S&P INDICES

Index Mathematics Methodology

January 2012

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Introduction

This document covers the mathematics of index calculations. To understand and successfully use indices for investment analyses, it is important to know how they are calculated and how adjustments are made when constituents change or when different kinds of corporate actions occur. While actual index calculations are done almost entirely by computer, utilizing a wide range of programs, algorithms and routines, the underlying math is fairly straightforward. This document assumes some acquaintance with mathematical notation and simple operations, and little more. At the same time, the calculations are presented principally as equations, with a few examples or tables of results. The equations – which have largely been excluded from the individual index methodologies – are the most efficient way to present the information.

Different Varieties of Indices

A majority of S&P's equity indices are market cap weighted and float-adjusted, where each stock's weight in the index is proportional to its float-adjusted market value. The first section covers these indices. A second group of indices are equal weighted, where each stock is weighted equally in the index. These are discussed in the second section. A third group of indices are weighted by other factors, such as maximum weight restrictions or certain attributes used to choose the stocks. These are discussed in sections three, four and five. A fourth group of indices are leverage and inverse indices, which return positive or negative multiples of their underlying indices. These are discussed in section six.

There are also calculations that operate on an index as a whole rather than on the individual stocks. These include calculations of total returns and index fundamentals. These are covered in sections seven and eight below.

Fund strategy indices include portfolios of funds. These indices are covered in section nine.

Dividend indices, which track the total dividend payments of index constituents, and excess return indices, which calculate the return on an investment in an index where the investment was made through the use of borrowed funds, are covered in sections 10 and 11, respectively.

Finally, information on the methodology for risk control and currency hedged indices are covered in the 12^{th} and 13^{th} sections of this document.

The Index Divisor

Throughout all the calculations there is one concept that is crucially important to understanding how indices are calculated – the index divisor.

The simplest capitalization weighted index can be thought of as a portfolio consisting of all available shares of the stocks in the index. While one might track this portfolio's value in dollar terms, it would probably be an unwieldy number – for example, the S&P 500 float-adjusted market value is roughly US$ 11.0 trillion. Rather than deal with ten or more digits, the figure is scaled to a more easily handled number, currently around 1200. Dividing the portfolio market value by a factor, usually called the divisor, does the scaling.

An index is not exactly the same as a portfolio. For instance, when a stock is added to or deleted from an index, the index level should not jump up or drop down; while a portfolio's value would usually change as stocks are swapped in and out. To assure that the index's value, or level, does not change when stocks are added or deleted, the divisor is adjusted to offset the change in market value of the index. Thus, the divisor plays a critical role in the index's ability to provide a continuous measure of market valuation when faced with changes to the stocks included in the index. In a similar manner, some corporate actions that cause changes in the market value of the stocks in an index should not be reflected in the index level. Adjustments are made to the divisor to eliminate the impact of these corporate actions.

Capitalization Weighted Indices

Most of S&P's indices, indeed most widely quoted stock indices, are capitalization-weighted indices. Sometimes these are called value-weighted or market cap weighted instead of capitalization weighted. Examples include the S&P 500, the S&P Global 1200 and the S&P BMI indices. Examples from other index providers (where some of the details may vary slightly from those described here) include MSCI's indices, FTSE's indices and Russell's indices. While Dow Jones does offer market cap weighted indices, the well-known Dow Jones Industrial Average is not cap-weighted.

In the discussion below most of the examples refer to the S&P 500 but apply equally to a long list of S&P cap-weighted indices.

Definition

The formula to calculate the S&P 500 is:

$$Index\ Level = \frac{\sum_i P_i * Q_i}{Divisor} \quad (1)$$

The numerator on the right hand side is the price of each stock in the index multiplied by the number of shares used in the index calculation. This is summed across all the stocks in the index. The denominator is the divisor. If the sum in the numerator is US$ 11.8 trillion and the divisor is US$ 9.4 billion, the index level would be 1250, close to the current levels of the S&P 500.

This index formula is sometimes called a "base-weighted aggregative" method.[1] The formula is created by a modification of a *LasPeyres* index, which uses base period quantities (share counts) to calculate the price change. A *LasPeyres* index would be:

$$Index = \frac{\sum_i P_{i,1} * Q_{i,o}}{\sum_i P_{i,0} * Q_{i,o}} \quad (2)$$

[1] This term is used in one of the earlier and more complete descriptions of S&P index calculations in Alfred Cowles, *Common Stock Indices*, Principia Press for the Cowles Commission of Research in Economics, 1939. The book refers to the "Standard Statistics Company Formula;" S&P was formed by the merger of Standard Statistics Corporation and Poor's Publishing in 1941.

In the modification to (2), the quantity measure in the numerator, Q_0, is replaced by Q_1, so the numerator becomes a measure of the current market value, and the product in the denominator is replaced by the divisor which both represents the initial market value and sets the base value for the index. The result of these modifications is equation (1) above.

Adjustments to Share Counts

S&P Indices' market cap-weighted indices are float adjusted – the number of shares outstanding is reduced to exclude closely held shares from the index calculation because such shares are not available to investors. S&P's rules for float adjustment are described in more detail in *S&P Indices' Float Adjustment Methodology* or in some of the individual index methodology documents. As discussed there, for each stock S&P calculates an Investable Weight Factor (IWF) which is the percentage of total shares outstanding that are included in the index calculation. When the index is calculated using equation (1), the variable Q_i is replaced by the product of outstanding shares and the IWF:

$$Q_i = IWF_i * Total\ Shares_i \qquad (3a)$$

At times there are other adjustments made to the share count to reflect foreign ownership restrictions or to adjust the weight of a stock in an index. These are combined into a single multiplier in place of the IWF in equation (3a). In combining restrictions it is important to avoid unwanted double counting. Let FA represent the fraction of shares eliminated due to float adjustment, FR represent the fraction of shares excluded for foreign ownership restrictions and IS represent the fraction of total shares to be excluded based on the combination of FA and FR.

$$\text{If } FA > FR \text{ then } IS = 1 - FA, \qquad (4a)$$
$$\text{If } FA < FR \text{ then } IS = 1 - FR \qquad (4b)$$

and equation (3a) can be written as:

$$Q_i = IS_i * Total\ Shares_i \qquad (3b)$$

Note that any time the share count or the IWF is changed, it will be necessary to adjust the index divisor to keep the level of the index unchanged.

Divisor Adjustments

The key to index maintenance is the adjustment of the divisor. Index maintenance – reflecting changes in shares outstanding, capital actions, addition or deletion of stocks to the index – should not change the level of the index. If the S&P 500 closes at 1250 and one stock is replaced by another, after the market close, the index should open at 1250 the next morning if all of the opening prices are the same as the previous day's closing prices. This is accomplished with an adjustment to the divisor.

Any change to the stocks in the index that alters the total market value of the index while holding stock prices constant will require a divisor adjustment. This section explains how the divisor adjustment is made given the change in total market value. The next section discusses what index changes and corporate actions lead to changes in total market value and the divisor.

Equation (1) is expanded to show the stock being removed, stock *r*, separately from the stocks that will remain in the index:

$$Index\ Level_{t-1} = \frac{(\sum_i P_i * Q_i) + P_r Q_r}{Divisor_{t-1}} \qquad (5)$$

Note that the index level and the divisor are now labeled for the time period t-1 and, to simplify this example, that we are ignoring any possible IWF and adjustments to share counts. After stock *r* is replaced with stock *s*, the equation will read:

$$Index\ Level_{t} = \frac{(\sum_i P_i * Q_i) + P_s Q_s}{Divisor_{t}} \qquad (6)$$

In equations (5) and (6) *t-1* is the moment right before company *r* is removed from and *s* is added to the index; *t* is the moment right after the event. By design, $Index\ Level_{t-1}$ is equal to $Index\ Level_t$. Combining (5) and (6) and re-arranging, the adjustment to the Divisor can be determined from the index market value before and after the change:

$$\frac{(\sum_i P_i * Q_i) + P_r Q_r}{Divisor_{t-1}} = Index\ Level = \frac{(\sum_i P_i * Q_i) + P_s Q_s}{Divisor_{t}} \qquad (7)$$

Let the numerator of the left hand fraction be called MV_{t-1}, for the index market value at (*t-1*), and the numerator of the right hand fraction be called MV_t, for the index market value at time *t*. Now, MV_{t-1}, MV_t and $Divisor_{t-1}$ are all known quantities. Given these, it is easy to determine the new divisor that will keep the index level constant when stock *r* is replaced by stock *s*:

$$Divisor_t = (Divisor_{t-1}) * \frac{MV_t}{MV_{t-1}} \qquad (8)$$

As discussed below, various index adjustments result in changes to the index market value. When these adjustments occur, the divisor is adjusted as shown in equation (8).

In some implementations, including the computer programs used in S&P's index calculations, the divisor adjustment is calculated in a slightly different, but equivalent, format where the divisor change is calculated by addition rather than multiplication. This alternative format is defined here. Rearranging equation (1) and using the term MV (market value) to replace the summation gives:

$$Divisor = \frac{MV}{Index\,Level} \qquad (9)$$

When stocks are added to or deleted from an index there is an increase or decrease in the index's market value. This increase or decrease is the market value of the stocks being added less the market value of those stocks deleted; define CMV as the Change in Market Value. Recalling that the index level does not change, the new divisor is defined as:

$$Divisor_{New} = \frac{MV + CMV}{Index\,Level} \qquad (10)$$

or

$$Divisor_{New} = \frac{MV}{IndexLevel} + \frac{CMV}{IndexLevel} \qquad (11)$$

However, the first term on the right hand side is simply the Divisor value before the addition or deletion of the stocks. This yields:

$$Divisor_{New} = Divisor_{Old} + \frac{CMV}{IndexLevel} \qquad (12)$$

Note that this form is more versatile for computer implementations. With this additive form, the second term (*CMV/Index Level*) can be calculated for each stock or other adjustment independently and then all the adjustments can be combined into one change to the Divisor.

Necessary Divisor Adjustments

Divisor adjustments are made "after the close" meaning that after the close of trading the closing prices are used to calculate the new divisor based on whatever changes are being made. It is, then, possible to provide two complete descriptions of the index – one as it existed at the close of trading and one as it will exist at the next opening of trading. If the same stock prices are used to calculate the index level for these two descriptions, the index levels are the same.

With prices constant, any change that changes the total market value included in the index will require a divisor change. For cataloging changes, it is useful to separate changes caused by the management of the index from those stemming from corporate actions of the constituent companies. Among those changes driven by index management are adding or deleting companies, adjusting share counts and changes to IWFs and other factors affecting share counts or stock prices.

Index Management Related Changes: When a company is added to or deleted from the index, the net change in the market value of the index is calculated and this is used to calculate the new divisor. The market values of stocks being added or deleted are based on the prices, shares outstanding, IWFs and any other share count adjustments. Specifically, if a company being added has a total market cap of US$ 1 billion, an IWF of 85% and, therefore, a float adjusted market cap of US$ 850 million, the market value for the added company used is US$ 850 million. The calculations would be based on either equation (8) or equation (12) above.

For most S&P indices, there are a few dates during the year when IWFs and share counts are updated. (Typically small changes in shares outstanding are reflected in indices once a quarter to avoid excessive changes to an index.) The revisions to the divisor resulting from these are calculated and a new divisor is determined. Equation (12) shows how the impact of a series of share count changes can be combined to determine the new divisor.

Corporate Action Related Changes: There are a large range of different corporate actions ranging from routine share issuances or buy backs to unusual events like spin-offs or mergers. These are listed on the table below with notes about the necessary changes and whether the divisor is adjusted.

Corporate Action	Comments	Divisor Adjustment
Company added/deleted	Net change in market value determines the divisor adjustment.	Yes
Change in shares outstanding	Any combination of secondary issuance, share repurchase or buy back – share counts revised to reflect change.	Yes
Stock split	Share count revised to reflect new count. Divisor adjustment is not required since the share count and price changes are offsetting.	No
Spin-off	If the spun-off company is not being added to the index, the divisor adjustment reflects the decline in index market value (i.e., the value of the spun-off unit).	Yes
Spin-off	Spun-off company added to the index, no company removed from the index.	No
Spin-off	Spun-off company added to the index, another company removed to keep number of names fixed. Divisor adjustment reflects deletion.	Yes
Change in IWF	Increasing (decreasing) the IWF increases (decreases) the total market value of the index. The divisor change reflects the change in market value caused by the change to an IWF.	Yes
Special Dividend	When a company pays a special dividend the share price is assumed to drop by the amount of the dividend; the divisor adjustment reflects this drop in index market value.	Yes
Rights offering	Each shareholder receives the right to buy a proportional number of additional shares at a set (often discounted) price. The calculation assumes that the offering is fully subscribed. Divisor adjustment reflects increase in market cap measured as the shares issued multiplied by the price paid.	Yes

With corporate actions where cash or other corporate assets are distributed to shareholders, the price of the stock will gap down on the ex-dividend day (the first day when a new shareholder is not eligible to receive the distribution.) The effect of the divisor adjustment is to prevent this price drop from causing a corresponding drop in the index.

For more information on the treatment of corporate actions, please refer to the S&P Corporate Actions Policies & Practices Methodology.

Capped Indices

At times it is desirable to set a maximum weight for some stocks in an index. In some markets regulations restrict the weight of the largest stock or group of stocks to be less than a certain percentage of a portfolio. This is done by a further adjustment to the share count, beyond the investable weight factor. Since the total weight of all stocks in the index will add up to 100%, reducing the weight of one stock will increase the weight of the others. It is possible that when the largest stock's weight is brought down below some limit, the weight of the next largest – or several next largest – stocks will exceed the limit. Therefore, the process must be iterative. Weights will change over time as stock prices move even if share counts remain constant. If a capped stock enjoys a price run-up, it may exceed the cap. In most cases buffer zones are used – e.g., if the maximum allowable weight is 10% the stock's shares are adjusted downward until its weight is 9% leaving a one-percentage point buffer before another adjustment is necessary. Various forms of capped indices are covered in the next three sections of this document.

Equal Weighted Indices

Definition

An equal weighted index is one where every stock has the same weight in the index, and a portfolio that tracks the index will invest an equal dollar amount in each security. As stock prices move, the weights will shift and exact equality will be lost. Therefore, an equal weighted index must be rebalanced from time to time to re-establish the proper weighting. (In contrast, a cap-weighted index requires no rebalancing as long as there aren't any changes to share counts, IWFs, returns of capital, or stocks added or deleted.)

The overall approach to calculate equal weighted indices is the same as in the cap-weighted indices; however, the constituents' market values are re-defined to be values that will achieve equal weighting at each rebalancing. Recall two basic formulae:

$$Index\ Level = \frac{Index\ Market\ Value}{Divisor} \qquad (13)$$

and

$$Index\ Market\ Value = \sum_i P_i * Shares_i * IWF_i \qquad (14)$$

To calculate an equal weighted index, the market capitalization for each stock used in the calculation of the index is redefined so that each index constituent has an equal weight in the index at each rebalancing date. In addition to being the product of the stock price, the stock's shares outstanding, and the stock's float factor (IWF), as written above – and the exchange rate when applicable – a new adjustment factor is also introduced in the market capitalization calculation to establish equal weighting.

$$AdjustedStock\ Market\ Value_i = P_i * Shares_i * IWF_i * FxRate_i * AWF_i \qquad (15)$$

where AWF_i is the adjustment factor of stock *i* assigned at each index rebalancing date, *t,* which makes all index constituents modified market capitalization equal (and, therefore, equal weight), while maintaining the total market value of the overall index. The *AWF* for each index constituent, *i*, at rebalancing date, *t*, is calculated by:

$$AWF_{i,t} = \frac{Z}{N * FloatAdjustedMarketValue_{i,t}} \qquad (16)$$

where N is the number of stocks in the index and Z is an index specific constant set for the purpose of deriving the AWF and, therefore, each stock's share count used in the index calculation (often referred to as modified index shares).

The index divisor is defined based on the index level and market value from equation (13). The index level is not altered by index rebalancings. However, since prices and outstanding shares will have changed since the last rebalancing, the divisor will change at the rebalancing.

So:

$$(Divisor)_{after\ rebalancing} = \frac{(Index\ Market\ Value)_{after\ rebalancing}}{(Index\ Value)_{before\ rebalancing}} \quad (16a)$$

where,

$$Index\ Market\ Value = \sum_i P_i * Shares_i * IWF_i * FxRate_i * AWF_i \quad (16b)$$

Corporate Actions and Index Adjustments

The tables on the following page show the necessary adjustments to the index and the divisor for managing an equal weighted index. One key issue is how to handle events when one stock is replaced by another. Given that stock prices move all the time, the index is only truly equally weighted at the rebalancing. Therefore, when stocks are added or deleted either the new stock must assume the actual weight of the old stock or the entire index must be rebalanced. Since index rebalancing generates trading costs for tracker funds, the design decision is usually made to have a new stock assume the weight of the stock being dropped until the next rebalancing. However, this is not always the case and may vary by index family.

For more information on the treatment of corporate actions, please refer to the S&P Corporate Actions Policies & Practices Methodology. For more information on the specific treatment within an index family, please refer to that index methodology.

Index Actions

S&P parent index action	Adjustment made to the equal weight index	Divisor adjustment for the S&P EWI
Constituent change – even number of adds and drops	The company entering the index goes in at the weight of the company coming out. This weight is used to compute the adjusted weight factor of the added stock, using Equation 15. If a company is being removed at a price of 0.00, the replacement goes in at the weight of the deleted company at the close on the day before the effective date. If more than one company is being replaced in the index on a single date, the replacements would be in the order stated in the press release for the parent index change.	None
Constituent change – deletion only	The weights of all stocks in the index will change, due to the absolute change in the number of index constituents. Relative weights will stay the same.	Yes
Share changes between quarterly share adjustments	None. The adjustment factor is changed to keep the index weight the same.	None
Quarterly share changes	There is no direct adjustment.	None

Corporate Actions

S&P parent index action	Adjustment made to the equal weight index	Divisor adjustment for the S&P EWI
Spin-off	The price is adjusted to the Price of the Parent Company minus (the Price of Spin-off company/Share Exchange Ratio). The adjustment factor changes according to Equation 16, to maintain the weight to be the same as the company had before the spin-off.	None
Rights Offering	The price is adjusted to the Price of the Parent Company minus (the Price of Rights Offering/Rights Ratio). The adjustment factor changes according to Equation 16, to maintain the weight to be the same as the company had before the rights offering.	None
Stock Split	Shares are multiplied by and the price is divided by the split factor.	None
Share Issuance or Share Repurchase	None.	None
Special Dividends	The price of the stock making the special dividend payment is reduced by the per share special dividend amount after the close of trading on the day before the ex-date.	A divisor adjustment is made to ensure the index level remains the same.

Modified Market Capitalization Weighted Indices

Definition

A modified market cap weighted index is one where index constituents have a user-defined weight in the index. This methodology is typically used for indices where some constituents are confined to a maximum weight, and the excess weight is distributed proportionately among the remaining index constituents. Between index rebalancings, corporate actions generally have no affect on index weights, as they are fixed through the processes defined below. As stock prices move, the weights will shift and the modified weights will change. Therefore, as in the case of an equal-weighted index, a modified market cap weighted index must be rebalanced from time to time to re-establish the proper weighting.

The overall approach to calculate modified market cap weighted indices is the same as in the cap-weighted indices; however, the constituents' market values are re-defined to be values that will achieve the user-defined weighting at each rebalancing. Recall two basic formulae:

$$Index\ Level = \frac{Index\ Market\ Value}{Divisor} \qquad (17)$$

and

$$Index\ Market\ Value = \sum_i P_i * Shares_i * IWF_i * FxRate \qquad (18)$$

To calculate a modified market cap weighted index, the market capitalization for each stock used in the calculation of the index is redefined so that each index constituent has the appropriate user-defined weight in the index at each rebalancing date.

In addition to being the product of the stock price, the stock's shares outstanding, and the stock's float factor (IWF), as written above – and the exchange rate when applicable – a new adjustment factor is also introduced in the market capitalization calculation to establish the appropriate weighting.

$$Adjusted Stock\ Market\ Value_i = P_i * Shares_i * IWF_i * FxRate_i * AWF_i \qquad (19)$$

where AWF_i is the adjustment factor of stock *i* assigned at each index rebalancing date, *t,* which adjusts the market capitalization for all index constituents to achieve the user-defined weight, while maintaining the total market value of the overall index.

The *AWF* for each index constituent, *i*, on rebalancing date, *t*, is calculated by:

$$AWF_{i,t} = \frac{Z}{FloatAdjustedMarketValue_{i,t}} * W_{i,t} \qquad (20)$$

where *Z* is an index specific constant set for the purpose of deriving the *AWF* and, therefore, each stock's share count used in the index calculation (often referred to as modified index shares). $W_{i,t}$ is the user-defined weight of stock *i* on rebalancing date *t*.

The index divisor is defined based on the index level and market value from equation (17). The index level is not altered by index rebalancings. However, since prices and outstanding shares will have changed since the last rebalancing, the divisor will change at the rebalancing.

So:

$$(Divisor)_{after\ rebalancing} = \frac{(Index\ Market\ Value)_{after\ rebalancing}}{(Index\ Value)_{before\ rebalancing}} \qquad (20a)$$

where,

$$Index\ Market\ Value = \sum_i P_i * Shares_i * IWF_i * FxRate_i * AWF_i \qquad (20b)$$

Corporate Actions and Index Adjustments

The tables below shows the necessary adjustments to the index and the divisor for managing a modified market cap weighted index. *For more information on the treatment of corporate actions, please refer to the S&P Corporate Actions Policies & Practices Methodology. For more information on the specific treatment within an index family, please refer to that index methodology.*

Index Actions

S&P parent index action	**Adjustment made to the modified market cap weighted index**	**Divisor adjustment for the index?**
Constituent change	The company entering the index goes in at the weight of the company coming out.	None
Delisting, acquisition or any other corporate action resulting in a constituent deletion.	The stock is dropped from the Index	Yes
Share changes between quarterly share adjustments	None. The adjustment factor is changed to keep the index weight the same.	None
Quarterly share changes	There is no direct adjustment.	None

Corporate Actions

S&P parent index action	Adjustment made to the modified market cap weighted index	Divisor adjustment for the index?
Spin-off	The price is adjusted to the Price of the Parent Company minus (the Price of Spin-off company/Share Exchange Ratio). The adjustment factor changes according to Equation 20, to maintain the weight to be the same as the company had before the spin-off.	None
Rights Offering	The price is adjusted to the Price of the Parent Company minus (the Price of Rights Offering/Rights Ratio). The adjustment factor changes according to Equation 20, to maintain the weight to be the same as the company had before the rights offering.	None
Stock Split	Shares are multiplied by and the price is divided by the split factor.	None
Share Issuance or Share Repurchase	None.	None
Special Dividends	The price of the stock making the special dividend payment is reduced by the per share special dividend amount after the close of trading on the day before the ex-date.	A divisor adjustment is made to ensure the index level remains the same.
Merger or acquisition	If the surviving company is already an index member, it is retained in the index. If the surviving company does not meet index criteria, it is removed.	Yes, if there is a removal.

Capped Market Capitalization Indices

Definition

A capped market cap weighted index is one where single index constituents or defined groups of index constituents are confined to a maximum weight and the excess weight is distributed proportionately among the remaining index constituents. As stock prices move, the weights will shift and the modified weights will change. Therefore, as in the case of an equal-weighted index and a modified market cap index, a capped market cap weighted index must be rebalanced from time to time to re-establish the proper weighting. The methodology for capped indices proceeds similarly to that for modified market cap weighted indices. The main difference between the two methods is the treatment of corporate actions between rebalancing periods. For modified market cap weighted indices most corporate actions which affect the market capitalization of a given stock are counterbalanced by a corresponding change in the AWF assigned to the stock in that index, thus resulting in no weight change to the stock and no index divisor change (see the prior section for reference). On the other hand, for capped indices no AWF change is made due to corporate actions between rebalancings and, thus, the weights of stocks in the index as well as the index divisor will change due to corporate actions

The overall approach to calculate capped market cap weighted indices is the same as in the pure market-cap weighted indices; however, the constituents' market values are re-defined to be values that will meet the particular capping rules of the index in question. Recall equations 17, 18 and 19 from the prior section:

$$Index\ Level = \frac{Index\ Market\ Value}{Divisor} \qquad (17)$$

and

$$Index\ Market\ Value = \sum_i P_i * Shares_i * IWF_i * FxRate \qquad (18)$$

To calculate a capped market cap weighted index, the market capitalization for each stock used in the calculation of the index is redefined so that each index constituent has the appropriate weight in the index at each rebalancing date.

In addition to being the product of the stock price, the stock's shares outstanding, and the stock's float factor (IWF), as written above – and the exchange rate when applicable – a new adjustment factor is also introduced in the market capitalization calculation to establish the appropriate weighting.

$$AdjustedStock\ MarketValue_i = P_i * Shares_i * IWF_i * FxRate_i * AWF_i \qquad (19)$$

where AWF_i is the adjustment factor of stock i assigned at each index rebalancing date, t, which adjusts the market capitalization for all index constituents to achieve the user-defined weight, while maintaining the total market value of the overall index.

The AWF for each index constituent, i, on rebalancing date, t, is calculated by:

$$AWF_{i,t} = \frac{CW_{i,t}}{W_{i,t}} \qquad (21)$$

where $W_{i,t}$ is the uncapped weight of stock i on rebalancing date t based on the float market capitalization of all index constituents; and $CW_{i,t}$ is the capped weight of stock i on rebalancing date t as determined by the capping rule of the index in question and the process for determining capped weights as described in Different Capping Methods below.

The index divisor is defined based on the index level and market value from equation (17). The index level is not altered by index rebalancings. However, since prices and outstanding shares will have changed since the last rebalancing, the divisor will change at the rebalancing.

So:

$$(Divisor)_{after\ rebalancing} = \frac{(Index\ Market\ Value)_{after\ rebalancing}}{(Index\ Value)_{before\ rebalancing}} \qquad (21a)$$

where,

$$Index\ Market\ Value = \sum_i P_i * Shares_i * IWF_i * FxRate_i * AWF_i \qquad (21b)$$

Corporate Actions and Index Adjustments

All corporate actions for capped indices affect the index in the same manner as in market cap weighted indices. For details on how each corporate action is treated please refer to the table for Capitalization Weighted Indices section above.

For more information on the treatment of corporate actions, please refer to the S&P Corporate Actions Policies & Practices Methodology.

Different Capping Methods

Capped indices arise due to the need for benchmarks which comply with diversification rules, and with funds and listed products when the general desire is for a benchmark which is highly concentrated in one or a small number of stocks. Capping may apply to single stock concentration limits or concentration limits on a defined group of stocks.

The standard S&P methodology for determining the weights of capped indices using the most popular capping methods are described below.

Single Stock Capping

In a single stock capping methodology no stock in an index is allowed to breach a certain pre-determined weight as of each rebalancing period. The procedure for assigning capped weights to each stock at each rebalancing is as follows:

1. With data reflected on the rebalancing reference date, each company is weighted by float-adjusted market capitalization.
2. If any company has a weight greater than X% (where X% is the maximum weight allowed in the index), that company has its weight capped at X%.
3. All excess weight is proportionally redistributed to all uncapped stocks within the index.
4. After this redistribution, if the weight of any other stock(s), then, breaches X%, the process is repeated iteratively until no stocks breach the X% weight cap.

Single Stock and Concentration Limit Capping

In a single stock and concentration limit capping no stock in an index is allowed to breach a certain pre-determined weight and all stocks with a weight greater than a certain amount are not allowed, as a group, to exceed a predetermined total weight. One example of this is 5/25/50 capping (B/A/C in the example below). No single stock is allowed to exceed 25% of the index and all stocks with a weight greater than 5% of the index cannot exceed, as a group, 50% of the index. The procedure for assigning capped weights to each stock at each rebalancing is as follows:

1. With data reflected on the rebalancing reference date, each company is weighted by float-adjusted market capitalization.
2. If any company has a weight greater than A% (where A% is the maximum weight allowed in the index), that company has its weight capped at A%.
3. All excess weight is proportionally redistributed to all uncapped stocks within the index.
4. After this redistribution, if the weight of any other stock(s), then, breaches A%, the process is repeated iteratively until no stocks breach the A% weight cap.
5. The weight of all stocks within the index which have a weight greater than B% are added together. If the total weight of these stocks is less than C% then the capping is completed.
6. If the total weight is greater than C%, then the stocks in question are ranked in descending order based on weight, summed cumulatively, and the first stock that brings the total weight of the group above C% is, then, capped. This stock is capped to a weight equal to the larger of (1) B% or (2) the difference between C% and the total weight of all the stocks larger than the stock in question.

7. All stocks with weights greater than B%, but with lower weights than the stock capped in step 6, are capped to a weight of B%.
8. All excess weight is proportionally redistributed to all stocks within the index with a weight less than B%
9. After this redistribution, if the weight of any stock(s) that was originally less than B% then breaches B%, the process is repeated iteratively until no stocks breach the B% weight cap.

Attribute Weighted Indices

In recent years various new approaches to weighting stocks in indices have appeared to supplement cap weighting and price weighting.[2] S&P's Pure Style Indices, introduced in 2005, are attribute weighted – a stock's weight depends on the measures of its growth or value attributes, the same measure that was used to select stocks for the index. The discussion here covers how these indices are calculated; the selection of stocks is covered in the S&P U.S. Style Indices methodology document.

There are both Pure Growth Style and Pure Value Style indices. Under the selection process, each stock has a growth score and a value score. These scores are used to identify pure growth stocks and pure value stocks. (A stock cannot be both pure growth and pure value; it can be neither pure growth nor pure value.) The Pure Growth index includes only pure growth stocks; a stocks weight in the index is determined by its growth score; likewise for pure value.

When the index is rebalanced, the relative weights of any two stocks should be proportional to their relative style scores. For instance, in the pure value index, at the rebalancing, a stock with a value score of 1.5 should have 50% more weight than a stock with a value score of 1.0 (since 1.5 is 50% greater than 1.0). Since the weight represents how much of the index's total market value is in each stock, the weight depends on both the price of the stock and the number of shares held. The design of the index calculation procedure determines the number of shares of each stock to be included while the market determines the price.

As in the case of equal weighted indices, the general structure follows that used for the cap-weighted indices, defining the basic equations as:

$$Index\ Level = \frac{Index\,Market\,Value}{Divisor} \qquad (22)$$

and

$$Index\,Market\,Value = \sum_i P_i * Shares_i \qquad (23)$$

[2] Price weighting means an unweighted or simple average of the stock prices. It is seldom used in new indices but continues to be used by some long-standing indices including the Dow Jones Industrial Average and the Nikkei 225 Stock Average. The Dow dates from 1896; the Nikkei from 1949.

For simplicity, the equations do not include the IWF term even though it is used in the actual calculations. The share count is modified so that the stock's weight reflects its style score rather than its market capitalization. Define *M_Shares* as the modified share count:

$$M\_Shares_i = Shares_i * PWF_i \qquad (24)$$

where PWF_i is the Pure Style Weight Factor for stock *i*. The PWF_i depends on the true share count (*Shares*), the style score and the stock's price value when the index is rebalanced. The PWF_i is defined as:

$$PWF_i = \frac{k * SV_i}{Shares_{i,t=reb} * Price_{i,t=reb}} \qquad (25)$$

$Shares_i$ is the true share count, not the modified share count. SV_i is the value score of stock *i*. The factor *k* is included to scale the numbers; the denominator is quite large and without *k* the PWF_i would be a very small number. *SV* is the capped value score (to prevent outliers from having high weights in the index, the scores are capped at 2, effectively chopping off a distribution at two standard deviations away from the mean.) The subscript *t=reb* indicates that the shares and price for stock *i* are evaluated at the rebalancing date. Note also that the IWF would be included in the actual calculation.

By substituting (25) into (24) and then (24) into (23):

$$Index\,Market\,Value = \sum_i \frac{k * SV_i * P_{i,t} * Shares_{i,t}}{P_{i,t=reb} * Shares_{i,t=reb}} \qquad (26)$$

Note that we have substituted for *M_shares* so that the Shares variable in (26) is the actual shares. Now, re-arranging terms on the right hand side:

$$= k \times \sum_i SV_i * \frac{P_{i,t} * Shares_{i,t}}{P_{i,t=reb} * Shares_{i,t=reb}} \qquad (27)$$

The fraction term inside the summation sign is one plus the proportional increase or decrease in the value of the stock since the rebalancing (in effect the "value relative"). Therefore, the right hand side of (27) represents the weighted average of the value relatives of the stocks where the weights are the value scores. Recalling (22), we can define the index as:

$$IndexLevel = \frac{k * \sum_i SV_i \dfrac{P_{i,t} * Shares_{i,t}}{P_{i,t=reb} * Shares_{i,t=reb}}}{Divisor} \qquad (28)$$

Users of S&P Indices' attribute weighted indices typically get the modified index shares and price in daily files from S&P Indices, and do not have to do the above calculations themselves. Having a simple share number in terms of the modified index shares helps fit these indices seamlessly into systems designed for market capitalization weighted indices.

Corporate Actions

Parent Index Action	Adjustment made to Pure Style Index	Divisor adjustment required?
Constituent change	If the constituent being dropped is a member of the Pure Style Index, it is removed from the Pure Style Index. The replacement stock can be added to either the pure value or the pure growth index, or to neither. The weight is simply the ratio of the capped style score of the added stock divided by the sum of style scores of all index constituents. The indicative weight is announced through *S&P Indices' IDP*.	Yes
Share changes between quarterly share adjustments	The weight of stocks is unchanged. (The share count follows the parent index share count. To keep weights of stocks unchanged following a share change the PWF is adjusted for the stock whose shares are being changed.)	No
Quarterly share changes	If the annual rebalancing date coincides with a quarterly share update, this is the only time when weights are revised.	Yes if it coincides, No otherwise.
Spin-off	The weight of stocks is unchanged. The price follows the parent index price change. To keep weights of stocks unchanged following price change, PWF is adjusted for the stock whose shares are being changed.	No
Rights Offering	The weight of stocks is unchanged. The price follows the parent index price change. To keep weights of stocks unchanged, PWF is adjusted for the stock whose shares are being changed.	No

Parent Index Action	Adjustment made to Pure Style Index	Divisor adjustment required?
Stock Split	Shares are multiplied by and the price is divided by the split factor.	No
Special Dividends	The price of the stock making the special dividend payment is reduced by the per share special dividend amount after the close of trading on the day before the ex-date.	Yes

For more information on the treatment of corporate actions, please refer to the S&P Corporate Actions Policies & Practices Methodology.

Leveraged and Inverse Indices

Leveraged Indices for Equities

S&P Indices' Leveraged Indices are designed to generate a multiple of the return of the underlying index in situations where the investor borrows funds to generate index exposure beyond his/her cash position. The approach is to first calculate the underlying index, then calculate the daily returns for the leveraged index and, finally, to calculate the current value of the leveraged index by incrementing the previous value by the daily return. There is no change to the calculation of the underlying index.

The daily return for the leveraged index consists of two components: (1) the return on the total position in the underlying index less (2) the borrowing costs for the leverage.

The formula for calculating the Leveraged Index is as follows:

$$Leveraged\ Index\ Return = K * \left(\frac{Underlying\ Index_t}{Underlying\ Index_{t-1}} - 1 \right) - (K - 1) * \left(\frac{Borrowing\ Rate}{360} \right) * D_{t,t-1} \quad (29)$$

In equation (29) the borrowing rate is applied to the leveraged index value because this represents the funds being borrowed. Given this, the Leveraged Index Value at time *t* can be calculated as:

$$Leveraged\ IndexValue_t = (Leveraged\ IndexValue_{t-1}) * (1 + Leveraged\ Index\ Return) \quad (30)$$

Substituting (29) into (30) and expanding the right hand side of (30) yields:

$$Leveraged\ IndexValue_t = Leveraged\ IndexValue_{t-1} * \left[1 + \left[K * \left(\frac{UnderlyingIndex_t}{UnderlyingIndex_{t-1}} - 1 \right) - (K - 1) * \left[\frac{Borrowing\ Rate}{360} \right] * D_{t,t-1} \right] \right] \quad (31)$$

where:

K (K ≥ 1) = Leverage Ratio

- K =1, no leverage
- K =2, Exposure = 200%
- K = 3, Exposure = 300%

Borrowing Rate = Overnight LIBOR in the U.S. or EONIA in Europe

$D_{t,\,t-1}$ = the number of calendar days between date *t* and *t-1*

In the absence of leverage (K=1),

$$Leveraged\ Index Value_t = Leveraged\ Index\ Value_{t-1} * \left[\frac{Underlying\ Index_t}{Underlying\ Index_{t-1}} \right] \tag{32}$$

The leverage position is rebalanced daily. This is consistent with the payoff from futures based replication.

Inverse Indices for Equities

S&P Indices' Inverse indices are designed to provide the inverse performance of the underlying index; this represents a short position in the underlying index. When an investor holds a short position he/she must pay dividends and interest for the borrowed stock. The calculation follows the same general approach as the leveraged index with certain adjustments: First, the return on the underlying index is reversed and is based on the total return of the underlying index so that dividends and price movements are included. Second, while the costs of borrowing the securities are not included, there is an adjustment to reflect the interest earned on both the initial investment and the proceeds from selling short the securities in the underlying index. These assumptions reflect normal industry practice. (Straightforward adjustments can be made to either to include the costs of borrowing securities or to exclude the interest earned on the shorting proceeds and the initial investment.)

The general formula for the return to the inverse index is

$$Inverse\ Index\ Re\,turn = -K * \left(\frac{Underlying\ IndexTotal\ Re\,turn_t}{Underlying\ IndexTotal\ Re\,turn_{t-1}} - 1 \right) + (K+1) * \left(\frac{Lending\ \ Rate}{360} \right) D_{t,t-1} \tag{33}$$

Where the first right hand side term represents the total return on the underlying index and the second right hand side term represents the interest earned on the initial investment and the shorting proceeds.

Expanding this as done above for the leveraged index yields:

$$InverseIndexValue_t = Inverse\ IndexValue_{t-1} * \left[1 - \left[K * \left(\frac{UnderlyingIndexTR_t}{UnderlyingIndexTR_{t-1}} - 1 \right) - (K+1) * \left[\frac{LendingRate}{360} \right] * D_{t,t-1} \right] \right] \tag{34}$$

where:

K (K ≥ 1) = Leverage Ratio

- K =1, Exposure = -100%
- K =2, Exposure = -200%
- K = 3, Exposure = -300%

Lending Rate = Overnight LIBOR in the U.S. or EONIA in Europe

$D_{t,\,t-1}$ = the number of calendar days between date t and $t-1$

In the absence of leverage (K =1),

$$InverseIndexValue_t = Inverse\ IndexValue_{t-1} * \left[1 - \left[\left(\frac{UnderlyingIndexTR_t}{UnderlyingIndexTR_{t-1}} - 1\right) - (2) * \left[\frac{LendingRate}{360}\right] * D_{t,t-1}\right]\right] \quad (35)$$

The inverse position is rebalanced daily. This is consistent with the payoff from futures based replication.

Leveraged and Inverse Indices for Futures

S&P Indices' futures-based Leveraged Indices are designed to generate a multiple of the return of the underlying futures index in situations where the investor borrows funds to generate index exposure beyond his/her cash position.

S&P Indices' futures-based Inverse indices are designed to provide the inverse performance of the underlying futures index; this represents a short position in the underlying index.

The approach is to first calculate the underlying index, then calculate the daily returns for the leveraged or inverse index. There is no change to the calculation of the underlying futures index.

The leveraged or inverse index may be rebalanced daily or periodically.

Daily Rebalanced Leverage or Inverse Futures Indices

If the S&P futures-based leveraged or inverse index is rebalanced daily, the index excess return is the multiple of the underlying index's excess return and calculated as follows:

$$IndexER_t = IndexER_{t-1} * \left(1 + \left(K * \left(\frac{UnderlyingIndexER_t}{UnderlyingIndexER_{t-1}} - 1\right)\right)\right) \quad (36)$$

where:

K (K ≠ 0) = Leverage/Inverse Ratio

- K = 1, no leverage
- K = 2, leverage exposure = 200%
- K = 3, leverage exposure = 300%
- K = -1, inverse exposure = -100%

A total return version of each of the Indices is calculated, which includes interest accrual on the notional value of the index based on the 91-day US Treasury rate, as follows:

$$IndexTR_t = IndexTR_{t-1} * \left(\left(\frac{IndexER_t}{IndexER_{t-1}} \right) + TBR_t \right) \quad (37)$$

where:

$IndexTR_{t-1}$ = The Index Total Return on the preceding business day.

TBR_t = Treasury Bill Return, as determined by the following formula:

$$TBR_t = \left[\frac{1}{1 - \frac{91}{360} * TBAR_{t-1}} \right]^{\frac{Delta_t}{91}} - 1 \quad (38)$$

$Delta_t$ = the number of calendar days between the current and previous business days.

$TBAR_{t-1}$ = the most recent weekly high discount rate for 91-day US Treasury bills effective on the preceding business day. Generally the rates are announced by the US Treasury on each Monday. On Mondays that are bank holidays, Friday's rates apply

Periodically Rebalanced Leverage or Inverse Futures Indices

If the S&P futures-based leveraged or inverse index is rebalanced periodically (e.g. weekly, monthly, or quarterly), the index excess return is the multiple of the underlying index excess return since last rebalancing business day and shall be calculated as follows:

$$IndexER_t = IndexER_{t\_LR} * \left(1 + \left(K * \left(\frac{UnderlyingIndexER_t}{UnderlyingIndexER_{t\_LR}} - 1 \right) \right) \right) \quad (39)$$

where:

$IndexER_{t\_LR}$ = The Index Excess Return on the last rebalancing business day, t_LR.

$UnderlyingIndexER_{t\_LR}$ = The Underlying Index Excess Return value on the last rebalancing business day, t_LR.

t_LR = the last rebalancing business day.

K ($K \neq 0$) = Leverage / Inverse Ratio

- K = 1, no leverage
- K = 2, leverage exposure = 200%
- K = 3, leverage exposure = 300%
- K = -1, inverse exposure = -100%

A total return version of each of the Indices is calculated, which includes interest accrual on the notional value of the index based on the 91-day US Treasury rate. The formulae are the same as (37) and (38) above.

Total Return Calculations

The preceding discussions were related to price indices where changes in the index level reflect changes in stock prices. In a total return index changes in the index level reflect both movements in stock prices and the reinvestment of dividend income. A total return index represents the total return earned in a portfolio that tracks the underlying price index and reinvests dividend income in the overall index, not in the specific stock paying the dividend.

The total return construction differs from the price index and builds the index from the price index and daily total dividend returns. The first step is to calculate the total dividend paid on a given day and convert this figure into points of the price index:

$$TotalDailyDividend = \sum_i Dividend_i * Shares_i \qquad (40)$$

Where *Dividend* is the dividend per share paid for stock *i* and *Shares* are the shares. This is done for each trading day. $Dividend_i$ is generally zero except for four times a year when it goes ex-dividend for the quarterly dividend payment. Some stocks do not pay a dividend and *Dividend* is always zero. *TotalDailyDividend* is measured in dollars. This is converted to index points by dividing by the divisor for the underlying price index:

$$IndexDividend = \frac{TotalDailyDividend}{Divisor} \qquad (41)$$

The next step is to apply the usual definition of a total return from a financial instrument to the price index. Equation (40) gives the definition, equation (41) applies it to the index:

$$Total\ Return = (\frac{P_t + D_t}{P_{t-1}}) - 1 \qquad (42)$$

and

$$DTR_t = (\frac{IndexLevel_t + IndexDividend_t}{IndexLevel_{t-1}} - 1) \qquad (43)$$

where the *TotalReturn* and the daily total return for the index (*DTR*) is stated as a decimal. The *DTR* is used to update the total return index from one day to the next:

$$Total\ Return\ Index_t = (Total\ Return\ Index_{t-1}) * (1 + DTR_t) \qquad (44)$$

Index Fundamentals

Indices are often used to measure market conditions or gauge valuations among markets or between stocks and indices through measures like earnings per share (EPS), price-earnings ratios, dividend yields and so forth. These are calculated by using the divisor as if it represents shares for a company. The basic format is illustrated for the EPS for an index:

$$Index\,EPS = \frac{\sum_i eps_i * shares_i}{Divisor} \qquad (45)$$

where *IndexEPS* is the EPS for the overall index, eps_i is the EPS for stock *i* and $shares_i$ are the shares used to calculate the index with any adjustments such as the IWF incorporated into the figure. If the calculation refers to an equal weighted or attribute weighted index, the calculation use the shares defined for those indices (*C_shares* or *M_shares*, as appropriate).

The price-earnings (PE) ratio for the index is simply the ratio of the index level (or price) to the index EPS. For a cap-weighted index, this can also be calculated directly from the stock level data by dividing the total market cap of the index by total earnings of all companies in the index. In this calculation, the Divisor terms in the denominator drop out:

$$IndexPE = \frac{\dfrac{\sum P_i * Shares_i}{Divisor}}{\dfrac{\sum eps_i * Shares_i}{Divisor}} \qquad (46)$$

The same general approach can be used for various index fundamentals and ratios such as book value per share, price-to-book, dividend-to-price (i.e. dividend yield) and so forth.

Fund Strategy Indices

A fund strategy index is a portfolio of funds that is designed to deliver excess returns over a benchmark within an established risk framework. A fund strategy index can include a portfolio of funds for a single asset class (i.e. U.S. equities) or cover multiple asset classes (i.e. asset allocation programs). Fund strategy indices are created with U.S. mutual funds for U.S. investors or International (offshore) funds for non-U.S. investors.

Each fund's weight within the strategy index portfolio is determined by a rules-based portfolio construction methodology. The calculation process for fund strategy indices are based upon the divisor method commonly used for S&P equity indices. However, the share count is redefined to be a number that will achieve the fund's allocation weighting at the rebalancing date.

$$Index\ Level = \frac{Index\ Market\ Value}{Divisor} \qquad (47)$$

and

$$Index\ Market\ Value = \sum_i NAV_i * Shares_i \qquad (48)$$

where NAV is the Net Asset Value, a fund's share price.

To calculate a fund index, shares are redefined. Rather than being the actual count of shares multiplied by the investable weight factors (IWF) or other such adjustment factors, the *Shares* number is calculated to establish the allocation weighting at the rebalancing date. For clarity, this section will refer to the "shares" figure as *fund_shares*. Not only are these not the true share count, they have essentially no relation to a fund's true share count. Since fund_shares are being used instead of true shares, the Index Market Value defined in (44) is not the actual market value of the index. In a fund strategy index the Index Market Value is an arbitrary or nominal value for the portfolio used when the fund_shares figure is established. The fund_shares are calculated at each rebalancing:

$$fund\_shares_{i,\ rebalancing\ date} = \frac{w_{i, rebalancing\ date}}{NAV_{i, rebalancing\ date}} \qquad (49)$$

where w_i is the new allocation weight and NAV_i is the price of the $fund_i$ at the rebalancing date.

At each index rebalancing, in order to maintain index series continuity, it is also necessary to adjust the divisor.

$$(Index\ Level)_{before\ rebalancing} = (Index\ Level)_{after\ rebalancing} \quad (50)$$

Therefore,

$$(Divisor)_{after\ rebalancing} = \frac{(Index\ Market\ Value)_{after\ rebalancing}}{(Index\ Level)_{before\ rebalancing}} \quad (51)$$

where,

$$Index\ Market\ Value = \sum_i NAV_i * fund\_shares_i$$

Total Return and Synthetic Price Indices

Some funds, primarily international (offshore), provide investors with accumulating or distributing share classes. The accumulating share class does not declare nor pay out net income and/or net realized capital gains. Net income and/or net realized capital gains are retained and included in the net asset value per share. The distributing share class declares and pays out net income and net capital gains. For consistency, S&P publishes total return and synthetic price return indices.

Total Return Index

The total return index assumes dividend and capital gain distributions are reinvested in the index. Note that the results will be different if an investor held all funds in an index in the correct proportions and instructed each fund company to reinvest distributions into the fund paying those distributions. On any given date *t*:

$$Total\ Return\ Index_t =$$

$$Total\ Return\ Index_{t-1} * Total\ Return\ Multiplier_t$$

where

$$Total\ Return\ Multiplier_t =$$

$$\frac{Index\ Level_t + Index\ Distribution\ Points_t}{Index\ Value_{t-1}} \quad (52)$$

and,

Index Distribution Points$_t$ =

$$\frac{\sum_{i=1}^{N} fund\_shares_{i,t} * Distributions_{i,t}}{Divisor_t} \qquad (53)$$

Synthetic Price Index

The synthetic price index is calculated from the total return strategy index and is based upon the ratio of the benchmark's total return index to its price index from the base date of the fund strategy index.

Price Index$_t$ =

$$\frac{Price\ Index_{base\ date} * \left(\frac{Total\ Return\ Index_t}{Total\ Return\ Index_{base\ date}}\right)}{\left(\frac{Benchmark\ Price\ Index_{base\ date}}{Benchmark\ Price\ Index_t}\right) * \left(\frac{Benchmark\ TR\ Index_t}{Benchmark\ TR\ Index_{base\ date}}\right)} \qquad (54)$$

Index and Corporate Actions

S&P fund index action	Adjustment made to fund index	Divisor adjustment for the fund index?
Constituent Change	The fund entering the index goes in at the weight of the fund coming out. This weight is used to compute fund_shares of the added fund, using Equation 49.	No.
Fund added/deleted	Net change in fund index market value determines divisor adjustment.	Yes.

Corporate Action	Adjustment made to strategy index	Divisor adjustment for strategy index
Fund Closing	Funds must be open to new investments. Fund will be removed from the index.	Yes, if fund removed without replacement. No, if fund replaced.
Fund Split	Fund_shares are multiplied by and NAV is divided by the split factor.	No.
Fund Merger – fund is the acquiring fund.	Assets will be added to the fund at NAV. No change in fund_shares.	No.
Fund Merger – fund is merged into another fund.	Fund no longer available. Fund will be removed from the index.	Yes, if fund removed without replacement. No, if fund replaced.
Fund Liquidation	Fund no longer available. Fund will be removed from the index.	Yes, if fund removed without replacement. No, if fund replaced.

Dividend Indices

S&P Indices' Dividend Indices are designed to track the total dividend payments from the constituents of an underlying index. The level of the index is based on a running total of dividends of the constituents of the underlying index. The index resets to zero on a periodic basis, generally quarterly or annually. Thus, the index measures the total dividends paid in the underlying index since the previous rebalancing date. For quarterly indices, the index resets to zero after the close on the third Friday of the last month of the quarter, to coincide with futures and options expiration. For annual indices, the index resets to zero after the close on the third Friday of December, to coincide with futures and options expiration.

The formula for calculating the dividend index on any date, *t,* for a given underlying index, *x*, is:

$$DividendIndex_{t,x} = \sum_{i=r+1}^{t} ID_{i,x} \qquad (55)$$

where:

$ID_{i,x}$ = The index dividend of the underlying index *x* on day *i.*

t = The current date.

r+1 = The trading date immediately following the reset date of the index.

The index dividend (*ID*) of the underlying index is calculated on any given day as the total dividend value for all constituents of the index divided by the index divisor. The total dividend value is calculated as the sum of dividends per share multiplied by index shares outstanding for all constituents of the index which have a dividend going ex on the date in question. For more detail concerning the calculation of index dividends please refer to the *Total Returns Calculation* section above.

Excess Return Indices

S&P Indices' Excess Return Indices are designed to track an unfunded investment in an underlying index. In other words, an excess return index calculates the return on an investment in an index where the investment was made through the use of borrowed funds. Thus the return of an excess return index will be equal to that of the underlying index less the associated borrowing costs.

The formula for calculating the Excess Return Index is as follows:

$$Excess\ Return = \left(\frac{Underlying\ Index_t}{Underlying\ Index_{t-1}} - 1 \right) - \left(\frac{Borrowing\ Rate}{360} \right) * D_{t,t-1} \qquad (56)$$

The Excess Return Index Value at time t can be calculated as:

$$Excess\ Return\ IndexValue_t = (\ Excess\ Return\ IndexValue_{t-1}\ ) * (\ 1 + Excess\ Return\ ) \qquad (57)$$

Substituting (52) into (53) and expanding the right hand side of (53) yields:

$$Excess\ Return\ IndexValue_t = Excess\ Return\ IndexValue_{t-1} * \left[1 + \left[\left(\frac{UnderlyingIndex_t}{UnderlyingIndex_{t-1}} - 1 \right) - \left[\frac{Borrowing\ Rate}{360} \right] * D_{t,t-1} \right] \right] \qquad (58)$$

where:

Borrowing Rate = The investment funds borrowing rates, which will differ for each excess return index. Generally an overnight rate, such as overnight LIBOR in the U.S. or EONIA in Europe, will be used. However, in some cases other interest rates may be used. A 360-day year is assumed for the interest calculations in accordance with U.S. banking practices.

$D_{t,\ t-1}$ = The number of calendar days between date t and $t-1$

Risk Control Indices

S&P Indices' Risk Control Indices are designed to track the return of a strategy that applies dynamic exposure to an underlying index in an attempt to control the level of volatility.

The Index includes a leverage factor that changes based on realized historical volatility. If realized volatility exceeds the target level of volatility, the leverage factor will be less than one; if realized volatility is lower than the target level, the leverage factor may be greater than one, assuming the index allows for a leverage factor of greater than one. A given Risk Control Index may have a maximum leverage factor that cannot be exceeded. There are no guarantees that the Index shall achieve its stated targets.

The return of the Index consists of two components: (1) the return on the position in the underlying index and (2) the interest cost or gain, depending upon whether the position is leveraged or deleveraged.

A leverage factor greater than one represents a leveraged position, a leverage factor equal to one represents an unleveraged position, and a leverage factor less than one represents a deleveraged position. The leverage factor may change periodically, on a set schedule, or may change when volatility exceeds or falls below predetermined volatility thresholds.

For equity indices, the leverage factor will not change at the close of any index calculation day in which stocks representing 15% or more of the total weight of the underlying index are not trading due to an exchange holiday. At each underlying index's rebalancing, and using each stock's weight at that time, a forward looking calendar of such dates is determined and posted on S&P Indices' Web site at www.indices.standardandpoors.com.

The formula for calculating the Risk Control Index is as follows:

$$Risk\ Control\ Index\ Return_t = K_{rb} * \left(\frac{Underlying\ Index_t}{Underlying\ Index_{rb}} - 1 \right) + (1 - K_{rb}) * \left[\prod_{i=rb+1}^{t} (1 + InterestRate_{i-1} * D_{i-1,i} / 360) - 1 \right] \quad (59)$$

The Risk Control Index Value at time *t* can, then, be calculated as:

$$RiskControlIndexValue_t = (RiskControlIndexValue_{rb}) * (1 + RiskControlIndex\ Return_t) \quad (60)$$

Substituting equation (59) into (60) and expanding yields:

Risk Control Index Value$_t$ =

$$Risk\ Control\ Index\ Value_{rb} *$$

$$\left[1+\left[K_{rb}*\left(\frac{Underlying\ Index_t}{Underlying\ Index_{rb}}-1\right)+(1-K_{rb})*\left[\prod_{i=rb+1}^{t}(1+InterestRate_{i-1}*D_{i-1,i}/360)-1\right]\right]\right] \quad (61)$$

where:

$UnderlyingIndex_t$ = the level of the underlying index on day *t.*

$UnderlyingIndex_{rb}$ = the level of the underlying index as of the previous rebalancing date.

rb = the last index rebalancing date. The inception date of each Risk Control Index is considered the first rebalancing date of that index.

K_{rb} = the leverage factor set at the last rebalancing date, calculated as:

$$Min(Max\ K,\ Target\ Volatility/Realized\ Volatility_{rb-d})$$

Max K = the maximum leverage factor allowed in the Index

d = the number of days between when volatility is observed and the rebalancing date. For example, if *d* = 2 , the historical volatility of the Underlying Index as of the close two days prior to the rebalancing date will be used to calculate the leverage factor K_{rb}

Target Volatility = the target level of volatility set for the Index.

Realized Volatility$_{rb-d}$ = the historical realized volatility of the Underlying Index as of the close of *d* trading days prior to the previous rebalancing date, *rb,* where a trading day is defined as a day on which the underlying index is calculated.

Interest Rate$_{i-1}$ = the interest rate set for the index. The interest rate may be an overnight rate, such as LIBOR or EONIA, or a daily valuation of a rolling investment in a 3-month interest rate, or zero. A 360-day year is assumed for the interest calculations in accordance with U.S. banking practices.

For indices that replicate a rolling investment in a 3-month interest rate the above formula is altered to:

Risk Control Index Value$_t$=

$$Risk\ Control\ Index\ Value_{rb} *$$

$$\left[1+\left[K_{rb} * \left(\frac{Underlying\ Index_t}{Underlying\ Index_{rb}} - 1\right) + (1-K_{rb}) * \left[\prod_{i=rb+1}^{t}(1+InterestRate_{i-1}) - 1\right]\right]\right] \quad (62)$$

where:

$$InterestRate_{i-1} = (D_{i-1,i} * IR3M_{i-1} - (IR3M_{i-1} - IR3M_{i-2} - D_{i-1,i} * (IR3M_{i-1} - IR2M_{i-1}) * (1/30)) * 90)/360$$

where:

$D_{i-1,\,t}$ = the number of calendar days between day *i-1* and day *t*

$IR3M_{i-1}$ = 3-month interest rate on day i-1

$IR2M_{i-1}$ = 2-month interest rate on day i-1

For indices that are rebalanced daily, the leverage factor is not recalculated at the close of any index calculation day when stocks representing 15% or more of the total weight of the underlying index are not trading due to an exchange holiday. If *rb* is a holiday, then K_{rb} is calculated as follows:

$$K_{rb} = K_{rb-1} * \left(\frac{Underlying\ Index_{rb}}{Underlying\ Index_{rb-1}}\right) \Big/ \left(\frac{RiskContorlIndexValue_{rb}}{RiskcontrolIndexValue_{rb-1}}\right)$$

This shows what the effect will be on *rb*, given that no adjustment of positions is allowed to occur on such days. The leverage factor will adjust solely to account for market movements on that day.

For periodically rebalanced risk control indices, K_{rb} is calculated at each rebalancing and held constant until the next rebalancing.

For large position moves, some investors like to rebalance risk control indices intra-period, when the periodicity is longer than daily. This feature is incorporated in the risk-control framework by introducing a barrier, K_b, on the leverage factor. Intra-period rebalancing is allowed only if the absolute change of the equity leverage factor K_t, at time *t*, is larger than the barrier K_b from the value at the last rebalancing date.

The equity leverage factor K_t is calculated as:

$$K_t = Min(Max\ K,\ Target\ Volatility/Realized\ Volatility_{t\text{-}d})$$

If no barrier is provided for the index, then intra-period rebalancing is not allowed.

Exponentially-Weighted Volatility

The realized volatility is calculated as the maximum of two exponentially weighted moving averages, one measuring short-term and one measuring long-term volatility.

$$RealizedVolatility_t = Max\left(RealizedVolatility_{S,t}, RealizedVolatility_{L,t}\right)$$

where:

S,t = The short-term volatility measure at time t, calculated as:

$$RealizedVolatility_{S,t} = \sqrt{\frac{252}{n} * Variance_{S,t}}$$
$$for\ \ t > T_0$$
$$Variance_{S,t} = \lambda_S * Variance_{S,t-1} + (1 - \lambda_S) * \left[ln\left(\frac{UnderlyingIndex_t}{UnderlyingIndex_{t-n}}\right)\right]^2$$
$$for\ \ t = T_0$$
$$Variance_{S,T_0} = \sum_{i=m+1}^{T_0} \frac{\alpha_{S,i,m}}{WeightingFactor_S} * \left[ln\left(\frac{UnderlyingIndex_i}{UnderlyingIndex_{i-n}}\right)\right]^2 \tag{63}$$

L,t = The long-term volatility measure at time t, calculated as:

$$RealizedVolatility_{L,t} = \sqrt{\frac{252}{n} * Variance_{L,t}}$$
$$for\ \ t > T_0$$
$$Variance_{L,t} = \lambda_L * Variance_{L,t-1} + (1 - \lambda_L) * \left[ln\left(\frac{UnderlyingIndex_t}{UnderlyingIndex_{t-n}}\right)\right]^2$$
$$for\ \ t = T_0$$
$$Variance_{L,T_0} = \sum_{i=m+1}^{T_0} \frac{\alpha_{L,i,m}}{WeightingFactor_L} * \left[ln\left(\frac{UnderlyingIndex_i}{UnderlyingIndex_{i-n}}\right)\right]^2 \tag{64}$$

where:

T_0 = the start date for a given Risk Control Index

n = the number of days inherent in the return calculation used for determining volatility. If $n = 1$ daily returns are used, while if $n = 2$ two day returns are used, and so forth.

m = the N^{th} trading date prior to T_0

N = the number of trading days observed for calculating the initial variance as of the start date of the index.

λ_S = The short-term decay factor used for exponential weighting. The decay factor is a number greater than zero and less than one that determines the weight of each daily return in the calculation of historical variance.

λ_L = The long-term decay factor used for exponential weighting. The decay factor is a number greater than zero and less than one that determines the weight of each daily return in the calculation of historical variance.

$\alpha_{S,m,i}$ = Weight of date *t* in the short-term volatility calculation, as calculated based on the following formula:

$$\alpha_{S,t} = (1-\lambda_S) * \lambda_S^{\ N+m-i}$$

$$WeightingFactor_S = \sum_{i=m+1}^{T_0} \alpha_{S,i,m}$$

$\alpha_{L,m,i}$ = Weight of date *t* in the long-term volatility calculation, as calculated based on the following formula:

$$\alpha_{L,t} = (1-\lambda_L) * \lambda_L^{\ N+m-i}$$

$$WeightingFactor_L = \sum_{i=m+1}^{T_0} \alpha_{L,i,m}$$

The interest rate, maximum leverage, target volatility and the lambda decay factors are defined in relation to each index and are generally held constant throughout the life of the index. The leverage position changes at each rebalancing based on changes in realized volatility. There is a two-day lag between the calculation of the leverage factor, based on the ratio of target volatility to realized volatility, and the implementation of that leverage factor in the index.

The above formulae can be used for simpler models by the appropriate choice of parameters. For example, if the short-term and long-term decay factors, λ_S and λ_L are set to the same value (e.g. 5%) than there are no separate considerations for short-term and long-term volatility.

Simple-Weighted Volatility

The realized volatility is calculated as the maximum of two simple-weighted moving averages, one measuring short-term and one measuring long-term volatility.

$$RealizedVolatility_t = Max\left(RealizedVolatility_{S,t}, RealizedVolatility_{L,t}\right)$$

where:

S,t = The short-term volatility measure at time *t*, calculated as:

$$RealizedVolatility_{S,t} = \sqrt{\frac{252}{n} * Variance_{S,t}}$$

$$Variance_{S,t} = 1/N_S * \sum_{i=t-Ns+1}^{t} \ln\left(\frac{UnderlyingIndex_i}{UnderlyingIndex_{i-n}}\right)^2 \quad (65)$$

L,t = The long-term volatility measure at time *t*, calculated as:

$$RealizedVolatility_{L,t} = \sqrt{\frac{252}{n} * Variance_{L,t}}$$

$$Variance_{L,t} = 1/N_L * \sum_{i=t-Nl+1}^{t} \ln\left(\frac{UnderlyingIndex_i}{UnderlyingIndex_{i-n}}\right)^2 \quad (66)$$

where:

n = the number of days inherent in the return calculation used for determining volatility. If $n = 1$ daily returns are used, while if $n = 2$ two day returns are used, and so forth.

N_S = the number of trading days observed for calculating variance for the short-term volatility measure.

N_L = the number of trading days observed for calculating variance for the long-term volatility measure.

Futures-Based Risk Control Indices

When the underlying index is based on futures contracts, most of the Risk Control methodology follows the details on the prior six pages. However, there are some differences as detailed below, particularly as it relates to the cash component of the index.

For such an index, it includes a leverage factor that changes based on realized historical volatility. If realized volatility exceeds the target level of volatility, the leverage factor will be less than one; if realized volatility is lower than the target level, the leverage factor may be greater than one. A given Risk Control Index may have a maximum leverage factor that cannot be exceeded.

For equity risk control indices, the return consists of two components: (1) the return on the position in the underlying S&P index and (2) the interest cost or gain, depending upon whether the position is leveraged or deleveraged. For futures-based risk control indices, there is no borrowing or lending to achieve investment objectives in the underlying index. Therefore, the cash component of the Index does not exist.

Again, a leverage factor greater than one represents a leveraged position, a leverage factor equal to one represents an unleveraged position, and a leverage factor less than one represents a deleveraged position. The leverage factor may change at regular intervals, in response to changes in realized historical volatility, or when the expected volatility exceeds or falls below predetermined volatility thresholds, if such thresholds were in place.

The formula for calculating the Risk Control Excess Return Index largely follows that detailed beginning with equation (59). However, since there is no funding for such indices (as opposed to the case with equity excess return indices, where it is assumed the initial investment is borrowed and excess cash is invested), the interest rate used in the calculation is eliminated:

$$Risk\ Control\ Excess\ Return\ Index\ Return_t = K_{rb} * \left(\frac{Underlying\ Index_t}{Underlying\ Index_{rb}} - 1 \right) \qquad (67)$$

The Risk Control Excess Return Index Value at time *t* can, then, be calculated as:

$$RiskControlExcessReturnIndexValue_t =$$
$$(RiskControlExcessReturnIndexValue_{rb}) * (1 + RiskControlExcessReturnIndexReturn_t) \quad (68)$$

The formula for calculating the Risk Control Total Return Index, which includes interest earned on Treasury Bills, is as follows:

$$Risk\ Control\ Total\ Return\ Index\ Return_t =$$
$$K_{rb} * \left(\frac{Underlying\ Index_t}{Underlying\ Index_{rb}} - 1 \right) + \left[\prod_{i=rb+1}^{t} (1 + InterestRate_{i-1} * D_{i-1,i} / 360) - 1 \right] \quad (69)$$

The Risk Control Total Return Index Value at time *t* can, then, be calculated as:

$$RiskControlTotalReturnIndexValue_t =$$
$$(RiskControlTotalReturnIndexValue_{rb}) * (1 + RiskControlTotalReturnIndexReturn_t) \quad (70)$$

Substituting equation (69) into (70) and expanding yields:

$$Risk\ Control\ Total\ Return\ Index\ Value_t =$$

$$Risk\ Control\ Index\ Value_{rb} *$$
$$\left[1 + \left[K_{rb} * \left(\frac{Underlying\ Index_t}{Underlying\ Index_{rb}} - 1 \right) + \left[\prod_{i=rb+1}^{t} (1 + InterestRate_{i-1} * D_{i-1,i} / 360) - 1 \right] \right] \right] \quad (71)$$

where all variables in equations (67)-(71) are the same as those defined for (59)-(61) except:

Interest Rate$_{i-1}$ = the interest rate set for the index. In accordance with the S&P GSCI approach, the interest rate for these indices is the 91-day US Treasury rate. A 360-day year is assumed for the interest calculations in accordance with U.S. banking practices.

Exponentially-Weighted Volatility

Please refer to pages 41-43 for information on Exponentially-Weighted Volatility. However, for futures-based risk control indices there is a three (3)-day lag between the calculation of the leverage factor, based on the ratio of target volatility to realized volatility, and the implementation of that leverage factor in the index.

Risk Control 2.0 Indices

S&P Indices' Risk Control 2.0 Indices are the next generation Risk Control indices, where the cash portion of the investment in the standard Risk Control strategy is replaced with a liquid Bond Index.

The index portfolio consists of two assets, the index for a risky asset *A*, with weight *W,* and the corresponding bond index *B*, with weight of (1-*W*). Weight *W* lies between 0 and 100%. There is no shorting or leverage allowed in the strategy.

Constituent Weighting

The formula to assign weights to the underlying indices is determined by the following:

$$W^2 * \sigma_A^2 + (1-W)^2 * \sigma_B^2 + 2 * W * (1-W) * \rho * \sigma_A * \sigma_B = \sigma_{Target}^2 \qquad (72)$$

where:

W = the weight of the risky asset *A*;
σ_A = the volatility of the risky asset *A*;
σ_B = the volatility of the bond index *B*;
ρ = the correlation of Index *A* and *B*;
σ_{Target} = the target volatility.

The calculation of volatility and correlation follows the same procedure and conventions as outlined in the prior section for the standard Risk Control strategy.

The quadratic equation above has two solutions to the weight allocated the index A:

$$W_1 = (b + \sqrt{b^2 - a * c}) / a$$
$$W_2 = (b - \sqrt{b^2 - a * c}) / a \qquad (73)$$

where:

$a = \sigma_A^2 + \sigma_B^2 - 2 * \rho * \sigma_A * \sigma_B$;

$b = \sigma_B^2 - \rho \sigma_A \sigma_B$

$c = \sigma_B^2 - \sigma_{Target}^2$

The fall-back mechanism for the solutions of weight *W*:

1. If none of the solutions in equation (73) above falls between 0 and 100%, then the strategy full back to standard Risk Control, where the maximum leverage is capped at 100%.
2. If both solutions to the equation (73) are valid weights that lie between 0 and 100%, then the larger of the two, *max(W1, W2),* becomes the weight of the risky asset *A*.

The final weights of the underlying assets are determined using the following steps:

Step 1: Determine the weights under the short term parameters

a) Determine the short-term variance for assets *A* and *B* using the short term exponential parameter with the same formulae as described in Equation 63 under the section *Risk Control Indices,* with the returns for assets *A* and *B* used in determining the short-term variance for assets *A* and *B*.

b) Determine the short-term covariance for assets *A* and *B* using similar formulae as described for short-term covariance calculations in Equation 63 under the section *Risk Control Indices,* but replacing the squared equity returns with the product of the risky assets *A* and *B*.

c) Determine the short-term volatility measure for the risky assets *A* and *B* from their respective variance measures in the same manner as described in Equation 63 under the section *Risk Control Indices*.

d) Determine the short-term correlation of *A* and *B* from the short-term covariance and the short-term volatility measures.

e) Determine the possible levels for the weights for *A* and *B* using Equation 72 & 73 above.

Step 2: Determine the weights under the long term parameters

Repeat (*a*) to (*e*) in Step 1 above with long-term parameters as described in Equation 64 under the section *Risk-Control Indices*.

Step 3: Determine the final weight W.

The weight for risky asset *A* is set equal to the lower of the weight of *A* as determined in Step 1 and Step 2.

The excess return of the Risk Control 2.0 Indices is calculated as:

$$RiskControl2.0ExcessReturn_t = W * Index_A ExcessReturn + (1-W) * Index_B ExcessReturn \quad (74)$$

and the Risk Control 2.0 Index value is:

$$RiskControl2.0IndexValue_t = RiskControl2.0IndexValue_{rb} * (1 + RiskControl2.0ExcessReturn_t) \quad (75)$$

where

$RiskControl2.0IndexValue_{rb}$ is the value of the index at the last rebalancing.

Risk Control 2.0 total return indices are calculated in a similar way, where the total return is a weighted sum of total returns of the underlying indices.

Risk Control 2.0 is an extension of standard Risk Control described in detail in the previous section. The parameters used in Risk Control 2.0 follow exactly the way they are calculated in the standard Risk Control methodology.

Currency Hedged Indices

A currency-hedged index is designed to represent returns for those global index investment strategies that involve hedging currency risk[3], but not the underlying constituent risk.

Investors employing a currency-hedged strategy seek to eliminate the risk of currency fluctuations and are willing to sacrifice potential currency gains. By selling foreign exchange forward contracts, global investors are able to lock in current exchange forward rates and manage their currency risk. Profits (losses) from the forward contracts are offset by losses (profits) in the value of the currency, thereby negating exposure to the currency.

Return Definitions

S&P Indices' standard currency hedged indices are calculated by hedging beginning-of-period balances using rolling one-month forward contracts. The amount hedged is adjusted on a monthly basis.

Returns are defined as follows:

$$\textit{Currency Return} = \left(\textit{End Spot Rate} \big/ \textit{Beginning Spot Rate} \right) - 1$$

$$\textit{Unhedged Return} = \left(1 + \textit{Local Total Return}\right) * \left(1 + \textit{Currency Return}\right) - 1$$

$$\textit{Currency Return on Unhedged Local Total Return} = \left(\textit{Currency Return}\right) * \left(1 + \textit{Local Total Return}\right)$$

$$\textit{Forward Return} = \left(\textit{Beginning one-month Forward Rate} \big/ \textit{Beginning Spot Rate} \right) - 1$$

$$\textit{Hedge Return} = \textit{HedgeRatio} * \left(\textit{Forward Return} - \textit{Currency Return}\right)$$

[3] By currency risk, we simply mean the risk attributable to the security trading in a currency different from the investor's home currency. This definition does not incorporate risks that exchange rate changes can have on an underlying security's price performance.

Hedged Index Return =
Local Total Return + Currency Return on Unhedged Local Total Return + Hedge Return

Hedged Index Level =
*Beginning Hedged Index Level * (1 + Hedged Index Return)*

S&P Indices also offers daily currency hedged indices for clients who require benchmarks with more frequent currency hedging. The daily currency hedged indices differ from the standard currency hedged indices by adjusting the amount of the forward contracts, that mature at the end of month, on a daily basis according to the performance of the underlying index. This further reduces the currency risk from under-hedging or over-hedging resulting from index movement between two monthly rolling periods.

Details of the formulae used in computing S&P Indices' currency-hedged indices are below.

The Hedge Ratio

The hedge ratio is simply the proportion of the portfolio's currency exposure that is hedged.

- **Standard Currency-Hedged Index.** In a standard currency-hedged index, we simply wish to eliminate the currency risk of the portfolio. Therefore, the hedge ratio used is 100%.
- **No Hedging.** An investor who expects upside potential for the local currency of the index portfolio versus the home currency, or does not wish to eliminate the currency risk of the portfolio, will use an unhedged index. In this case, the hedge ratio is 0, and the index simply becomes the standard index calculated in the investor's home currency. Such indices are available in major currencies as standard indices for many of S&P's indices.

In contrast to a 100% currency-hedged standard index, which seeks to eliminate currency risk and has passive equity exposure, over- or under-hedged portfolios seek to take active currency risks to varying degrees based on the portfolio manager's view of future currency movements

- **Over Hedging.** An investor who expects significant upside potential for the home currency versus the local currency of the index portfolio might choose to double the currency exposure. In this case, the hedge ratio will be 200%.
- **Under Hedging.** An investor who expects some upside potential for the local currency of the index portfolio versus the home currency, but wishes to eliminate some of the currency risk, might choose to have half the currency exposure hedged using a 50% hedge ratio.
- **Optimal Hedging.** In order to minimize variability and, therefore, risk in the value of the currency-hedged portfolio, standard variance minimization suggests the following hedge ratio:

Hedge Ratio= COV(Portfolio Return to Forward Return)/VAR(Forward Return)

S&P Indices calculates indices with hedge ratio different from 100% as custom indices.

Calculating a Currency-Hedged Index

Using the returns definitions on prior pages, the Hedged Index Return can be expressed as:

Hedged Index Return =
Local Total Return + Currency Return(1 + Local Total Return) + Hedge Return*

Rearranging yields:

Hedged Index Return =
*(1 + Local Return) * (1 + Currency Return) -1 + Hedge Return* (76)

Again, using the returns definitions on prior pages with a hedge ratio of 1 (100%), the expression yields:

Hedged Index Return = Unhedged Index Return + Hedge Return

Hedged Index Return =
Unhedged Index Return + Forward Return - Currency Return (77)

This equation is more intuitive since when you do a 100% currency hedge of a portfolio, the investor sacrifices the gains (or losses) on currency in return for gains (or losses) in a forward contract.

From the equation above, we can see that the volatility of the hedged index is a function of the volatility of the unhedged index return, the forward return, and the currency return, and their pair-wise correlations. These variables will determine whether the hedged index return series' volatility is greater than, equal to, or less than the volatility of the unhedged index return series

Currency Hedging Outcomes

The results of a currency-hedged index strategy versus that of an unhedged strategy vary depending upon the movement of the exchange rate between the local currency and home currency of the investor.

S&P Indices' standard currency hedging process involves eliminating currency exposure using a hedge ratio of 1 (100%).

1. The currency-hedged index does not necessarily give a return exactly equal to the return of the index available to local market investor. This is because there are two additional returns -- currency return on the local total return and hedge

return. These two variables usually add to a non-zero value because the monthly rolling of forward contracts does not result in a perfect hedge. Further, the local total return between two readjustment periods remains unhedged. However, hedging does ensure that these two returns remain fairly close.

2. The results of a currency-hedged index strategy versus that of an unhedged strategy varies depending upon the movement of the exchange rate between the local currency and home currency of the investor. For example, a depreciating euro in 1999 resulted in an unhedged S&P 500 return of 40.0% for European investors, while those European investors who hedged their USD exposure experienced a return of 17.3%. Conversely, in 2003 an appreciating euro in 2003 resulted in an unhedged S&P 500 return of 5.1% for European investors, while those European investors who hedged their USD exposure experienced a return of 27.3%.

Index Computation

Monthly Return Series (For Monthly Currency Hedged Indices)

m = the month in the calculation, represented as 0, 1, 2, etc..

SPI_EH_m = the S&P Currency-Hedged Index level at the end of month m

SPI_EH_{m-1} = the S&P Currency-Hedged Index level at the end of the prior month

SPI_E_m = the S&P Index level, in foreign currency, at the end of month m

SPI_E_{m-1} = the S&P Index level, in foreign currency, at the end of the prior month

SPI_EL_{m-1} = the S&P Index level, in local currency, at the end of the prior month, $m-1$

HR_m = the hedge return (%) over month m

S_m = the spot rate in foreign currency per local currency (*FC/LC*) , at the end of month m

F_m = the forward rate in foreign currency per local currency (*FC/LC*), at the end of month m

For the end of month m = 1,

$$SPI\_EH_1 = SPI\_EH_0 * \left(\frac{SPI\_E_1}{SPI\_E_0} + HR_1 \right)$$

For the end of month m,

$$SPI\_EH_m = SPI\_EH_{m-1} * \left(\frac{SPI\_E_m}{SPI\_E_{m-1}} + HR_m \right)$$

The hedge return for monthly currency hedged indices is:

$$HR_m = \frac{S_{m-1}}{F_{m-1}} - \frac{S_{m-1}}{S_m}$$

Daily Return Series (For Monthly Currency Hedged Indices and Daily Currency Hedged Indices)

The daily return series are computed by interpolating between the spot price and the forward price.

For each month m, there are d = 1, 2, 3…D calendar days.

md is day d for month m and $m0$ is the last day of the month $m-1$.

F_I_{md} = the interpolated forward rate as of day d of month m

AF_{md} = the adjustment factor for daily hedged indices as of day d of month m

$$F\_I_{md} = S_{md} + \left(\frac{D-d}{D}\right) * \left(F_{md} - S_{md}\right)$$

$$AF_{md} = SPI\_EL_{md-1} / SPI\_EL_{m0}$$

For the day d of month m,

$$SPI\_EH_{md} = SPI\_EH_{m0} * \left(\frac{SPI\_E_{md}}{SPI\_E_{m0}} + HR_{md}\right)$$

The hedge return for monthly currency hedged indices is:

$$HR_{md} = \frac{S_{m0}}{F_{m0}} - \frac{S_{m0}}{F\_I_{md}}$$

The hedge return for daily currency hedged indices is calculated as follows:

when day d is not the last business day of month m,

$$HR_{md} = \sum_{i=1}^{d} AF_{mi} * \left(F\_I_{mi} / F\_I_{mi-1} - 1 \right)$$

when day d is the last business day of month m,

$$HR_{md} = \sum_{i=1}^{d-1} AF_{mi} * \left(F\_I_{mi} / F\_I_{mi-1} - 1 \right) + AF_{md} * \left(S_{md} / F\_I_{md-1} - 1 \right)$$

Domestic Currency Return Index Calculation

Background

Domestic Currency Return (DCR) calculations give the same results as divisor based calculations. Moreover, adjustments for corporate actions, additions and deletions of securities and other changes can be done using DCR.

In DCR one calculates the period-to-period percentage change of the index from the weighted percentage change of each security price and then constructs the index levels from the percentage changes. In a divisor based index the process is reversed: the index level is calculated as total market value divided by the divisor and the period-to-period percentage change is calculated from the index levels. Both approaches require an initial base period or divisor value for normalization. Both approaches give the same results. The choice depends on which approach is more convenient for a particular index. When an index of indices or an index with securities in different currencies is constructed the DCR method may be preferred.

In the DCR calculation, we calculate the percentage change in each security price, weight the percentage changes by the security's weight in the index at the start of the period and then combine the weighted price changes to calculate the index price change for the time period. The change in the index is, then, applied to the index level in the previous period to determine the current period index level.

Equivalence of DCR and Divisor Calculations

The equivalence of the two approaches – DCR and divisor based – can be understood in two ways. First, except for the initial base value of an index, it can be defined by either the index levels or the percentage change from one period to the next. If we defined an index by a time series of index levels (100, 101.2, 103, 105…) we can derive the period to period changes (1.2%, 1.78%, 1.94%...). Given these changes and assuming the index base is a value of 100 allows us to calculate the index levels. Except for the base, the two series are equivalent. DCR calculates the changes; the divisor approach calculates the levels.

The can be shown mathematically:

The divisor calculation approach defines an index as:

$$\frac{\sum_{i} price_{i,t} * share_{i}}{divisor} \tag{78}$$

Since the initial divisor is defined by the base value and date of the index,. we can replace the it with the value of the index market cap at time *t=0*:

$$\frac{\sum_{i} price_{i,t} * share_{i}}{\sum_{i} price_{i,0} * share_{i}} \tag{79}$$

Now we can multiply and divide the term in the summation in the numerator by the price at time *t=0* without changing its value.

$$\frac{\sum_{i} \frac{price_{t}}{price_{0}} * price_{0} * shares_{i}}{\sum price_{i,o} * shares_{i}} \tag{80}$$

If we look at the term in the numerator for a single stock in the index (i.e, no summation, as there is only one stock) and rearrange we get:

$$\left(\frac{price_{i,t}}{price_{i,0}}\right) * \frac{price_{i,0} * shares_{i}}{\sum price_{i,0} * share_{i}} \tag{81}$$

which is equivalent to the relative price performance for each stock multiplied by its weight in the index. When this is combined across all constituent stocks, the result is the price performance for the index.

The DCR approach uses the summation of equation (81) across all the stocks in the index to calculate the daily price performance of the index. Once the daily index performance is calculated, the index level can be updated from the previous day's index level.

DCR Calculation

$$Index_t = (Index_{t-1}) * \sum_i \frac{P_{i,t}}{P_{i,t-1}} * weight_{i,t-1} \qquad (82)$$

where:

$Index_t$ = index level at date t
P_t = security price at the close of date t
$weight_t$ = security weight in the index at close of date t

and

$$weight_{i,t-1} = \frac{P_{i,t-1} * S_{i,t-1} * FX_{i,t-1}}{\sum_i P_{i,t-1} * S_{i,t-1} * FX_{i,t-1}} \qquad (83)$$

where:

$S_{i,t-1}$ = shares of stock i
$FX_{i,t-1}$ = exchange rate of stock i for currency conversion.

Essential Adjustments

The share count ($S_{i,t-1}$) includes the adjustment for float by multiplying by the investable weight factor (*IWF*) and for index weight by multiplying by the additional weight factor (*AWF*) where necessary. Further, when an adjustment to shares is made due to a secondary offering, share buyback or any other corporate action, this adjustment must be included in $S_{i,t-1}$ if the adjusted share count takes effect on date t. A price adjustment due to a corporate action which takes effect on date t should be reflected in $P_{i,t-}$

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