Workshop on an Optical Clock Mission in ESA‘s Cosmic Vision Program
Düsseldorf 8. - 9. 3. 2007

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Gravitational Physics with Optical Clocks in Space

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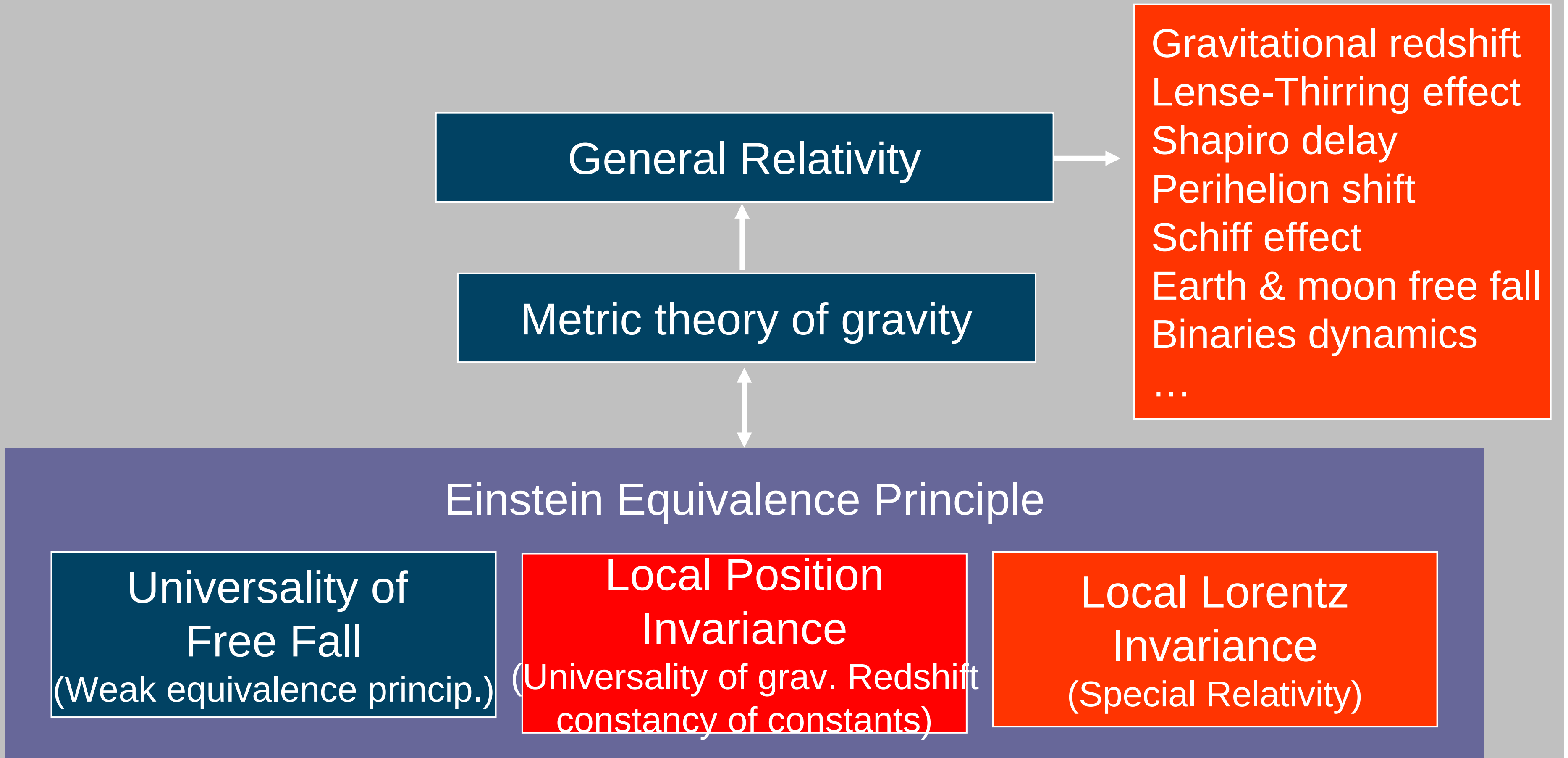
Contents

Gravity and its foundations

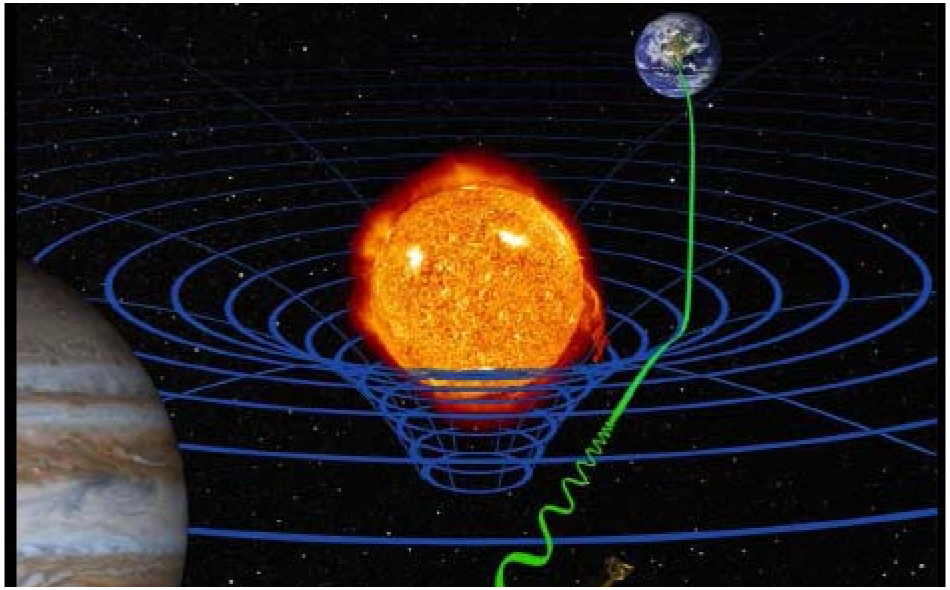


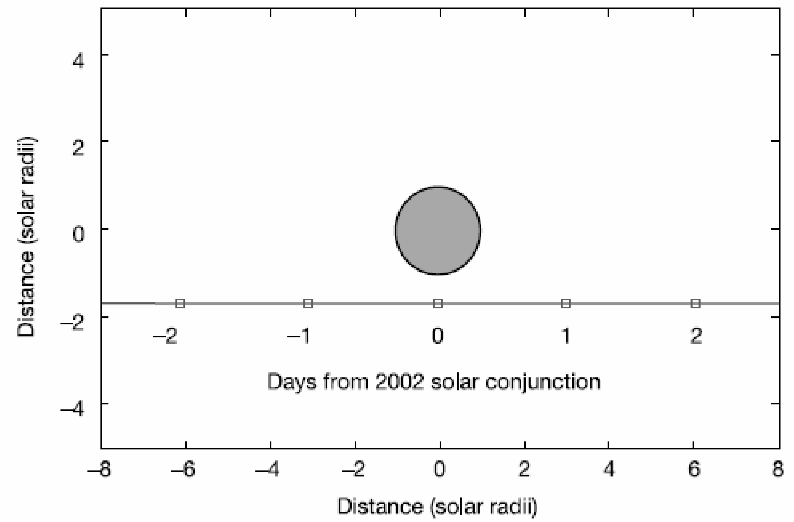
Shapiro time delay

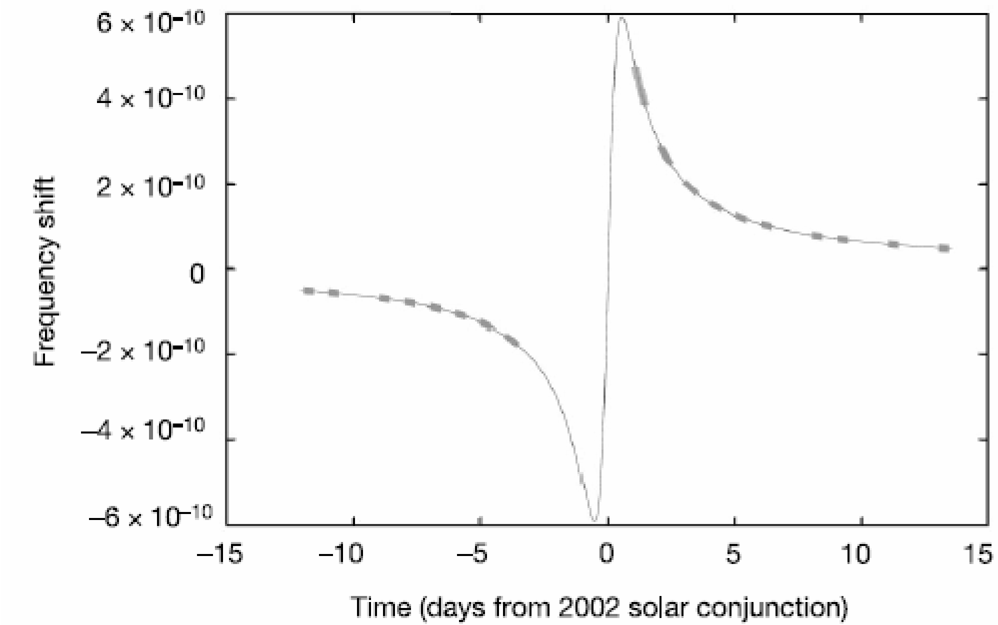
- Cassini Mission (Tortora et al 2003)

$$y_{gr} = \frac{d\Delta t}{dt} = -2(1+\gamma)\frac{GM_S}{c^3 b}\frac{db}{dt}$$
$$= -(1\times10^{-5}\,s)(1+\gamma)\frac{1}{b}\frac{db}{dt}$$

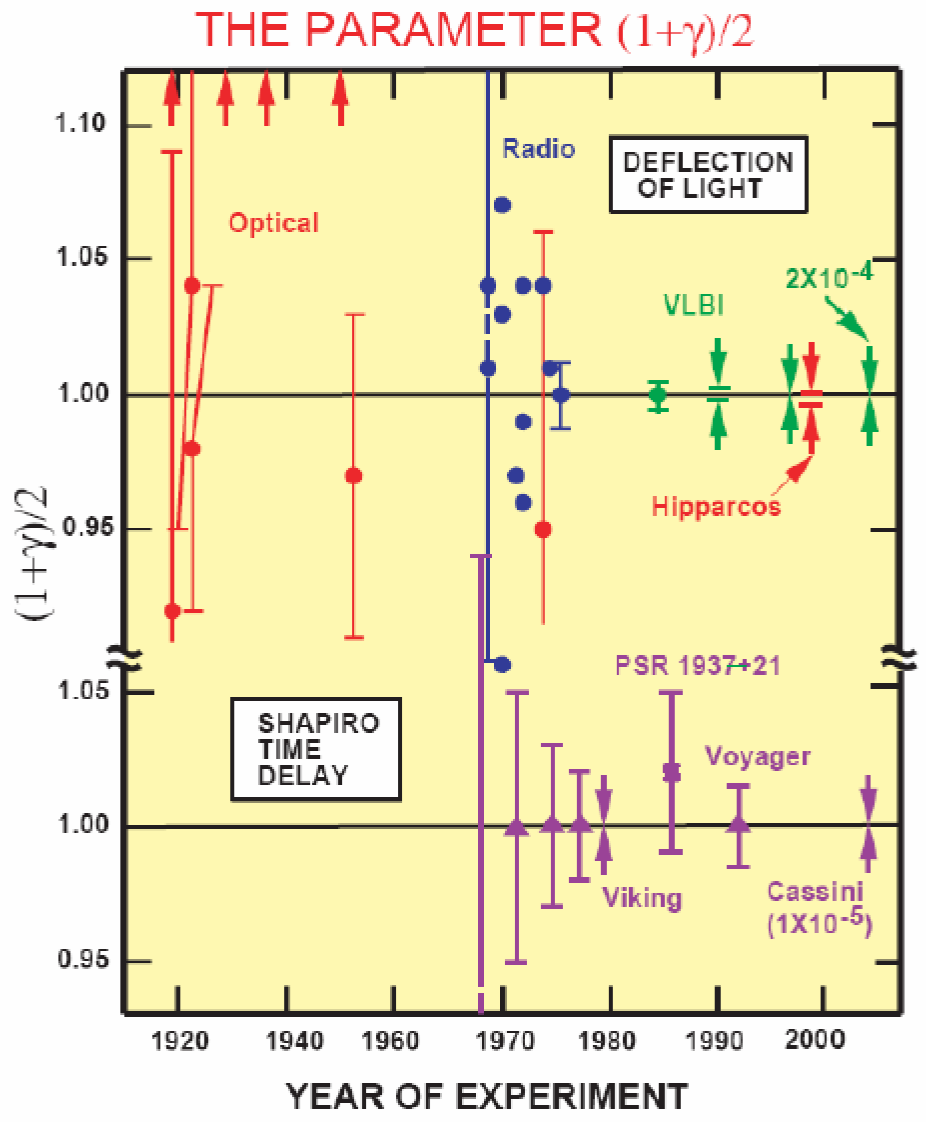
- Achieved accuracy: $|1-\gamma| < 2.10^{-5}$







Time delay and deflection of light



From: C. Will (2006)

Nonlinearity of gravity

- Nonlinearity of metric

$$g_{00} = 1 + 2\frac{U}{c^2} + 2\beta\frac{U^2}{c^4} + \ldots$$

- From Lunar Laser Ranging results and assuming only β and γ nonzero: $|1-\beta| < 2.10^{-4}$

Testing the Gravitational Redshift of Clocks

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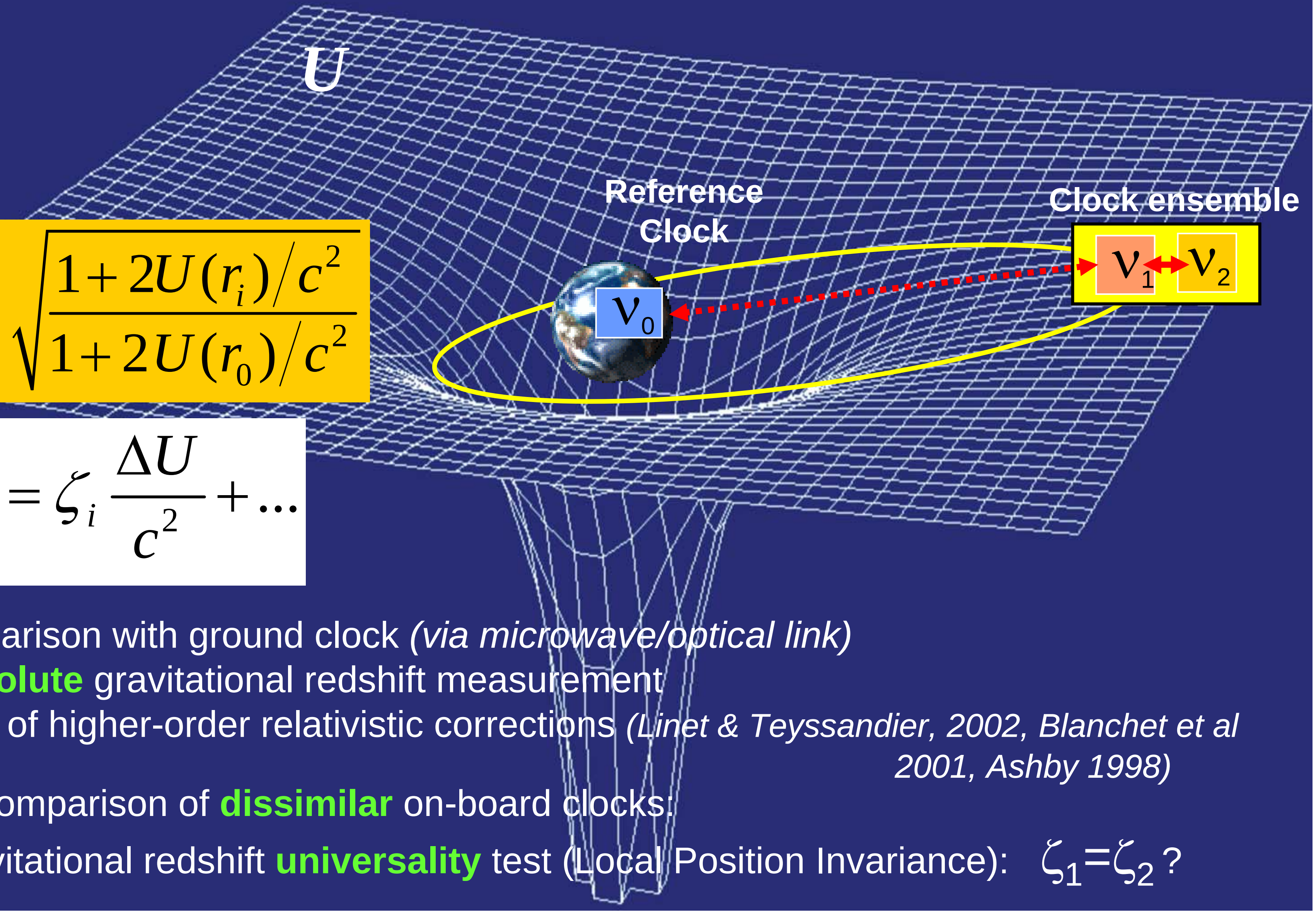


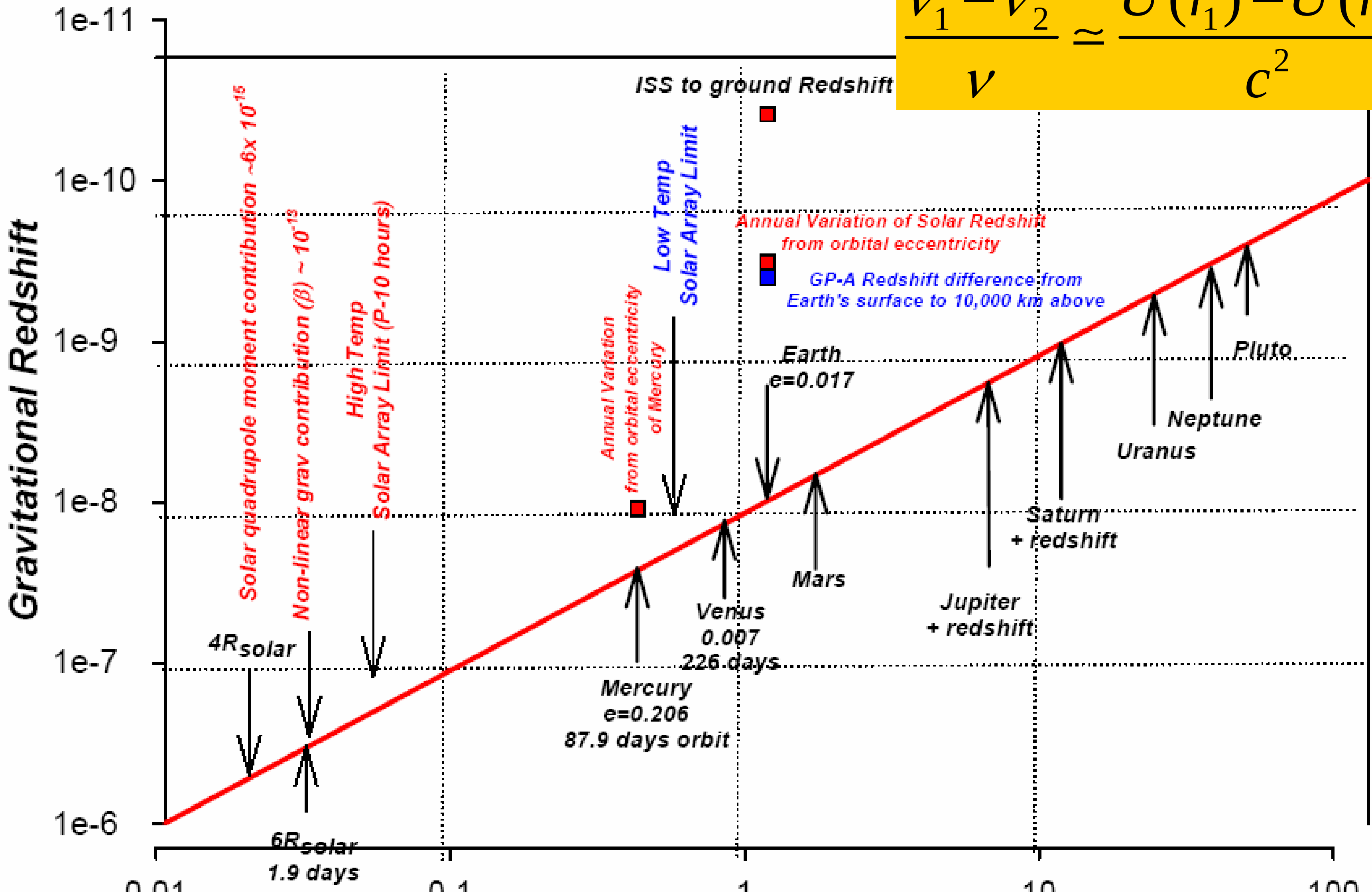
$$\nu_i / \nu_0 = \sqrt{\frac{1+2U(r_i)/c^2}{1+2U(r_0)/c^2}}$$

$$\frac{\Delta \nu_i}{\nu_0} = \zeta_i \frac{\Delta U}{c^2} + \ldots$$

- Comparison with ground clock *(via microwave/optical link)*
  - **Absolute** gravitational redshift measurement
  - Test of higher-order relativistic corrections *(Linet & Teyssandier, 2002, Blanchet et al 2001, Ashby 1998)*
- Intercomparison of **dissimilar** on-board clocks:
  - Gravitational redshift **universality** test (Local Position Invariance): $\zeta_1 = \zeta_2$?

Gravitational Redshift

$$\frac{\nu_1-\nu_2}{\nu} \simeq \frac{U(r_1)-U(r_2)}{c^2}$$



From: J. Prestage and L. Maleki, JPL

TESTS OF LOCAL POSITION INVARIANCE

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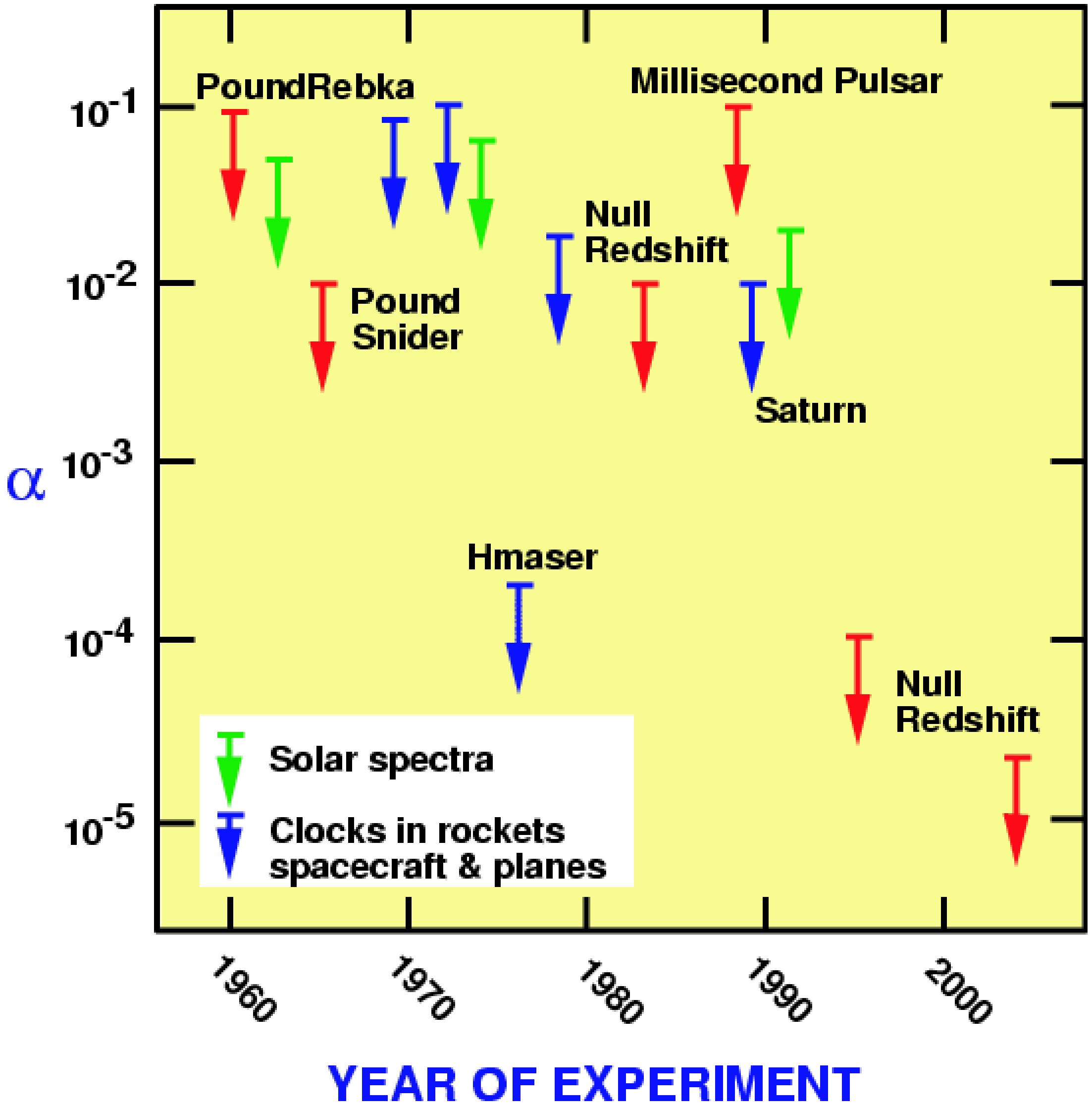

From: C. Will (2006)

$$\Delta\nu/\nu = (1+\alpha)\Delta U/c^2$$

Gravitational redshift: Past & upcoming missions

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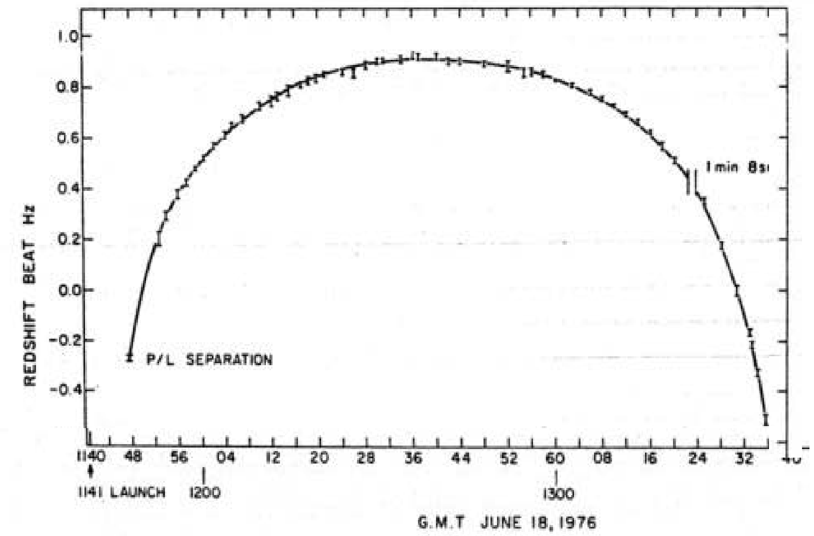
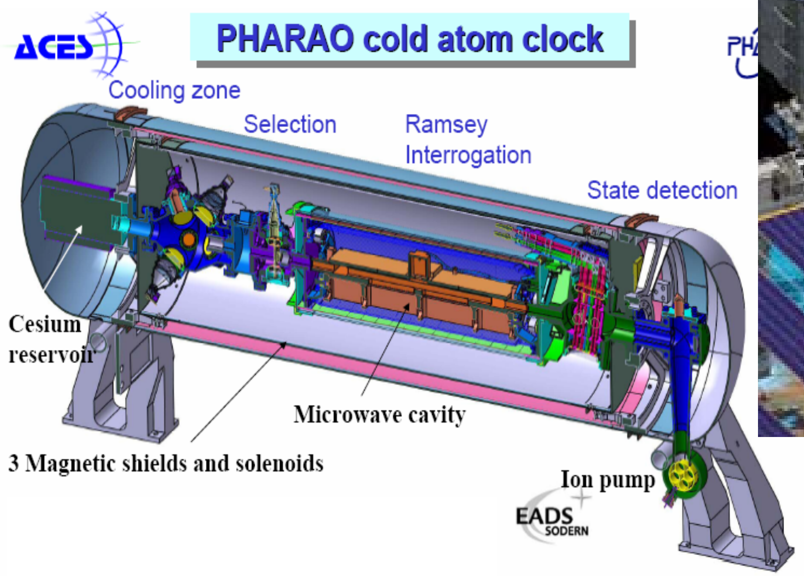
Gravity Probe A: hydrogen maser (1976)

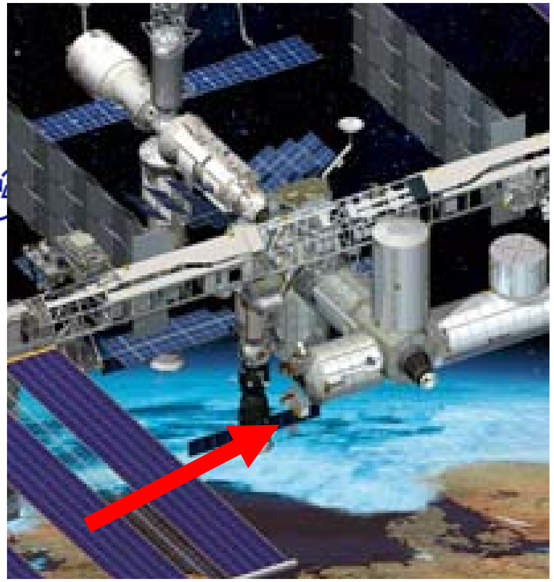
- rocket flight to 10 000 km altitude
- tested relativistic Doppler effect and gravitational redshift to 70 ppm



ACES: Atomic clock ensemble (2012)

- PHARAO: cold atom microwave clock
- instability $1 \cdot 10^{-13}$ at 1 s, $4 \cdot 10^{-16}$ at 50 000 s
- accuracy ~ $1 \cdot 10^{-16}$
- redshift test at 2 ppm
- technology demonstrator
- world-wide time dissemination and comparisons
- test of special and general relativity





Scientific Goals

- How scientifically powerful?
- „The most powerful test of gravitational theory"

  Gravity Probe A: 7.10^{-5} (redshift)
  Cassini: 2.10^{-5} (γ)
  Gravity Probe B: goal 1.10^{-5} (γ)
  ACES: goal 2.10^{-6} (redshift)
- **Proposals:**
  - Mercury Radioscience Orbiter Experiment: $\Delta\gamma \sim 2.10^{-6}$, $\Delta\beta \sim 5.10^{-6}$
  - GAIA 5.10^{-7} (spacecraft at Lagrange-point L1)
  - ASTROD I: γ at 1.10^{-7}, β at 1.10^{-7} (1 spacecraft, drag-free)
  - Gravitational Time Delay Mission: γ at 2.10^{-8} (2 spacecraft, drag-free)
  - LATOR: γ at 2.10^{-9} (3 spacecraft, incl. ISS, not drag-free)
  - ASTROD: γ at 1.10^{-9} (3 spacecraft, drag-free)
- **Earth-based tests:** Local Position Invariance
  (U/c^2 daily amplitude: $\sim 4.10^{-13}$, yearly amplitude $\sim 2.10^{-10}$)
  Bauch and Weyers (2002), upcoming results with Cs & optical clocks

Theoretical Models

- *Damour and Nordtvedt (1993), Damour (1999), Damour, Piazza, Veneziano (2002):*
  existence of scalar fields (dilaton) that violate EP, strength: γ -1
  - model takes into account inflation and WMAP measurements;
  - γ is time-dependent, =0 in early universe, nearly 1 now; $1-\gamma \sim 5.10^{-5} – 5.10^{-8}$
  - *Within the dilaton model, the earth-based Equivalence Principle tests have already shown $|1-\gamma|<2.10^{-7}$ (to be improved by MICROSCOPE), and predict $d \ln \alpha/dt < 10^{-20}/yr$*
  - *But EP tests and γ measurement only probe hadronic matter and Coulomb energy; hyperfine and molecular clocks also probe leptonic matter (electron mass)*

- Alternative explanation to Dark Energy: extension of GR in the low-energy regime *(Carroll et al. 2004)*, $1-\gamma \sim 10^{-9} – 5.10^{-7}$
- *Sandvik et al (2002):* Local Position Invariance for α may be violated at level $\sim 10^{-4}$ (ruled out now)
- *See also Lämmerzahl (2006), Turyshev et al., in Dittus et al. (2007)*

Second-order effects

- Achievable values of U/c^2 in the solar system are of order

  1.10^{-8} for a spacecraft going to Mercury or outer planets

  3.10^{-7} for a spacecraft approaching sun

  1.10^{-6} for a wave grazing the sun

- Clocks of 1.10^{-18} accuracy, would allow a test of GR at 10^{-10} level

- Effects of second order in U/c^2 (still in „weak-field" regime)
- Achievable values of U/c^2 in our solar system imply that resolution of measurement must be 1.10^{-12} or better

  - ASTROD, LATOR, Gravitiational Time Delay would be sensitive to second-order effects (probe sun field and aim at relative accuracies of measured PN parameter beyond 1.10^{-6})
- Clocks could also allow a sensitive test of second-order effects

„The most precise test of general relativity"

$$\nu_i / \nu_0 = \sqrt{\frac{1 + 2U(r_i)/c^2}{1 + 2U(r_0)/c^2}}$$

Gravitational Redshift and PN formalism

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- Contribution to redshift from the two PN parameters β, γ in a fully conservative metric theory without preferred location effects (*see Teyssandier et al (2007)*)

$$\frac{\nu_1 - \nu_2}{\nu} = \ldots - (1+\gamma)\left(\frac{U(r_1)}{c^2}\frac{v_1^2}{c^2} - \frac{U(r_2)}{c^2}\frac{v_2^2}{c^2} \right) + (\beta - 1)\left(\frac{U(r_1)^2}{c^4} - \frac{U(r_2)^2}{c^4} \right)$$

- Present accuracy of β (2.10^{-4}) and γ (2.10^{-5}) rules out any effects observable with clocks for solar-system level U
- Clocks test a different sector of the theory:

  LPI violation, theories beyond PN theory

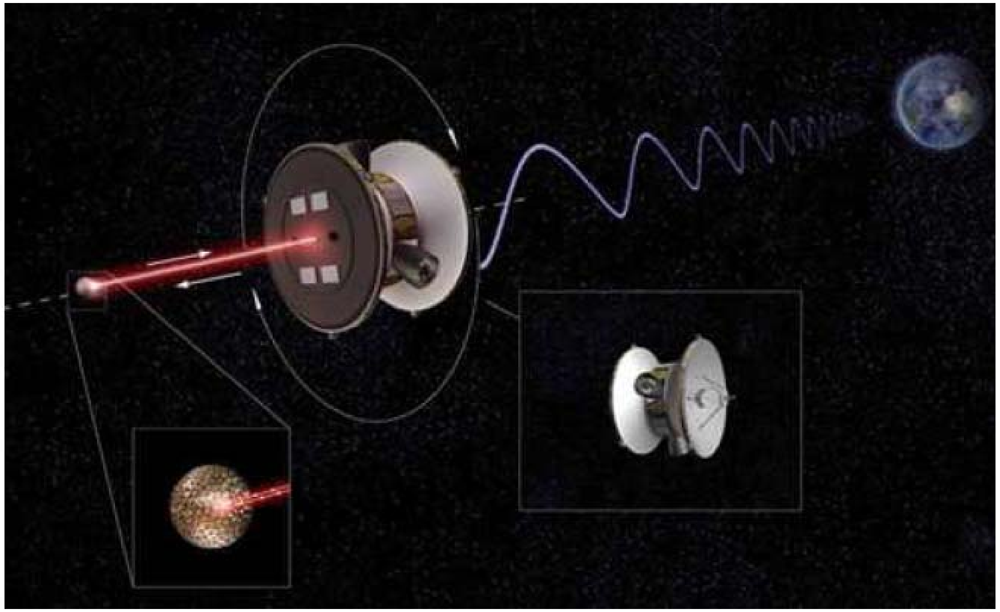
Complexity and Cost

- Drivers:
  - Number of spacecrafts: 1, 2 or 3
  - Distance of travel from earth
  - Type and number of dissimilar clocks
  - Frequency link to earth or between spacecrafts
    - no link: only Local Position Invariance test
    - link: also absolute gravitational redshift
    - link to earth: limited by inaccuracy of gravitational potential on earth ($\sim 10^{-18}$, similar to expected clock accuracy)
  - Drag-free
  - Additional measurements (e.g. Pioneer anomaly, Lorentz Invariance, geophysics, orbit dynamics)

Mission to outer solar system - Pioneer anomaly

Dittus et al., Firenze (2006)

- Main measurement: ranging of spacecraft while at large distance from earth (main s/c + free-flyer)
- Clock on board to sense anomalous acceleration: to achieve 1% accuracy in the anomalous acceleration, need a clock of 10^{-15} long-term (~ 10 years) accuracy



- Additional payload: optical clock would enable accurate measurement of gravitational redshift over planetary distance (first section of voyage) ($\Delta U/c^2 \sim 1.10^{-8}$ relative to earth; earth gravitational potential limits accuracy at 1.10^{-18}, allowing 1% of second-order contribution)
- Link at 1.10^{-18} over inter-planetary distances possible?
- Need to know distances to sun with $\Delta r_{earth\text{-}sun} \sim 15$ m, $\Delta r_{s/c\text{-}sun} \sim 140$ m, achievable
- LPI test could test second order-effect at 1%

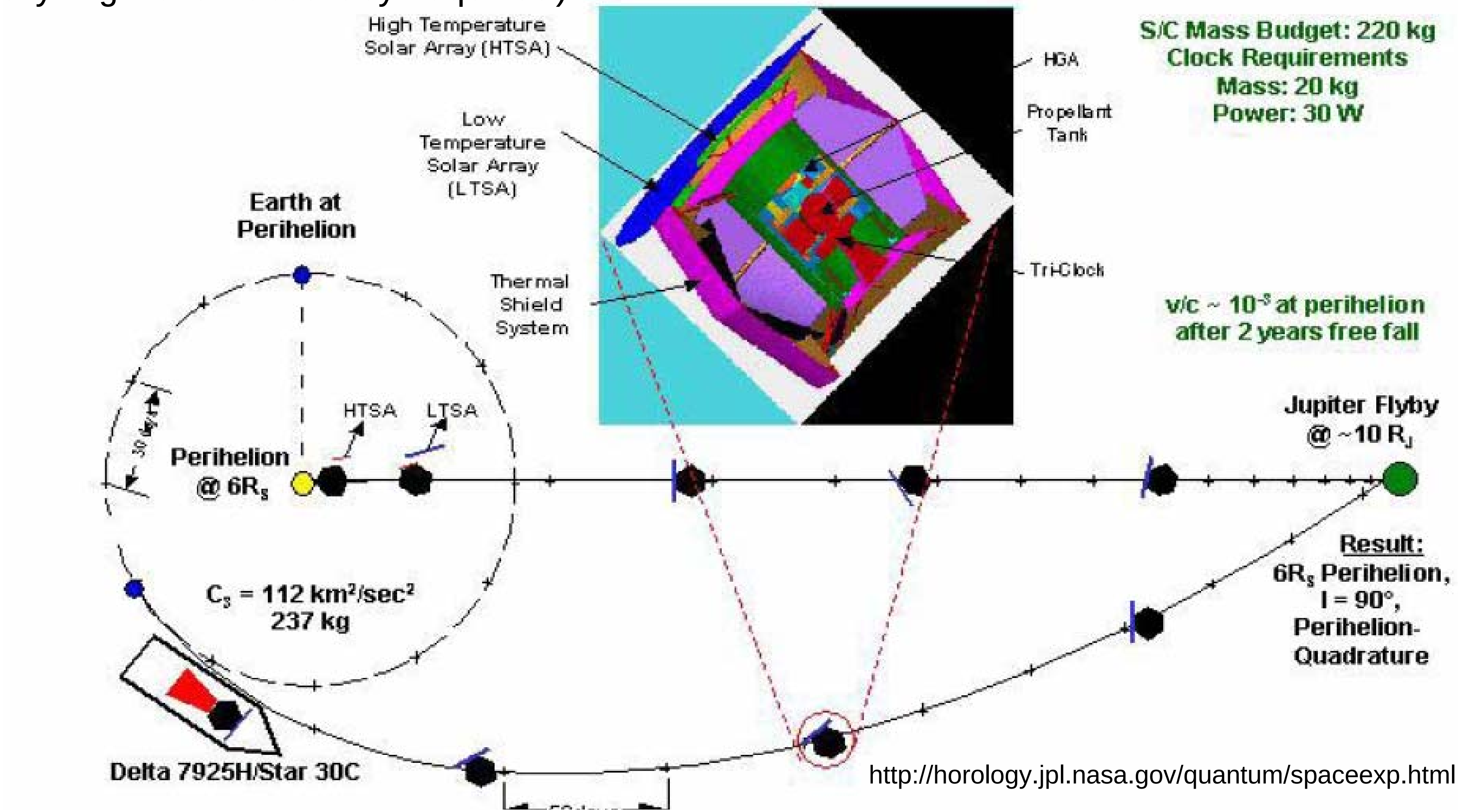
Mission to inner solar system

- Flight to mercury provides $\Delta U/c^2 \sim 1.5\ 10^{-8}$, $\frac{1}{2}\ \Delta(U/c^2)^2 \sim 2.10^{-16}$.
  - Comparison with earth clock can test second-order effects at 1% level
  - LPI test at 1 % of second-order
  - Interplanetary link at 1.10^{-18} possible?
- Need to know distances to sun with $\Delta r_{earth\text{-}sun} \sim 15$ m, $\Delta r_{s/c\text{-}sun} \sim 2$ m, achievable
- Additional science goal: combine with time delay measurement when s/c is in conjunction
  - From ground: ASTROD I-type mission, $\Delta\gamma$ at $\sim 1.10^{-7}$, $\Delta\beta$ at $\sim 1.10^{-7}$
  - add second spacecraft; GTD-type mission, $\gamma \sim 1.10^{-8}$

Solar Fly-by (SpaceTime Mission)

Maleki et al. - JPL

- Flyby at 6 solar radii gives a potential variation ~ 3.10^{-7} along orbit;
  LPI test using microwave ion clocks (room-temperature)
- Optical clocks would be an alternative, allowing LPI test at 10^{-11} level
- Gravitational redshift measurement making use of the full $\Delta U/c^2$ seems too difficult (very high orbit accuracy required)



http://horology.jpl.nasa.gov/quantum/spaceexp.html

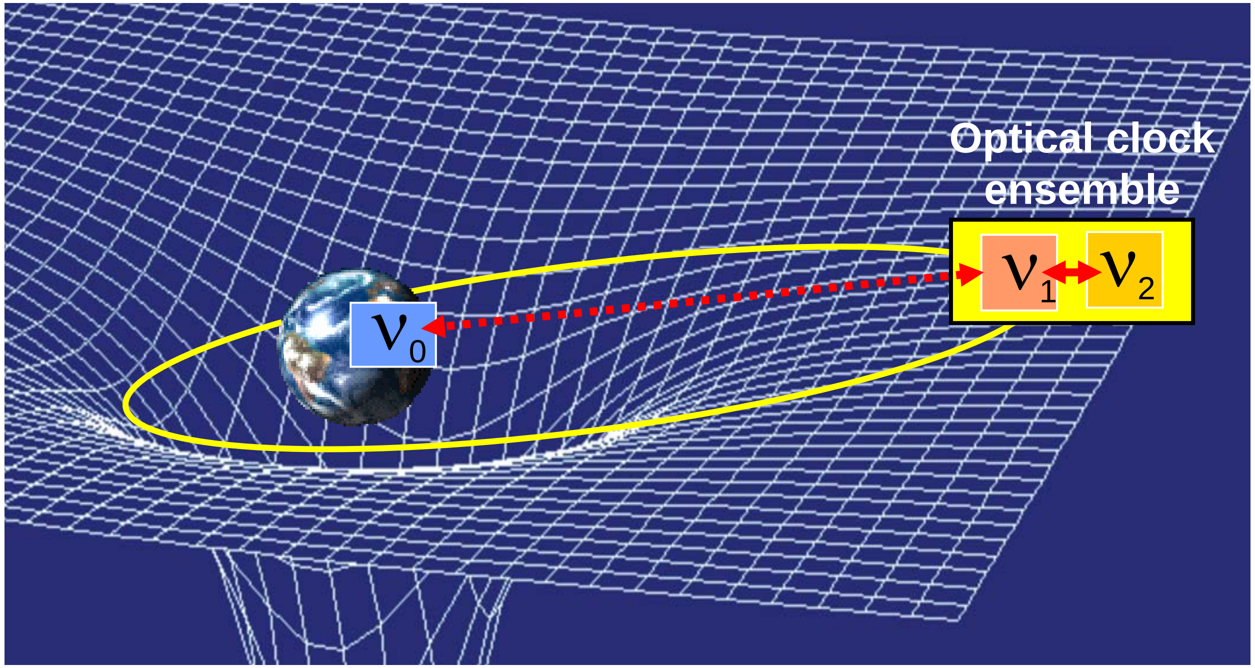
Earth orbit mission

- A constant distance, high-altitude earth orbit, e.g. geostationary: $\Delta U/c^2 \sim 6.10^{-10}$, ½ $\Delta(U/c^2)^2 \sim 2.10^{-19}$
  - But: current uncertainty in earth gravitational potential (~ 1 cm) implies a $\sim 1.10^{-18}$ uncertainty
  - Future clocks and improved earth models could measure 2nd-order effect
  - Highly ellipitic orbit: avoids earth gravitational potential uncertainty, as long as earth potential is constant to fraction of % over orbital period (~ 0.5 d);
  - Such an orbit also allows LPI test
  - variation in U is few 10^{-10}, so test barely at second-order level (averaging over many orbits)

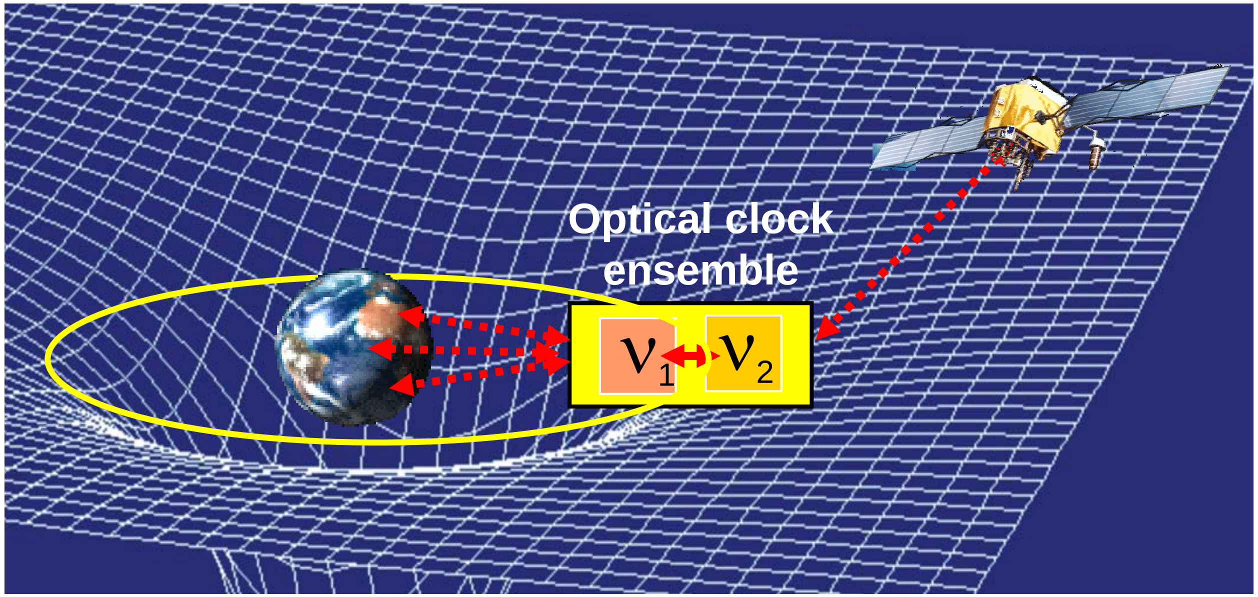
Gravity Explorer

Schiller et al, (2005, 2007)

- Orbital phase I (~ 1 year duration, highly elliptic orbit)
  - Test of Local Position Invariance and of grav. Redshift (2.10^{-10} amplitude)

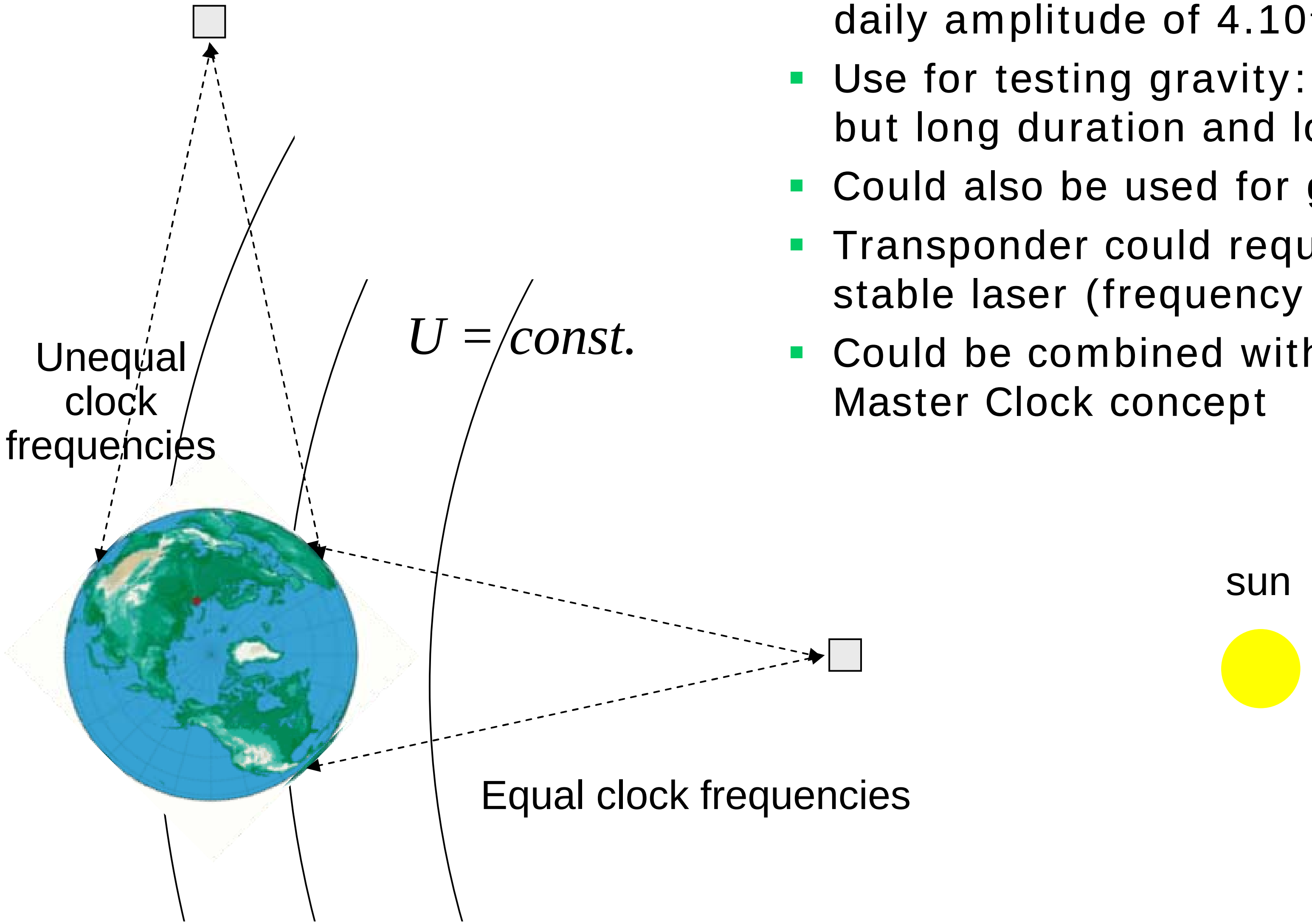


- Orbital phase II (geostationary, several years duration)
  - Master clock for earth and space users
  - Geophysics
  - Ground clock comparison (sun redshift ampl. 4.10^{-13})
  - LPI & Redshift in sun field (amplitude 2.10^{-12})



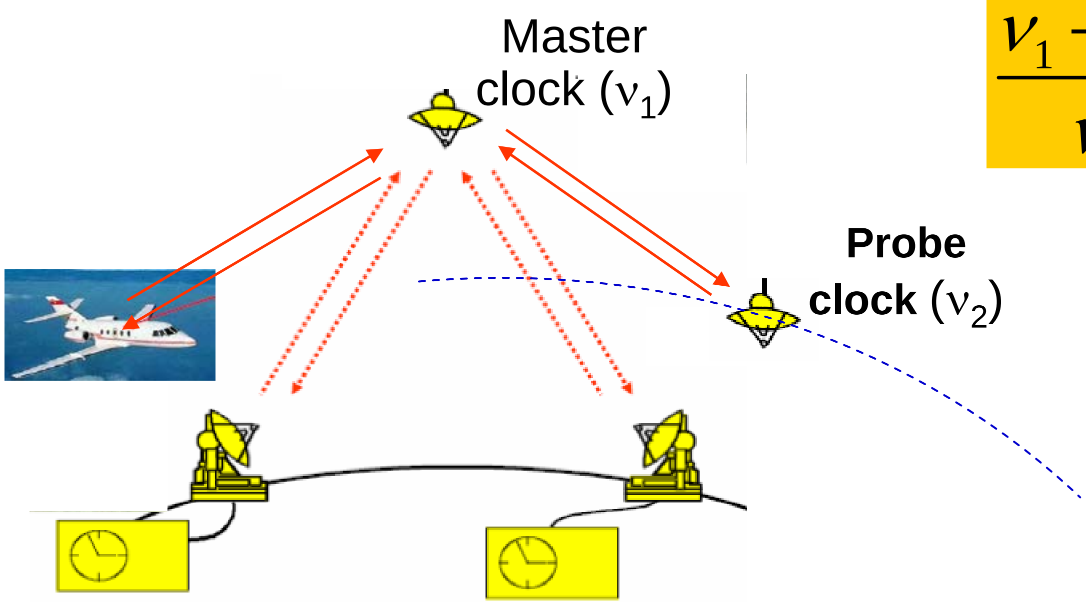
Ground clock comparisons

- Transponder satellite for terrestrial clock comparison
- Solar clock redshift has daily amplitude of 4.10^{-13}
- Use for testing gravity: small effect but long duration and lower cost
- Could also be used for geophysics
- Transponder could require a stable laser (frequency comb?)
- Could be combined with Master Clock concept



Clock comparisons

$$\frac{\nu_1 - \nu_2}{\nu} = \frac{U(r_1) - U(r_2)}{c^2}$$



- Comparison between master and probe clock *(via microwave/optical link)*
  - yields information about gravitational potential U

- Features: - non-local measurement (cf. with two-satellite measurements such as GRACE or SSI)
  - measurement time to reach 10^{-18} ~ 10 h (? - limited by link)
  - independent of satellite acceleration

Gravity (geopotential) measurements

A clock accuracy $\frac{\Delta \nu}{\nu} = 10^{-18}$

yields $\Delta\left(\frac{U(r)}{c^2}\right) = 10^{-18} \Rightarrow \frac{\Delta U}{U} \simeq 10^{-9}$ near earth's surface
(equivalent to 1 cm height change)

This requires a position accuracy:

$$\Delta r = \begin{cases} 30\,\text{cm} & \text{for a geostationary orbit} \\ 1\,\text{cm} & \text{for a LEO orbit} \end{cases}$$

Orbitography at a level of 3 cm via GPS is available,
- potential for improvement using laser ranging?

Compare:

GOCE: Measurement of *g* with 1 mGal = 10^{-5} m/s^2 resolution, i.e. $\Delta g/g \sim 10^{-6}$
Absolute (corner-cube) gravimeters: resolution $\Delta g/g \sim 10^{-9}$
Superconducting gravimeters (stationary): resolution $\Delta g/g \sim 10^{-11}$

Geophysics perspectives

- Desirable is a geoid measurement with better than 1 mm accuracy (Sumatra earthquake produced geoid variations of -6 to +12 mm across the fault).
- This requires clocks with accuracy $$\frac{\Delta\nu}{\nu} \simeq 10^{-19}$$

Further possible goals of a clock mission

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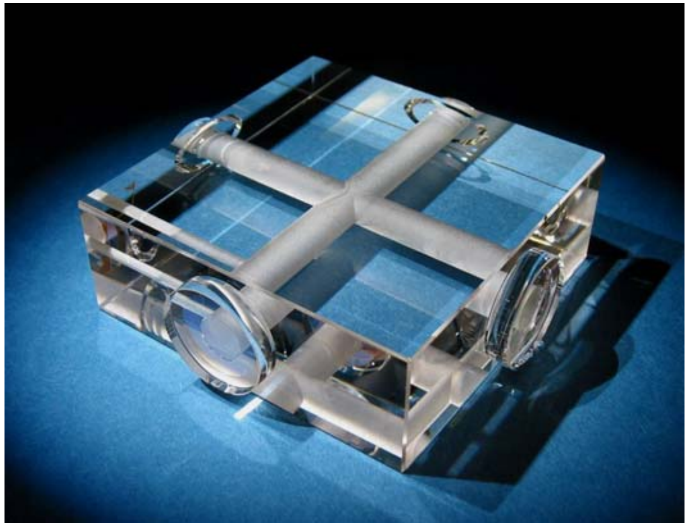
- Test of isotropy of space (Michelson-Morley expt.):
  requires an additional optical cavity & optoelec. components
- Test of constancy of speed of light (Kennedy-Thorndike expt.):
  no additional components
- Test of Lorentz Invariance (large s/c velocity helpful)
- For particular earth orbits: test of Lense-Thirring effect (laser ranging retroreflectors; requires drag-free satellite)
- *See OPTIS proposal (Lämmerzahl et al 2001, 2004, Iorio et al 2004)*



C. Eisele, A. Nevsky, M. Okhapkin, S.S.

Clocks for Local Position Invariance Tests

- Frequencies depend on fundamental constants

$$\nu_i = \nu_i(\alpha, \alpha_S, G_F, m_e, m_N, g_N, ...)$$

- Gravitational redshift experiments test whether some of these constants depend on the gravitational potential

$$\beta_j = \beta_j(U)? \quad \Rightarrow \quad \zeta_i = 1 + \sum_j \left(\nu_i^{-1} \frac{d\nu_i}{d\beta_j} \right) \left(\frac{d\beta_j}{d(U/c^2)} \right)$$

- Some constants can be related to more fundamental constants:

$$\alpha$$

Electromagnetic interaction

$$m_p \propto \Lambda_{QCD} + corrections\,(m_q, m_s)$$

$$g_N = g_N(m_q / \Lambda_{QCD}, m_s / \Lambda_{QCD})$$

Strong interaction

$$m_e \propto \langle \phi \rangle \quad = \quad Higgs\ vacuum\ field$$

Weak interaction

$$\frac{\Delta(m_N/m_p)}{m_N/m_p} = c_\alpha \frac{\Delta\alpha}{\alpha} + c_\phi \frac{\Delta\phi}{\phi} + c_\Lambda \frac{\Delta\Lambda_{QCD}}{\Lambda_{QCD}} \qquad c_\alpha, c_\phi, c_\Lambda \simeq O(10^{-3})$$

Optical clock types

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Completeness:

Scaling of transition energies (in units of Rydberg energy)

- Hyperfine energies [1,2] $g \frac{m_e}{m_p} \alpha^2 F(Z\alpha)$
- Electronic energies (incl. relativistic effects) $G(\alpha)$ →
- Vibrational energies in molecules [3,4] $\sqrt{m_e/m_N}$
- Rotational energies in molecules m_e/m_N
- Cavity frequency α^{-1}
- Nuclear energies [5] $H(\alpha, m_q/\Lambda_{QCD}, m_s/\Lambda_{QCD})$

Yb:	0.31
Sr:	0.06
Yb^+:	(0.9, - 5.3)

Electronic transitions do not furnish a complete test!
Complement with hyperfine, nuclear, or molecular clock

[1] Microwave cold atom clocks (PHARAO)

[2] (near-optical) range: highly charged atomic ions (S. Schiller, 2007)

[3] L. Hilico et al. (2000)

[4] S. Schiller and V. Korobov (2005)

[5] V. Flambaum (2006)

Clock choice

- A comparison of an atomic optical clock to a molecular optical clock is (within the Standard Model) sensitive to largest number of fundamental constants

- In gauge unification theories the dependencies of α and Λ_{QCD} on U are correlated
(Damour 1999, Langacker et al, Calmet & Fritzsch. 2002)

$$\frac{\Delta\Lambda_{QCD}}{\Lambda_{QCD}} \simeq 40\frac{\Delta\alpha}{\alpha}$$

- **Enhancement effects are desirable**
  - Sensitivity of nuclear transition frequencies to α, m_s, m_q are predicted to have ~10^5-fold enhancement (Flambaum, 2006)
  - Other systems?

Space suitability

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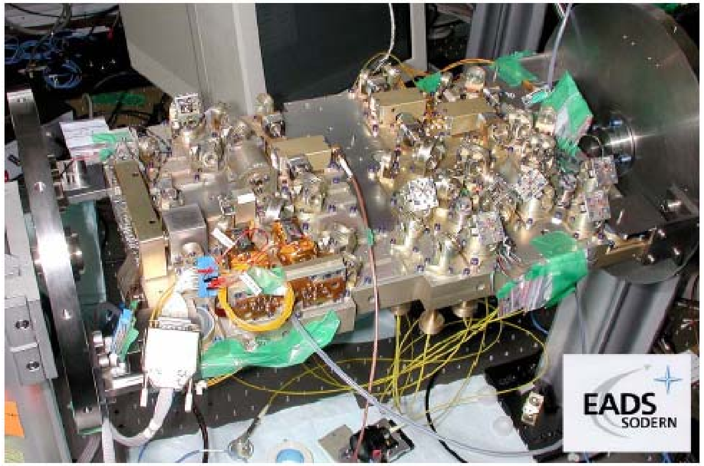
- Important optical clock components are already space-qualified
  - Single-frequency diode lasers (PHARAO)
  - Ultracold atom sources (PHARAO)
  - Opto-electronic components
  - Solid-state lasers and amplifiers (TESAT Spacecom)
  - Optical resonators (TESAT Spacecom)
  - Phase-locking (TESAT Spacecom)



Mass 22 kg, power 65 W

- Further optical technology on LISA Pathfinder
- Studies toward space qualification and space uses of frequency combs are under way (DLR, ESA)
- Ultracold atoms in free fall studies (Bose-Einstein Condensate) at ZARM Bremen (DLR)
- High-precision time transfer between satellites and earth to be tested in upcoming missions (ACES on ISS, T2L2 on JASON 2)
- Optical link experiments (LCT TerraSAR, LOLA,...)
- Quantum information research is likely to produce important technology also for compact optical clocks

Summary

- Different scenarios – different science output
  & costs
- LPI test only: „simplest“
- A complete test should include an absolute redshift measurement
  new component: link
- A powerful test requires measuring the 2nd-order contributions
  spacecraft needs to fly far away from earth
  higher cost
- Combination with additional science goals
  spacecraft bigger and more expensive
  larger community

Cost, complexity, science output

Gravity Explorer
(earth orbit)

Gravity Explorer
(interplanetary orbit)

Optical Clocks +
ASTROD I

Optical Clocks +
Deep Space
Gravity Probe

Summary – Gravity Explorer proposal

S.S. et al. arxiv:gr-qc/0608081

- **Fundamental physics goals:**
  (using clocks/links with 10^{-18} instability/accuracy)
  - Measure gravitational redshift with ~ 10^4 higher accuracy
  - Test higher-order relativistic effects in frequency comparison
  - Measure 2nd order Doppler effect with ~ 10^2 higher accuracy
  - Test independence of fine structure constant α on U with 10^2 higher accuracy*
  - Test independence of m_e/m_p on U with 10^2 higher accuracy*
  - **Additional possibilities**
    - With drag-free satellite, measure Lense-Thirring effect and perigee advance, ~10 times more accurately
    - Contribution to tests of time-independence of fundamental constants
    - Test of isotropy of speed of light (requires rotating satellite)
    - Other Local Lorentz Invariance tests
- **Gravity mapping**
  - Enable gravitational potential measurements at 2.10^{-10} resol. (1 mm equiv.), requires clocks at 10^{-19} accuracy
- **Master clock for earth and space applications**
- **Enable distant ground clock comparisons**
- **Technology demonstration and validation**

*compared to future terrestrial experiments