{if} = R_s + R_i + R_3. \quad (4.16)$$

It can be seen that R_{if} is the input resistance of the circuit in Fig. 4.4 which has been used for obtaining the gain of the forward path.

4.5.3 Output Resistance with feedback

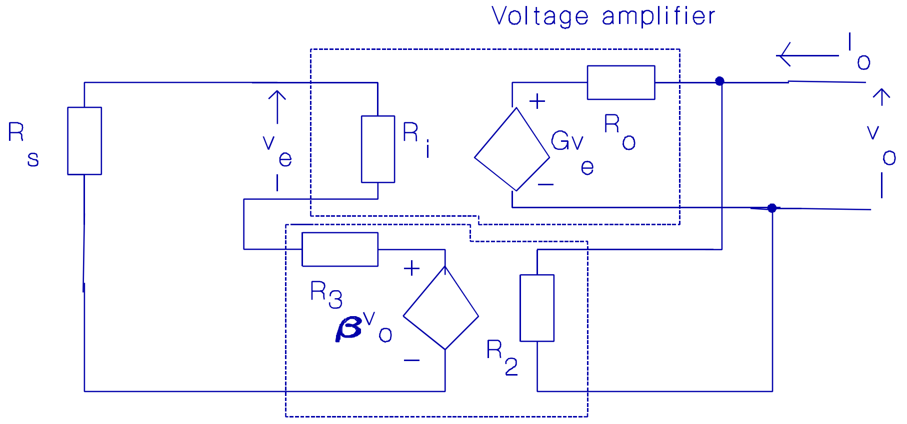


Fig.4 .5 - Finding Output Resistance

An ideal voltage amplifier has zero output resistance. In practice, it has a small and finite output resistance, which is denoted as R_o in Fig. 4.2. The output resistance is the resistance of the circuit seen by the load. That is, we can replace the load resistance by a voltage source. The resistance as seen by this source with the input source V_s removed is usually defined as the output resistance. By definition, output resistance R_y is:

$$\text{Output resistance}, R_y = \left| \frac{\Delta V_o}{\Delta I_o} \right|_{(\Delta V_s = 0)} \quad (4.17).$$

For the sake of convenience, the output resistance is defined here to be (V_o/I_o) with $V_s = 0$, where the load resistor is replaced by a source,say V_o. The circuit that results is shown in Fig. 3.5 and the expression obtained for R_y is:

$$R_y = \frac{1}{\left(\frac{1}{R_o} + \frac{1}{R_L} + \frac{1}{R_2}\right) * (1 + A\beta) - \frac{1}{R_L}}. \quad (4.18)$$

It is easy to remember the above equation. It is known that the output resistance gets reduced to feedback. It means that the output conductance gets magnified due to feedback. Obtain the output conductance of the circuit used for getting the forward gain. With R_L in circuit, the output conductance ($1/R_{of}$) of circuit in Fig. 4.4 is

$$\frac{1}{R_{of}} = \frac{1}{R_o} + \frac{1}{R_L} + \frac{1}{R_2}. \quad (4.19)$$

The above expression is obtained with the input source V_s replaced by a short-circuit. When V_s has zero value, the dependent source GV_e also has zero value and can hence be replaced by a short-circuit. To obtain the output conductance of the circuit with feedback, multiply ($1/R_{of}$) by ($1 + A\beta$) and remove from the product the conductance $1/R_L$ due to load resistor.

The expression for R_o can be derived as follows. From Fig. 4.5,

$$I_o = \frac{V_o - GV_e}{R_o} + \frac{V_o}{R_2}. \quad (4.20)$$

The ratio of V_o/ I_o gives R_o. Then if can express V_e in terms of V_o, an expression for R_o can be obtained. From Fig. (3.5), we get that

$$V_e = -\frac{\beta V_o * R_i}{R_i + R_3 + R_s}. \quad (4.21)$$

Replace V_e in equation (4.20) by its expression in equation (4.21). On re-arranging, we get that

$$R_y = \frac{1}{\frac{1}{R_o}\left[1 + \frac{G\beta * R_i}{R_i + R_s + R_3}\right] + \frac{1}{R_2}}. \quad (4.22)$$

Usually $R_i >> (R_s + R_3)$. Hence ($R_i + R_s + R_3$) is nearly equal to R_i. Also R_2, the input resistance of the feedback network tends to be far greater than $R_o/(1 + G\beta)$. Thus

$$R_y \simeq \frac{R_o}{1 + G\beta} \quad (4.23)$$

It is seen that the output resistance with feedback is much smaller than the output resistance of the voltage amplifier, since Gβ » 1.

It is preferable to find an alternative expression for output resistance R_y in terms of the forward path gain A. From equation (4.5), we get that

$$\frac{G * R_i}{R_i + R_s + R_3} = A * \frac{(R_o + R_p)}{R_p} \quad . \quad (4.24)$$

From equations (4.22) and (4.24), we get that

$$R_y = \frac{1}{\frac{1}{R_o}\left[1 + A\beta * \frac{(R_o + R_p)}{R_p}\right] + \frac{1}{R_2}} \quad . \quad (4.25)$$

Since $R_p = (R_2 \parallel R_L)$, the above equation can be stated as:

$$R_y = \frac{1}{\frac{1}{R_o} + \frac{A\beta}{R_p} + \frac{A\beta}{R_o} + \frac{1}{R_2}} = \frac{1}{\left(\frac{1}{R_o} + \frac{1}{R_2} + \frac{1}{R_L}\right)(1 + A\beta) - \frac{1}{R_L}} \quad . \quad (4.26)$$

We can get a more compact expression by using equation (4.19). Then

$$R_y = \frac{1}{\frac{1}{R_{of}} * (1 + A\beta) - \frac{1}{R_L}} \quad . \quad (4.27)$$

4.5.4 Closed-Loop Gain

Next an expression for the closed-loop gain is obtained. Let the closed-loop gain be T. The expression for T shown in equation (3.1) is obtained as follows. From Fig. 3.2,

$$V_o = \frac{G * R_p}{R_o + R_p} V_e, \; V_e = V_s - I_s(R_s + R_3) - \beta V_o, \; I_s = \frac{V_e}{R_i} \quad . \quad (4.28)$$

Substitute for I_s in the expression for V_e. Then

$$V_e = \frac{V_s - \beta V_o}{1 + \frac{R_3 + R_s}{R_i}}, V_o = \frac{G * R_p * R_i}{(R_p + R_o)} * \frac{V_s - \beta V_o}{(R_i + R_3 + R_s)}. \quad (4.29)$$

On using equation (3.2) and re-arranging, we get the expression (4.1) for closed-loop gain.

4.6 SUMMARY

Given a circuit which has the series-shunt topology, first obtain its forward gain A, and then obtain the input resistance R_{if} and the output resistance R_{of} the circuit drawn for obtaining the gain A. From the values of A and β, we can get the closed-loop gain T.

With a practical circuit, the analysis is slightly more complicated. The given circuit has to be reduced to the form in Fig. 4.2 before we can apply the results derived in this lesson.

5 SHUNT-SERIES TOPOLOGY

5.1 IDEAL NETWORK

The block diagram of a circuit with shunt-series feedback topology is shown in Fig. 5.1. It can be seen that the error current I_e , obtained as the difference between I_s and I_f, is amplified by the current amplifier. The feedback signal I_f is obtained from the load current I_o.

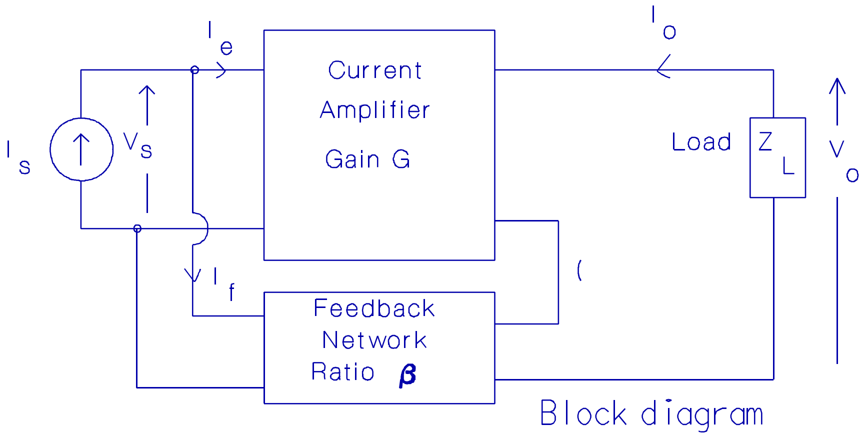


Fig. 5.1: Shunt-series Topology

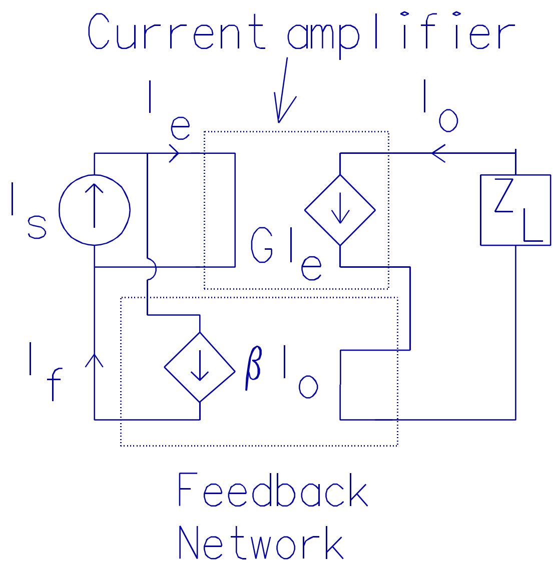


Fig. 5.2: Ideal Network

The ideal network with this topology can be represented by an equivalent circuit as shown in Fig. 5.2. This topology is the dual of series-shunt topology considered in the previous section.This topology uses a current amplifier in the forward path.

The feedback network also has a current signal as its input and output signal. Ideally,

i. the input resistance of a current amplifier is zero,
ii. the output resistance of current amplifier is infinity,
iii. the input resistance of feedback network is zero and
iv. the output resistance of feedback network is infinity.

Since the feedback network provides a current signal derived from another current signal, it behaves essentially as a current amplifier and it also has the same ideal parameters. Again it is worth reiterating that the feedback may have only passive components and no active device. Hence its gain, shown as the feedback ratio β, is less than unity. In practice, the source, the current amplifier and the feedback network are not ideal and the equivalent circuit representing the nonideal shunt-series topology is shown in Fig. 5.3.

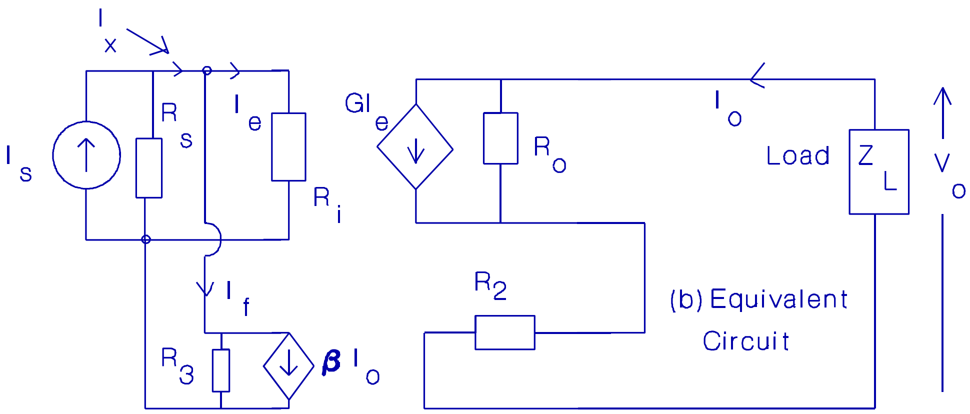


Fig. 5.3: Shunt-series Topology with nonideal parameters

5.3 EFFECTS OF FEEDBACK

Due to feedback, the closed-loop circuit has reduced sensitivity to changes in gain of the current amplifier, increased bandwidth and less distortion. In addition, the feedback leads to:

i. a large reduction in input impedance and
ii. a large increase in output impedance.

5.4 RESULTS OF ANALYSIS OF THE CIRCUIT WITH FEEDBACK

The closed-loop gain of the circuit in Fig. 5.3 is obtained to be :

$$T = \frac{I_o}{I_s} = \frac{A}{1 + A\beta}, \quad (5.1)$$

where A is the gain of the forward path. The value of A is obtained to be:

$$A = \left(\frac{I_o}{I_s} \right)_{(I_f = 0)} = \frac{GR_o}{R_o + R_L + R_2} * \frac{\frac{1}{R_i}}{\frac{1}{R_i} + \frac{1}{R_s} + \frac{1}{R_3}}, \quad (5.2)$$

The value of A is not usually very much less than G, the gain of the current amplifier.

The input resistance of the circuit with feedback is obtained to be:

$$R_x = \frac{1}{\left(\frac{1}{R_i} + \frac{1}{R_3} + \frac{1}{R_s} \right) * (1 + A\beta) - \frac{1}{R_s}}. \quad (5.3)$$

From equation (4.3), it can be seen that R_x is much smaller than R_i.
The output resistance of the circuit with feedback is found to be equal to:

$$R_y = (R_o + R_L + R_2) * (1 + A\beta) - R_L. \quad (5.4)$$

It can be seen from equation (4.4) that $R_y \gg R_o$.

5.5 DERIVATIONS

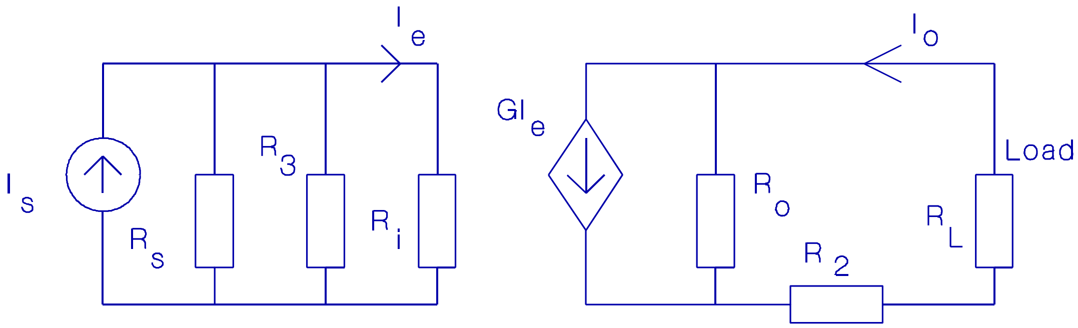


Fig.5 .4: - Finding the forward current gain A for Shunt-series Topology

5.5.1 Obtaining an expression for the gain of the forward path

As shown in equation (4.2), the gain A of the forward path is obtained subject to the condition that $I_f = 0$. It means that the dependent current source βI_o is replaced by an open-circuit. The circuit that results is shown in Fig. 4.4. From Fig. 4.4, we get that

$$A = \left(\frac{I_o}{I_s}\right)_{(I_f=0)} = \left(\frac{I_o}{I_e}\right) * \left(\frac{I_e}{I_s}\right) = \frac{GR_o}{R_o + R_L + R_2} * \frac{\frac{1}{R_i}}{\frac{1}{R_i} + \frac{1}{R_s} + \frac{1}{R_3}}.$$

5.5.2 Input Resistance with Feedback

An expression for input resistance R_x with feedback is shown in equation (5.2). That the input resistance R_x gets reduced means that the input conductance increases. This input conductance, $1/R_x$ can be expressed as follows. Let $1/R_{if}$ be the input conductance of the circuit used for obtaining A. From Fig. 5.4,

$$\frac{1}{R_{if}} = \left(\frac{1}{R_i} + \frac{1}{R_3} + \frac{1}{R_s}\right). \quad (5.5)$$

Then

$$\frac{1}{R_x} = \frac{1}{R_{if}} * (1 + A\beta) - \frac{1}{R_s}. \quad (5.6)$$

Now an expression for the input resistance of the circuit in Fig. 5.3 is shown below. By definition,

$$\text{Input resistance}, R_x = \frac{V_s}{I_x}, \quad (5.7)$$

where V_s is the voltage across the current source I_s. Note that I_x is not the same as I_s. From Fig. 4.3,

$$I_x = \frac{V_s}{R_i} + \frac{V_s}{R_3} + \beta I_o = \frac{V_s}{R_i} + \frac{V_s}{R_3} + G\beta I_e * \frac{R_o}{R_o + R_2 + R_L} = \frac{V_s}{R_i} + \frac{V_s}{R_3} + G\beta\frac{V_s}{R_i} * \frac{R_o}{R_o + R_2 + R_L}. \quad (5.8)$$

Therefore,

$$R_x = \frac{1}{\frac{1}{R_i} * \left(1 + G\beta \frac{R_o}{R_o + R_2 + R_L}\right) + \frac{1}{R_3}} . (5.9)$$

Usually, the output resistance of a dependent current source, R_o here, is large. We can state that $R_o >> R_2$ and that $R_o >> R_L$. In addition, $R_3 >> R_i/(1+G\beta)$. Hence

$$R_x \simeq \frac{R_i}{1 + G\beta} . \quad (5.10)$$

Equation (5.9) can be expressed using A instead of G, where A is the forward current gain of the network ignoring the effects of feedback. From equation (5.2), we get that

$$\frac{GR_o}{R_o + R_L + R_2} = A * \frac{\left(\frac{1}{R_i} + \frac{1}{R_s} + \frac{1}{R_3}\right)}{\frac{1}{R_i}} , (5.11)$$

From equations (5.9) and (5.11), we get that

$$R_x = \frac{1}{\left(\frac{1}{R_i} + \frac{1}{R_3} + \frac{1}{R_s}\right) * (1 + A\beta) - \frac{1}{R_s}} .$$

5.5.3 Output Resistance with Feedback

A current amplifier has a large output resistance and due to feedback, the closed-loop circuit will have a much larger output resistance. The resistance of the circuit as seen from the load terminals is the output resistance. The expression for the output resistance in equation (5.4) can easily be remembered this way. Let R_{of} be the resistance of the path for flow of current I_o in the circuit drawn for obtaining current gain of the forward path. In Fig. 4.3, if I_s be a steady value, GI_e is a dependent current source and to circuit external to it, it appears to have a resistance of R_o, its output resistance. If R_{of} be the resistance of the path for I_o, then from Fig. 5.3,

$$R_y = R_{of} * (1 + A\beta) - R_L, \ where \ R_{of} = (R_o + R_L + R_2) \ (5.12)$$

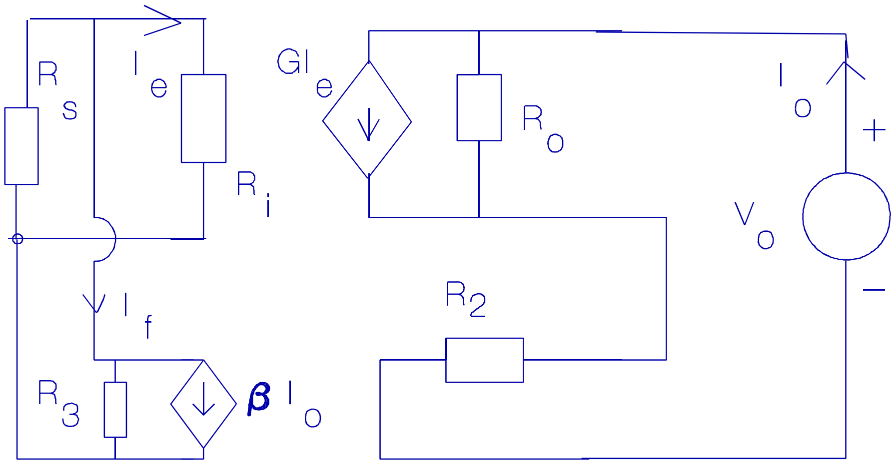


Fig.5 .5 :- Finding Output Resistance

Usually the output resistance is defined with the input source removed. To get a circuit that can be used for evaluating the output resistance, replace the load resistor R_L by a voltage source and the input current source is replaced by an open circuit. That is, the output impedance is evaluated with $I_s = 0$. From Fig. 5.5,

$$I_o = G * I_e + \frac{V_o - I_o R_2}{R_o} . \quad (5.13)$$

$$I_e = -\frac{\frac{1}{R_i}}{\frac{1}{R_i} + \frac{1}{R_s} + \frac{1}{R_3}} * \beta I_o . \quad (5.14)$$

$$\therefore Output\ \ Resistance, R_y = \left(\frac{V_o}{I_o}\right)_{(I_s = 0)} = \left[(R_o + R_2) + \frac{G\beta R_o * \frac{1}{R_i}}{\frac{1}{R_i} + \frac{1}{R_3} + \frac{1}{R_s}}\right] \quad (5.15)$$

Since $R_o \gg R_2$ and $(1/R_i) \gg (1/R_3 + 1/R_s)$, we can state that

$$R_y \simeq R_o * (1 + G\beta) . \quad (5.16)$$

Equation (4.15) can be expressed in terms of A instead of G. From equation (4.2), we get that

$$A*(R_o+R_L+R_2)=GR_o*\frac{\frac{1}{R_i}}{\frac{1}{R_i}+\frac{1}{R_s}+\frac{1}{R_3}}. \quad (5.17)$$

From equations (4.15) and (4.17), we get that

$$R_y=(R_o+R_L+R_2)*(1+A\beta)-R_L.$$

It is seen that the effect of feedback is to change the input and output resistance of current amplifier such that they tend to reach their ideal value.

An expression for closed-loop current gain can be obtained now.

5.5.4 Closed-Loop Gain

Closed-loop gain T is defined as

$$T = \frac{I_o}{I_s}. \quad (5.18)$$

We can derive an expression for T from Fig. 5.3. From Fig. 5.3,

$$I_o=\frac{GI_eR_o}{R_o+R_2+R_L}, \quad (5.19)$$

and

$$I_e=\frac{(I_s-\beta I_o)*\frac{1}{R_i}}{\frac{1}{R_i}+\frac{1}{R_s}+\frac{1}{R_3}}. \quad (5.20)$$

From equations (5.2), (5.18), (5.19) and (5.20), we get that

$$T = \frac{I_o}{I_s} = \frac{A}{1 + A\beta}.$$

5.6 SUMMARY

Given a circuit which has shunt-series topology, we first obtain the gain of its forward path, called A. Then we can get expressions for its input resistance, output resistance and closed-loop gain.

6. SOME EXAMPLES

An operational amplifier is a versatile device, because its properties are close to the ideal values. This lesson outlines some applications of operational amplifiers.

6.1 INTEGRATOR

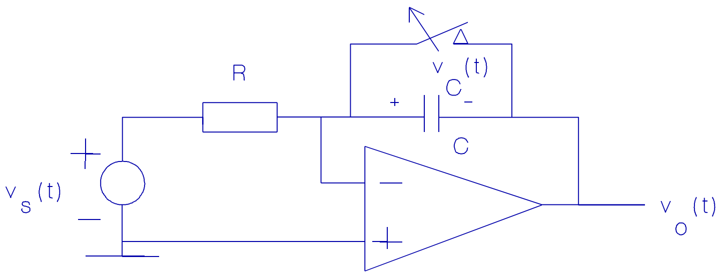


Fig. 6 .1: Integrator

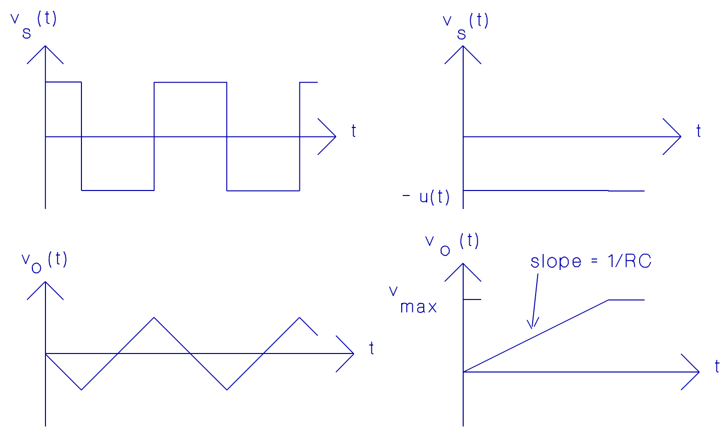


Fig. 6 .2: Waveforms for Integrator

6.1.1 Circuit Operation

The circuit of an integrator is shown in Fig.6.1. The switch across capacitor is opened up at t= 0. Then $v_C(0) = 0$ V. With the opamp being treated as ideal, we get the current through R as

$$i(t) = \frac{v_s(t)}{R}.$$

Also, $$v_C(t) = \frac{1}{C}\int i.dt + v_C(0) = \frac{1}{RC}\int v_S.dt$$

since $v_C(0) = 0$ V. From Fig. 10.1, we get that $v_o(t) = - v_C(t)$. Figure 10.2 sketches the input-output relationship of an ideal opamp. In terms of Laplace transforms,

$$\frac{V_o(s)}{V_s(s)} = - \frac{1}{sRC}.$$

6.1.2 Effect of input bias current

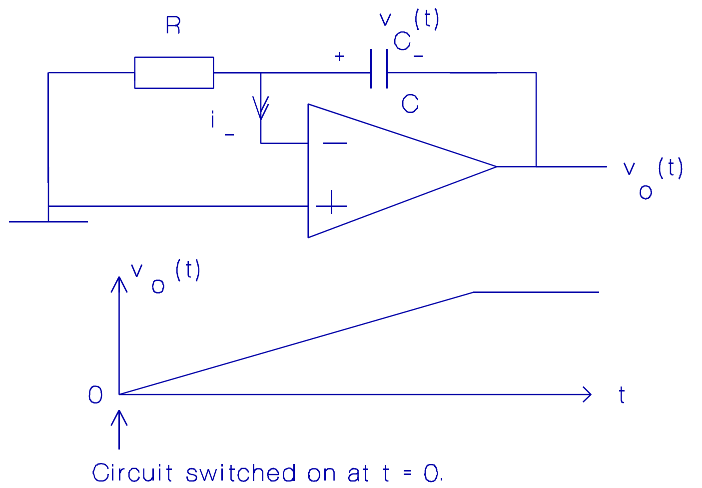


Fig. 6 .3: Effect of Bias Current

Given an opamp with npn transistors at the input stage, there would be some input current into both the inverting and non-inverting terminals. This current is known as the input bias current. The effect of this current can be explained with the help of circuit in Fig. 6.3. Let the current flowing into the inverting terminal be i_-. This current flows through capacitor and the output of opamp would reach its highest positive value some time after switching on the circuit. In order to reduce the effect of bias current, a resistor can be connected across C.

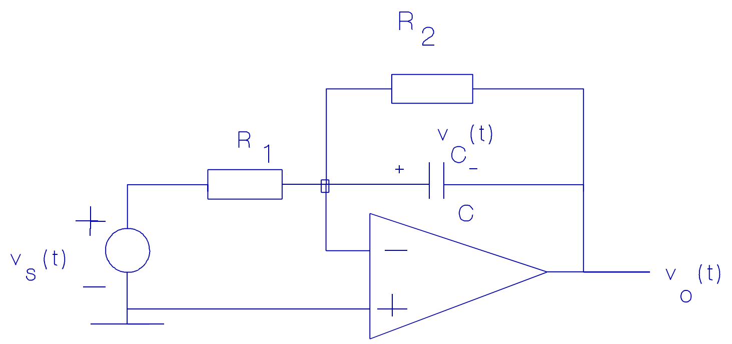


Fig. 6 .4: Modified Integrator Circuit

For the circuit in Fig. 6.4, the transfer function is given by

$$\frac{V_o(s)}{V_s(s)} = -\frac{\frac{R_2}{R_1}}{(1+sCR_2)} .$$

At frequencies below the corner frequency given by ($1/R_2C$), we can approximate the transfer function as:

$$\frac{V_o(s)}{V_s(s)} = -\frac{R_2}{R_1} .$$

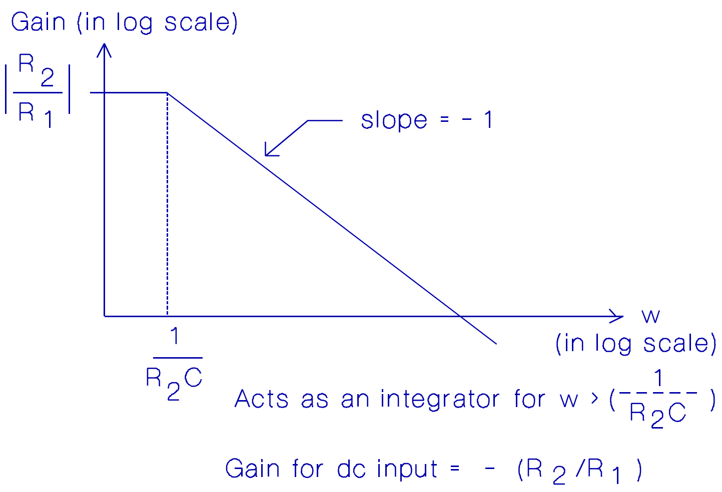


Fig. 6.5: Behaviour of Modified Integrator Circuit

At frequencies above the corner frequency, the transfer function is:

$$\frac{V_o(s)}{V_s(s)} = -\frac{1}{sR_1C} .$$

The Bode plot of the transfer function for this circuit is shown in Fig. 6.5. An opamp with FETs at its input stage can behave differently and this discussion may not be relevant to BIFET opamps.

It is to be noted that the value of resistor across C is to be much greater than R_1. Then the circuit starts operating as an integrator from a relatively low frequency. The integrator circuit is a basic building block and has many applications, such as in filters and waveshaping circuits.. It can also be used as an integral controller in a closed loop system. It was one of the principal blocks in the analogue computer. Now that we have studied what an integrator is, the next step is to find out how a differentiator can be built and analysed. Unlike an integrator, the use of differentiator is not common.

6.2 DIFFERENTIATOR

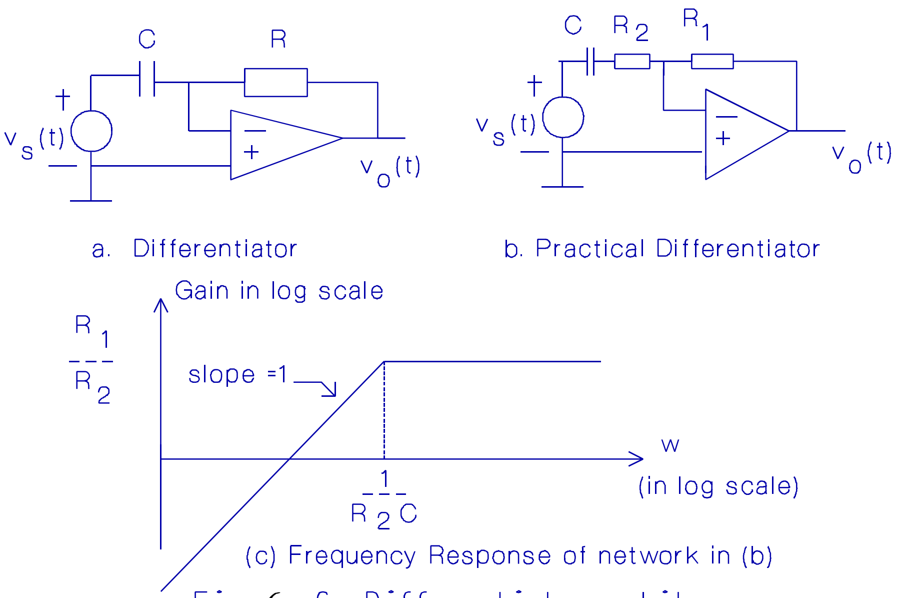


Fig.6 .6: Differentiator and its response

A circuit that works well as a differentiator under ideal conditions is shown in Fig. 6.6a. For this circuit, the transfer function is:

$$\frac{V_o(s)}{V_s(s)} = -sRC.$$

This circuit is not suitable for practical use because gain of the circuit increases with frequency. A noise signal contains high frequencies mostly. Hence noise-to-signal ratio in its output is too high. This drawback is remedied by the addition of a resistor in series with the capacitor as shown in Fig. 6.6b. Frequency response of this circuit is shown in Fig. 6.6c. For the circuit in Fig. 6.6b,the transfer function is:

$$\frac{V_o(s)}{V_s(s)} = -\frac{sR_1C}{1+sR_2C}.$$

At frequencies below $(1/R_2C)$, the circuit behaves as a differentiator. Above this frequency,this circuit has a fixed gain. It is to be noted that $R_1 >> R_2$.

6.3 OPAMP CIRCUITS USING DIODES AND ZENERS

6.3.1 Precision Diode

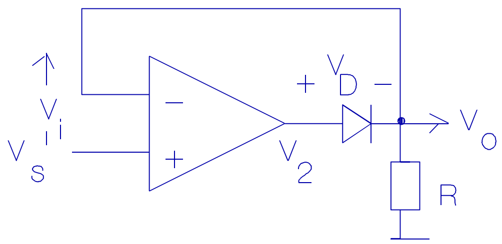


Fig. 6.7 - A Precision-Diode Circuit

Figure 6.7 shows the circuit of a precision diode. When V_s is positive,

$$V_o = V_s + v_i = V_s - \frac{V_o}{A_o}.$$

It is seen that V_o is less than V_s by (V_2/A_o) only, where A_o is the open loop gain of opamp. If only a diode be used, the output will be less by a diode drop which is approximately 0.7 V, but here the output is only 1 or 2 mV less than V_s. When V_s is negative, the diode is reverse-biased, and the output is at zero volts.

6.3.2 Full-wave Rectifier Circuit

Figure 6.8 shows a full-wave rectifier circuit using opamps. This circuit is known as a precision rectifier circuit. From Fig. 6.8, it is seen that

$$V_s = -V_s - 2*V_x.$$

When V_s is positive, the inverting input terminal of first opamp tends to be positive and its output becomes negative. Then $V_x = -V_s$ and $V_1 = -V_x - 0.7$, assuming that the voltage drop across diode D_1 in conduction is 0.7 V with D_2 remaining reverse-biased. Under these conditions, $V_o = V_s$.

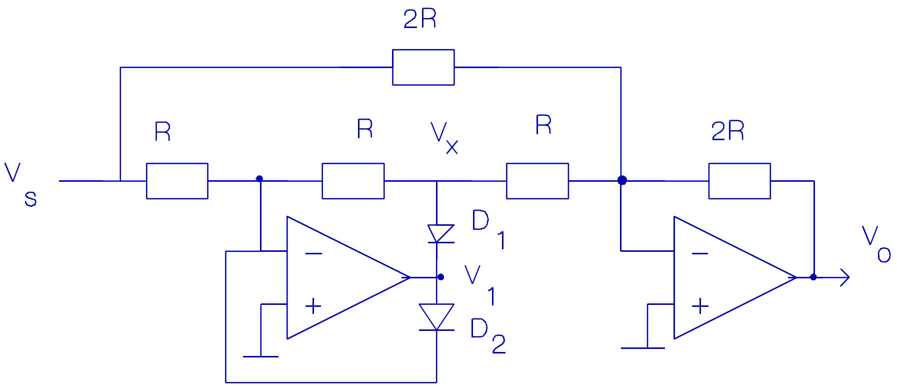


Fig. 6.8 : - Full-wave Rectifier

Let V_s be negative. The inverting input terminal of first opamp tends to be negative and its output becomes positive. Then $V_1 = 0.7$ V, the diode drop and $V_x = 0$ V. Now D_2 conducts and D_1 is reverse-biased. Under these conditions, $V_o = -V_s$. Hence we have that $V_o = |V_s|$. If diodes D_1 and D_2 are reverse-connected, $V_o = -|V_s|$.

6.4 WORKED EXAMPLES

WE 6.4.1 Indicate how each circuit in Fig. 6.9 works.

Solution:

Circuit (a) : Current amplifier or current-controlled current source.

$$\frac{I_L}{I_s} = \frac{R_1}{R_2}.$$

Circuit (b) : Transconductance amplifier / voltage-controlled current source

$$\frac{I_L}{V_s} = \frac{1}{R_2}.$$

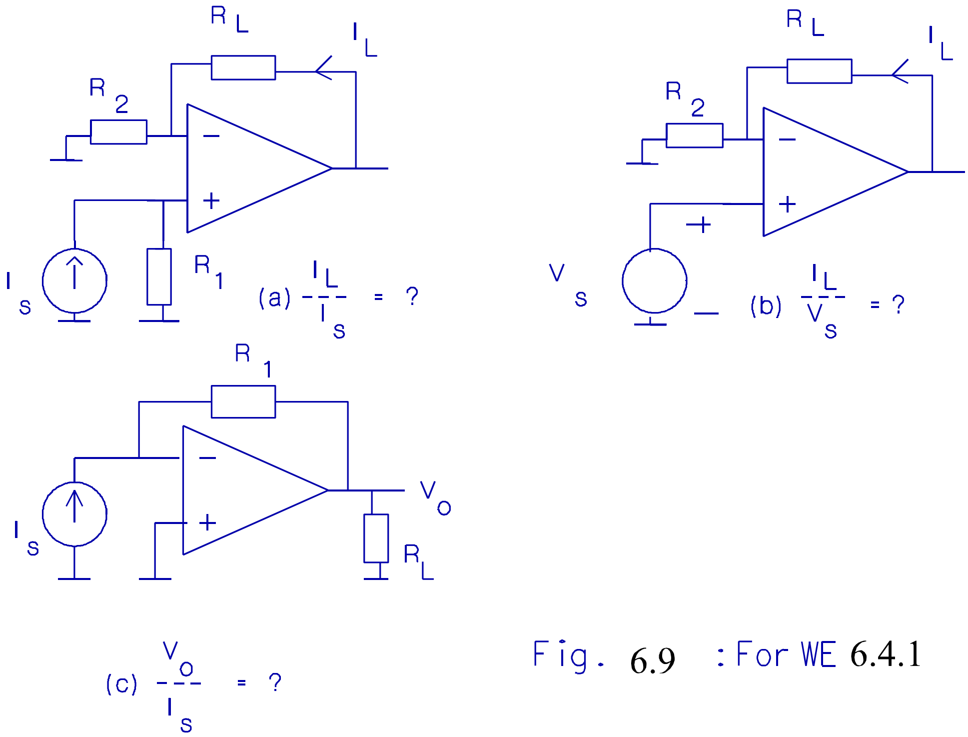


Fig. 6.9 : For WE 6.4.1

Circuit (c) :Transresistance amplifier / current-controlled voltage source

$$\frac{V_o}{I_s} = -R_1 .$$

WE 6.4.2: Figure 6.10 shows an application for difference amplifier. Resistor R_f can be a transducer the quiescent value of which is R_2. The transducer here could be a strain gauge, the resistance of which changes about R_2 as it is bent or twisted. It can be a thermistor or a magnetic field-dependent resistor. Find the output of circuit in Fig. 6.10 when $R_f = R_2 + \delta R$.

Solution:

From Fig. 6.10,

$$\text{Current through } R_1,\ I_1 = \frac{E}{R_1 + R_2} .$$

$$V_o = E - I_1 * (R_1 + R_f) = E - I_1 * (R_1 + R_2 + \delta R) = -E * \frac{\delta R}{R_1 + R_2} .$$

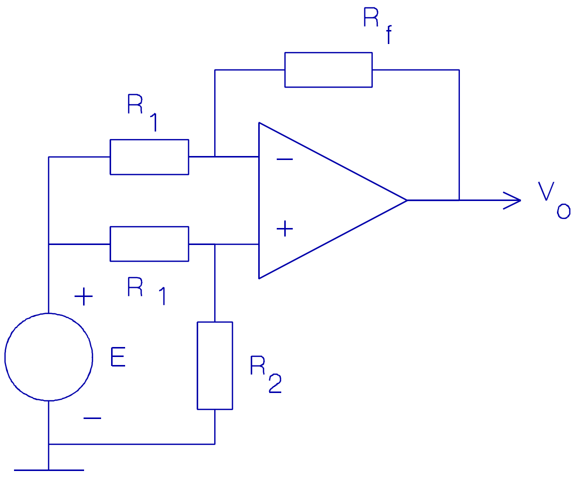


Fig. 6.10 : For WE 6.4.2

WE 6.4.3 Get an expression for output of circuit in Fig. 6.11.

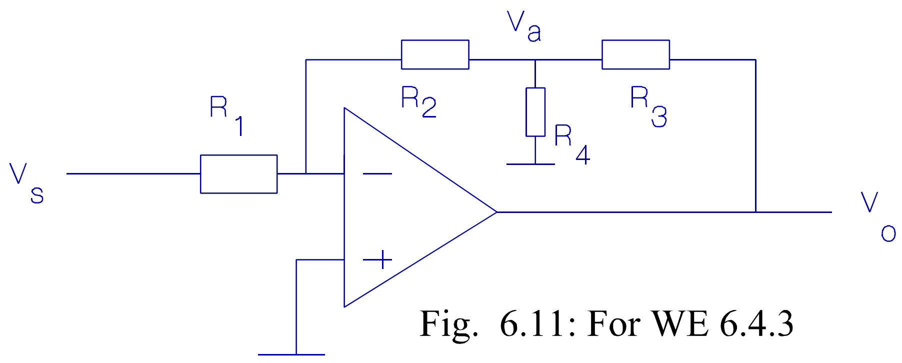


Fig. 6.11: For WE 6.4.3

Solution: From Fig. 6.11,

$$\frac{V_s}{R_1} + \frac{V_a}{R_2} = 0.$$

Eliminating V_a, we get that

$$V_a * \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) = \frac{V_o}{R_3}.$$

$$\frac{V_o}{V_s} = -\frac{1}{R_1} * (R_2 + R_3 + \frac{R_2 R_3}{R_4}) .$$

WE 6.4.4: Find the CMRR of the circuit in Fig. 6.12.

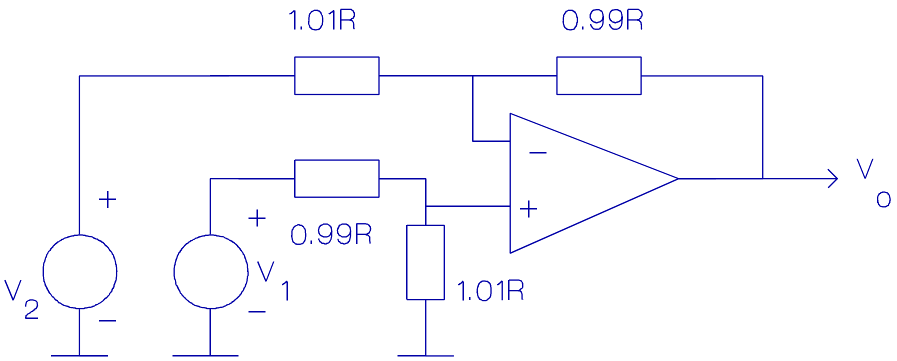


Fig: 6.12: For WE 6.4.4

Solution: For the circuit in Fig. 6.12,

$$V_o = V_1 - \frac{0.99}{1.01} * V_2 .$$

We can express the output as:

$$V_o = A_{cm} * \frac{(V_1 + V_2)}{2} + A_{dm} * (V_1 - V_2) .$$

By comparing the coefficients of V_1 and V_2, we get that

A_{dm} = 1/1.01 and A_{cm} = 0.02/1.01. Then CMRR = A_{dm}/A_{cm} = 50.

Comment: This problem can be re-phrased as follows: A difference amplifier is constructed with four resistors of same nominal value. If these resistors have a tolerance of 1%, find out the worst-case CMRR of this circuit.

EXAMPLE 6.4.5:

Obtain the transfer function of the circuit shown below.

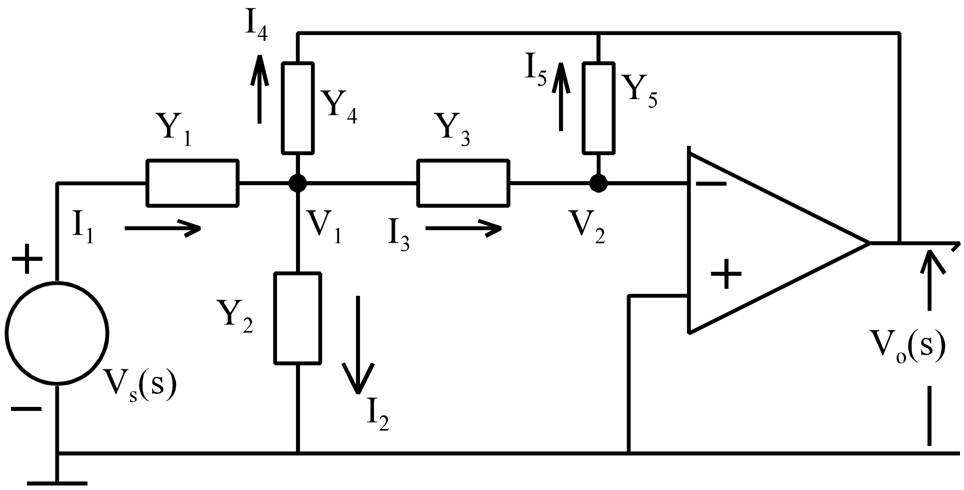


Fig. 6.13: Example 6.4.5

SOLUTION:

From the circuit in Fig. 6.13, we get that

$$V_s = V_1 + \frac{I_1}{Y_1} \ .$$

$$I_1 = I_2 + I_3 + I_4 \ .$$

$$I_2 = Y_2 V_1$$

$$I_3 = Y_3 V_1 = - Y_5 V_o, \text{ since } V_2 = 0.$$

$$I_4 = Y_4 (V_1 - V_o)$$

We know V_1 in terms of V_o. Then I_2 can be expressed in terms of V_o. We can express I_1 and I_4 in terms of V_o. We then get

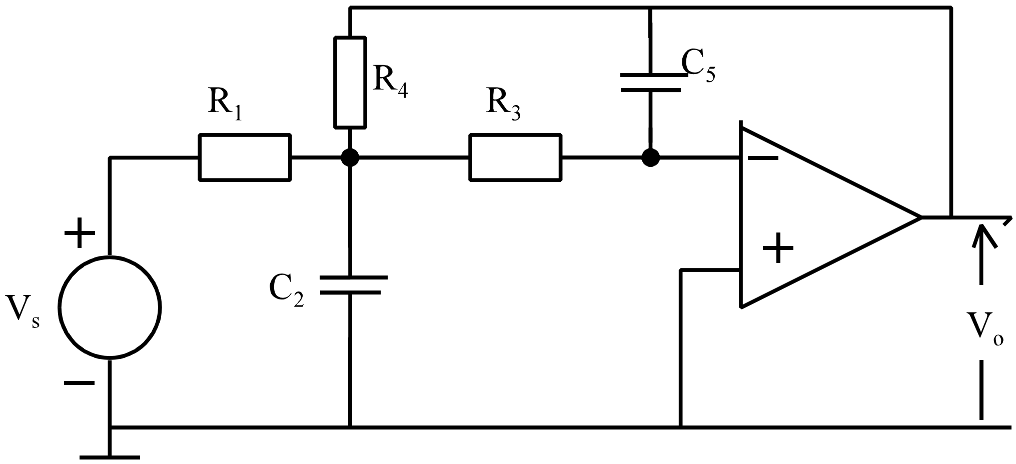


Fig. 6.14: Example 6.4.5

$$\frac{V_o(s)}{V_s(s)} = \frac{-Y_1Y_3}{Y_3Y_4 + Y_5(Y_1 + Y_2 + Y_3 + Y_4)}$$

For example, let

$$Y_1 = \frac{1}{R_1}, Y_3 = \frac{1}{R_3}, Y_4 = \frac{1}{R_4}, Y_2 = -sC_2 \text{ and } Y_5 = sC_5$$

Then

$$\frac{V_o(s)}{V_s(s)} = \frac{-\frac{1}{R_1R_3} \times \frac{1}{C_2C_5}}{s^2 + s\frac{1}{C_2}\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4}\right) + \frac{1}{R_3R_4C_2C_5}}$$

With the components as specified, the network functions as second-order low-pass filter. The above equation can be presented as follows.

$$\frac{V_o}{V_s} = \frac{-A_o w_o^2}{s^2 + 2\xi w_n s + w_n^2} \cdot$$

Then

$$w_n^2 = \frac{1}{R_3 R_4 C_2 C_5}$$

$$A_o = \frac{R_4}{R_1}, \quad 2\xi w_n = \frac{1}{C_2} * \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \right)$$

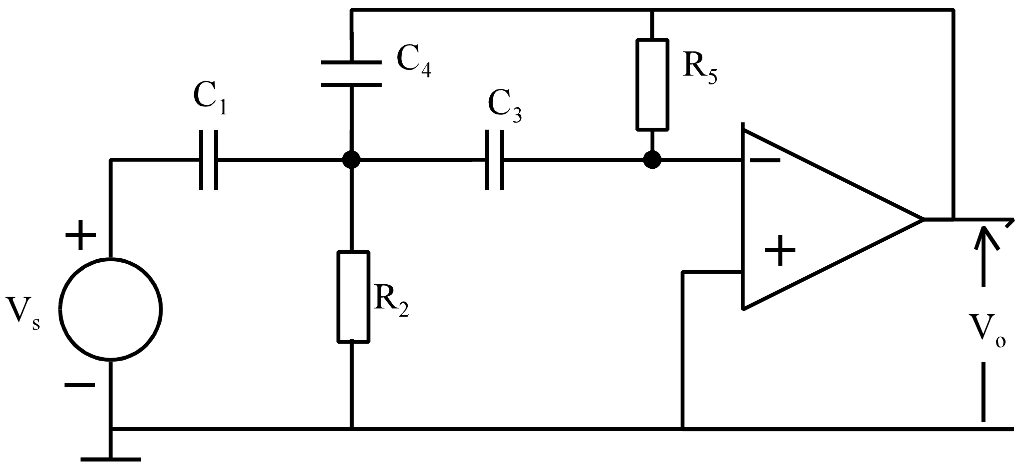


Fig. 6.15: Example 6.4.5

For the circuit in Fig. 6.15, the transfer function is obtained as:

$$\frac{V_o(s)}{V_S(s)} = -\frac{s^2 \frac{C_1}{C_4}}{s^2 + s\left(C_1 + C_3 + C_4\right) \cdot \frac{1}{C_3 C_4 R_5} + \frac{1}{C_3 C_4 R_2 R_5}}$$

The above transfer function represents a second-order high-pass filter. Let

$$\frac{V_o(s)}{V_s(s)} = \frac{-A_o s^2}{s^2 + 2\xi w_n s + w_n^2}$$

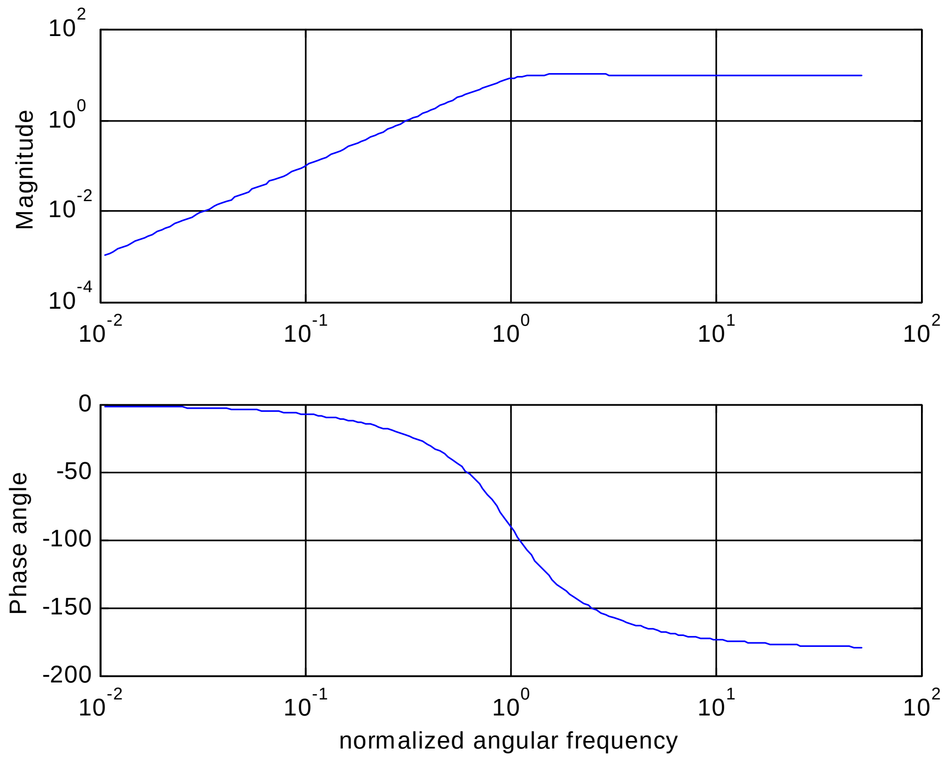
Then

$$A_o = \frac{C_1}{C_4}, \quad w_n^2 = \frac{1}{C_3 C_4 R_2 R_5}, \quad 2\xi w_n = \frac{C_1 + C_3 + C_4}{C_3 C_4 R_5}$$

A Matlab program is used to obtain the response. The program is presented below.

```
% Second-order High Pass Filter
clear;
Ao=10;
df=0.6;
for n=1:175;
 af(n)=(1.05^n)/100;
 num(n)=Ao*(af(n)*af(n));
 den1(n)=1-(af(n)*af(n));
 den2(n)=2*df*af(n);
 mag(n)=num(n)/(den1(n)+j*den2(n));
 phase(n)=180/pi*angle(mag(n));
end;
subplot(2,1,1)
loglog(af,abs(mag))
ylabel('Magnitude')
grid on;
subplot(2,1,2)
semilogx(af,phase)
ylabel('Phase angle')
xlabel('normalized angular frequency')
grid on;
```

The response obtained have been presented next.



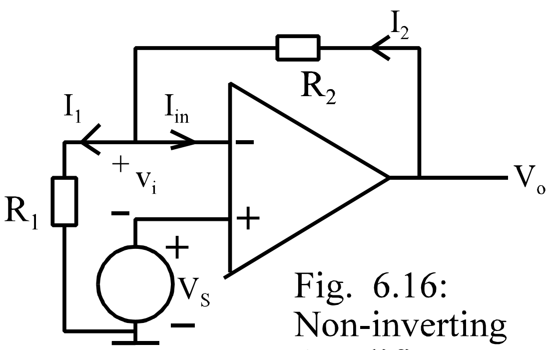


Fig. 6.16: Non-inverting Amplifier

EXAMPLE 6.4.6:

The opamp used for the non- inverting amplifier has an open-loop gain with a single pole. It is defined as:

$$A(jw) = \frac{A_o}{1 + \frac{jw}{w_H}}$$

Obtain the transfer function for the inverting amplifier and sketch its frequency response.

SOLUTION:

The gain for the non-inverting amplifier has been obtained as:

$$\left(\frac{V_o}{V_s}\right) = \frac{\left(1 + \frac{R_2}{R_1}\right)}{1 + \frac{1}{A_o} * \left(1 + \frac{R_2}{R_1}\right)}.$$

Now the above expression becomes:

$$\frac{V_o}{V_S} = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1 + j\frac{w}{w_H}}{A_o}\left(1 + \frac{R_2}{R_1}\right)}$$

If $\left(A_o\right) >> \left(1 + \frac{R_2}{R_1}\right)$, then

$$\frac{V_o}{V_S} = \frac{1 + \frac{R_2}{R_1}}{1 + j\frac{w}{A_o w_H}\left(1 + \frac{R_2}{R_1}\right)} = \frac{1 + \frac{R_2}{R_1}}{1 + j\frac{w}{w_T}\left(1 + \frac{R_2}{R_1}\right)}, \; w_T = A_o w_H$$

The frequency response of the non-inverting amplifier is sketched in Fig. 6.17.

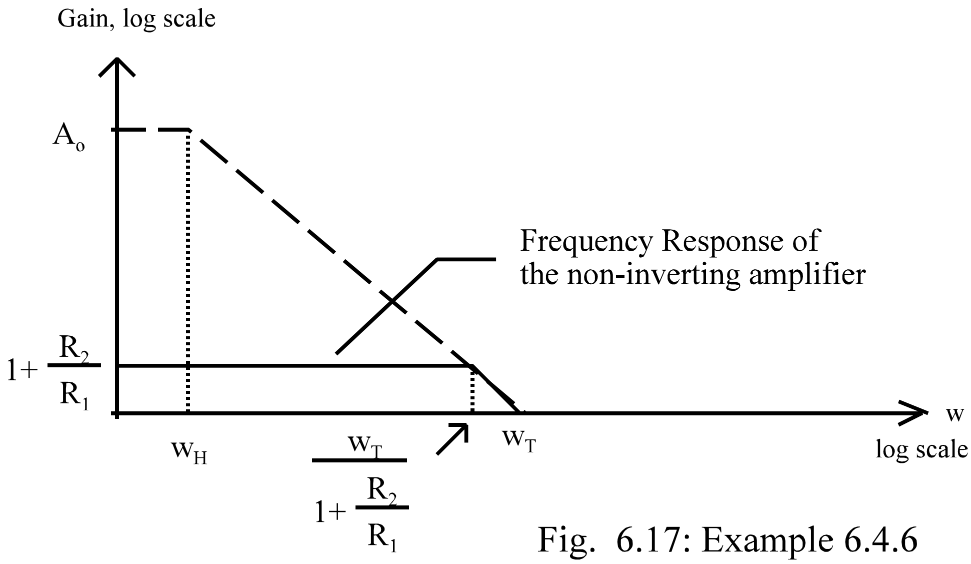


Fig. 6.17: Example 6.4.6

7ACTIVE FILTERS

The commonly used types of filter responses are :

Butterworth and

Chebyshev.

The **Butterworth Filter response** is characterized by a flat frequency upto the critical frequency, followed by a smooth roll-off of 20 db per decade per pole. The phase shift, however, varies nonlinearly with frequency. This means that different frequency components will experience different time delays as they pass through the filter. This will cause distortion and ringing on square and pulse waves. For pulse waves it is better to use **Bessel filter response**, which has a linear variation of phase with frequency . Bessel filters have a somewhat slower initial roll-off than Butterworth filters and consequently poorer for linear applications such as audio circuits. The **Chebyshev filter response** has a faster initial roll-off than a Butterworth filter, allowing the design of a much sharper cutoff filter with the same number of poles. The price of this faster roll-off is increased nonlinear phase shift and ripples in the amplitude response of the filter passband.

The filter function H(s) can be expressed as:

$$H(s) = \frac{A(s)}{B(s)} \qquad (7.1)$$

7.1 BUTTERWORTH POLYNOMIALS

The first-order normalized Butterworth polynomial for the denominator B(s) can be expressed to be:

$$B(s) = (s+1)$$

The corresponding normalized transfer function is:

$$H(s) = \frac{1}{s+1} \qquad (7.2)$$

The above transfer function can be realized by the circuit shown in Fig. 7.1

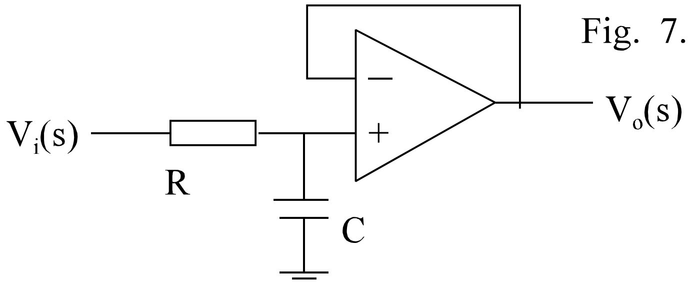


Fig. 7.1

For the circuit in Fig. 7.1, the transfer function is $H(s) = \dfrac{1}{sRC+1}$ (7.3)

In order to obtain the normalized transfer function, we can let R = 1 Ω and C = 1 F. To design a first-order low-pass filter with a cut-off frequency at 1 kHz,

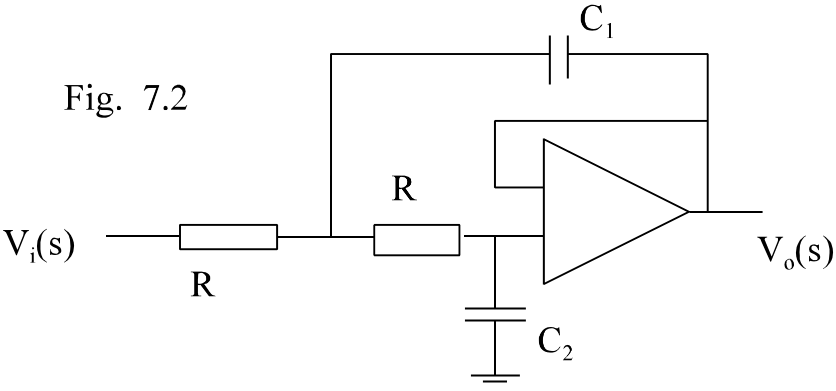
$$RC = \frac{1}{2\pi \times 1000}.\ Let\ R = 10\ k\Omega.\ Then\ C = \frac{50}{\pi}\quad nF$$

The normalized Butterworth polynomial B(s) is $\left(s^2 + \sqrt{2}s + 1\right)$. The corresponding transfer function is:

$$H(s) = \frac{1}{s^2\ \sqrt{2}s + 1} \quad (7.4)$$

The 2-pole filter circuit is shown in Fig. 7.2.

Fig. 7.2



For the circuit in Fig. 7.2, the transfer function is:

$$H(s) = \frac{1}{s^2 R^2 C_1 C_2 + sRC_2 + 1} \quad (7.5)$$

To obtain the normalized transfer function, let

$$R_n = 1\,\Omega,\ C_{1n} = \sqrt{2}\ F\ \ and\ \ C_{2n} = \frac{1}{\sqrt{2}}\ F$$

The suffix n indicates that these are normalized values.

To design a unity-gain low pass filter with a cut-off frequency at 1000 Hz, the components can be selected as follows. For design, two scale factors have to be defined. Let frequency scaling constant be K_f.

$$K_f = w_c = 2\pi\, f_c = 2000\pi$$

Divide the capacitor values by this scaling constant. The capacitor and the resistor values still do not lie in the acceptable range for an opamp circuit. Let us define another constant, known as the impedance scaling constant, K_m. Let

K_m = 10000.

Then each of the resistors has the value of 10 kΩ. Also

$$C_1 = \frac{C_{1n}}{K_m K_f} = 22.5\,nF,$$

$$C_2 = \frac{C_{2n}}{K_m K_f} = 11.25\,nF.$$

Choose the nearest commercially available values.

The Butterworth polynomial for a 3-pole filter is:

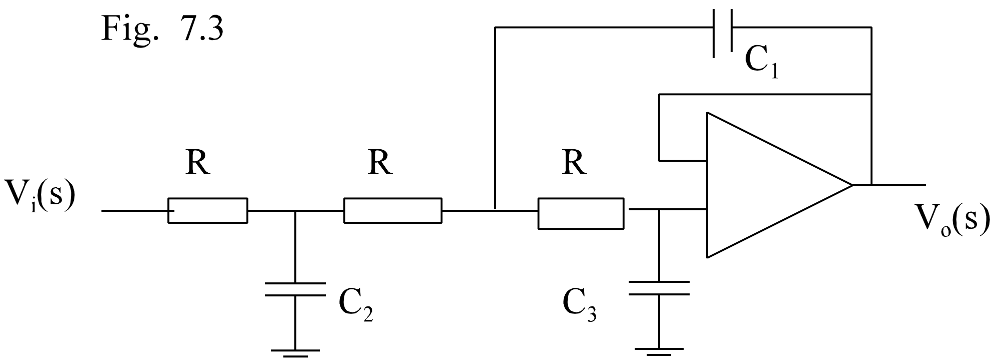
$B(s) = (s + 1)(s^2 + s + 1) = s^3 + 2s^2 + 2s + 1.$

The corresponding transfer function is:

$$H(s) = \frac{1}{s^3 + 2s^2 + 2s + 1} \quad (7.5)$$

The 3-pole filter circuit is shown in Fig. 7.3.

Fig. 7.3



The transfer function for the circuit in Fig. 3 is:

$$H(s) = \frac{1}{s^3 + \left(2T_1T_3 + 2T_2T_3\right)s^2 + \left(3T_3 + T_2\right)s + 1} \quad (7.6)$$

where

$T_1 = RC_1$, $T_2 = RC_2$, $T_3 = RC_3$.

To normalize, let

$$R = 1\,\Omega, C_1 = 3.546\,F,\ C_2 = 1.392\,F,\ C_3 = 0.2024\,F. \quad (7.7)$$

Example: Design a unity-gain Butterworth filter with a critical frequency of 3000 Hz and a roll-off of 60 db per decade.

Solution:

The frequency scaling constant is: $K_f = 2\pi f_C = 2\pi \times 3000 = 18{,}849.6\ rad/s$
After applying the frequency scaling constant, we get that

$$R = 1\,\Omega, C_1 = 188.12\,\mu F,\ C_2 = 73.848\,\mu F,\ C_3 = 10.738\,\mu F.$$

To make C1 be equal to 10 nF, the amplitude scale factor is 18,812. With this scale factor,

$$R = 18\ k\Omega, C_1 = 10\,nF,\ C_2 = 3.926\,nF,\ C_3 = 560\ pF.$$

4-th order Butterworth polynomial

B(s) = (s^2 + 0.765s + 1).(s^2 + 1.848s + 1)

The corresponding normalized transfer function is:

$$H(s) = \frac{1}{\left(s^2 + 0.765s + 1\right).\left(s^2 + 1.848s + 1\right)} \quad (7.8)$$

The circuit for 4-pole filter is presented below in Fig. 7.4

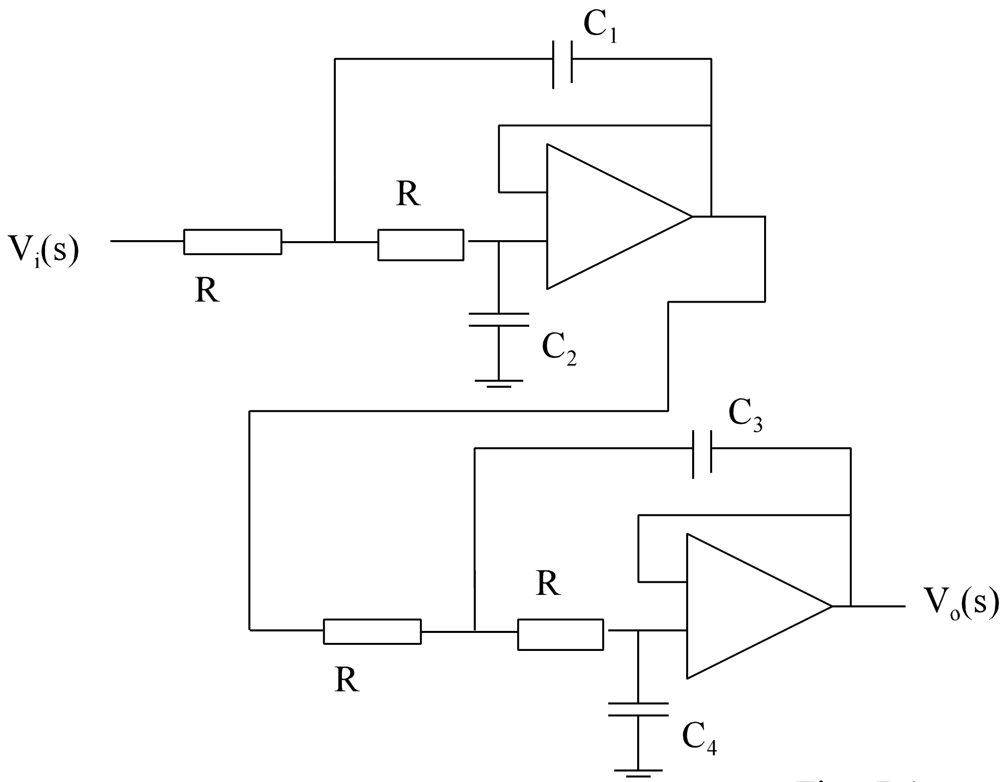


Fig. 7.4

The transfer function for the above circuit is:

$$H(s) = \frac{1}{\left(s^2 T_1 T_2 + sT_2 + 1\right).\left(s^2 T_3 T_4 + sT_4 + 1\right)} \quad (7.9)$$

where

$T_1 = RC_1$, $T_2 = RC_2$, $T_3 = RC_3$, $T_4 = RC_4$.

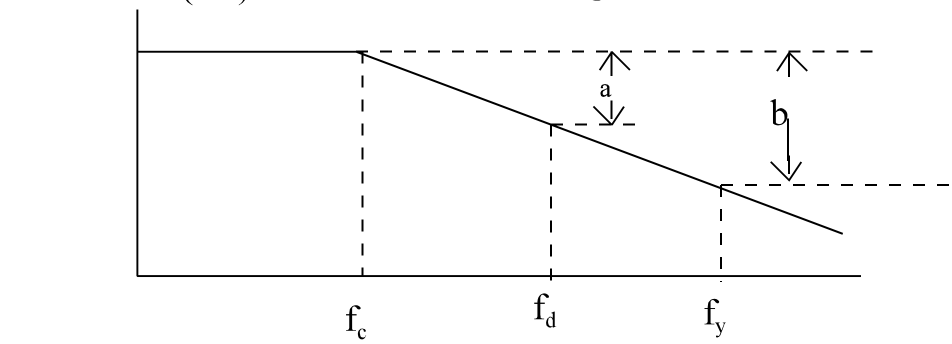
The transfer function can be normalized if:

$$R = 1\,\Omega, C_1 = 1.082\,F,\ C_2 = 0.9241\,F,\ C_3 = 2.614\,F,\ C_4 = 0.3825\,F \quad (7.10)$$

Example

Design a unity-gain low-pass Butterworth filter with a critical frequency of 15 kHz. The attenuation should at least 300 at 20,000 kHz.

Fig. 7.5



Using Bode Plot to determine attenuation at any frequency

Solution:

In Fig. 7.5, let

f_c = critical frequency, f_d = decade frequency, $10 \times f_c$,

f_y = another frequency.

Let

a = attenuation at frequency, f_d

b = attenuation at frequency, f_y

Then

$$\frac{a}{b} = \frac{f_d - f_c}{f_y - f_c}$$

For the given problem, b = 300, f_c = 15 kHz, f_d = 150 kHz, f_y = 20 kHz. Then

a = 8100

In terms of dB, attenutation in dB per decade = 20 log (8100) = 78.17 dB. It means that a roll-off of at elast 80 dB decade is required. A 4-pole active filer will suffice. For a 4-pole filter, we have that

$$R = 1\,\Omega, C_1 = 1.082\,F, C_2 = 0.9241\,F, C_3 = 2.614\,F, C_4 = 0.3825\,F$$

We have that

$$K_f = 2\pi f_C = 2\pi \times 15000 = 94{,}247.8\ rad\ /\ s$$

Then we get that

$$R = 1\,\Omega, C_1 = 11{,}480\,\mu F, C_2 = 9{,}805\,\mu F, C_3 = 27{,}725\,\mu F, C_4 = 4{,}0585\,\mu F$$

Let the amplitude scale factor be 11480. Then

$$R = 11.48\ k\Omega, C_1 = 10\,nF, C_2 = 8.541\,nF, C_3 = 24.15\,nF, C_4 = 3.535\,nF$$

Choose the nearest values.

CHEBYSHEV FILTERS

For a Chebyshev Filter

$$H^2(jw) = \frac{(H_0)^2}{1+\varepsilon^2 C_n^2\left(\frac{w}{w_C}\right)}$$

$$where\ C_n\left(\frac{w}{w_C}\right)\ is$$

$$C_n\left(\frac{w}{w_C}\right) = \cos\left(n.\cos^{-1}\left(\frac{w}{w_C}\right)\right)\ for\ 0 \le \frac{w}{w_C} \le 1$$

$$C_n\left(\frac{w}{w_C}\right) = \cos\left(n.\cosh^{-1}\left(\frac{w}{w_C}\right)\right)\ for\ \frac{w}{w_C} > 1$$

The normalized polynomials for low pass filter are defined as follows.

$$s+1$$

$$s^2 + 1.425s + 1.516$$

$$(s+0.626)\left(s^2 + 0.845s + 0.356\right)$$

$$\left(s^2 + 0.351s + 1.064\right)\left(s^2 + 0.845s + 0.356\right)$$

CONCLUSION

High-pass filters, band-pass filters and notch filters can also be realized using the well-known filter configurations.

Reference:

1. Microelectronics, Millman and Grabel, Second Edition. MacGraw-Hill.
2. Linear Integrated Circuits, Winzer,Saunders College Publishing
3. Active Filter Design, Alan Waters, Macmillan Education
