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References: 1. Baird, K. M.: Appl. Opt. **2**, 471 (1963). — 2. Hart, K. H., and K. M. Baird: Can. J. Phys. **39**, 781 (1961). — 3. Carré, P.: Metrologia **2**, 13 (1966). — 4. Hoffrogge, C., u. H. Rummert: Metrologia **4**, 68 (1968). — 5. Ciddor, P. E., u. C. F. Bruce: Metrologia **3**, 109 (1967). — 6. Dunn, A. F.: Rev. Sci. Instr. **30**, 203 (1959). — 7. Baird, K. M.: Rev. Sci. Instr. **32**, 549 (1961). — 8. Astrop, A. W.: Machinery (London) **99**, 944 (1961). — 9. Procès-Verbaux des Séances, Comité Intern. Poids Mesures, 2^{e} serie **28**, 71 (1960). — 10. Dobrowolski, J. A.: J. Opt. Soc. Am. **49**, 794 (1959). — 11. Candler, C.: Modern Interferometers, p. 202. London, Eng.: Hilger and Watts Ltd. 1951. — 12. Baird, K. M.: J. phys. radium **19**, 384 (1958). — 13. Baird, K. M.: J. Opt. Soc. Am. **53**, 717 (1963). — 14. Procès-Verbaux des Séances, Comité Intern. Poids Mesures, 2^{e} serie **31**, 26 (1963). — 15. Edlén, B.: J. Opt. Soc. Am. **43**, 339 (1953). — 16. Barrell, H., and J. E. Sears: Phil. Trans. Roy. Soc. London Ser. A. **238**, 1 (1939). — 17. Bruce, C. F.: Optica Acta (Paris) **4**, 127 (1957). — 18. Thwaite, E. G.: Australian J. Phys. **18**, 401 (1965).

Time Scales

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Summary

Time scales have traditionally provided the time of day and the season of the year, as well as time interval, and if it is to be of universal use the atomic scale must be coordinated with astronomical scales. Two major steps in the coordination have already been taken: the atomic unit has been defined in terms of the second of Ephemeris Time, and the scales were made to agree on 1st January 1958. If the rotation of the Earth were constant the scales would continue to agree, but because of the variations in the rate of rotation the atomic scale will diverge from the astronomical scale unless some adjustments are made to bring them together. The present method of adjustment is to apply a frequency offset from the nominal value to the oscillator from which the 1s timing pulses are derived and when necessary a step adjustment to the epoch of the signals. This method possesses serious disadvantages and it is suggested that all standard transmissions should operate at their nominal values provision being made to supply those users needing astronomical time with the difference between atomic and astronomical time.

1. Introduction

The adoption of a definition of the second in terms of an atomic transition emphasises the need to consider the associated problems of the maintenance of a scale of atomic time, its distribution throughout the world and its coordination with astronomical time. The coordination is regarded as essential because time scales have always served the three quite distinct functions, of giving the time of day, the season of the year, and also a measure of time interval or duration. Any new scale must continue to serve these purposes, if it is to be of universal use and although an atomic clock can provide a very precise scale by simply counting and recording the number of seconds that have elapsed since some arbitrary zero, the time of day and the season of the year can be obtained only by astronomical measurements. It would of course be possible to use separate and independent scales of atomic and astronomical time but this possibility already seems to have been rejected, and rightly so in my view, since it would lead to confusion and duplication of effort. It remains therefore to determine the best way of achieving the desired coordination and to this end it may be useful to give first a brief survey of the main features of astronomical and atomic time.

2. Astronomical Time

Some of the problems now facing us are quite similar to those which have faced and have been overcome by astronomers in the past. The most precise astronomical measurement is that of a star transit resulting from the rotation of the Earth but a simple scale formed by recording the number of rotations, that is, the number of sidereal days, would not give the information required for civil purposes. The sidereal day is therefore converted to the mean solar day, and the days are counted in terms of months and years. The three periodicities concerned are not commensurate and in order to keep them roughly in phase the lengths of the months are made unequal and step adjustments in the form of leap years have to be made to the length of the year. The time of day is given by the position of the Sun in the daytime and by the stars at night. In the seventeenth century the need to navigate at sea provided a great incentive to the improvement of timekeeping. Observatories were founded with the express purpose of obtaining the necessary astronomical data for predicting star positions; and fortunately they were helped in this work by the development of the pendulum which was the basis of the first precise man-made clock. It provided a scale on which the times of observation could be recorded, and also a reliable means of subdividing the day into equal seconds. The emphasis during this period however was on the position of the Earth relative to the Sun, Moon and stars and not on equal intervals of time. The laws of their apparent motion were determined from measurements extending over a long period and star positions could be predicted with adequate precision for navigation.

The advent of radio and the need to measure frequencies quickly and accurately not only demanded still more accurate timekeeping but moved the emphasis from position to uniformity of the scale and constancy of the unit. Radio engineering itself provided one of the most important tools, the quartz clock, which enabled the astronomical observations to be smoothed and averaged for periods as long as a year. With the help of quartz clocks and the detailed astronomical knowledge gained in the past astronomers were able to derive a scale uniform to about 1 part in 10^8, or 1 ms per day. The averaging process could have been taken further but the limit was placed by the secular and seasonal variations in the rate of rotation of the Earth itself. Although the uniform scale departed from true astronomical time or position, it remained sufficiently close to it for many purposes and corrections were published for those needing them. There were a number of scales in use as listed below, and of these the most uniform scale known as $UT2$ was the one made available by time signals.

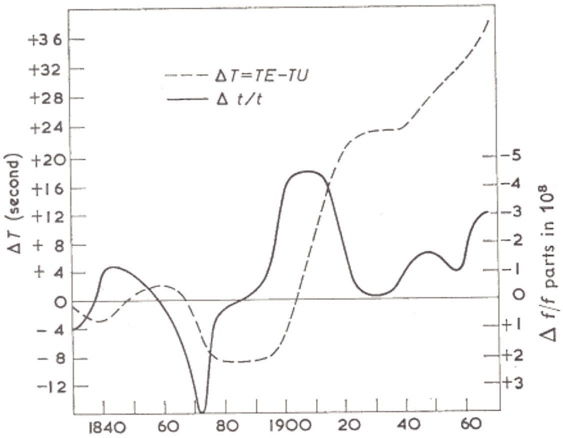

Fig. 1. Relationship between Universal Time and Ephemeris Time

1. Sidereal time.

2. Universal time $UT0$ is sidereal time converted to mean solar time from the law of the Sun's apparent motion.

3. $UT1$ is $UT0$ corrected for the movement of the poles.

4. $UT2$ is $UT1$ corrected also for the seasonal variation.

5. Ephemeris Time (ET) is the independent variable of the differential equation describing the motion of the Sun. As the equation is based on observations extending over a very long period, of about 200 years, it is in fact a kind of averaged UT. The second of ET, which was adopted as the unit of time in 1956 is thus an average value of the second of UT. The smoothed difference between ET and UT is shown in Fig. 1.

6. UTC. This scale is now used by many time services and its main features are:

a) that the transmission times of the 1s pulses are all in agreement to within 1 ms,

b) that it is allowed to depart by as much as 100 ms from $UT2$ and

c) that when a step adjustment is necessary it is made in steps of 100 ms at all the coordinated stations simultaneously. The UTC scale is a combined atomic and astronomical scale because it is in practice derived from atomic clocks but is deliberately offset from atomic time and thus brought into fairly close agreement with $UT2$.

3. Atomic Time

It was clear, as soon as a caesium standard was put into operation, that it was much more precise than any astronomical unit and could be tested only in terms of an atomic unit. Essen and Parry therefore defined such a unit as the duration of 9192631830 cycles of the transition frequency, which made it equal to the value of the second of $UT2$ at that time, and they maintained an atomic scale by means of the quartz clocks at the National Physical Laboratory. The clocks were stable to 1 part in 10^{10} per month when their steady drift was allowed for and they were checked by means of the transition at frequent intervals during the investigation. It seemed reasonable however in view of the variations of $UT2$ to define the new atomic unit in terms of its average value represented by the second of ET. This relationship took 3 years to determine and the value found in 1958 (Markowitz, Hall, Essen and Parry, 1958) was: —

Frequency of caesium transition $= 9192631770 \pm 20$ cycles per second of ET.

This value was supported by subsequent measurements and forms the basis of the present definition of the second. As it is well known atomic clocks now have an accuracy of better than 1 part in 10^{11} or 1 μs per day.

From 1955 the errors of the MSF service of standard frequency transmissions were determined and expressed in terms of the NPL caesium clock. They were used in particular at the BIH to measure and correct for the seasonal variation in the rate of rotation of the Earth. Astronomical time thus became more and more dependent on atomic clocks, which were used to divide periods of a year in the case of UT, and 3 years in the case of ET, into equal seconds.

The establishment of an atomic unit and scale involves the reverse process of integrating the very small periods of the frequency of the atomic transition. The individual oscillations are not counted, but the atomic transition is used to control the frequency of a quartz oscillator so that the relationship between them remains constant. The statement that a caesium standard is accurate to 1 part in 10^{11} implies that the frequency of the quartz oscillator is controlled and maintained with this accuracy by the atomic transition. Any errors in this process are thus already taken into account and the only other errors to consider are those associated with the integration of the seconds for long periods. For example there will be occasional counting errors and any individual clock will stop, perhaps once in every 2 years. There are in practice always a number of clocks available either in the same laboratory or through standard frequency transmissions controlled by distant clocks. Any miscount is quickly detected and allowed for and any gap in operation can be bridged by reference to another clock with a precision of 1 μs. Although there is a fundamental difference between the astronomical clock and the atomic clock in so far as the former never stops, this difference has little effect in the practical construction of the time scales. If the quartz clocks at an observatory stop the gap is bridged much more accurately by reference to other clocks than by reference to the stars.

It is however important that the reliability of atomic scales should be checked experimentally. Although the

National Physical Laboratory published corrections to the frequency of the MSF transmission in terms of the atomic clock it did not integrate the frequency to form a time scale. The first atomic time scale was established by the U.S. Naval Observatory (U.S. Naval Observatory 1959) and the times of the WWV signals were given in terms of this scale from January 1959, being extrapolated back to 1956. The zero of the scale was fixed so that on 1st January 1958 at $0^h\,0^m\,0^s$ *UT*2 that value of atomic time called *A*1 was $0^h\,0^m\,0^s$. The National Bureau of Standards and the Observatory of Neuchâtel formed independent time scales and compared them for a number of years (Bonanomi et al. 1964, Barnes et al. 1965). From 1960 onwards the divergence between the scales corresponded to an average frequency difference of 1 part in 10^{11}. The most comprehensive results have been obtained at the Bureau International de l'Heure. Independent time scales were formed for the various atomic standards as they came into use. The results are expressed in terms of a scale known as *A*3 formed from the average of 3 clocks which are given at the head of Table 1. These three are independently made caesium standards, differing in many practical details. The NPL and LSRH clocks employed a long beam giving a narrow spectral line and a similar long beam was used at the NBS after 1963. Since 1967 *A*3 has been formed from a larger group of standards.

The time deviations show that the integrated time from all the standards agreed rather better than the accuracy which was claimed for them. It is particularly noteworthy that since January 1966 the clocks have operated with average frequencies agreeing to within a few parts in 10^{12}. The table shows clearly that atomic time scales extending over long periods have the full accuracy of the clocks which is now a few parts in 10^{12}.

4. The Distribution of Atomic Time

Atomic time has been introduced into time signal services with the following aims in mind:

a) To make the full accuracy of the atomic clock widely and immediately available.

b) To avoid confusion by having two separate systems of time signals.

c) To maintain the time scale made available by the signals sufficiently near to *UT*2 for navigators who might not have ready access to published corrections.

The MSF service based on the NPL standard met these requirements between 1955 and 1958 because it was based on a provisional atomic unit which was made equal to the unit of *UT*2 in 1955. It was not anticipated that the use of a unit based on *ET* would introduce much difficulty because in 1955 it did not differ greatly from that of *UT*2. However by 1958 when the value was published *UT*2 had deviated considerably. It was therefore announced at a symposium of the International Astronomical Union (IAU) in 1958 (Essen 1959) that the MSF service would be based on the new unit but that the frequencies would be offset by a stated amount in order to bring the signals close to the scale of *UT*2. The amount of the offset was decided after consultation with the observatories and the values used in 1959 and 1960 respectively were -170 parts in 10^{10} and -150 parts in 10^{10}. The International Scientific Radio Union (URSI) recommended in 1960 that the amount of offset for each year should be announced by the BIH and this proposal was supported by the IAU in 1961. The values adopted have been as follows:

1961	-150 parts in 10^{10}	1965	-150 parts in 10^{8}
1962	-130 parts in 10^{10}	1966	-300 parts in 10^{8}
1963	-130 parts in 10^{10}	1967	-300 parts in 10^{8}
1964	-150 parts in 10^{10}	1968	-300 parts in 10^{8}.

Table 1. *Time scales based on atomic standards derived at the BIH*

Laboratory	$A3 - A$ Laboratory unit 1×10^{-4} s					
	Jan 1961	Jan 1964	Jan 1965	Jan 1966	Jan 1967	June 1967
NPL Teddington	0	+ 25	+ 32	+28	+27	+26
NBS Boulder	0	− 1	− 6	− 5	− 5	− 5
LSRH Neuchâtel	0	− 24	− 27	−23		
NRL Washington	0	+ 68	+ 63	+63		
USNO Washington	0	+122	+131	− 1	− 1	− 1
Bagneux	0	+181	+176	− 3	− 7	
Tokyo			+ 1	− 1	+ 1	0
NRC Ottawa				+ 1	0	0
IRNO Stockholm				0	0	− 1
Paris				0	+ 3	+ 5

Important steps in the coordination of standard frequency transmissions and observatory time signal transmissions were taken in 1959 in both the U.S.A. and the U.K. (NBS 1960, NPL 1960, Essen, 1960). In the U.K. this was fairly easy to achieve because both transmissions are made from the Post Office Radio Station at Rugby. It was therefore agreed between the NPL and the Royal Greenwich Observatory that all the 1s time signals for both transmissions should be obtained from the quartz standards controlling the standard frequency service. This closer coordination involved a further change in the mode of operation because even with the offset applied there was a possibility that the time might drift inconveniently far from *UT*2. It was therefore agreed that a step adjustment of 50 ms could be made when necessary on the first day of the month. It was also agreed between the U.K. and U.S.A. authorities that the nominal frequencies should be maintained within ±1 part in 10^{10}, that the emission times should be maintained in close agreement, and that any step adjustments should be by an agreed amount at an agreed time. The coordinated system of operation has now been adopted by about 30 services. In 1964 the IAU recommended that the amount of the step adjustments should be 100 ms and also that the offset should change only in steps of 50×10^{-10}. This recommendation has been widely accepted and represents the current practice of the services adopting the coordinated system described in section 2.

In addition to the offset the following step adjustments have been required:

Date	Adjustment	Date	Adjustment
1. 11. 1963	−100 ms	1. 3. 1965	−100 ms
1. 4. 1964	−100 ms	1. 7. 1965	−100 ms
1. 9. 1964	−100 ms	1. 9. 1965	−100 ms
1. 1. 1965	−100 ms	1. 2. 1968	+100 ms.

The large number of adjustments made in 1965 were not due to any unusual changes in the rate of rotation of the Earth but to the reluctance to change the offset more often than necessary. There are obviously conflicting interests and some kind of compromise must be reached.

5. Proposals for a New Method of Operation

We have seen that atomic time has been introduced into the time signal services gradually without fundamentally altering their nature. They now define interval with far greater accuracy than before, but they are allowed to deviate further from *UT*2 and deviate considerably from *AT*. We should consider whether this compromise is the best that can be achieved in the light of present conditions, which differ in a number of important respects from those existing at the beginning of this development. The unit of time is now defined in terms of the atomic transition and atomic clocks and atomic time scales have been thoroughly tested. There no longer seems to be any need, or any useful purpose in the observatories attempting to derive a uniform astronomical time scale since the atomic uniform scale is now available. The transmissions are very widely used in many branches of physics and engineering and in new navigational systems depending on accurate carrier frequencies. The signals are becoming increasingly used for operating automatic equipment and such applications are important economically. The transmissions give all these users the information they require but they have the following disadvantages:

a) Corrections have to be applied. The user must know the value of the offset, and the way in which it is applied to the particular transmission being used. He must also note the step adjustments that are made.

b) Equipment must be modified when the offset is changed, which may happen once each year.

c) In one method of applying the offset the carrier wave and the timing pulses are not coherent.

d) It is illogical to apply offsets to a unit which is constant in order to make it agree with astronomical periods which are not constant. The fact that the offset is not logical leads the user to think that no correction is required.

e) Automatic equipment might be upset by the step adjustments.

These disadvantages affect all users but it may be that for some, probably a small minority, the disadvantages are outweighed by the fact that the signals are near enough to *UT*2 to be used without corrections. After consideration of these problems the IAU recommended in 1964 that studies should be undertaken with a view to the adoption of a system of operation allowing the transmission of the epoch of *UT*2 and the unit of time interval without step adjustments or frequency changes, and still preserving a known relationship between the frequency and the time signals. In 1966 however the International Radio Consultative Committee recommended procedures of operation which conform with those already described here and involve both step adjustments and frequency changes. This recommendation reflects the difficulty of achieving the objects outlined by the IAU. URSI, also in 1966, expressed the opinion that all the methods of operation so far proposed contained defects which would create increasing difficulties, and that the services must inevitably develop towards a system of uniform atomic time and constant frequency. A meeting called by URSI in Brussels in 1967 (URSI 1967, Essen 1967) to consider the coordination of standard frequency services in the European area, included among its conclusions the view that the advantages of dispensing with frequency offsets and step adjustments were overwhelming.

Table 2. *Time deviation in seconds between AT and UT2*

Date	$UT2 - A3$	Date	$UT2 - A3$
Jan 7, 1956	+0.8186	Jan 10, 1963	
Jan 1, 1957	+0.4973	Jan 5, 1964	
Jan 6, 1958	−0.0036	Jan 4, 1965	
Jan 1, 1959	−0.4962	Jan 4, 1966	
Jan 6, 1960	−0.9827	Jan 4, 1967	
Jan 10, 1961	−1.4215	Jan 4, 1968	
Jan 5, 1962			

Although such a step is in accord with a part of the IAU recommendation it cannot be implemented without allowing the epoch of the signals to deviate more than at present from *UT*2. The deviation between *AT* and *UT*2 since 1956 is given in Table 2.

If atomic time was transmitted without any adjustments it would be a great convenience to most users and it would cause little inconvenience to those requiring *UT*2 to closer than 0.1 s since they already need to apply corrections. There remain the users of *UT*2 who can tolerate an error of 0.1 s and do not therefore at present need to apply corrections. Their number may not be great but it is essential nevertheless to provide them with the information they need. It is important to know therefore the accuracy which they can use. In view of the type of instrument used in navigation it is probable that the accuracy required is less than 0.1 s. It was for example suggested at the IAU meeting in 1967 that the time deviation might be allowed to increase to 0.2 s in order to reduce the number of step adjustments. It is suggested therefore that the possibility should be explored of incorporating the predicted deviation with the astronomical data which the navigators already use. If it is found that this cannot be done with sufficient accuracy any gross error could be avoided by moving the minute marker along by 1 sec on a prearranged date, possible 1 January and 1 June when the deviation exceeds 0.5 s. It is possible that another objection to changing to atomic time is that a certain amount of automatic equipment already in operation is adjusted to *UT*2. This argument is one which will always apply and the situation will become worse the longer the change is delayed. Although it is not regarded as a valid reason for preserving the transmission of *UT*2 it is a valid reason for giving adequate notice of any change that is contemplated.

It is suggested therefore that the major steps in the coordination of astronomical and atomic time have already been taken in the definition of the atomic second and the adoption of the zero of the atomic scale as 1 January, 1958, $0^h\,0^m\,0^s$. Standard transmissions should operate on *AT* for both frequency and time from say 1 January 1970. If it is necessary the minute markers should be moved by 1s at a prearranged date if the deviation between *AT* and *UT*2 exceeds 0.5s.

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References: Essen, L.: Astronomical Journal 64, Symposium No. 11, p. 120 (1959). — USNO: Time service notice No. 6, 1 Jan. 1959. — Bonanomi, J., P. Kartaschoff, J. Newman, J. A. Barnes, and W. R. Atkinson: Proc. IEEE 52, 439 (1964). — Barnes, J. A., D. H. Andrews, and D. W. Allan: IEEE Trans. Instr. Meas. **IM-14**, 228 (1965). — National Physical Laboratory Announcement: Electronic Technology 37, 90 (1960). — National Bureau of Standards Announcement: Proc. I.R.E. 48, 105 (1960). — Essen, L.: Nature 187, 452 (1960). — Markowitz, W., R. G. Hall, L. Essen, and J. V. L. Parry: Phys. Rev. Letters 1, 105 (1958). — URSI: Information Bulletin No. 164, p. 8 (1967). — Essen, L.: Telecommunication Journal 34, 468 (1967).

A Re-Evaluation of the N. R. C. Long Cesium Beam Frequency Standard

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Abstract

The accuracy of the National Research Council 2.1 m cesium beam frequency standard has been re-evaluated using the two NRC hydrogen masers as stable comparison standards. At the time of re-evaluation, the various uncertainties gave rise to an error limit of $\pm 3.8 \times 10^{-12}$, with a most probable value of $\pm 1.5 \times 10^{-12}$ if they were assumed to be random. After application of all corrections, the agreement with several other cesium standards, as determined from monthly VLF comparisons during January to March, 1968, appears generally to be within the narrower limits of $\pm 1.5 \times 10^{-12}$.

Introduction

With the completion of a pair of hydrogen masers at the National Research Council [1] it has become possible to re-evaluate the accuracy of the NRC 2.1 m cesium beam frequency standard. This standard, in operation since 1965 [2], has exhibited much closer agreement [3, 4, 5] with other cesium frequency standards than the original estimate of accuracy, $\pm 1.9 \times 10^{-11}$, might indicate. This initial estimate was based on experimental comparisons with a stable crystal oscillator, and appears to have been influenced mainly by the limitation of frequency stability imposed by this crystal rather than that of the standard itself. Use of the hydrogen masers, which possess frequency stabilities over short periods far in excess of that of the cesium standard, has eliminated this primary source of frequency instability, and has appreciably improved the precision of estimating the accuracy of the cesium standard.

Method of Re-Evaluation

The technique used has been to observe changes in the relative frequency of the hydrogen and cesium standards when certain modifications to the cesium standard alone are made. The output of the 5 MHz crystal locked to the cesium resonance is used to drive a multiplier and synthesizer system used to detect the maser signal. The beat frequency between these two signals at 1420 MHz is then monitored, and changes in this, and in the synthesizer settings, are used to calculate the corresponding changes in the apparent cesium resonant frequency as determined by the cesium standard.

The frequency f_x of the 5 MHz crystal oscillator locked to the cesium resonance can be expressed as follows:

$$9192632000\ \frac{f_x}{5 \times 10^6} = 9192631770 + \delta f_H + \sum \delta f_i \qquad (1)$$

where 9192632000 refers to the nominal frequency synthesized, 9192631770 Hz is the defined value of the cesium hyperfine transition frequency, $\sum \delta f_i$ represents the sum of systematic errors δf_i, $i = 1, \ldots, n$, and δf_H is the C field correction.

In the NRC standard, δf_H is determined from measurements of the $(4, -4) \longleftrightarrow (4, -3)$ transition frequencies, ν_k, $k = 1, \ldots 8$, over eight adjacent sections of the drift space between the two oscillating field regions.

$$\delta f_H = 427.18\ \overline{\left(\frac{\nu_k}{349746}\right)^2}\ . \qquad (2)$$

The values chosen for the constants are those given by Mockler [6]. Experimentally, it is only the ratio of these constants which can be determined, and within the limits of experimental error, to be described later, these values appear appropriate.

From Eq. (1), the frequency offset from nominal, $\frac{\Delta f}{f}$, of the locked 5 MHz oscillator, given by

$$\frac{f_x}{5 \times 10^6} = 1 - \frac{\Delta f}{f} \qquad (3)$$

is

$$\frac{\Delta f}{f} = \frac{230 - \delta f_H - \Sigma \delta f_i}{9192632000} \qquad (4)$$

and any change in this offset, $\mathrm{d}\left(\frac{\Delta f}{f}\right)$ due to changes in δf_H, or in $\sum \delta f_i$, is

$$\mathrm{d}\left(\frac{\Delta f}{f}\right) = \frac{-\,\mathrm{d}\delta f_H - \underset{i}{\Sigma}\,\mathrm{d}\delta f_i}{9192632000} \qquad (5)$$