Racial Discrimination and Redlining in Cities*

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Abstract

We propose a model where employers have two types of prejudices: racial and spatial discriminations. Because of the first one, black workers have less chance than white workers to find a job. Because of the second one, workers living closer to the city-center have less chances than suburban workers to find a job. In this context, we show that depending on the importance of access costs to employment centers two urban equilibria may emerge. In Equilibrium 1 (the access cost for blacks is quite large), black and white workers are totally separated while in Equilibrium 2 (the access cost for blacks is relatively small), workers are separated by their employment status (the unemployed versus the employed). We then study the labor market equilibrium and its interactions with the land market. We show in particular that both "race" and "space" matter to explain high unemployment rates among blacks.

Key words: urban equilibrium, access costs, spatial discrimination, urban unemployment.

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1 Introduction

The "race versus space" debate is extremely important if one wants to understand why minorities especially blacks suffer from economic disadvantages. This debate attempts to measure whether these economic disadvantages endured by blacks are the results of labor market discrimination or their residential location. The aim of this paper is to show that both race and space matter for explaining the high unemployment rate among blacks.

The theoretical literature has given several answers to these two types of problems, i.e., labor discrimination and urban segregation. In labor economics, Becker (1957) assumes a 'taste for discrimination' for employers so that discriminated workers will not invest very much in human capital because the returns are too low. In urban economics, Rose-Ackerman (1975), Yinger (1976), Courant and Yinger (1977) (among others) assume that whites do not want to live close to blacks because it affects negatively their utility level (negative externalities). They show that, in equilibrium, the two communities will be clearly separated, blacks living closer to the city-center and whites closer to the outskirts of the city. More recently, Benabou (1993, 1996) shows that, even though education is a local public good and generates externalities, the only stable urban equilibrium is the one in which the two communities are totally separated.

In all these studies, the direct link between labor discrimination and urban segregation is not explicit. The more natural link has been introduced empirically by Kain (1968). He proposed the *spatial mismatch* hypothesis to explain the high rates of poverty and unemployment among African American inner city residents. Residing in segregated areas distant from and poorly connected to major centers of employment growth (located in general in the U.S. in suburbs), African Americans were said to face strong geographic barriers to finding and keeping well-paid jobs. For example Zax and Kain (1996) show that the suburbanization of employment tends to reduce black opportunities and increase black unemployment. Indeed, when firms decide to delocate themselves to the periphery of the city (which is an important phenomenon in the US; see e.g., Garreau, 1991), segregation forces some African Americans to quit their jobs rather to follow their employer. This is due to the fact that discrimination in the housing market (which is common fact in the U.S., see e.g. Gordon, 1987) limits the residential locations of blacks workers and implicitly restricts black employment to workplaces which are within acceptable commuting distances of black residential areas. This also shows that blacks have more difficult access to employment centers than whites because they are more sensitive to commuting costs than whites. For example, Raphael (1998) shows that the differential of accessibility explains 30 to 50% of the neighborhood employment rate differential between white and black male Bay-Area youths (San Fransisco-Oakland-San Jose consolidated Metropolitan Statistical Area for the year 1990). Ihlanfeldt (1980, 1993) and Ihlanfeldt and Sjoquist (1990, 1991) find similar results for other MSAs. Although there is a huge empirical literature testing the spatial mismatch hypothesis (see the two survey articles written by Holzer, 1991, and Kain, 1992), its modelling is still in its infancy (exceptions include Arnott, 1998, Brueckner and Martin, 1997, Coulson, Laing and Wang, 1997, Wasmer and Zenou, 1998).

In terms of policies, the U.S. Affirmative action tries to fight against discrimination by giving a preferential treatment for African Americans, women or other minority groups. However, affirmative action is one of the most controversial government interventions in the labor market since its effects are ambiguous and not clear. In particular, the wage gaps between blacks and whites and the access to employment are still very important (Leonard, 1990, 1996, O'Neill, 1990) and major criticisms stipulate that affirmative action does not work and should be suppressed.

Although the Kerner Commission (U.S. National Advisor Commission on Civil Disorders, 1968) singled out the black ghetto as a fundamental factor behind urban poverty in the United States, residential segregation has remained strangely absent from policy debates and discussions in recent years. *We believe that, in order to be efficient, policies such as affirmative action must be accompanied by measures aimimg at fighting spatial segregation.* In particular, affirmative action must take into account the problem of residential segregation since a large share of African Americans remains spatially segregated on the basis of race and because life chances are so decisively influenced by where one lives (see Akerlof, 1997, Benabou, 1993, 1996, Montgomery, 1991, Sigelman, Bledsoe, Welch, Combs, 1996, Wial, 1991, who point out the importance of local contacts and networks in getting jobs and good education). As a result of residential segregation, African Americans must endure an extraordinary harsh and intensely disadvantaged environment where poverty, crime, single parenthood, welfare dependency, and educational failure are not only common, but the norm (Wilson, 1996).

There is obviously an important link between labor discrimination and urban residential segregation and it seems that the federal government should consider both. For example, Massey (1994) advocates the elimination of residential segregation by requiring the direct involvement of the federal government (in particular the Housing and Urban Development department).

In Europe, there is a rising problem of ghettos located in the suburbs and composed of minorities (see e.g., Brun and Rhein 1994, Dubet, 1995, Wilson, 1996, ch. 6 and 8) but no policies such as affirmative actions have been contemplated due to the fact that it is a burning and taboo topic (it is for example very difficult to

obtain data on religions or ethnicity). However, recently, policies aiming at fighting spatial segregation or redlining have been considered, in particular in France. Any firm which desires to set up in a 'priority district' (a district that needs to be helped because of high unemployment rates, low income, high criminality...) is tax free but 20% of its workforce must be composed of local workers. Typically, this type of policy acts on residential segregation and redlining phenomena but not on labor discrimination. The same type of policy (the enterprise zone programs) has been implemented in the US but their effects are controversial (see e.g. Papke, 1994 or Boarnet and Bogart, 1996).

In the present paper, we consider a model that combines both labor discrimination and urban segregation. We first present the urban land use model (section 2). There are four types of workers, blacks and whites, employed and unemployed. Because the employed are obviously richer than the unemployed, they consume more land and prefer peripheral locations. Because blacks have more difficulty to access to the employment center (the transportation network is in general not very good and blacks are thus more sensitive than whites to it since they tend to have in average less car per capita) they prefer central locations. This highlights what we have said above about the empirical results of the spatial mismatch hypothesis. We show that depending on the importance of this access cost two urban equilibria may emerge. In Equilibrium 1 (the access cost is quite large), black and white workers are totally separated while in Equilibrium 2 (the access cost is relatively small), workers are separated by their employment status (the unemployed versus the employed). We then study the labor market equilibrium. In section 3 where only racial discrimination is introduced (employers have racial prejudices so that they hire less and fire more blacks compared to whites), we show that there is complete separation between the housing market and the labor market analyses. Whatever the urban equilibrium (Equilibrium 1 or 2), the unemployment level is the same. In this context, we show that the latter is higher for black than for white workers. In section 4, we introduce both racial discrimination and spatial discrimination or redlining. The latter means that irrespective of race employers have spatial prejudices so that they are reluctant to hire workers living in 'bad areas' close to the city-center. In fact, employers draw a red line between central and peripheral locations and discriminate workers living in central locations. In this case, the labor market equilibrium depends strongly on the urban equilibrium. We show that urban Equilibrium 1 leads to the worse labor outcomes. Unemployment level as well as unemployment duration have the highest values so that black unemployed workers are getting stuck in ghettos. This confirms that the explanation of black unemployment is due to both "race and space".

2 Urban land use equilibrium

There is a continuum of black workers (B) whose mass is given exogeneously by $\overline{N}_B$ and a continuum of white workers (W) whose mass is given exogeneously by $\overline{N}_W$ (with $\overline{N}_B + \overline{N}_W = \overline{N}$). Each worker can be either unemployed (where U_k , $k = B, W$ is the mass of the unemployed) or employed (where L_k , $k = B, W$ is the mass of the employed). We have:

$$\overline{N}_k = U_k + L_k \qquad k = B, W \tag{1}$$

The city is *monocentric*, i.e., all jobs and services are located at the city-center called the CBD (Central Business District), *closed,* (i.e., the workers' utility level is endogeneous whereas the number of workers is exogeneous), and *circular*. All land is owned by absentee landlords. Land is a normal good so that richer workers consume more land than poorer ones.

Each worker enjoys housing consumption q and a (non-spatial) composite good z (taken as the numeraire) so that the utility function writes $V(q, z)$. We assume that this function is well behaved (increasing and concave in its arguments). Moreover, the budget constraint for an unemployed worker of type k is given by:

$$w - c.\tau.x = z + q\ R(x) \qquad k = B, W \tag{2}$$

while the one for an employed worker of type k is equal to:

$$b - c.\tau.x = z + q\ R(x) \qquad k = B, W \tag{3}$$

where w is the downward rigid (exogeneous) wage which is assumed to be greater that the market clearing wage so that unemployment prevails in equilibrium,[1], b, the (exogeneous) unemployment benefit with $b < w$, $R(x)$, the equilibrium land rent at a distance x from the CBD, τ, the commuting cost per unit of distance. Concerning commuting costs, we have an important but realistic assumption captured by $c > 1$ for blacks and $c = 1$ for whites. Indeed, we assume that black workers have higher commuting cost per unit of distance than whites ($c.\tau$ versus τ). This highlights a well established fact that access to employment centers is more difficult for blacks than for whites. For example, in the U.S., large cities have poor transportation networks so that workers have to take frequently their cars to go to work. It is well known that blacks possess less cars than whites so that access to jobs is more difficult (see for example Zax and Kain, 1996 or Raphael, 1998 among others).

[1]This wage could have been endogeneized by using for example the efficiency wage theory (as in Zenou and Smith, 1995) but this would have complicated the analysis without changing the main results since our focus is not on the formation of wages and unemployment in cities but on labor discrimination and urban segregation.

Observe that we could also have differentiated the unemployed and the employed in terms of CBD-trips by assuming that the former go less often to the CBD than the latter. However, we assume here that, even if there is no explicit job search behavior, the unemployed go to the CBD as often as the employed because they go there for shopping, acquiring job information and interviews. Relaxing this assumption will not change the main results but will complicate the analysis (see the discussion below).

It is important to observe that we don't need the labor market analysis (next section) to determine the urban land use equilibrium. In fact, we can solve the locational equilibrium *prior* to the labor market equilibrium because there is no job search and locational choices for both employed and unemployed workers involved only fixed transportation costs and consumption of housing and composite goods which affect only short run utilities. This is similar to the study of Zenou and Smith (1995). However, if locational choices would had involved explicit tradeoffs between accessibility to the job market through search (affecting long run utilities) and consumption of housing and composite goods (affecting short run utilities), then expected lifetime utilities would have played a fundamental role and the two market equilibria (land and labor) would then had to be solved for *simultaneously* (as in Wasmer and Zenou, 1998 and Smith and Zenou, 1999).

Let us denote by I_k^l the net income of a worker of type $k = B, W$ with employment status $l = U, L$ (this is the worker of type k, l). We have: $I_B^U = b - c.\tau.x$ is the net income of an unemployed black worker, $I_B^L = w - c.\tau.x$ for an employed black worker, etc.). Each worker of type k, l chooses q that maximizes $V(q, I_k^l - qR(x))$. First order condition yields:[2]

$$V_q(.) - R(x).V_z(.) = 0 \tag{4}$$

which defines implicitly the Marshallian demand for land $q(I_k^l, R, x)$ for a worker of type k, l. In the (steady state) urban equilibrium, all workers of the same type k and the same employment status l will reach the same utility level, v_k^l. By using (4), the indirect utility function can be written as:

$$V(q(I_k^l, R, x), I_k^l - q(I_k^l, R, x)R) \equiv v_k^l \qquad k = B, W \qquad l = U, L \tag{5}$$

By taking the inverse of the indirect utility function, we can determine workers' bid rents $\Psi_k^l(x, v_k^l)$ as a function of distance, net income and equilibrium utility. Its properties are:

$$\frac{\partial \Psi_B^l(x, v_B^l)}{\partial x} = \frac{-c.\tau}{q(I_B^l, R, x)} < 0 \qquad l = U, L \tag{6}$$

[2]The second order condition is always satisfied because of the concavity of $V(.)$.

$$\frac{\partial \Psi_W^l(x, v_B^l)}{\partial x} = \frac{-\tau}{q(I_W^l, R, x)} < 0 \qquad l = U, L \tag{7}$$

Lemma 1 *Irrespective of race, the employed workers have flatter bid rents than the unemployed so that they reside farther away from the CBD.*

Proof. Let us denote by $\overline{x}$ the bid rents' intersection point. Then $\Psi_k^L(\overline{x}, v_k^L) = \Psi_k^U(\overline{x}, v_k^L) \equiv \overline{R}$ for $k = B, W$. We must show that:

$$-\frac{\partial \Psi_k^U(\overline{x}, v_k^L)}{\partial x} > -\frac{\partial \Psi_k^L(\overline{x}, v_k^U)}{\partial x} \qquad k = B, W$$

Since land is a normal good, $q(I_k^L, \overline{R}, x) > q(I_k^U, \overline{R}, x)$, $k = B, W$. Then by using (6) and (7), it is easily checked that the inequality above is always true. ■

Lemma 2 *Irrespective of employment status, white individuals have flatter bid rents than black people so that they reside farther away from the CBD.*

Proof. We must show that at $\overline{x}$ we have:

$$-\frac{\partial \Psi_B^l(\overline{x}, v_B^l)}{\partial x} > -\frac{\partial \Psi_W^l(\overline{x}, v_W^l)}{\partial x} \qquad l = U, L$$

Since $c > 1$, $I_W^l > I_B^l$ so that (because land is a normal good) $q(I_W^l, \overline{R}, x) > q(I_B^l, \overline{R}, x)$, $l = U, L$. Then by using (6) and (7), it is easily checked that the inequality above is always true. ■

These two results are easy to understand. Because the employed are richer than the unemployed they want to consume more land so that they prefer peripheral locations where land is cheaper. This result lies on the hypothesis that the unemployed have the same number of CBD-trips. If this were not the case, there would be a trade off between commuting costs and housing consumption. However, it would always be possible to find conditions to ensure that the unemployed have steeper bid rents than the employed (see e.g. Brueckner and Zenou, 1999). The second result (Lemma 2) is also quite intuitive. Since blacks have difficulty to access the employment center (we have already discussed this assumption in the introduction with the spatial mismatch hypothesis), they bid away whites at the outskirts of the city. In Detroit, Zax and Kain (1996) show that blacks are indeed very sensitive to commuting costs. They study the case of a large firm that delocates to the periphery. They show that a large fraction of black workers prefer to stay unemployed rather to have long commuting trips. Of course, it is important to observe that in our model there is free mobility and thus no discrimination in

the housing market so that blacks are able to locate closer to jobs and to bid away whites. However, this is not always true since some land owners are reluctant to rent apartments to black workers in certain areas of the city.

Because of Lemma 1 and 2, it is easily verified that only two urban equilibria can exist. The first one (Equilibrium 1) is when, beginning from the CBD, we have the following groups: (BU, BL, WU, WL) where BU means the unemployed black workers, BL, the employed black workers, WU, the unemployed white workers and WL, the employed white workers (see Figure 1). In this case, there is a total separation between blacks and whites. The second one (Equilibrium 2) is when we have starting from the CBD: (BU, WU, BL, WL) (see Figure 2) where the separation is now in terms of employment status and not in terms of race.

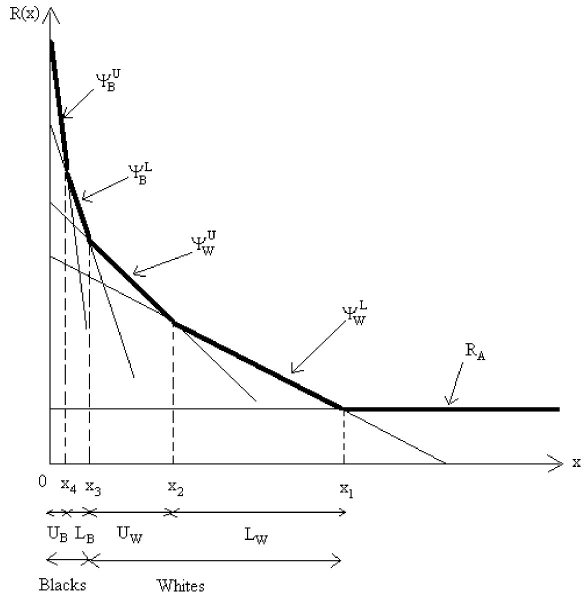

Figure 1

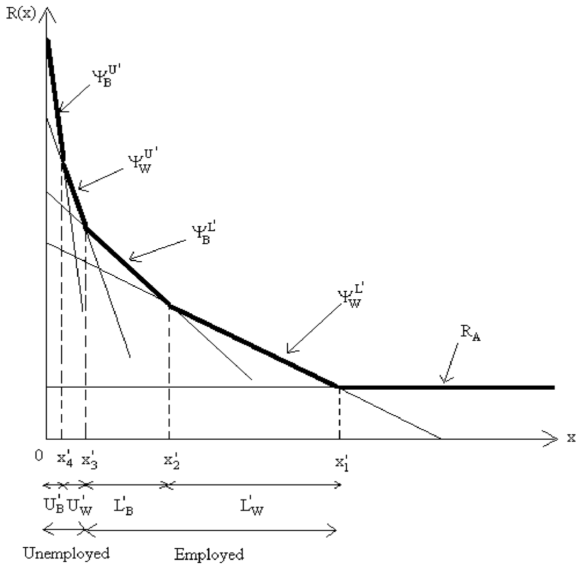

Figure 2

<u>**Proposition 1**</u>

- (i) *If the access cost c is sufficiently large, then the unique urban equilibrium configuration is Equilibrium* 1 *where blacks and whites are spatially separated* (BU, BL, WU, WL).
- (ii) *If the access cost c is sufficiently small, then the unique urban equilibrium configuration is Equilibrium* 2 *where workers are separated by their employment status* (BU, BL, WU, WL).

Proof. For (i), we want to show that:

$$-\frac{\partial \Psi_B^U(\overline{x}, v_B^U)}{\partial x} > -\frac{\partial \Psi_B^L(\overline{x}, v_B^L)}{\partial x} > -\frac{\partial \Psi_W^U(\overline{x}, v_W^U)}{\partial x} > -\frac{\partial \Psi_W^L(\overline{x}, v_w^L)}{\partial x}$$

while for (ii) we must have:

$$-\frac{\partial \Psi_B^U(\overline{x}, v_B^U)}{\partial x} > -\frac{\partial \Psi_W^U(\overline{x}, v_B^L)}{\partial x} > -\frac{\partial \Psi_B^L(\overline{x}, v_W^U)}{\partial x} > -\frac{\partial \Psi_W^E(\overline{x}, v_w^E)}{\partial x}$$

Because of Lemma 1 and 2, we just have to verify that:

$$-\frac{\partial \Psi_B^L(x, v_B^U)}{\partial x} \mathrel{\substack{>\\<}} -\frac{\partial \Psi_W^U(x, v_W^L)}{\partial x}$$

i.e., to have Equilibrium 1 the sign $>$ must hold whereas to have Equilibrium 2 the sign $<$ must prevail. By using (6) and (7), this equation can be rewritten as:

$$c \mathrel{\substack{>\\<}} \frac{q(I_B^L, \overline{R}, x)}{q(I_W^U, \overline{R}, x)}$$

Since $c > 1$, we must know if $q(I_B^L, \overline{R})/q(I_W^U, \overline{R})$ is greater or less than 1. Since land is a normal good, we have:

$$w - c.\tau.x \mathrel{\substack{>\\<}} b - \tau.x \Longleftrightarrow q(I_B^L, \overline{R}, x) \mathrel{\substack{>\\<}} q(I_W^U, \overline{R}, x)$$

Now if c is large enough we have obviously (i) while if it is small enough we obtain (ii) ■

So blacks are totally separated from whites (Equilibrium 1, *segregated city*) when the access cost is too large. This is because there is trade off for the black employed workers between land consumption and access costs. If the latter are very high, then even their net revenue if affected so that they bid away white employed workers. If c is quite small, then their net income increase and they prefer more peripheral locations because they can consume more land. The value of c is thus crucial to determine which equilibrium prevails.

2.1 Equilibrium 1

The urban land use equilibrium conditions for Equilibrium 1 (see Figure 1) are given by:

$$\Psi_B^U(x_4, v_B^U) = \Psi_B^L(x_4, v_B^L) \tag{8}$$

$$\Psi_B^L(x_3, v_B^L) = \Psi_W^U(x_3, v_W^U) \tag{9}$$

$$\Psi_W^U(x_2, v_W^U) = \Psi_W^L(x_2, v_W^L) \tag{10}$$

$$\Psi_W^L(x_1, v_W^L) = R_A \tag{11}$$

where x_4, x_3, x_2 are respectively the borders between BU and BL, BL and WU, WU and WL, x_1 is the city-fringe and R_A the land rent outside the city (the agricultural land rent). We need also the population constraints which are equal to:

$$\int_0^{x_4} \frac{2\pi x}{q(I_B^U, v_B^U, x_4)} dx = U_B \tag{12}$$

$$\int_{x_4}^{x_3} \frac{2\pi x}{q(I_B^L, v_B^L, x_3)} dx = L_B \equiv \overline{N} - U_B \tag{13}$$

$$\int_{x_3}^{x_2} \frac{2\pi x}{q(I_W^U, v_W^U, x_2)} dx = U_W \tag{14}$$

$$\int_{x_2}^{x_1} \frac{2\pi x}{q(I_W^L, v_W^L, x_1)} dx = L_W \equiv \overline{N} - U_W \tag{15}$$

Since we have a closed-city model, we have 8 unknowns (the endogeneous variables) which are the four utility levels $(v_B^U, v_B^L, v_W^U, v_W^L)$ and the four borders (x_1, x_2, x_3, x_4), and eight equations (8)–(15). By Proposition 1, Equilibrium 1 exists if c is large enough so that:

$$w - \tau.x > b - \tau.x > w - c.\tau.x > b - c.\tau.x$$

implying the following distribution of net incomes:

$$I_W^L > I_W^U > I_B^L > I_B^U \tag{16}$$

We are therefore embedded with the standard income classes framework (see e.g., Hartwick, Schweizer and Varaiya, 1976 or Fujita, 1989, ch.4) so that there exists a unique urban equilibrium (see Fujita, 1989). In this context, each of the eight equilibrium values depends on the exogeneous parameters $I_W^L, I_W^U, I_B^L, I_B^U, U_B, U_W$. Observe that U_W and U_B will be determined at the labor market equilibrium (see next sections).

Let us now perform a comparative statics analysis by focusing on the impact of the size of the unemployed workers and of net incomes on the endogeneous variables. Actually, we can use theorems 1 and 2 of Hartwick, Schweizer and Varaiya (1976) since our framework is similar to them. Indeed to prove these two theorems they need three assumptions. The first one (p.398) imposes that marginal commuting costs are always positive which is always true here since we assume linear commuting costs. The second one (p.399) states that housing is a normal good and that facing the same land rent richer individuals demand more housing that poorer ones, an hypothesis that we also assume. The third assumption (p.407) which is in fact a condition reduces to $1 > 0$ in our framework since we assume linear commuting costs and a circular city (see their note 1 p.407).

Before deriving the results it is important to observe that our framework differs slightly from the one of Hartwick et al. (1976) since in the present model the size of one class is not always independent of the size of the other. Obviously, since $\overline{N}_k$ $(k = B, W)$ is fixed exogeneously, the size of $L_k \equiv \overline{N}_k - U_k$ is exactly the residual size of U_k. So when U_k increases, contrary to Hartwick et al. (1976), we have to take into account the fact that L_k decreases by exactly $\overline{N}_k - U_k$. However, since all workers don't consume the same amount of land, when for example U_W rises, the

resulting increase of the area's size of the unemployed white workers $(x_2 - x_3)$ is less important than the resulting decrease of the area's size of the employed white workers $(x_1 - x_2)$ since the latter are richer and thus consume more land than the former. This is why in the following table we have differentiated the effect of U_k from the one of $L_k \equiv \overline{N}_k - U_k$ on the endogeneous variables and then study the net effect (with is fact the real effect), denoted by U_k^{Net}.

Table 1a: Comparative Statics Analysis for Equilibrium 1 with racial discrimination

	U_B	U_W	L_B	L_W	U_B^{Net}	U_W^{Net}	$\overline{N}_B$	$\overline{N}_W$
v_B^U	$-$	$-$	$-$	$-$	?	?	$-$	$-$
v_B^L	$-$	$-$	$-$	$-$	?	?	$-$	$-$
v_W^U	$-$	$-$	$-$	$-$	?	?	$-$	$-$
v_W^L	$-$	$-$	$-$	$-$	?	?	$-$	$-$
x_1	$+$	$+$	$+$	$+$	$-$	$-$	$+$	$+$
x_2	$+$	$+$	$+$	$-$	$-$	$+$	$+$	
x_3	$+$	$-$	$+$	$-$	$-$	$+$	$+$	$-$
x_4	$+$	$-$	$-$	$-$	$+$	$+$		$-$

Our comments are the following. Let us start with the first four columns. First, irrespective of race, when the size of the unemployed population increases (this is also true for the employed population) all utilities are reduced. This is because both the size of the ghetto (where only black unemployed workers live) and of the white unemployed area increase thus enlarging the city and consequently leading to higher commuting costs and lower net wages and utilities for all workers in the city. Second, the effect of the unemployed workers on the different borders are easy to understand. When the size of U_B increases all workers are pushed away while when the population of white unemployed workers rises only people living on the right of U_W are pushed away. The general message is the following. When a class of workers increases in size, then the outer classes are pushed away from the CBD whereas the inner ones are squeezed towards it. Indeed, since housing consumption is endogeneous, more workers means more space and thus less space for workers closer to the CBD.

Let us now focus on the effects of U_B^{Net} and U_W^{Net} on the endogeneous variables. First, all signs are ambiguous for the utilities. This is easy to understand since there are two opposite forces at work. For example when U_B increases it reduces all utilities in the city but at the same time it implies a decrease in L_B (since $\overline{N}_B = L_B + U_B$), which in turns affects positively utilities. The net effect is thus ambiguous. Second, this is no longer true for the borders. Indeed, as stated above, the unemployed workers consume less land that the employed workers of the same type $k = B, W$. So for example when U_B increases, the resulting increase of the area's size of the unemployed black workers x_4 is less important than the resulting decrease of the area's size of the employed black workers $(x_3 - x_4)$ since the latter are richer and thus consume more land than the former. In this context, it is interesting to see that when the number of unemployed rises (black or white) the city size is reduced. Last, in order to avoid the previous problem, we have divided the city in two, blacks and whites, irrespective of their employment status. It is easy to see that increasing the size of $\overline{N}_B$ or $\overline{N}_W$ reduces all utilities in the city while it has different effects on borders.

Table 1b: Comparative Statics Analysis for Equilibrium 1 with racial discrimination

	I_B^U	I_B^L	I_W^U	I_W^L
v_B^U	$+$	$+$	$+$	$+$
v_B^L	$-$	$+$	$+$	$+$
v_W^U	$-$	$-$	$+$	$+$
v_W^L	$-$	$-$	$-$	$+$
x_1	$+$	$+$	$+$	$+$
x_2	$+$	$+$	$+$	$+$
x_3	$+$	$+$	$+$	$+$
x_4	$+$	$+$	$+$	$+$

The following comments are in order. When net incomes rise all borders in the city increase and the city becomes larger so that all classes are more suburbanized. This is because housing is a normal good and more (net) income means more space. Moreover, when the net income of any class of workers increases, then the outer classes suffer a reduction of their net income and thus of utilities whereas the inner classes enjoy an increase. This is due to the fact that when the (net) income of a class rises the land rent of the outer classes increases while the land rent of the inner classes decreases (see Theorem 3, p.411 of Hartwick et al., 1976).

To sum up, when the net income of a particular class increases some workers are worse off and others are better off depending on their location. However, in both cases the city expands.

2.2 Equilibrium 2

The urban land use equilibrium conditions for Equilibrium 2 (see Figure 2) are given by:[3]

$$\Psi_B^{U'}(x_4', v_B^{U'}) = \Psi_W^{U'}(x_4', v_W^{U'}) \tag{17}$$

$$\Psi_W^{U'}(x_3', v_W^{U'}) = \Psi_B^{L'}(x_3', v_B^{L'}) \tag{18}$$

$$\Psi_B^{L'}(x_2', v_B^{L'}) = \Psi_W^{L'}(x_2', v_W^{L'}) \tag{19}$$

$$\Psi_W^{L'}(x_1', v_W^{L'}) = R_A \tag{20}$$

where x_4', x_3', x_2' are respectively the borders between BU and WU, WU and BL, BL and WL, x_1' is the city-fringe. We need also the population constraints which are equal to:

$$\int_0^{x_4'} \frac{2\pi x}{q(I_B^{U'}, v_B^{U'}, x_4')} dx = U_B' \tag{21}$$

$$\int_{x_4'}^{x_3'} \frac{2\pi x}{q(I_B^{L'}, v_W^{U'}, x_3')} dx = U_W' \tag{22}$$

$$\int_{x_3'}^{x_2'} \frac{2\pi x}{q(I_W^{U'}, v_B^{L'}, x_2')} dx = L_B' \tag{23}$$

$$\int_{x_2'}^{x_1'} \frac{2\pi x}{q(I_W^{L'}, v_W^{L'}, x_1')} dx = L_W' \tag{24}$$

We have here also eight unknowns which consist of four utility levels $(v_B^{U'}, v_B^{L'}, v_W^{U'}, v_W^{L'}$ and 4 borders (x_1', x_2', x_3', x_4'), and 8 equations (17)−(24). Because of Proposition 1, Equilibrium 2 exists if c is quite small $(1 < c' << c)$ so that we have:

$$w - \tau.x > w - c'.\tau.x > b - \tau.x > b - c'.\tau.x$$

which is equivalent to:

$$I_W^{L'} > I_B^{L'} > I_W^{U'} > I_B^{U'} \tag{25}$$

We are therefore embedded with a standard income classes framework so that there exists a unique urban equilibrium 1 (see Fujita, 1989). The comparative statics analysis is easy to derive and we can also use here the results of Hartwick, Schweizer and Varaiya (1976). They are given by (a superscript star indicates a different result compared to Equilibrium 1):

[3] All variables with a prime correspond to Equilibrium 2.

Table 2a: Comparative Statics Analysis for Equilibrium 2 with racial discrimination

	U'_B	U'_W	L'_B	L'_W	U'^{Net}_B	U'^{Net}_W
$v_B^{U'}$	$-$	$-$	$-$	$-$	?	?
$v_B^{L'}$	$-$	$-$	$-$	$-$	?	?
$v_W^{U'}$	$-$	$-$	$-$	$-$	?	?
$v_W^{L'}$	$-$	$-$	$-$	$-$	?	?
x'_1	$+$	$+$	$+$	$+$	$-$	$-$
x'_2	$+$	$+$	$+$	$-$	$-$	$+$
x'_3	$+$	$+^*$	$-^*$	$-$	$+^*$	$+$
x'_4	$+$	$-$	$-$	$-$	$+$	$+$

The general comments of this table are similar to the ones of Table 1a. The main difference lies on the fact that workers are now separated by their employment status while in Equilibrium 1 there were separated by their race. This explains for example why when U'^{Net}_B increases x'_3 increases while it decreased before. In both cases, x_3 is the border between BL and WL. This is not true for all other borders.

Table 2b: Comparative Statics Analysis for Equilibrium 2 with racial discrimination

	$I_B^{U'}$	$I_B^{L'}$	$I_W^{U'}$	$I_W^{L'}$
$v_B^{U'}$	$+$	$+$	$+$	$+$
$v_B^{L'}$	$-$	$+$	$-^*$	$+$
$v_W^{U'}$	$-$	$+^*$	$+$	$+$
$v_W^{L'}$	$-$	$-$	$-$	$+$
x'_1	$+$	$+$	$+$	$+$
x'_2	$+$	$+$	$+$	$+$
x'_3	$+$	$+$	$+$	$+$
x'_4	$+$	$+$	$+$	$+$

The comments of this table are also similar to the ones of Table 1b. The only difference lies on the fact that the white unemployed reside now closer to the CBD so that more workers are pushed away from the CBD and less are squeezed towards. So for example when the net income of the white unemployed rises (i.e. $I_W^{U'}$), the outer classes, which are now all the employed workers (while in Equilibrium 1 it

was just the white employed workers) suffer a reduction of their utilities ($v_B^{L'}$ and $v_W^{L'}$) whereas the inner classes, which are now all the unemployed workers (while in Equilibrium 1 it implied all the unemployed plus the black employed workers), enjoy an increase of their utilities ($v_B^{U'}$ and $v_W^{U'}$). It is important to observe that here the interpretation of the results is in terms of employment status while in Equilibrium 1 it was in terms of race (see Figures 1 and 2).

3 Labor market equilibrium with racial discrimination

3.1 Racial discrimination

Racial discrimination is introduced in the following way. Employers have racial prejudice so that they are more reluctant to hire blacks than whites (identified respectively by B and W) and once employed, black workers have a greater chance to be fired than whites. Formally, let θ_k be the (exogeneous) probability that a worker of type $k = B, W$ looses his job during the current period and δ_k, the probability that a worker of type $k = B, W$ finds a job during the current period. Racial discrimination implies that $\theta_W < \theta_B$, i.e., a black worker is more likely to loose his job than of white worker and $\delta_W > \delta_B$, i.e., a white unemployed person has a greater chance to find a job than a black unemployed person. This formulation implicitly assume that blacks are not discriminated within the job (blacks and whites earn the same wage w) or outside of the job (blacks and whites earn the same unemployment benefit b) but at the entry of the job (through the probability of finding a job) or at the exit of the job (through the probability of loosing a job). For simplicity and without loss of generality, we assume that there is a racial factor r which is equal to 1 for whites and equal to $r > 1$ for blacks. We have therefore:

$$\theta_B = r\theta \quad , \quad \delta_B = \frac{\pi}{r} \tag{26}$$

$$\theta_W = \theta \quad , \quad \delta_W = \pi \tag{27}$$

3.2 Dynamic job turnover

Each worker of type $k = B, W$ is characterized by his employment status (U or L). We assume that changes in employment status are governed by a time homogeneous Markov process with finite state space, $S = \{U, L\}$ representing the employment status.[4] At each period of time, any worker can be either employed or unemployed. We know from the probability theory of Markov stochastic process (see e.g. Kulkarni, 1995) that, given the employment-status history of individuals up to time t_0, their employment status X_t at any subsequent time, t_0+t, depends only on their status at time t_0, i.e.,

$$P\left(X_{t_0+t} \mid X_\eta\ ,\ \eta \leq t_0\right) = P\left(X_{t_0+t} \mid X_{t_0}\right)$$

Moreover, we also know that these conditional probabilities depend only on the elapsed time, t, so that for all $i,j \in S$ and $t \geq 0$,

$$P\left(X_{t_0+t} = j \mid X_{t_0} = i\right) = P\left(X_t = i\right) \equiv P_{t,k}(i,j)$$

where $P_{t,k}(i,j)$ are the *transition probabilities* of being in employment state j at time t given state i at time zero for a worker of type $k=B,W$. For example, $P_{t,k}(L,U)$ is the transition probability for workers of type $k=B,W$ of being unemployed at time t given that they have been employed at time zero. In this context, the *stochastic or transition matrix* P_B for a black worker is given by:

$$\begin{array}{cc} & \begin{array}{cc} U & L \end{array} \\ \begin{array}{c} U \\ L \end{array} & \left(\begin{array}{cc} 1-\delta/r & \delta/r \\ r.\theta & 1-r.\theta \end{array}\right) \end{array} \equiv P_B$$

while the one (P_W) for a white worker is equal to:

$$\begin{array}{cc} & \begin{array}{cc} U & L \end{array} \\ \begin{array}{c} U \\ L \end{array} & \left(\begin{array}{cc} 1-\delta & \delta \\ \theta & 1-\theta \end{array}\right) \end{array} \equiv P_W$$

This is a standard problem that can be easily solved. By using the same methodology as in Zenou and Smith (1995), we easily obtain the following probabilities for a worker of type $k=B,W$:

$$P_{t,k}(U,L) = \frac{\delta}{(r)^2\theta+\delta} - \frac{\delta}{(r)^2\theta+\delta}e^{-(r\theta+\frac{\delta}{r})} \tag{28}$$

$$P_{t,k}(L,L) = \frac{\delta}{(r)^2\theta+\delta} + \frac{r^2\theta}{(r)^2\theta+\delta}e^{-(r\theta+\frac{\delta}{r})} \tag{29}$$

[4] People who are black cannot become white and vice versa. We can therefore study the stochastic process for each population of workers (B and W) separately.

Observe that $P_{t,k}(U,U) = 1 - P_{t,k}(U,L)$ and $P_{t,k}(L,U) = 1 - P_{t,k}(L,L)$ so that we obtain:

$$P_{t,k}(U,U) = \frac{(r)^2\theta}{(r)^2\theta+\delta} + \frac{\delta}{(r)^2\theta+\delta}e^{-(r\theta+\frac{\delta}{r})} \tag{30}$$

$$P_{t,k}(L,U) = \frac{(r)^2\theta}{(r)^2\theta+\delta} - \frac{(r)^2\theta}{(r)^2\theta+\delta}e^{-(r\theta+\frac{\delta}{r})} \tag{31}$$

3.3 Steady state relations and labor market equilibrium

In this paper, we focus only on steady state equilibrium. Thus in steady state (when $t \to +\infty$), because of the *ergodic* properties of Markov stochastic processes, we have:

$$\lim_{t\to+\infty} P_{t,k}(U,L) = \lim_{t\to+\infty} P_{t,k}(L,L) = \frac{\delta}{(r)^2\theta+\delta}$$

$$\lim_{t\to+\infty} P_{t,k}(U,U) = \lim_{t\to+\infty} P_{t,k}(L,U) = \frac{(r)^2\theta}{(r)^2\theta+\delta}$$

which means that the probability of being unemployed or employed does not depend on the initial employment status (at time zero). In this context, we obtain the following steady state relations:

$$\left(\frac{\delta}{(r)^2\theta+\delta}\right)\overline{N}_k = L_k \qquad k=B,W \tag{32}$$

$$\left(\frac{(r)^2\theta}{(r)^2\theta+\delta}\right)\overline{N}_k = U_k \qquad k=B,W \tag{33}$$

Since $\overline{N}_k = U_k + L_k$, we can determine the steady state unemployment level for black workers. It is equal to:

$$U_B = \frac{(r)^2\theta}{\delta+(r)^2\theta}\overline{N}_B \tag{34}$$

with

$$\frac{\partial U_B}{\partial r} > 0 \quad ; \quad \frac{\partial U_B}{\partial \theta} > 0 \quad ; \quad \frac{\partial U_B}{\partial \delta} < 0 \tag{35}$$

while the one for white workers is given by:

$$U_W = \frac{\theta}{\delta+\theta}\overline{N}_W \tag{36}$$

with

$$\frac{\partial U_W}{\partial \theta} > 0 \quad ; \quad \frac{\partial U_W}{\partial \delta} < 0 \tag{37}$$

The total level of unemployment in this economy is thus equal to:

$$U \equiv U_B + U_W = \theta \left[\frac{(r)^2}{\delta + (r)^2 \theta} \overline{N}_B + \frac{1}{\delta + \theta} \overline{N}_W \right] \tag{38}$$

The values of $\overline{N}_B$ and $\overline{N}_W$ are exogenous so that since:

$$\frac{(r)^2 \theta}{\delta + (r)^2 \theta} > \frac{\theta}{\delta + \theta}$$

we have in general:

$$U_B > U_W$$

Thus if in a city the number of blacks is greater or equal than the one of whites, then the unemployment level is also greater.

We are thus able to explain *intra-urban unemployment rate differences.* Indeed, in Equilibrium 1, where access costs for blacks are important, unemployment rates are different within the city; they are in fact higher in the city-center (U_B) than in suburbs (U_W). This is no longer true in Equilibrium 2 (access costs for blacks are low) ince the unemployment rate in the center is U and given by (38) while there is no unemployment is the suburbs.

If for simplicity we assume that $\overline{N}_B = \overline{N}_W = 1$, then the differences in unemployment between the center and the suburbs is given by:

$$\Delta U \equiv U_B - U_W = \left[\frac{(r)^2 - 1}{\delta + (r)^2 \theta} \right] \left(\frac{\delta\, \theta}{\delta + \theta} \right) \tag{39}$$

Furthermore, by using Table 1a, (35) and (37), we easily obtain for Equilibrium 1:

Table 3a: Comparative Statics Analysis for Equilibrium 1 with racial discrimination

	U_B^{Net}	U_W^{Net}	r	θ	δ
x_1	$-$	$-$	$-$	$-$	$+$
x_2	$-$	$+$	$-$	?	?
x_3	$-$	$+$	$-$	?	?
x_4	$+$	$+$	$+$	$+$	$-$

while, by using Table 1b, (35) and (37), we have for Equilibrium 2:

Table 3b: Comparative Statics Analysis for Equilibrium 2 with racial discrimination

	U_B'	U_W'	r	θ	δ
x_1'	$-$	$-$	$-$	$-$	$+$
x_2'	$-$	$+$	$-$	?	?
x_3'	$+$	$+$	$+^*$	$+^*$	$-^*$
x_4'	$+$	$+$	$+$	$+$	$-$

The following comments are in order. First, we did not consider the impact of r, θ or δ on equilibrium utilities since the effects are all ambiguous. Second, since the racial discrimination factor r affects only U_B, its effects on the endogeneous variables are unambiguous. For example,when r increases, then more blacks become unemployed so that in both urban equilibria *the city shrinks and the size of the ghetto (measured by all the black unemployed workers living close to the city-center) increases.* This result is interesting since it shows that racial discrimination don't only affect unemployment but also ghettos. Observe also that when r rises, x_3 (the border between BL and WU) decreases in Equilibrium 1 while x_3' (the border between WU and BL) increases in Equilibrium 2. This is because r affects the number of black unemployed which in turn affects x_3 or x_3' but the position of BU are different in the two urban equilibria. Last, when δ (the job acquisition rate) increases or θ (the job destruction rate) decreases the size of the city increases while the size of the ghetto, x_4 or x_4' is reduced.

Proposition 2

In steady state,

- *the unemployment level is higher for blacks than for whites. This is true whatever the equilibrium urban spatial structure;*
- *intra-urban unemployment differences exist only when access costs for blacks are sufficiently high. In this case, unemployment is higher in the center than in suburbs.*

The following comments are in order. First r affects only black workers. When employers are more prejudice, then U_B increases while U_W is not affected. Second, for both blacks and whites, the unemployment level depends on θ and δ. When the job acquisition rate increases or when the job separation rate decreases, then U_B and U_W are reduced. Third, intra-urban unemployment differences appear only when Equilibrium 1 prevails i.e. when the access cost for blacks is large enough. In the other Equilibrium, there is no unemployment in the suburbs because the unemployed (black or white) are squeezed towards the city center. Last these results and in particular the impact of r on U_B are *independent of the urban equilibrium.* So whatever the location of workers in the city the labor market equilibrium stays the same. This will be no longer true when we introduce redlining.

4 Labor market equilibrium with spatial and racial discriminations

So far, only labor discrimination was introduced in our model. We have seen that two types of urban land use equilibria are possible. We want to see now how *redlining* (i.e., the fact that employers discriminate workers in the labor market upon their urban location) affects the labor market equilibrium. For that we split the city in two by drawing a (red) line between the center (from zero to x_3 or x_3') and the suburbs (from x_3 to x_1 or from x_3' to x_1'). In this context, redlining signifies that, irrespective of race, employers are more reluctant to hire workers residing close to the CBD (i.e., in the ghetto) than workers living in the suburbs. In our framework, *the red line is endogeneous* since it divides in two the city between the four types of workers and thus depends of the type of urban equilibrium. For equilibrium 1, this means that all blacks will suffer from redlining while in Equilibrium 2 all employed workers will be affected by spatial discrimination.

The timing is the following. As in the previous section, workers locate in the city according to Equilibrium 1 or 2 (don't forget that racial discrimination as well as redlining does not affect workers' location but only probabilities to find or to loose a job). Then we determine the labor market equilibrium.

As for racial discrimination, there is a 'double' spatial discrimination (at the entry and at the exit of the job) so that for workers residing in 'bad' areas (in the city-center), it takes more time to find a job (if unemployed) and it is easier to lose a job (if employed) than those residing in suburbs. Different reasons which are not in the model are possible to justify this type of prejudice. First, it is well documented that in ghettos (located in general close to the city-center), crime rate is quite important so that employers are more reluctant to employ "potential" criminals (see e.g. Rasmusen, 1996). Second, the suburban schools are in general better so workers from the suburbs might have a superior education, irrespective of race.

Spatial discrimination means that the probability of finding a job is more difficult for a central worker (C) than for a suburban one (S). Formally, if we denote

by θ^m ($m = C, S$), the probability of loosing a job for an employed worker residing in m and δ^m, the probability of finding a job for an unemployed worker residing in m. Spatial discrimination implies that $\theta^C > \theta^S$ and $\delta^C < \delta^S$. Here also in order to simplify the analysis, we assume that there is a spatial factor s which is equal to 1 for the people residing in the suburbs and greater than 1 for the ones residing in the central part of the city so that:

$$\theta^C = s\theta \quad , \quad \delta^C = \frac{\delta}{s} \tag{40}$$

4.1 Equilibrium 1

Let us start with Equilibrium 1 when black workers are both racially and spatially discriminated against. Thus the probabilities of losing a job for the employed workers are:

$$\theta_k^m = r.s.\theta \qquad k = B, W \qquad m = C, S \tag{41}$$

while the probability of finding a job for the unemployed workers are given by:

$$\delta_k^m = \frac{\delta}{r.s} \qquad k = B, W \qquad m = C, S \tag{42}$$

In this context, the transition matrix P_B^1 for (central) black workers writes now:

$$\begin{array}{cc} & \begin{array}{cc} U & L \end{array} \\ \begin{array}{c} U \\ L \end{array} & \begin{pmatrix} 1 - \delta/(r.s) & \delta/(r.s) \\ r.s.\theta & 1 - r.s.\theta \end{pmatrix} \end{array} \equiv P_B^1$$

whereas for (suburban) white workers, the transition matrix is the same as before ($P_W^1 = P_W$) and is thus equal to (whites are not affected by this policy):

$$\begin{array}{cc} & \begin{array}{cc} U & L \end{array} \\ \begin{array}{c} U \\ L \end{array} & \begin{pmatrix} 1 - \delta & \delta \\ \theta & 1 - \theta \end{pmatrix} \end{array} \equiv P_W^1$$

As above, we easily obtain the following probabilities for a worker of type $k = B, W$ ($r = s = 1$ for whites):

$$P_{t,k}^1(U, L) = \frac{\delta}{(r.s)^2\theta + \delta} - \frac{\delta}{(r.s)^2\theta + \delta} e^{-(rs\theta + \frac{\delta}{rs})} \tag{43}$$

$$P_{t,k}^1(L, L) = \frac{\delta}{(r.s)^2\theta + \delta} + \frac{(r.s)^2\theta}{(r.s)^2\theta + \delta} e^{-(rs\theta + \frac{\delta}{rs})} \tag{44}$$

$$P^1_{t,k}(U,U) = \frac{(r.s)^2\theta}{(r.s)^2\theta+\delta} + \frac{\delta}{(r.s)^2\theta+\delta} e^{-(rs\theta+\frac{\delta}{rs})} \quad (45)$$

$$P^1_{t,k}(L,U) = \frac{(r.s)^2\theta}{(r.s)^2\theta+\delta} - \frac{(r.s)^2\theta}{(r.s)^2\theta+\delta} e^{-(rs\theta+\frac{\delta}{rs})} \quad (46)$$

In steady state (when $t \to +\infty$), we have:

$$\lim_{t\to+\infty} P^1_{t,k}(U,L) = \lim_{t\to+\infty} P^1_{t,k}(L,L) = \frac{\delta}{(r.s)^2\theta+\delta}$$

$$\lim_{t\to+\infty} P^1_{t,k}(U,U) = \lim_{t\to+\infty} P^1_{t,k}(L,U) = \frac{(r.s)^2\theta}{(r.s)^2\theta+\delta}$$

In this context, we obtain the following steady state relations:

$$\left(\frac{\delta}{(r.s)^2\theta+\delta}\right)\overline{N}_k = L^1_k \qquad k = B, W \quad (47)$$

$$\left(\frac{(r.s)^2\theta}{(r.s)^2\theta+\delta}\right)\overline{N}_k = U^1_k \qquad k = B, W \quad (48)$$

Since $\overline{N}_k = U^1_k + L^1_k$, we can determine the steady state unemployment level for black workers. It is equal to:

$$U^1_B = \frac{(r.s)^2\theta}{\delta+(r.s)^2\theta}\overline{N}_B \quad (49)$$

with

$$\frac{\partial U^1_B}{\partial r} > 0 \quad ; \quad \frac{\partial U^1_B}{\partial s} > 0 \quad ; \quad \frac{\partial U^1_B}{\partial \theta} > 0 \quad ; \quad \frac{\partial U^1_B}{\partial \delta} < 0 \quad (50)$$

while the one for white workers is given by:

$$U^1_W = \frac{\theta}{\delta+\theta}\overline{N}_W \quad (51)$$

with

$$\frac{\partial U^1_W}{\partial \theta} > 0 \quad ; \quad \frac{\partial U^1_W}{\partial \delta} < 0 \quad (52)$$

The total level of unemployment in this economy is thus:

$$U^1 \equiv U^1_B + U^1_W = \theta\left[\frac{(r.s)^2}{\delta+(r.s)^2\theta}\overline{N}_B + \frac{1}{\delta+\theta}\overline{N}_W\right]$$

The values of $\overline{N}_B$ and $\overline{N}_W$ are exogenous so that we have in general:

$$U^1_B > U^1_W$$

More precisely, the unemployment differences (assume that $\overline{N}_B = \overline{N}_W = 1$) is equal to:

$$\Delta U^1 \equiv U^1_B - U^1_W = \left[\frac{(r.s)^2-1}{\delta+(r.s)^2\theta}\right]\left(\frac{\delta\,\theta}{\delta+\theta}\right) \quad (53)$$

It is easy to compare (53) with the case with no redlining. Since $s > 1$ and since:

$$\frac{(r.s)^2-1}{\delta+(r.s)^2\theta} > \frac{(r)^2-1}{\delta+(r)^2\theta}$$

the unemployment differences between blacks and whites and thus between the center and the suburbs has increased (see (39)). This is due to the fact that whites are not affected by the redlining policy so that their unemployed level stays the same while blacks suffering from their central location see an increase in their unemployment level.

Furthermore, by using Tables 1a, (50) and (52), we easily obtain the following comparative statics results for Equilibrium 1:

Table 4a: Comparative Statics Analysis for Equilibrium 1 with racial and spatial discriminations

	U_B^{Net}	U_W^{Net}	r	s	θ	δ
x_1	$-$	$-$	$-$	$-$	$-$	$+$
x_2	$-$	$+$	$-$	$-$	?	?
x_3	$-$	$+$	$-$	$-$	?	?
x_4	$+$	$+$	$+$	$+$	$+$	$-$

In Equilibrium 1 where there is a total separation between blacks and whites, then obviously the situation is not good for blacks. In particular, other things being equal, when r (racial discrimination factor) or s (spatial discrimination factor) increases, the city shrinks, all borders are reduced except the size of the ghetto, x_4, which increases. This means that proportionally the black unemployed workers are even more segregated because they are both racially and spatially discriminated against and have therefore a lower probability to get a job and to leave the ghetto. We have similar effects when the job acquisition rate δ decreases or the job destruction rate θ increases.

Proposition 3

Compared to the case with no redlining, the urban equilibrium 1 *(blacks and whites are separated) is characterized by a higher unemployment level for blacks and no changes for whites. This implies that the differences in unemployment between the center and the periphery increase.*

Observe that contrarily to the model with only labor discrimination, the labor market equilibrium depends strongly on the urban equilibrium one. This is obviously a bad equilibrium for black workers since they are both discriminated on the basis of their race and of their location. Because of this double discrimination, their unemployment level becomes more important since r and s affect only black workers. There is a kind of vicious cercle. Because of high access costs, blacks tend to locate at the vicinity of the CBD. Because of this (central) location because redline them. The resulting unemployment is now higher and the intra-urban unemployment differences is even more pronounced, implying that both "race and space" matter.

4.2 Equilibrium 2

In Equilibrium 2 workers are separated by their employment status. The red line is thus between the unemployed and the employed and not between blacks and whites as before. In this context, *blacks are racially discriminated and the unemployed workers are spatially discriminated.* Thus the transition matrix P_B^2 for black workers writes:

$$\begin{array}{cc} & \begin{array}{cc} U & L \end{array} \\ \begin{array}{c} U \\ L \end{array} & \begin{pmatrix} 1-\delta/(r.s) & \delta/(r.s) \\ r.\theta & 1-r.\theta \end{pmatrix} \end{array} \equiv P_B^2$$

This means that when a black worker is unemployed, we know that (at any moment of time) he lives in the central part of the city and he is thus spatially discriminated (see the first row of P_B^2 where s is always present) while if he is employed, (at any moment of time) he lives in the suburbs and thus he is not spatially discriminated (in the second row of P_B^2, s is absent). With the same type of argument, we find easily the transition matrix P_W^2 for white workers:

$$\begin{array}{cc} & \begin{array}{cc} U & L \end{array} \\ \begin{array}{c} U \\ L \end{array} & \begin{pmatrix} 1-\delta/s & \delta/s \\ \theta & 1-\theta \end{pmatrix} \end{array} \equiv P_W^2$$

As before, we easily obtain the following probabilities for black workers:

$$P_{t,B}^2(U,L) = \frac{\delta}{(r)^2 s.\theta+\delta} - \frac{\delta}{(r)^2 s.\theta+\delta} e^{-(rs\theta+\frac{\delta}{r})} \tag{54}$$

$$P_{t,B}^2(L,L) = \frac{\delta}{(r)^2 s.\theta+\delta} + \frac{(r)^2 s.\theta}{(r)^2 s.\theta+\delta} e^{-(rs\theta+\frac{\delta}{r})} \tag{55}$$

as well as the ones for white workers:

$$P_{t,W}^2(U,L) = \frac{\delta}{s.\theta+\delta} - \frac{\delta}{s.\theta+\delta} e^{-(s\theta+\delta)} \tag{56}$$

$$P_{t,W}^2(L,L) = \frac{\delta}{s.\theta+\delta} + \frac{s.\theta}{s.\theta+\delta} e^{-(s\theta+\delta)} \tag{57}$$

In steady state (when $t \to +\infty$), we have:

$$\lim_{t\to+\infty} P_{t,B}^2(U,L) = \lim_{t\to+\infty} P_{t,B}^C(L,L) = \frac{\delta}{(r)^2 s.\theta+\delta}$$

$$\lim_{t\to+\infty} P_{t,B}^2(U,U) = \lim_{t\to+\infty} P_{t,B}^S(L,U) = \frac{(r)^2 s.\theta}{(r)^2 s.\theta+\delta}$$

$$\lim_{t\to+\infty} P_{t,W}^2(U,L) = \lim_{t\to+\infty} P_{t,W}^C(L,L) = \frac{\delta}{s.\theta+\delta}$$

$$\lim_{t\to+\infty} P_{t,W}^2(U,U) = \lim_{t\to+\infty} P_{t,W}^S(L,U) = \frac{s.\theta}{s.\theta+\delta}$$

In this context, we obtain the following steady state relations:

$$\left(\frac{\delta}{(r)^2 s.\theta+\delta}\right) N_B^2 = L_B^2 \tag{58}$$

$$\left(\frac{(r)^2 s.\theta}{(r)^2 s.\theta+\delta}\right) N_B^2 = U_B^2 \tag{59}$$

$$\left(\frac{\delta}{s.\theta+\delta}\right) N_W^2 = L_W^2 \tag{60}$$

$$\left(\frac{s.\theta}{s.\theta+\delta}\right) N_W^2 = U_W^2 \tag{61}$$

Since $\overline{N}_k = U_k + L_k$, we can determine the steady state unemployment level for black workers. It is equal to:

$$U_B^2 = \frac{(r)^2 s.\theta}{\delta + (r)^2 s.\theta} \overline{N}_B \tag{62}$$

with

$$\frac{\partial U_B^2}{\partial r} > 0 \quad ; \quad \frac{\partial U_B^2}{\partial s} > 0 \quad ; \quad \frac{\partial U_B^2}{\partial \theta} > 0 \quad ; \quad \frac{\partial U_B^2}{\partial \delta} < 0 \tag{63}$$

while the one for white workers is given by:

$$U_W^2 = \frac{s.\theta}{\delta + s.\theta} \overline{N}_W \tag{64}$$

with

$$\frac{\partial U_W^2}{\partial s} > 0 \quad ; \quad \frac{\partial U_W^2}{\partial \theta} > 0 \quad ; \quad \frac{\partial U_W^2}{\partial \delta} < 0 \tag{65}$$

The total level of unemployment in this economy is thus equal to:

$$U^2 \equiv U_B^2 + U_W^2 = s.\theta \left[\frac{(r)^2}{\delta + (r)^2 s.\theta} \overline{N}_B + \frac{1}{\delta + s.\theta} \overline{N}_W \right]$$

The values of $\overline{N}_B$ and $\overline{N}_W$ are exogenous so that we have (in general):

$$U_B^2 > U_W^2$$

Observe that compared to the case with no redlining (section 3), the white unemployment level has increased because they are spatially segregated. This is also true for the black unemployed workers so that the prediction on the unemployment difference in ambiguous. Indeed, we have (assume that $\overline{N}_B = \overline{N}_W = 1$):

$$\Delta U^2 \equiv U_B^2 - U_W^2 = \left[\frac{(r)^2 - 1}{\delta + (r)^2 \ s \ \theta} \right] \left(\frac{s \ \delta \ \theta}{\delta + s \ \theta} \right) \tag{66}$$

so compare to (39), we obtain:

$$\Delta U^2 \gtrless \Delta U \tag{67}$$

which is equivalent to:

$$r^2 \gtrless \frac{(\delta)^2 (s-1)}{s \ \theta \ (s \ \theta - 1)} \equiv A$$

whith

$$\frac{\partial A}{\partial s} < 0$$

So the sign of (67) depends on the value of r compare to the one of s. If space and race matter, i.e. spatial discrimination s as well as racial discrimination r are important, then the unemployment difference between blacks and whites is augmented

as soon as firms redline. If race and space does not matter, then this difference is reduced with redlining. If race matters more than space and vice versa, then the sign becomes ambiguous. The important element here is that redlining affects both black and white unemployed workers so that the chance to leave unemployment is reduced compared to the case with no redlining. However, black employed workers are not affected by redlining so that they probability to become unemployed is the same as in the case of no redlining by lower than Equilibrium 1 with redlining.

Furthermore, by using Table 1b, (63) and (65), we have:

Table 4b: Comparative Statics Analysis for Equilibrium 2 with racial and spatial discriminations

	U_B'	U_W'	r	s	θ	δ
x_1'	$-$	$-$	$-$	$-$	$-$	$+$
x_2'	$-$	$+$	$-$	?	?	?
x_3'	$+$	$+$	$+$	$+$	$+$	$-$
x_4'	$+$	$+$	$+$	$+$	$+$	$-$

Compared to Equilibrium 1, black *and* white unemployed workers are discriminated against; blacks because of r and s and whites because of s. In this context, when r or s increases the city shrinks but both x_3' and x_4' rise. The interesting point here is that when redlining is introduced, the urban equilibrium affects strongly the labor market equilibrium so that urban equilibrium 1 becomes very problematic for blacks while equilibrium 2 is better. It is thus quite intuitive that we can rank the different types of urban equilibria. The best one is when there is no redlining (Equilibrium 1 and 2 are the same in terms of labor market equilibrium). Then it is Equilibrium 2 with redlining and the worse one is Equilibrium 1 with redlining. This is accordance with the empirical study of Cutler and Glaeser (1997) who found that blacks in more segregated areas have significantly worse outcomes than blacks in less segregated areas.

Proposition 4

In the urban Equilibrium 2 with spatial discrimination (redlining), the unemployment level increases for black workers compared to the case with no redlining. For white workers, the unemployment level increases because whites are spatially discriminated against.

5 Concluding remarks

In this paper, we have proposed a model dealing with both racial and spatial discriminations. Because of the first one, blacks have less chance than whites to find a job and are thus segregated at the vicinity of the city-center. Because of the second one, whatever of their origin, workers living in 'ghettos' have less chance than suburban workers to find a job.

It is easy to see the impact of a policy such as the Affirmative Action (which aims at fighting only against racial discrimination) on the labor market equilibrium. In our model, this policy amounts at reducing r thus reducing the unemployment rate for black workers. However, as we have seen, this is not sufficient since firms tend to redline workers and reducing r does not change the location of black unemployment workers.

The second policy aims at fighting only against spatial discrimination. In our model this means reducing s. This will reduce unemployment but can have a perverse effect by increasing the segregation of black people around the city center.

Therefore, the government should both decrease r (for example affirmative action) but also improve the accessibility of black workers to employment centers (by reducing c). This could change the configuration of the city and thus avoid spatial discrimination for black workers. For example, the enterprise zone program is a good way to handle this problem but it should be accompanied by another policy that fights racial discrimination.

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