ASTRONOMICAL INFORMATION SHEET No. 58

Prepared by

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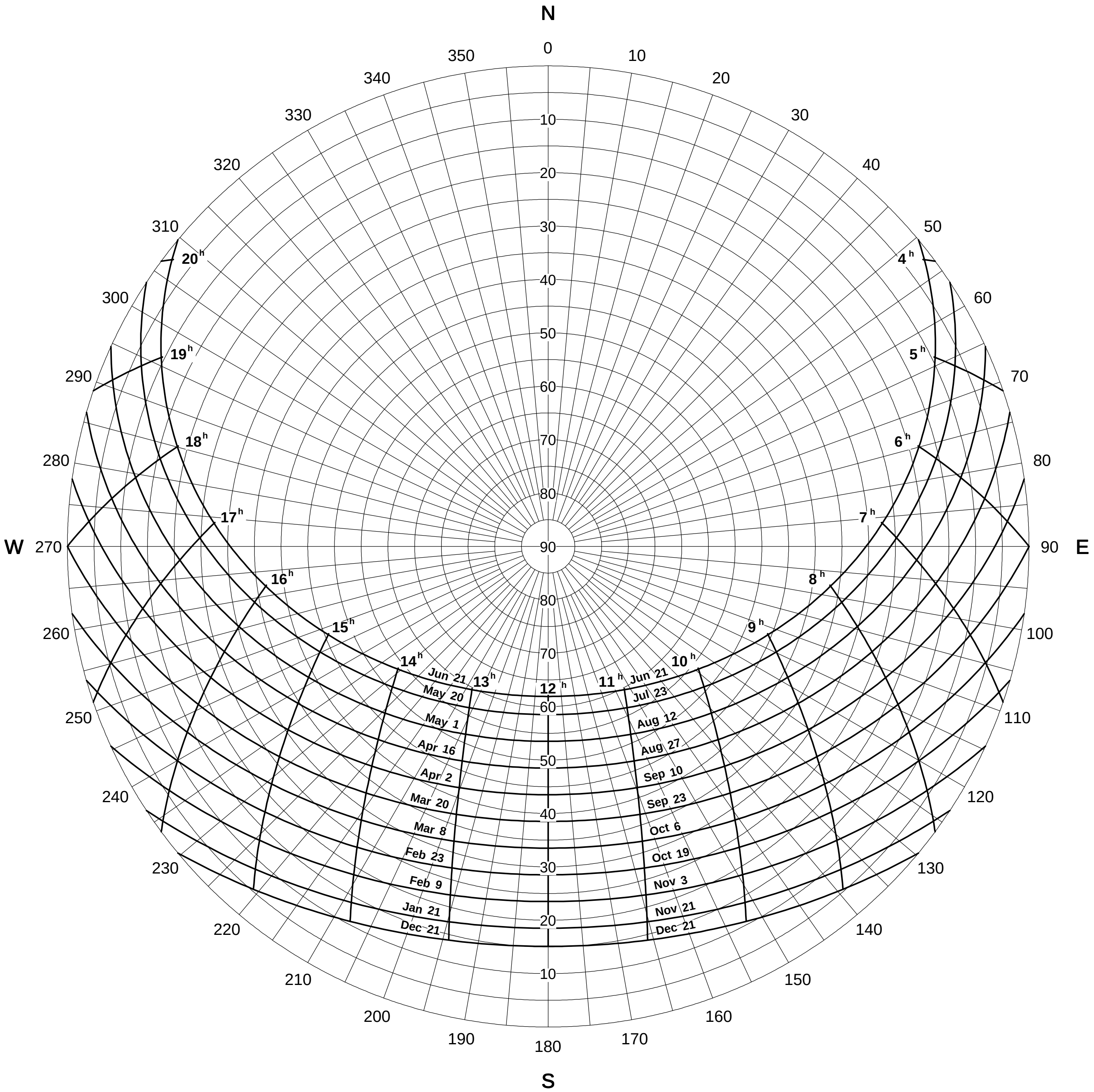
SOLAR LOCATION DIAGRAM

Altitude and Azimuth of the Sun for Latitude N51 30



The diagram shows the altitude and azimuth of the Sun for a range of local apparent solar times from sunrise to sunset at latitude N 51° 30′ (which is the latitude of London) for a series of dates throughout the year. A separate diagram for a specific latitude, which may be needed for better accuracy, will be supplied upon request.

The concentric circles indicate the altitude at an interval of 5°, from 0° on the horizon to 90° at the zenith. Refraction, a maximum value of 0°.6 at altitude 0°, has been ignored. The radial lines indicate the azimuth at 5° intervals measured 0° to 360° around the horizon from North (N) through East (E), South (S) and back to North.

The dark curves represent the apparent path of the Sun on the dates shown. These curves are drawn when the Sun's declination ranges between N20° to S20° in steps of 5°, as well as the maximum and minimum declinations of N23°.4 and S23°.4 which occur on the solstices, June 21 and December 21, respectively. The period for which the Sun is above the horizon, and thus the number of curves drawn will depend on the latitude of the diagram. If the latitude of your diagram is above or below about 66°.5 North or South, then the Sun will not be visible at its most negative or positive declinations. The table below may be used to obtain the declination (Dec) of the Sun on any date.

The dark curves marked with hours indicate the local apparent time *LAT*. In order to calculate *LAT* from the local standard time *LST* use the equation

$$LAT = LST + Long + E \tag{1}$$

conversely, to calculate *LST* from *LAT* use the equation

$$LST = LAT - Long - E \tag{2}$$

where *Long* is the longitude east of the standard meridian, measured in minutes of time (1° = 4 minutes of time), and E, the Equation of Time, is a correction from mean to apparent times which is given in the table below.

Solar Coordinates

Date	λ	*E*	*RA*	Dec	Date	λ	*E*	*RA*	Dec	Date	λ	*E*	*RA*	Dec	Date	λ	*E*	*RA*	Dec
	°	m	h	°		°	m	h	°		°	m	h	°		°	m	h	°
Dec.27	275	−1		−23	Mar. 26	5	−6		+2	June 27	95	−3		+23	Sep. 28	185	+9		−2
Jan. 1	280	−3		−23	31	10	−4		+4	July 2	100	−4		+23	Oct. 3	190	+11		−4
6	285	−6	19	−23	Apr. 5	15	−3	1	+6	7	105	−5	7	+23	8	195	+12	13	−6
10	290	−7		−22	10	20	−1		+8	13	110	−6		+22	14	200	+14		−8
15	295	−9		−21	15	25	0		+10	18	115	−6		+21	19	205	+15		−10
20	300	−11	20	−20	20	30	+1	2	+11	23	120	−6	8	+20	24	210	+16	14	−12
25	305	−12		−19	25	35	+2		+13	28	125	−6		+19	29	215	+16		−13
30	310	−13		−18	May 1	40	+3		+15	Aug. 2	130	−6		+18	Nov. 3	220	+16		−15
Feb. 4	315	−14	21	−16	6	45	+3	3	+16	8	135	−6	9	+16	8	225	+16	15	−16
9	320	−14		−15	11	50	+4		+18	13	140	−5		+15	13	230	+16		−18
14	325	−14		−13	16	55	+4		+19	18	145	−4		+13	18	235	+15		−19
19	330	−14	22	−11	21	60	+3	4	+20	23	150	−3	10	+12	22	240	+14	16	−20
24	335	−13		−10	26	65	+3		+21	28	155	−1		+10	27	245	+13		−21
Mar. 1	340	−12		−8	June 1	70	+2		+22	Sep. 3	160	0		+8	Dec. 2	250	+11		−22
6	345	−11	23	−6	6	75	+1	5	+23	8	165	+2	11	+6	7	255	+9	17	−23
11	350	−10		−4	11	80	+1		+23	13	170	+4		+4	12	260	+7		−23
16	355	−9		−2	16	85	−1		+23	18	175	+6		+2	17	265	+4		−23
Mar.21	0	−7	0	0	June 22	90	−2	6	+23·4	Sep. 23	180	+7	12	0	Dec.22	270	+2	18	−23·4

The above table gives the apparent ecliptic longitude of the Sun (λ), on those dates when the longitude is a multiple of 5°, together with the Equation of Time (E), to the nearest minute of time, and the declination of the Sun, to the nearest degree. The apparent right ascension (*RA*) is given on those dates when it is a whole number of hours. The *RA* increase by 4 minutes of time per day, thus making it possible to calculate it on any day. The table is valid from 1900 to 2100.

In the UK, the standard meridian is the Greenwich meridian and *LST* is universal time UT (also known as GMT). When British summer time (BST) is in force BST = UT + 1^{h}.

Formulae (1) and (2), together with the table may be used with diagrams for other geographic latitudes. In general the error in using this diagram for places 220 km north or south of N51° 30′ will not exceed 2° in altitude or azimuth.

The worked examples which follow are for use with the diagram for N51° 30′. They illustrate how to use the diagram in conjunction with the table on page 2 to solve practical problems that are often met for example by photographers, building constructors and surveyors. Examples 4 and 5 are also intended for Muslims to calculate prayer times.

Example 1 Find the altitude and azimuth of the Sun at $15^h\ 50^m$ BST on October 11 at a place near latitude N51° 30′ and longitude 1° 45′ West of Greenwich.

$$\text{Universal Time} = \text{BST} - 1^h = 14^h\ 50^m$$

In decimals of a degree, the longitude $Long = -1^\circ\!.75$. Convert to minutes of time by multiplying by 4, then $Long = -7^m$.

From the table, the equation of time, interpolating to October 11, is $E = +13^m$

Using equation (1) $LAT = 14^h\ 50^m - 7^m + 13^m = 14^h\ 56^m$

Using the diagram, interpolate for date (October 11) and the local apparent time ($LAT = 14^h\ 56^m$), then

$$\text{Altitude} = 21^\circ \quad \text{and} \quad \text{Azimuth} = 228^\circ$$

Example 2 Find the universal time when the Sun is on the meridian at a place 1° east of Greenwich on February 28.

In units of time the longitude $Long = +4^m$.

Using the table, the equation of time, on February 28 is $E = -12^m$.

The Sun is on the local meridian when $LAT = 12^h\ 00^m$.

Therefore from equation (2)

$$\text{UT} = LST = 12^h\ 00^m - 4^m + 12^m = 12^h\ 08^m$$

Example 3 Find the dates when the altitude and azimuth of the Sun is, 25° and 219°, at N51° 30′, E1°. This position corresponds to the position of the BSB satellite for that location.

Look at the diagram, on the radial line of azimuth 220°, 5 concentric circles from the edge and the dates are between February 23 and March 8, and October 6 and October 19. Interpolating by eye for date and time gives March 4 and October 10 at $14^h\ 20^m$ local apparent time (LAT). The longitude $Long = \text{E}\,1^\circ = +4^m$ and E from the table for March 4 and October 10 are -11^m and $+13^m$. Using equation (2) to convert to LST gives

$$\begin{aligned} \text{Mar. 4:}\quad & LST = 14^h\ 20^m - 4^m - (-11^m) = 14^h\ 27^m \\ \text{Oct.10:}\quad & LST = 14^h\ 20^m - 4^m - (+13^m) = 14^h\ 03^m \end{aligned}$$

For this location, on March 4 at $14^h\ 27^m$ UT and on October 10 at $15^h\ 03^m$ BST the Sun will be at the same altitude and azimuth as the BSB satellite.

Example 4 Find the length of the shadow cast by a vertical stick, h metres long, at the equinoxes, March 20 and September 23, for a location N51° 30′, W1° 25′, i) at meridian passage, ii) at $15^h\ 00^m$ UT.

i) The length of a shadow, x is calculated from

$$x = h/\tan a \qquad (3)$$

where h is the height of the stick, x is the length of the shadow (both x and h are in the same unit of measure), and the angle a, in degrees, is the altitude of the Sun, obtained from the diagram (ignoring refraction), at the time for which the shadow length is required.

The Sun is due South (azimuth 180°) at meridian transit and *LAT*= $12^{\text{h}}\ 00^{\text{m}}$. From the diagram at the equinoxes, the altitude of the Sun, $a = 38°$. Thus the length of the shadow is $x = h/\tan 38 = 1{\cdot}3h$ metres.

ii) Convert the standard time ($15^{\text{h}}\ 00^{\text{m}}$ UT) to apparent Solar time (*LAT*) using equation (1) *i.e.*

$$LAT = LST + Long + E$$

$$\begin{aligned} \text{On March 20:} \quad E &= -7^{\text{m}} \text{ from the table on page 2} \\ Long &= \text{W}\,1°\ 25' = -1\overset{\circ}{.}42 = -6^{\text{m}} \quad \text{multiplying degrees by 4} \\ LST &= 15^{\text{h}}\ 00^{\text{h}}\ \text{UT} \\ \text{and } LAT &= 15^{\text{h}}\ 00^{\text{m}} - 6^{\text{m}} - 7^{\text{m}} = 14^{\text{h}}\ 47^{\text{m}} \end{aligned}$$

From the diagram the altitude is 28° and the azimuth is 229°.

The length of the shadow $x = h/\tan 28 = 1{\cdot}9h$ metres.

$$\begin{aligned} \text{On September 23:} \quad E &= +7^{\text{m}} \text{ from the table on page 2} \\ Long &= \text{W}\,1°\ 25' = -1\overset{\circ}{.}42 = -6^{\text{m}} \quad \text{multiplying degrees by 4} \\ LST &= 15^{\text{h}}\ 00^{\text{h}}\ \text{UT} = 16^{\text{h}}\ 00^{\text{m}}\ \text{BST} \\ \text{and } LAT &= 15^{\text{h}}\ 00^{\text{m}} - 6^{\text{m}} + 7^{\text{m}} = 15^{\text{h}}\ 01^{\text{m}} \end{aligned}$$

From the diagram the altitude is 26° and the azimuth is 232°.

The length of the shadow $x = h/\tan 26 = 2{\cdot}1h$ metres.

Example 5 Find the time, in the afternoon, when the length of the shadow cast by a vertical stick is equal to the length of the shadow at meridian transit (ignoring refraction) plus the length of the stick, on March 20 for a location at N51° 30′, W1° 25′.

Equation (3) in example 4 gives the length of the shadow at transit $x = h/\tan a$, at some other time, t, the length of the shadow is $x_t = h/\tan a_t$. We need to find the altitude a_t, which satisfies the equation

$$\begin{aligned} x_t &= x + h \\ h/\tan a_t &= h/\tan a + h \end{aligned}$$

Hence this simplifies to

$$a_t = \tan^{-1}\left(\frac{1}{1 + 1/\tan a}\right)$$

From the previous example, at transit on March 20, $a = 28°$ and hence $a_t = 19°$. Looking along the curve on the diagram labelled with Mar 20, the two places where the altitude is 19° are $08^{\text{h}}\ 09^{\text{m}}$ and $15^{\text{h}}\ 51^{\text{m}}$. Now use equation (2) to convert the apparent solar time to standard time, thus

$$\begin{aligned} LST &= LAT - Long - E \\ &= 15^{\text{h}}\ 51^{\text{m}} + 6^{\text{m}} + 7^{\text{m}} = 16^{\text{h}}\ 04^{\text{m}} \end{aligned}$$

Since BST is not in force the time when the shadow is the required length is $16^{\text{h}}\ 04^{\text{m}}$ UT.