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DAVID PINGREE

**PTOLEMY** (or **Claudius Ptolemaeus**) (*b. ca.* A.D. 100; *d. ca.* A.D. 170), *mathematical sciences, especially astronomy.*

Our meager knowledge of Ptolemy's life is based mostly on deductions from his surviving works, supplemented by some dubious information from authors of late antiquity and Byzantine times. The best evidence for his dates is the series of his observations reported in his major astronomical work, the *Almagest*: these are all from the reigns of the Roman emperors Hadrian and Antoninus, the earliest 26 March 127 and the latest 2 February 141.[1] Since he wrote several major works after the *Almagest*, this evidence fits well with the statement of a scholiast attached to works of Ptolemy in several late manuscripts—that he flourished under Hadrian and lived until the reign of Marcus Aurelius (161–180).[2] The only other explicit date is that of the "Canobic Inscription": this is found in manuscripts of Ptolemy's astronomical works and purports to be a copy of an inscription dedicated by Ptolemy to the "Savior God" at Canopus, a town at the western mouth of the Nile, in the tenth year of Antoninus (A.D. 147/148).[3] It consists mostly of lists of astronomical parameters determined by Ptolemy; although most of its contents are extracted from the *Almagest* and other genuine works of Ptolemy, I doubt its authenticity. A statement by the sixth-century philosophical commentator Olympiodorus that Ptolemy practiced astronomy for forty years in "the so-called wings at Canopus," and hence set up there the inscription commemorating his astronomical discoveries,[4] is probably a fictional elaboration on the "Canobic Inscription." In fact the only place mentioned in any of Ptolemy's observations is Alexandria, and there is no reason to suppose that he ever lived anywhere else. The statement by Theodore Meliteniotes that he was born in Ptolemais Hermiou (in Upper Egypt) could be correct,[5] but it is late (*ca.* 1360) and unsupported. The belief that he came from Pelusium is a Renaissance misinterpretation of the title "Phelud(i)ensis" attached to his name in medieval Latin texts, which in turn comes from a corruption of the Arabic "qalūdī," a misunderstanding of Κλαύδιος.[6] His name "Ptolemaeus" indicates that he was an inhabitant of Egypt, descended from Greek or hellenized forebears, while "Claudius" shows that he possessed Roman citizenship, probably as a result of a grant to an ancestor by the emperor Claudius or Nero.

It is possible to deduce something about the order of composition of Ptolemy's surviving works from internal evidence. The *Almagest* is certainly the earliest of the major works: it is mentioned in the introductions to the *Tetrabiblos*, *Handy Tables*, and *Planetary Hypotheses*, and in book VIII, 2, of the *Geography*; a passage of the *Almagest* looks forward to the publication of the *Geography*.[7] One can occasionally trace development: in the *Handy Tables* many tables are presented in a form more convenient for practical use than the corresponding sections of the *Almagest*, and some parameters are slightly changed. The *Planetary Hypotheses* exhibits considerably more change in parameters, and introduces a notable improvement in the theory of planetary latitudes and an entirely new system for calculating the absolute sizes and distances of the planets. The prime meridian of the *Geography* is not Alexandria, as is promised in the *Almagest*, but a meridian through the "Blessed Isles" (the Canaries) at the extreme west of the ancient known world (which had the advantage that all longitudes were counted in the same direction). The phenomenon of the apparent enlargement of heavenly bodies when they are close to the horizon is explained in the *Almagest* as due to physical causes (the dampness of the atmosphere of the earth),[8] whereas in the *Optics* Ptolemy gives a purely psychological explanation.[9] Presumably in the interval between the composition of the two works he had discovered that there is no measurable enlargement. Similarly the *Optics* discusses the problem of astronomical refraction,[10] which is never considered in the *Almagest* despite its possible effect on observation. It is hardly possible, however, to trace Ptolemy's scientific development, except in his astronomical work; and even there the later works contribute only minor modifications to the masterly synthesis of the *Almagest*.

We know nothing of Ptolemy's teachers or associates, although it is a plausible conjecture that the Theon who is said in the *Almagest* to have "given" Ptolemy observations of planets from between 127 and 132 was his teacher.[11] Many of the works are addressed to an otherwise unknown Syrus. It would be hasty, but not absurd, to conclude from the fact that the *Geography* and the *Harmonica* are not addressed to Syrus that they are later than all the works so addressed (that is, all the other extant works except for the dubious Περὶ κριτηρίου and possibly the *Optics* and the *Phaseis*, the relevant sections of which are missing). Living in Alexandria must have been a great advantage to Ptolemy in his work (and perhaps his education). Although much declined from its former greatness as a center of learning, the city still maintained a scholarly tradition and must at the least have provided him with essential reference material from its libraries.

Ptolemy's chief work in astronomy, and the book on which his later reputation mainly rests, is the *Almagest*, in thirteen books. The Greek title is μαθηματικὴ σύνταξις, which means "mathematical [that is, astronomical] compilation." In later antiquity it came to be known informally as ἡ μεγάλη σύνταξις or ἡ μεγίστη σύνταξις ("the great [or greatest] compilation"), perhaps in contrast with a collection of earlier Greek works on elementary astronomy called ὁ μικρὸς ἀστρονομούμενος ("the small astronomical collection").[12] The translators into Arabic transformed ἡ μεγίστη into "al-majisṭī," and this became "almagesti" or "almagestum" in the medieval Latin translations. It is a manual covering the whole of mathematical astronomy as the ancients conceived it. Ptolemy assumes in the reader nothing beyond a knowledge of Euclidean geometry and an understanding of common astronomical terms; starting from first principles, he guides him through the prerequisite cosmological and mathematical apparatus to an exposition of the theory of the motion of those heavenly bodies which the ancients knew (sun, moon, Mercury, Venus, Mars, Jupiter, Saturn, and the fixed stars, the latter being considered to lie on a single sphere concentric with the earth) and of various phenomena associated with them, such as eclipses. For each body in turn Ptolemy describes the type of phenomena that have to be accounted for, proposes an appropriate geometric model, derives the numerical parameters from selected observations, and finally constructs tables enabling one to determine the motion or phenomenon in question for a given date.

In order to appreciate Ptolemy's achievement in the *Almagest*, we ought to know how far Greek astronomy had advanced before his time. Unfortunately the most significant works of his predecessors have not survived, and the earlier history has to be reconstructed almost entirely from secondary sources (chiefly the *Almagest* itself). Much remains uncertain, and the following sketch of that history is merely provisional.

The first serious attempt by a Greek to describe the motions of the heavenly bodies by a mathematical model was the system of "homocentric spheres" of Eudoxus (early fourth century B.C.). Although mathematically ingenious, this model was ill-suited to represent even the crude data on which it was based; and the approach proved abortive (it would have vanished from history had it not been adopted by Aristotle).[13] Equally insignificant for the development of astronomy were works on "spherics" that treat phenomena such as the risings and settings of stars in terms of spherical geometry (these appear from the fourth century B.C. on). The heliocentric theory of Aristarchus of Samos (early third century B.C.), perhaps developing ideas of Heraclides Ponticus (*ca.* 360 B.C.), was purely descriptive and also without consequence. After Eudoxus, however, the epicyclic and eccentric models of planetary motion were developed; and the equivalence of the two was proved by Apollonius of Perga (*ca.* 200 B.C.), if not earlier. Apollonius made an elegant application of these models to the problem of determining the stationary points of a planet.[14]

Meanwhile astronomical observations were being made in the Greek world from the late fifth century B.C. The earliest were mostly of the dates of solstices, but by the early third century Aristyllus and Timocharis in Alexandria were attempting to determine the positions of fixed stars and observing occultations. These observations, however, were few and unsystematic; no firmly based theory was possible until records of the observers in Babylon and other places in Mesopotamia, reaching back to the eighth century, became available to Greeks. It seems likely that period relations derived by the Babylonian astronomers from these observations were known as early as Eudoxus, but the first Greek who certainly used the observations themselves was Hipparchus; and it is no accident that Greek astronomy was established as a quantitative science with his work. Hipparchus (active from *ca.* 150 to 127 B.C.) used Babylonian eclipse records and his own systematic observations to construct an epicyclic theory of the sun and moon that produced reasonably accurate predictions of their positions. Hence he was able to predict eclipses. He measured the lunar parallax and evolved the first practical method for determining the distances of sun and moon. By comparing his own observations of the position of the star Spica with those of Timocharis 160 years earlier, he discovered the precession of the equinoxes. He employed plane trigonometry and stereographic projection. In the latter techniques and in the observational instruments he used he may have been a pioneer, but we know too little of his predecessors to be sure. Ptolemy expressly informs us that Hipparchus did not construct a theory of the five planets but contented himself with showing that existing theories did not satisfy the observations.[15] Between Hipparchus and Ptolemy the only advance was the work of Menelaus (*ca.* A.D. 100) on spherical trigonometry.

Greek astronomy, then, as Ptolemy found it, had evolved a geometric kinematic model of solar and lunar motion that successfully represented the phenomena, at least as far as the calculation of eclipses was concerned, but had produced only unsatisfactory planetary models. It had developed both plane and spherical trigonometry and had adopted the Babylonian sexagesimal place-value system not only for the expression of angles but also (although not systematically) for calculation. As a mathematical science it was already sophisticated. From the point of view of physics it was not a science at all: such physical theories as were enunciated were mere speculation. But there was available a fairly large body of astronomical observations, of which the most important, both for completeness and for the length of time it covered, was the series of eclipses observed in Mesopotamia. Ptolemy made no radical changes in the system he took over; but by intelligent use of available observations and ingenious modification of the basic principle of all existing kinematic models (uniform circular motion), he extended that system to include the five planets and significantly improved the lunar model.

Books I and II of the *Almagest* are devoted to preliminaries. Ptolemy begins by stating and attempting to justify his overall world picture, which is that generally (although not universally) accepted from Aristotle on: around a central, stationary, spherical earth the sphere of the fixed stars (situated at a distance so great that the earth's diameter is negligible in comparison) revolves from east to west, making one revolution per day and carrying with it the spheres of sun, moon, and planets; the latter have another, slower motion in the opposite sense, in or near a plane (the ecliptic) inclined to the plane of the first motion. He then develops the trigonometry that will be used throughout the work. The basic function is the chord (which we denote Crd) subtended by the angle at the center of a circle of radius 60. This is related to the modern sine function by

$$\sin \alpha = \frac{1}{2 \cdot 60} \text{Crd } 2\alpha.$$

The values of some chords (for instance, Crd 60°) are immediately obtainable by elementary geometry. By geometrically developing formulas for Crd $(\alpha + \beta)$, Crd $(\alpha - \beta)$, Crd $\frac{1}{2}\alpha$, where Crd $\alpha$ and Crd $\beta$ are known, and then finding Crd 1° by an approximation procedure, Ptolemy produces a table of chords, at intervals of 1/2° and to three sexagesimal places, which serves for all trigonometric calculations. There was probably nothing new in his procedure (Hipparchus had constructed a similar table).[16]

Since the sine function is sufficient for the solution of all plane triangles, the chord function too was sufficient, although the lack of anything corresponding to a tangent table often made the solution laborious, involving the extraction of square roots. For trigonometry on the surface of the sphere Ptolemy uses a figure that we call a Menelaus configuration. It is depicted in Figure 1, where all the arcs $AB$, $BE$, and so on are segments of a great circle less than 90°.

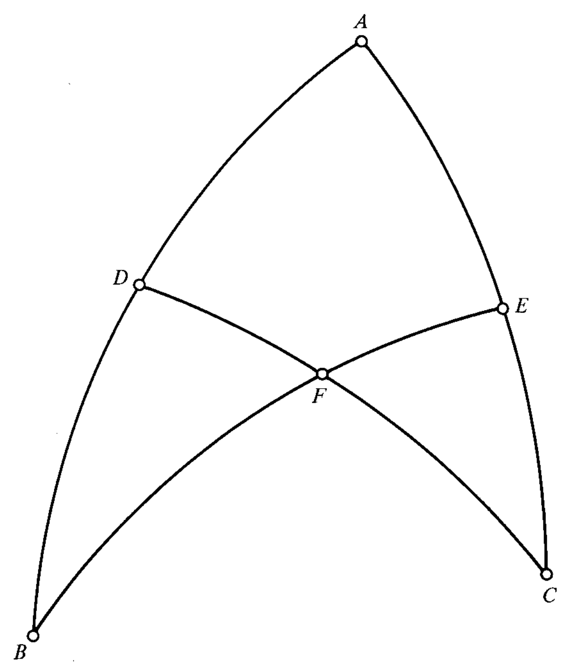


FIGURE 1

Ptolemy proves (following Menelaus) that

$$(1) \quad \frac{\text{Crd } 2EC}{\text{Crd } 2EA} = \frac{\text{Crd } 2CF}{\text{Crd } 2FD} \cdot \frac{\text{Crd } 2DB}{\text{Crd } 2BA},$$

$$(2) \quad \frac{\text{Crd } 2AC}{\text{Crd } 2AE} = \frac{\text{Crd } 2CD}{\text{Crd } 2DF} \cdot \frac{\text{Crd } 2FB}{\text{Crd } 2BE}.$$

It can be shown that the four basic formulas used in modern trigonometry for the solution of right spherical triangles are directly derivable from (1) and (2). Hence all problems soluble by means of the former can also be solved by the use of Menelaus configurations. Thus Ptolemaic trigonometry, although sometimes cumbersome, is completely adequate.

This trigonometry is applied in books I and II to various phenomena connected with the annual variation in solar declination. The sole numerical parameter used is the inclination of the ecliptic ($\epsilon$). This is also the amount of the greatest declination of the sun from the equator, and that is the basis of the two simple instruments for measuring $\epsilon$ which Ptolemy describes (I, 12). He reports that he measured $2\epsilon$ as between 47; 40° and 47; 45°. Since the estimation of Eratosthenes and Hipparchus, that $2\epsilon$ is 11/83 of a circle, also falls between these limits, Ptolemy too adopts the latter, taking $\epsilon$ as 23; 51, 20°. His failure to find a more accurate result, and hence to discover the slow decrease in the inclination, is explained by the crudity of his instruments. He can now construct a table of the declination of the sun as a function of its longitude, which is a prerequisite for solving problems concerning rising times. The rising time of an arc of the ecliptic is the time taken by that arc to cross the horizon at a given terrestrial latitude. Most of book II is devoted to calculating tables of rising times for various latitudes. Such tables are useful astronomically, for instance, for computing the length of daylight for a given date and latitude (important in ancient astronomy, since the time of day or night was reckoned in "civil hours," one civil hour being 1/12 of the varying length of day or night); but the space devoted to the topic is disproportionate to its use in the *Almagest*. It is essential in astrology, however (for example, in casting horoscopes); and this is one of the places where astrological requirements may have influenced the *Almagest* discussion (although they are never explicitly mentioned).

Book III treats the solar theory. By comparing his own observations of the dates of equinoxes with those of Hipparchus, and his observation of a solstice with one made by Meton and Euctemon in 432 B.C., Ptolemy confirms Hipparchus' estimate of the length of the tropical year as 365 1/4 − 1/300 days. This estimate is notoriously too long (the last fraction should be about 1/128), and the error was to have multiple consequences for Ptolemaic astronomy. It is derivable from the data only because Ptolemy made an error of about one day in the time of each of his observations. This is a gross error even by ancient standards and is the strongest ground of those modern commentators (such as Delambre) who maintain that Ptolemy slavishly copied Hipparchus, to the point of forging observations to obtain agreement with Hipparchus' results. The conclusion is implausible, but it is likely that in this case Ptolemy was influenced by his knowledge of Hipparchus' value to select such of his own observations as best agreed with it. For Hipparchus' solar and lunar theory represented the known facts (that is, eclipse records) very well, and Ptolemy would be reluctant to tamper with those elements of it that would seriously affect the circumstances of eclipses. Using the above year length, Ptolemy constructs a table of the mean motion of the sun that is the pattern for all other mean motion tables: the basis is the Egyptian calendar, in which the year has an unvarying length of 365 days (twelve thirty-day months plus five epagomenal days). The motion is tabulated to six sexagesimal places, for hours, days, months, years, and eighteen-year periods.

The main problem in dealing with all planets is to account for their "anomaly" (variation in velocity). In the case of the sun this variation is apparent from the fact that the seasons are of unequal length—for instance, in Ptolemy's day the time from spring equinox to summer solstice was longer than that from summer solstice to autumn equinox. Ptolemy proposes a general model for representing anomalistic motion. The "eccentric" version is depicted in Figure 2, where

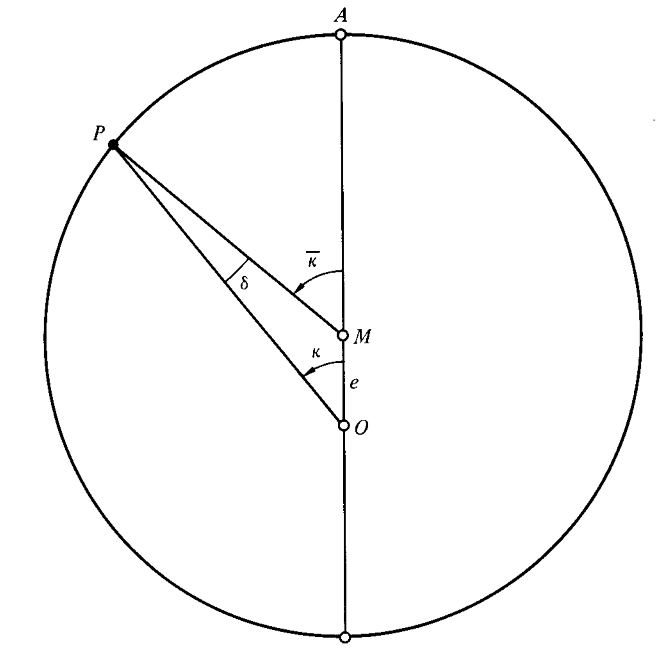


FIGURE 2

$O$ represents the central earth. The body $P$ moves with uniform angular velocity on a circle whose center $M$ is distant from $O$ by an amount $e$ (the eccentricity). It is clear that if the motion of $P$ appears uniform from $M$, it will appear nonuniform from $O$, being slowest at its greatest distance, the apogee $A$, and fastest at its least distance, the perigee $Z$. The angle $\kappa$ which $P$ has traveled from the apogee is derivable from its mean motion $\bar{\kappa}$ by the formula

$$\kappa = \bar{\kappa} \pm \delta.$$

$\delta$ is called by Ptolemy the πϱοσθαφαίϱεσις and by us, following medieval usage, the "equation." The same



motion can also be represented by an epicyclic model. In Figure 3 an epicycle, center $C$, moves with uniform

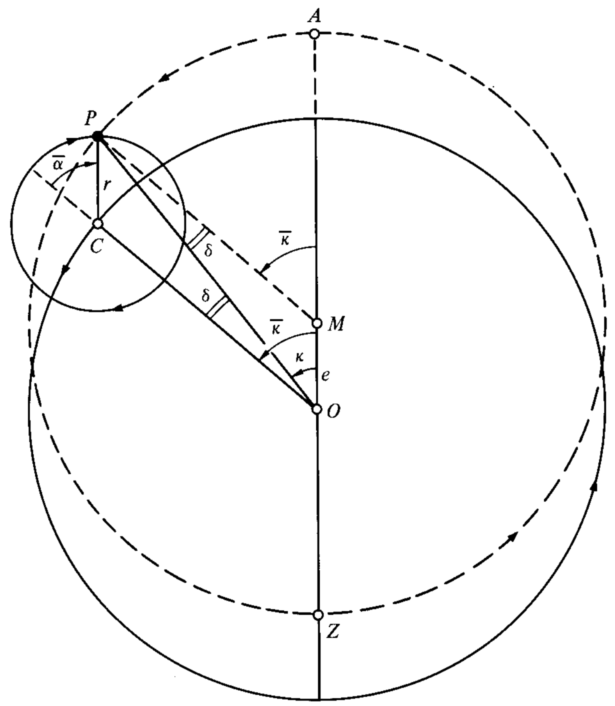


FIGURE 3

angular velocity on the circle (known as the deferent) about $O$, while the body $P$ moves uniformly about $C$ in the opposite sense. Provided that the angular velocities of body and epicycle are equal and the radius of the epicycle equals the eccentricity, the two models are completely equivalent, as can be seen from Figure 3, where $\bar{\alpha} = \bar{\kappa}$ and $r = e$. Such is the case of Ptolemy's solar model. But the angular velocity of $P$ may be different from that of $C$, as in the lunar theory (this is equivalent to a rotation of the apogee in the eccentric model); or the rotation of $P$ may be in the same sense as $C$, as in Ptolemy's planetary theory. The general model is, therefore, extremely versatile.

From his observations of solstices and equinoxes Ptolemy found the same length of seasons as Hipparchus. He therefore concluded that the apogee of the sun is tropically fixed and that its motion can be represented by the simple eccentric of Figure 2. He determined its parameters, as Hipparchus had, from the observed length of the seasons. In Figure 4 the sun is in $T$ at spring equinox, in $X$ at summer solstice, and in $Y$ at autumn equinox. Ptolemy states that it moves from $T$ to $X$ in 94 1/2 days and from $X$ to $Y$ in 92 1/2 days. Thus the angles at $M$ (the center of uniform motion), $TMX$ and $XMY$, can be calculated from the mean motion of the sun; and the angles at $O$, the earth, are right. Hence one can determine the





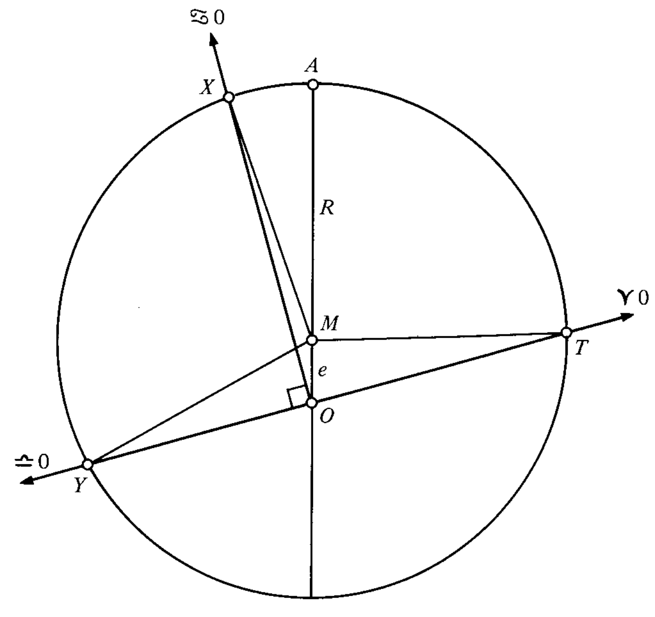


FIGURE 4

eccentricity $OM$ and the longitude of the apogee, $<TOA$. Since Ptolemy uses the same data and method as Hipparchus, he gets the same results: an eccentricity of 2;30 (where $R$, the standard radius, is 60) and an apogee longitude of 65;30°. In fact the eccentricity had decreased and the apogee longitude increased since Hipparchus' time, but this could not be detected through equinox and solstice observations made with crude ancient instruments.

Using the parameters determined, Ptolemy shows how the equation $\delta$ can be calculated trigonometrically for a given mean anomaly $\bar{\kappa}$ and sets up a table giving $\delta$ as a function of $\bar{\kappa}$. The only thing now lacking in order to calculate the position of the sun at any date is its mean position at some given date. From his equinox observation of A.D. 132 Ptolemy calculates its position at the date he has chosen as epoch, Thoth 1 (first day of the Egyptian year), year 1 of the Babylonian king Nabonassar (26 February 747 B.C.). To complete the solar theory he discusses the equation of time. Since the sun travels in a path inclined to the equator and with varying velocity, the interval between two successive meridian transits of the sun (the true solar day) will not be uniform but will vary slightly throughout the year. Since astronomical calculations employ uniform units (that is, mean solar days), whereas local time, in an age when the sundial is the main chronometer, is reckoned according to the true solar day, one must be able to convert one to the other. This conversion is done by means of the equation of time, the calculation of which Ptolemy explains.

The lunar theory is the subject of books IV and V.



According to Ptolemy one must distinguish three periods connected with the moon: the time in which it returns to the same longitude, the time in which it returns to the same velocity (period of anomaly), and the time in which it returns to the same latitude. In addition one must consider the synodic month, the time between successive conjunctions or oppositions of the sun and moon. He quotes a number of previous attempts to find a period containing an integer number of each of the above (such a period would, clearly, be an eclipse cycle); in particular, from Hipparchus:

(1) In 126,007 days, 1 hour, there occur 4,267 synodic months, 4,573 returns in anomaly, and 4,612 sidereal revolutions less 7 1/2° (hence the length of a mean synodic month is 29; 31, 50, 8, 20 days).
(2) In 5,458 synodic months there occur 5,923 returns in latitude.

Ptolemy says that Hipparchus established these "from Babylonian and his own observations." We now know from cuneiform documents that these parameters and period relations had all been established by Babylonian astronomers.[17] At best Hipparchus could have "confirmed" them from his own observations. Such confirmation could be carried out only by comparison of the circumstances of eclipses separated by a long interval. Ptolemy gives an acute analysis of the conditions that must then obtain in order for one to detect an exact period of return in anomaly. On the basis of his own comparison of eclipses he accepts the above relations, with very slight modifications to the parameters for returns in anomaly and latitude (justified later). Thus he is able to construct tables of the lunar mean motion in longitude, anomaly, argument of latitude (motion with respect to the nodes in which the orbit of the moon intersects the ecliptic), and elongation (motion with respect to the mean sun).

The next task is to determine the numerical parameters of the lunar model. Ptolemy first assumes (although he knows better) that the moon has a single anomaly, that is, that its motion can be represented by a simple epicycle model (or eccenter with rotating apogee); this was the system of Hipparchus. To determine the size of the epicycle he adopts a method invented by Hipparchus. He takes a set of three lunar eclipses: the time of the middle of each eclipse can be calculated from the observed circumstances. Hence the true longitude of the moon at eclipse middle is known, since it is exactly 180° different from the true longitude of the sun (calculated from the solar theory). Furthermore, the time intervals between the three eclipses are known; hence one can calculate the travel in mean longitude and mean anomaly between the three points. Thus one has the situation of Figure 5: $P_1$, $P_2$, $P_3$ represent the positions of the moon on the epicycle at the three eclipses. The angles $\delta_1$, $\delta_2$ (the equational differences as seen from the earth $O$) are found by comparing the intervals in true longitude with the intervals in mean longitude; the angles $\theta_1$, $\theta_2$ are found by taking the travel in mean anomaly modulo 360°. From these one can calculate trigonometrically the size of the epicycle radius $r$ in terms of the deferent radius $R = OC$, and the angle $ACP_1$ (which gives an epoch value for the anomaly). Ptolemy makes calculations for two sets of eclipses—the first early Babylonian, the second observed by himself—and gets almost identical results. He finally adopts the value $r = 5;15$, and on this basis he constructs an equation table. Although he borrowed the above procedure from Hipparchus, Ptolemy's result seems to be a distinct improvement on his predecessor's. He tells us that through small slips in calculating intervals Hipparchus found two discrepant results from two eclipse triples—namely, 327 2/3 : 3144 and 247 1/2 : 3122 1/2. Independent evidence shows that Hipparchus adopted the latter eccentricity, although it is a good deal too small.[18]

In the preceding calculations the moon has been treated as if it lay in the plane of the ecliptic. In fact its orbit is inclined to that plane, but the angle of inclination is so small that one is justified in neglecting it in longitude calculations. Accurate knowledge of the latitude is, however, essential for eclipse calculations. The inclination of 5° which Ptolemy accepts was probably an established value. But his procedure for finding the epoch value and mean motion in argument of latitude from two carefully selected eclipses is both original and a great improvement over Hipparchus' method, which involved estimating the apparent diameter of the moon and of the shadow of the earth at eclipse, both of which are difficult to measure accurately.

The simple lunar model of book IV is essentially that of Hipparchus. When Ptolemy compared observed positions of the moon with those calculated from the model, he found good agreement at conjunction and opposition (when elongation of the moon

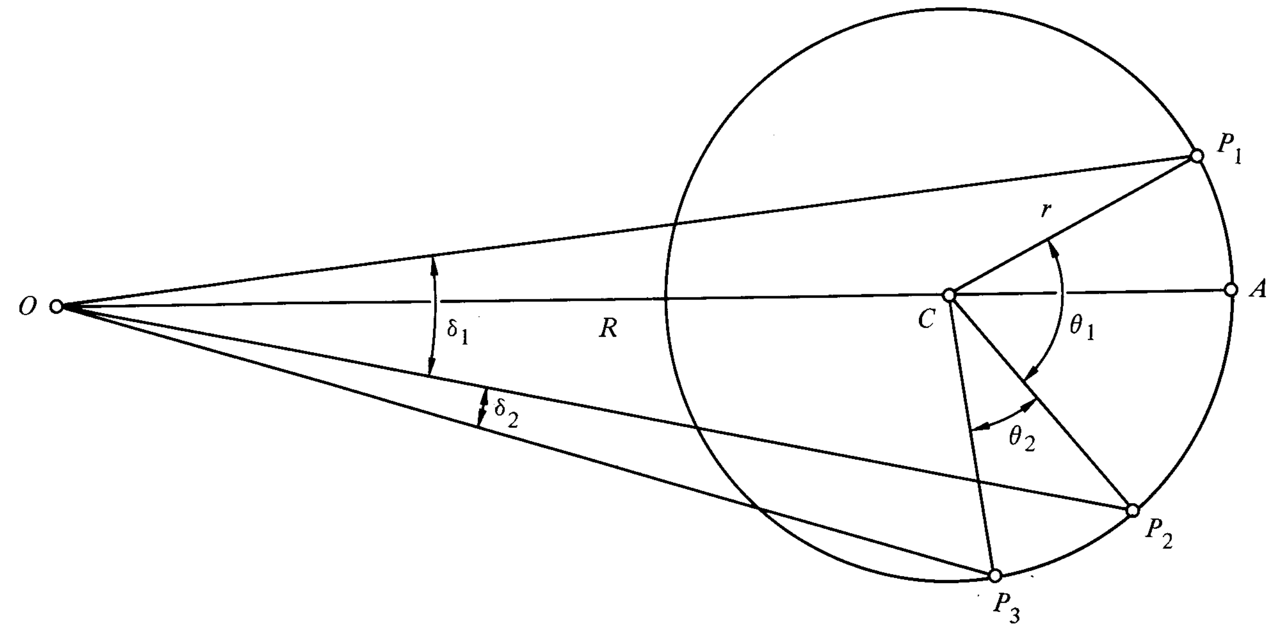
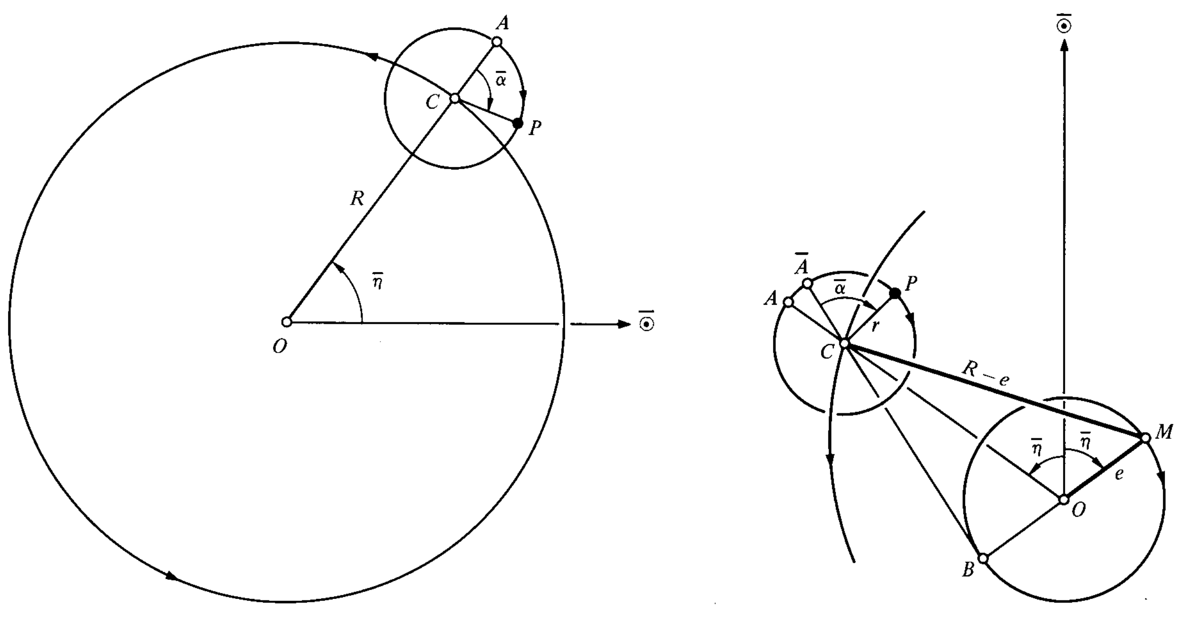



FIGURE 5










FIGURES 6A AND 6B

from the sun is 0° or 180°); this agreement was to be expected, since the model was derived from eclipse observations. But he found serious discrepancies at intermediate elongations. Hipparchus had already mentioned such discrepancies but failed to account for them. In book V Ptolemy develops his own lunar theory. Analyzing his observations, he finds that they seem to indicate an increase in the size of the epicycle between opposition and conjunction that reaches a maximum at quadrature (90° elongation). This increase he represents by incorporating in the model a "crank" mechanism that "pulls in" the epicycle as it approaches quadrature, thus making it appear larger. Compare Figures 6A and 6B, which depict the same situation according to the simple and refined models, respectively. In the latter the epicycle center $C$ continues to move uniformly about the earth $O$, but it now moves on a circle the center of which is not $O$ but $M$. $M$ moves about $O$ in the opposite sense to $C$, so that its elongation $\bar{\eta}$ from the mean sun is equal to that of $C$. Since $OM + MC$ of Figure 6B is equal to $OC$ of Figure 6A, it is clear that the two models are identical when $\bar{\eta} = 0°$ or 180°, that is, at mean conjunction and opposition. At intermediate elongations, however, the refined model pulls the epicycle closer to $O$, thus increasing the effect of the anomaly. This increase is greatest at quadrature ($\bar{\eta} = 90°$). From two observations of the moon near quadrature by himself and Hipparchus, Ptolemy finds that the maximum equation increases from about 5° at conjunction to 7;40° at quadrature, and hence $e$ in Figure 6B is 10;19 (where $R = 60$). As a further refinement he shows that one can obtain better agreement with observation if one reckons the anomaly $\bar{\alpha}$ not from the true epicycle apogee $A$ but from a mean apogee $\bar{A}$ opposite the point $B$ ($B$ in turn being opposite $M$ on the small circle about $O$). Thus a third inequality is introduced, also varying with the elongation but reaching its maximum near the octants ($\bar{\eta} = 45°$ and 135°). Ptolemy can now construct a table to compute the position of the moon. In contrast with previous tables, the tabulated function depends on two variables ($\bar{\alpha}$ and $2\bar{\eta}$). Ptolemy's solution to tabulating such a function (which may have been his own invention) became standard: he computes the equation at extreme points (in this case at conjunction and quadrature) and introduces an interpolation function (here varying with $2\bar{\eta}$) to be used as a coefficient for intermediate positions.

Ptolemy's refined lunar model represents the longitudes of the moon excellently. It is a major improvement on Hipparchus' model, and yet it does not disturb it at the points where it was successful—namely, where eclipses occur. But one effect of the crank mechanism is to increase greatly the variation in the distance of the moon from the earth, so that its minimum distance is little more than half its maximum. If correct, this should be reflected by a similar variation in the apparent size of the moon, whereas the observable variation is much smaller. This objection

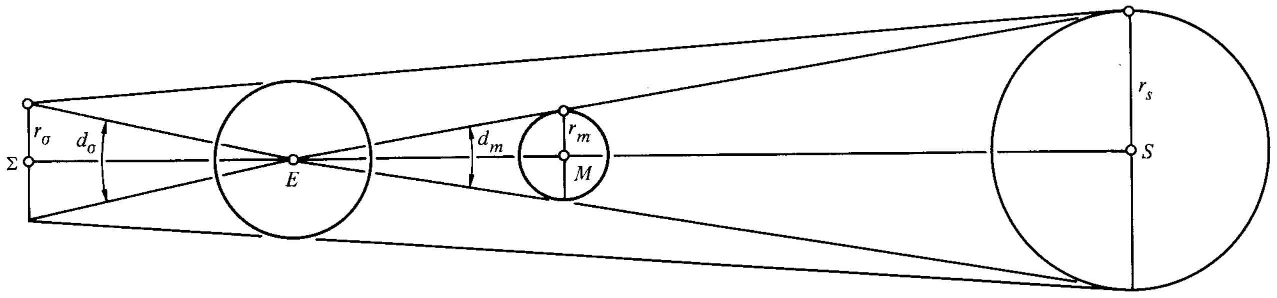



FIGURE 7

seems not to have occurred to Ptolemy: he treats the crank mechanism not merely as a convenient device for predicting the longitude but also as a real feature of the model (for instance, in his parallax computations). Fortunately the apparent size of the moon is of consequence only at eclipses, when the crank mechanism has no effect.

Having established a complete theory of the motion of sun and moon, Ptolemy can proceed to eclipse theory. But first he must deal with parallax, the angular difference between the true and apparent positions of a body that stems from the fact that we observe it not from the center of the earth but from a point on the surface. In practice only the parallax of the moon is significant, but the misconceptions of the ancients about the distance of the sun led them to estimate solar parallax as well. By comparing a suitable observed position of the moon with its computed position, Ptolemy obtains a value for the parallax. Since this value is equivalent to the angle under which the radius of the earth is seen from the moon in that position, he can immediately calculate the distance of the moon in earth radii, first at that position and then (from his model) at mean distance. His final figure, fifty-nine earth radii, is close to the truth but is reached by a combination of multiple errors in his data and model, which, by pure chance, cancel each other.

Ptolemy knew that solar parallax is too small to measure directly; but he calculates the distance of the sun, and hence its parallax, by a method invented by Hipparchus. The data required are the apparent diameters of moon and sun at distances such that their apparent diameters are equal, that distance for the moon, and the apparent diameter of the shadow of the earth at that distance of the moon. Ptolemy assumes that moon and sun have exactly the same apparent diameter when the moon is at greatest distance (which implies that annular eclipses cannot occur); he determines the apparent diameter of moon and shadow from two pairs of lunar eclipses by an ingenious method of his own (greatly improving on the figures which Hipparchus had obtained by direct measurement). Then, in Figure 7 he knows *EM* ($= E\Sigma$), the distance of the moon; $d_m$, the apparent diameter of moon and sun; and $d_\sigma$, the apparent diameter of the shadow. From these data it is simple to calculate $r_m$ and $r_s$, the radii of moon and sun, and *ES*, the distance of the sun. His value for the latter is 1,210 earth radii, too small by a factor of twenty. In fact, the method requires much too accurate a measurement of the apparent diameter of the sun to produce a reliable solar distance, but it continued to be used up to and beyond Copernicus. Ptolemy now constructs a table of solar and lunar parallaxes and explains how to compute the parallax for a given situation. This computation is both laborious and mathematically unsatisfactory. Ptolemaic parallax theory is perhaps the most faulty part of the *Almagest*. But it was not significantly improved until the late sixteenth century.

Eclipse theory, the topic of book VI, is easily derived from what precedes. Ptolemy sets up a table for calculating mean syzygies (conjunctions and oppositions), with the corresponding lunar anomaly and argument of latitude. He then determines (from the apparent sizes of the bodies) the eclipse limits, that is, how far from the node the moon can be at mean syzygy for an eclipse still to take place. The eclipse tables proper give the size in digits and duration of eclipses as a function of the distance of the moon from the node. Ptolemy explains minutely how to compute the size, duration, and other circumstances of both lunar and solar eclipses for any given place. But his method does not allow one to compute the path of a solar eclipse (a development of the late seventeenth century).[19]

Books VII and VIII deal with the fixed stars. The order of treatment is a logical one, since it is necessary to establish the coordinates of ecliptic stars to observe planetary positions. Ptolemy compares his own observations with those of Hipparchus and earlier Greeks to show that the relative positions of the fixed stars have not changed and that the sphere of the fixed





stars moves about the pole of the ecliptic from east to west 1° in 100 years with respect to the tropical points. He ascribes the discovery of the latter motion (the precession of the equinoxes) to Hipparchus, who had estimated it as *not less than* 1° in 100 years. This figure is too low (1° in seventy years would be more accurate); the error is mostly due to Ptolemy's wrong figure for the mean motion of the sun.[20] The bulk of these two books is composed of the "Star Catalog," a list of 1,022 stars, arranged under forty-eight constellations, with the longitude, latitude, and magnitude (from 1 to 6) of each. To compile this entirely from personal observation would be a gigantic task, and Ptolemy has often been denied the credit. Delambre, for instance, maintained that Ptolemy merely added 2;40° to the longitudes of "Hipparchus' catalog."[21] This particular hypothesis has been disproved.[22] In fact, the evidence suggests that no star catalog in this form had been composed by Hipparchus or anyone else before Ptolemy (the quotations from Hipparchus in *Almagest* VII, 1, show that Ptolemy had before him not a catalog but a description of the constellations with some numerical data concerning distances between stars). Modern computations have revealed numerous errors in Ptolemy's coordinates.[23] In general, the longitudes tend to be too small. This too is explained by the error in his solar mean motion, which is embedded in the lunar theory: the moon was used to fix the position of principal stars (the only practical method for an ancient astronomer).[24] Book VIII ends with a discussion of certain traditional Greek astronomical problems, such as the heliacal risings and settings of stars.

The last five books are devoted to planetary theory. Here, in contrast with the moon and sun, Ptolemy had no solid theoretical foundation to build upon and much less in the way of a body of observations. The most striking phenomenon of planetary motion is the frequent occurrence of retrogradation, which had been explained at least as early as Apollonius by a simple epicyclic model (see Figure 3) in which the sense of rotation of planet and epicycle is the same. Such a model, however, would produce a retrogradation arc of unvarying length and occurring at regular intervals, whereas observation soon shows that both arc and time of retrogradation vary. No geometric model had been proposed that would satisfactorily account for this phenomenon. Certain planetary periods, however, were well established, and so was the law for outer planets that

$$Y = L + A,$$

where (in integer numbers) $Y$ stands for years, $L$ for returns to the same longitude, and $A$ for returns in anomaly (Venus and Mercury have the same period of return in longitude as the sun, hence for them $Y = L$). Ptolemy quotes from Hipparchus such a period relation for each planet—for instance, for Saturn: "In 59 years occur 57 returns in anomaly and 2 returns in longitude." We now know that all the period relations quoted are in fact Babylonian in origin. From these Ptolemy constructs tables of mean motion in longitude and anomaly, first applying small corrections; it turns out, however, that the latter are in part based on the models he is going to develop, so he must have used the uncorrected period relations in the original development.

Analysis of observations revealed that each planet has two anomalies; the first varying according to the planet's elongation from the sun and a second varying according to its position in the ecliptic. Ptolemy isolated the first by comparing different planet-sun configurations in the same part of the ecliptic and the second by comparing the same planet-sun configurations in different parts of the ecliptic. He thus found that the first could be represented by an epicycle model in which the sense of rotation of planet and epicycle are the same, while the second was best represented by an eccenter (to avoid a double epicycle). The model he finally evolved is depicted in Figure 8. The planet $P$ moves on an

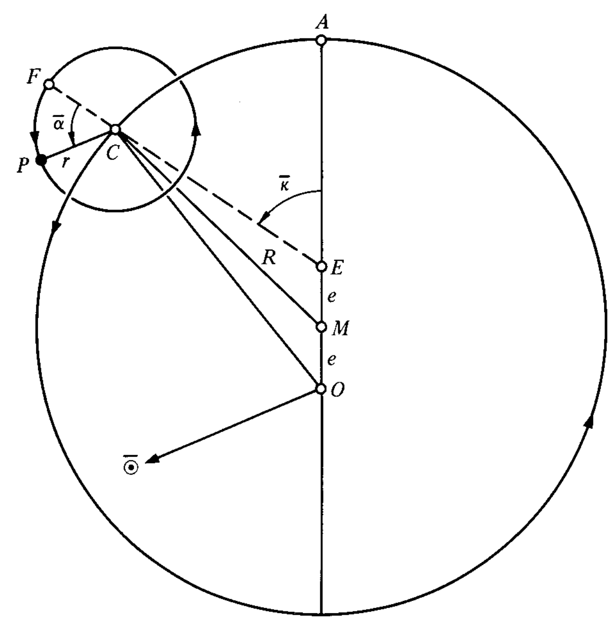



FIGURE 8

epicycle, center $C$. $C$ moves in the same sense on a circle the center of which, $M$, is distant from the earth $O$ by the eccentricity $e$. The uniform motion of $C$, however, takes place not about $M$ but about another point $E$ on the opposite side of $M$ from $O$

and distant from $M$ by the same amount $e$. The mean motion in anomaly is counted from a point $F$ on the epicycle opposite to $E$. Figure 8 depicts the situation for an outer planet, in which the line $CP$ always remains parallel to the line from $O$ to the mean sun (thus preserving the law $Y = L + A$). Except for this feature the model for Venus is identical. The Mercury model has an additional mechanism to produce a varying eccentricity but is otherwise the same as that for Venus.

The most original element in this model is the introduction of the point $E$ (known as the equant from medieval times). This element disturbed later theoreticians (including Copernicus), since it violated the philosophical principle that uniform motion should take place about the center of the circle of revolution. But it enabled Ptolemy to construct the first satisfactory planetary model. In heliocentric theory a Ptolemaic equant model with eccentricity $e$ produces planetary longitudes differing from those of a Kepler ellipse with the same eccentricity by less than 10′ (even for the comparatively large eccentricity of Mars).[25] Ptolemy deduces the existence of the eccenter directly from observations only in the case of Venus. For the other planets he merely assumes it and regards this as justified by the resulting agreement between observed and computed positions.

The determination of the parameters of the model for the individual planets, which occupies most of books IX–XI, reveals Ptolemy's brilliance. In his mastery of the choice and analysis of observations in conjunction with theory he has no peer until Kepler. For Venus and Mercury the problem is comparatively simple: the center of their epicycles coincides with the mean sun, and so one "sees" the epicycle by observing maximum elongations. The position of the apsidal line ($OMEA$ in Figure 8) can be determined by observing symmetrical positions of the epicycle, and then it is simple to calculate the size of epicycle and eccentricity. For the outer planets such "direct observation" of the epicycle is not possible. So Ptolemy eliminates the effect of the epicycle by choosing three observed oppositions; this choice gives him three positions of $C$ as seen from $O$. The problem of finding the eccentricity and apogee then resembles the problem of finding the size of the epicycle of the moon in book IV, but it is complicated by the existence of the equant. Ptolemy meets this complication with an ingenious iterative process: he first assumes that the equant coincides with the center of the eccentric; this produces an approximative apsidal line and eccentricity, which are used to compute corrections to the initial data; then the whole process is repeated as many times as is necessary for the results to converge. Such a procedure is alien to classical Greek mathematics, although not unique in the *Almagest*.

Book XII is devoted to establishing tables for computing the arcs and times of retrogradation and (for Venus and Mercury) the greatest elongation. In connection with the former, a theorem of Apollonius on stationary points is adapted to fit Ptolemy's refined planetary model. In book XIII he deals with planetary latitudes. All five planets have orbits slightly inclined to the ecliptic. From a heliocentric point of view the situation is simple, but a geocentric theory encounters considerable difficulties. The simplest approximation in Ptolemaic terms would be as follows: For an outer planet set the plane of the deferent at an inclination to the ecliptic equal to that of the heliocentric orbit of the planet, and keep the plane of the epicycle parallel to the ecliptic; for an inner planet keep the plane of the deferent in the ecliptic, and set the inclination of the epicycle to the deferent equal to the inclination of the heliocentric orbit of the planet. Ptolemy eventually reaches that solution in the *Planetary Hypotheses* (except that the deferents of Venus and Mercury are inclined to the ecliptic at a very small angle); but in the *Almagest*, misled by faulty observations and the eccentricity of the orbit of the earth, he devises a much more complicated theory in which the epicycles of outer planets and both epicycles and deferents of inner planets undergo varying inclinations as the epicycle moves round the deferent. The resultant tables do, however, represent the actual changes in latitude fairly well and are no mean achievement. The work ends with a discussion of the traditional problem of the heliacal risings and settings of the planets.

As a didactic work the *Almagest* is a masterpiece of clarity and method, superior to any ancient scientific textbook and with few peers from any period. But it is much more than that. Far from being a mere "systematization" of earlier Greek astronomy, as it is sometimes described, it is in many respects an original work. Without minimizing Ptolemy's debt to Hipparchus (which Ptolemy himself admiringly acknowledges), we may say confidently that Hipparchus' pioneering work would have had very little effect had it not found its completion in the *Almagest*. The Ptolemaic system is indeed named after the right man.

The *Almagest* contains all the tables necessary for astronomical computations, but they are scattered throughout the work. At a later date Ptolemy published the tables separately, together with an introduction explaining their use, under the title Πρόχειροι κανόνες (*Handy Tables*). The tables themselves are extant only in the revised version of Theon of Alexandria (*fl.* 360), but from Ptolemy's introduction it is clear that Theon changed nothing essential. Ptolemy





himself is responsible for the numerous differences from the *Almagest*. There are additions (such as a table of the longitudes and latitudes of principal cities, to enable one to convert from the meridian and latitude of Alexandria), changes in layout to facilitate computation, and even occasional improvements in basic parameters. The epoch is changed from era Nabonassar to era Philip (Thoth 1 = 12 November 324 B.C.). The tables became the standard manual in the ancient and Byzantine worlds, and their form persisted beyond the Middle Ages.

Later still Ptolemy published a "popular" résumé of the results of the *Almagest* under the title Ὑποθέσεις τῶν πλανωμένων (*Planetary Hypotheses*), in two books. Only the first part of book I survives in Greek, but the whole work is available in Arabic translation. It goes beyond the *Almagest* in several respects. First, it introduces changes in some parameters and even in the models, notably in the theory of planetary latitude already mentioned. Second, in accordance with Ptolemy's declaration in the introduction that one purpose of the work is to help those who aim to represent the heavenly motions mechanically (that is, with a planetarium), the models are made "physical," whereas in the *Almagest* they had been purely geometric. Ptolemy describes these physical models in detail in book II (most of book I is devoted to listing the numerical parameters). He argues that instead of assigning a whole sphere to each planet, it is sufficient to suppose that the mechanism is contained in a segment of a sphere consisting of a drum-shaped band extending either side of its equator. The most portentous innovation, however, is the system proposed at the end of book I for determining the absolute distances of the planets.

In the *Almagest* Ptolemy had adopted the (traditional) ascending order: moon, Mercury, Venus, sun, Mars, Jupiter, Saturn; but he admitted that this order was arbitrary as far as the planets are concerned, since they have no discernible parallax.[26] This order has no consequences, since the parameters of each planet are determined independently in terms of a conventional deferent radius of 60. In the *Planetary Hypotheses* Ptolemy proposes a system whereby the greatest distance from the earth attained by each body is exactly equal to the least distance attained by the body next in order outward (that is, the planetary spheres are touching, and there is no space wasted in the universe; this system conforms to Aristotelian thinking). He takes the distance of the moon in earth radii derived in *Almagest* V: its greatest distance is equal to the least distance of Mercury (if one assumes the above order of the planets). Using the previously determined parameters of the model for Mercury, he now computes the greatest distance of Mercury, which is equal to the least distance of Venus, and so on. By an extraordinary coincidence the greatest distance of Venus derived by this procedure comes out very close to the least distance of the sun derived by an independent procedure in *Almagest* V. Ptolemy takes this finding as a striking proof of the correctness of his system (and incidentally of the assumption that Mercury and Venus lie below the sun). He goes on to compute the exact distances of all the bodies right out to the sphere of the fixed stars in earth radii and stades (assuming the circumference of the earth to be 180,000 stades). Furthermore, taking some "observations" by Hipparchus of the apparent diameters of planets and first-magnitude stars, he computes their true diameters and volumes. This method of determining the exact dimensions of the universe became one of the most popular features of the Ptolemaic system in later times.

A work in two books named *Phases of the Fixed Stars* (Φάσεις ἀπλανῶν ἀστέρων) dealt in detail with a topic not fully elaborated in the *Almagest*, the heliacal risings and settings of bright stars. Only book II survives; and the greater part of this book consists of a "calendar," listing for every day of the year the heliacal risings and settings, as well as the weather prognostications associated with them by various authorities. Predicting the weather from the "phases" of well-known stars long predates scientific astronomy in Greece, and calendars like this were among the earliest astronomical publications (Ptolemy quotes from authorities as early as Meton and Euctemon). The chief value of the Φάσεις today is the information it contains on the history of this kind of literature.

Much greater scientific interest attaches to two small works applying mathematics to astronomical problems. The first is the *Analemma* (Περὶ ἀναλήμματος), surviving, apart from a few palimpsest fragments, only in William of Moerbeke's Latin translation from the Greek. It is an explanation of a method for finding angles used in the construction of sundials, involving projection onto the plane of the meridian and swinging other planes into that plane. The actual determination of the angles is achieved not by trigonometry (although Ptolemy shows how that is theoretically possible) but by an ingenious graphical technique which in modern terms would be classified as nomographic. Although the basic idea was not new (Ptolemy criticizes his predecessors, and a similar procedure is described by Vitruvius *ca.* 30 B.C.),[27] the sophisticated development is probably Ptolemy's. The other treatise is the *Planisphaerium* (the Greek title was probably Ἅπλωσις ἐπιφανείας σφαίρας).[28] This treatise survives only in Arabic translation; a revision of this translation was made by the Spanish Islamic astronomer Maslama al-Majrīṭī (*d.* 1007/1008) and was in turn translated into Latin by Hermann of Carinthia in 1143. It treats the problem of mapping circles on the celestial sphere onto a plane. Ptolemy projects them from the south celestial pole onto the plane of the equator. This projection is the mathematical basis of the plane astrolabe, the most popular of medieval astronomical instruments. Since the work explains how to use the mapping to calculate rising times, one of the main uses of the astrolabe, it is highly likely that the instrument itself goes back to Ptolemy (independent evidence suggests that it goes back to Hipparchus).[29] These two treatises are an important demonstration that Greek mathematics consisted of more than "classical" geometry.

To modern eyes it may seem strange that the same man who wrote a textbook of astronomy on strictly scientific principles should also compose a textbook of astrology (Ἀποτελεσματικά, meaning "astrological influences," or Τετράβιβλος, from its four books). Ptolemy, however, regards the *Tetrabiblos* as the natural complement to the *Almagest*: as the latter enables one to predict the positions of the heavenly bodies, so the former expounds the theory of their influences on terrestrial things. The introductory chapters are devoted to a defense of astrology against charges that it cannot achieve what it claims and that even if it can, it is useless. Ptolemy regards the influence of heavenly bodies as purely physical. From the obvious terrestrial physical effects of the sun and moon, he infers that all heavenly bodies must produce physical effects (that such an argument could be seriously advanced reflects the poverty of ancient physical science). By careful observation of the terrestrial manifestations accompanying the various recurring combinations of celestial bodies, he believes it possible to erect a system which, although not mathematically certain, will enable one to make useful predictions. Ptolemy is not a fatalist: at least he regards the influence of the heavenly bodies as only one of the determinants of terrestrial events. But, plausible as this introduction might appear to an ancient philosopher, the rest of the treatise shows it to be a specious "scientific" justification for crude superstition. It is difficult to see how most of the astrological doctrines propounded could be explained "physically" even in ancient terms, and Ptolemy's occasional attempts to do so are ludicrous. Astrology was almost universally accepted in the Roman empire, and even superior intellects like Ptolemy and Galen could not escape its dominance.

Book I explains the technical concepts of astrology, book II deals with influences on the earth in general ("astrological geography" and weather prediction), and books III and IV with influences on human life. Although dependent on earlier authorities, Ptolemy often develops his own dogma. The discussion in books III and IV is confined to what can be deduced from a man's horoscope: Ptolemy ignores altogether the branch of astrology known as catarchic, which answers questions about the outcome of events or the right time to do something by consulting the aspect of the heavens at the time of the question. This omission helps to explain why the *Tetrabiblos* never achieved an authority in its field comparable with that of the *Almagest* in astronomy.

The *Geography* (Γεωγραφικὴ ὑφήγησις), in eight books, is an attempt to map the known world. The bulk of it consists of lists of places with longitude and latitude, accompanied by very brief descriptions of the chief topographical features of the larger land areas. It was undoubtedly accompanied in Ptolemy's own publication by maps like those found in several of the manuscripts. But knowing how easily maps are corrupted in copying, Ptolemy takes pains to ensure that the reader will be able to reconstruct the maps on the basis of the text alone: he describes in book I how to draw a map of the inhabited world and lists longitudes and latitudes of principal cities and geographical features in books II–VII. Book VIII describes the breakdown of the world map into twenty-six individual maps of smaller areas. Ptolemy tells us that the *Geography* is based, for its factual content, on a similar recent work by Marinus of Tyre. But it seems to have improved on Marinus' work (for which the *Geography* is the sole source of our knowledge) in several ways. From I, 7–17 (in which various factual errors of Marinus are corrected), it appears that the bulk of Marinus' text was topographical description (giving, for instance, distances and directions between places), and that this was supplemented by lists of places with the same longest daylight and of places the same distance (in hours) from some standard meridian (book VIII of the *Geography*, which looks as if it is a remnant of an earlier version, uses a system similar to the latter). Ptolemy was probably the first to employ systematically listings by latitude and longitude. Here, as always, he shows a sound sense of what would be of most practical use to the reader.

Ptolemy also criticizes Marinus' map projection, a system of rectangular coordinates in which the ratio of the unit of longitude to that of latitude was 4:5. Ptolemy objects that this system distorts distances except near the latitude of Rhodes (36°). While accepting such a system for maps covering a small



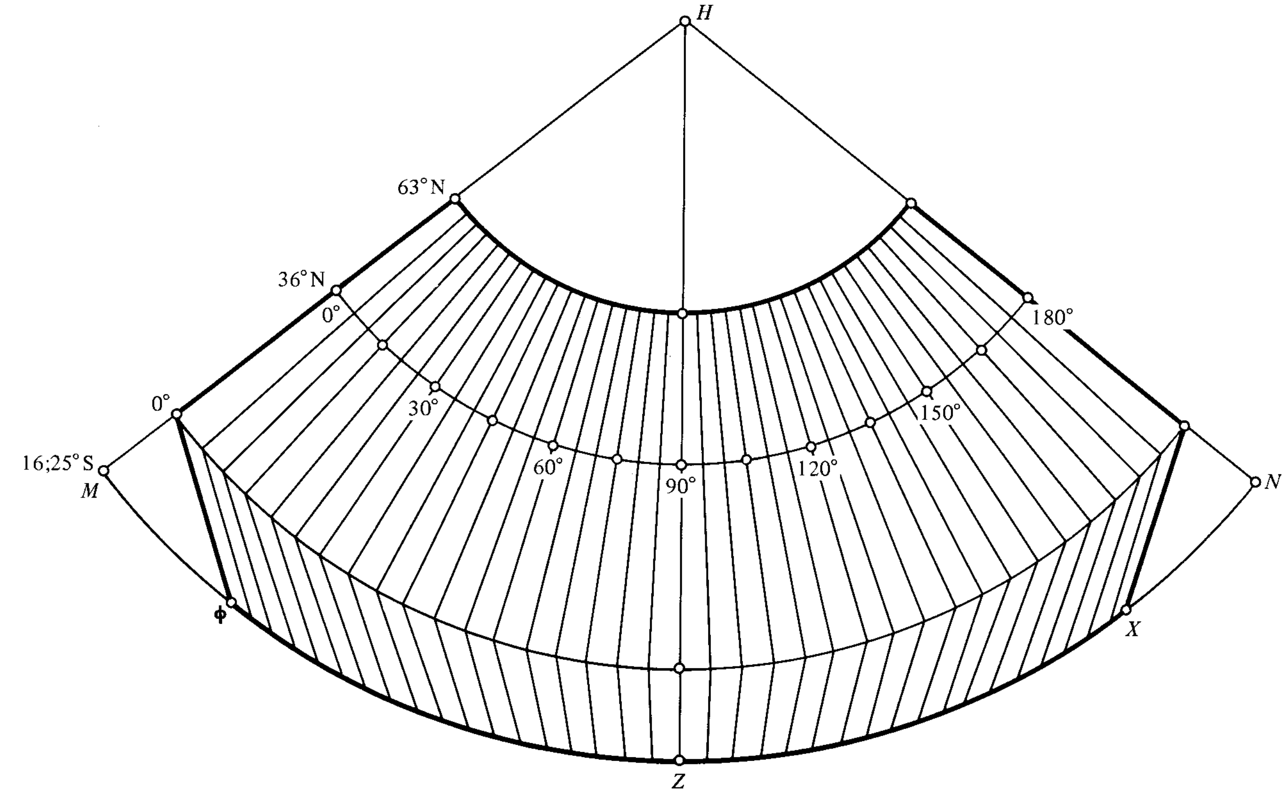


FIGURE 9

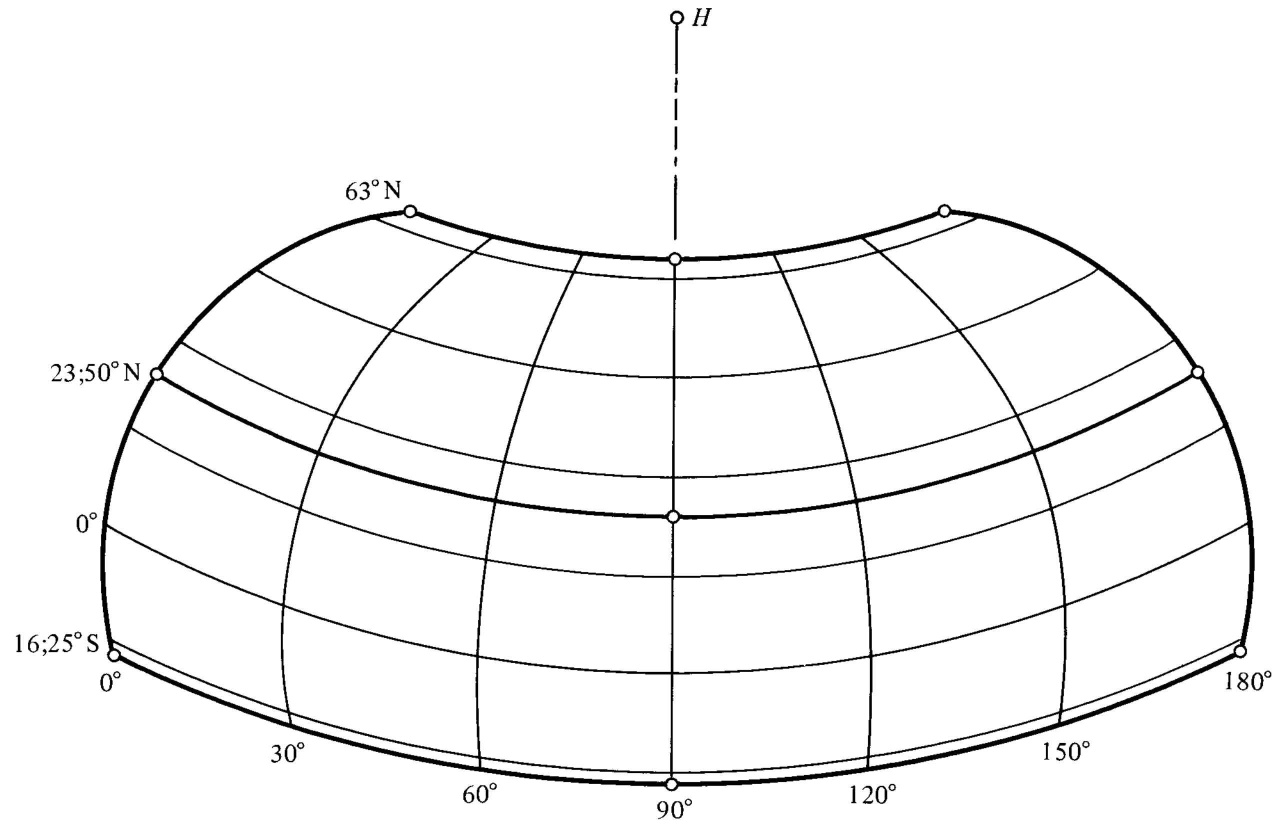


FIGURE 10

area, for the world map he proposes two alternative projection systems (I, 21–24). The known world, according to Ptolemy, covers 180° in longitude from his zero meridian (the Blessed Isles) and in latitude stretches from 16;25° south to 63° north. In his first projection (see Figure 9) the meridians are mapped as radii meeting in a point *H* (*not* the north pole), the parallels of latitude as circular arcs with *H* as center. Distances are preserved along the meridians and along the parallel of Rhodes, and the ratio of distances along the parallel of 63° to those along the parallel of the equator is preserved. These conditions completely determine the projection. Ptolemy modifies it south of the equator by dividing the parallel *MZN* as if it lay at 16;25° *north*, thus avoiding distortion at the expense of mathematical consistency. The second projection aims to achieve more of the appearance of a globe (see Figure 10). The parallels of latitude are again constructed as circular arcs, but now distances are preserved along three parallels: 63° north, 23;50° north, and 16;25° south. The meridians are constructed by drawing circular arcs through the points on these three parallels representing the same angular distance from the central meridian, which is mapped as a straight line along which distances are preserved. The first projection is (except for the modification south of the equator) a true conic projection; the second is not, but for the segment covered by the map is a remarkably good approximation to the true conic projection that was later developed from it (the Bonne projection, which preserves distances along all parallels). Ptolemy took a giant step in the science of mapmaking, but he had no successor for nearly 1,400 years.

The factual content of the *Geography* naturally is inaccurate. Only the Roman empire was well known; and Ptolemy's idea of the outline of, for instance, southern Africa or India is grossly wrong. Even within the boundaries of the empire there are some serious distortions. This was inevitable: although the latitude of a place could be fixed fairly accurately by astronomical observation, very few such observations had been made. Ptolemy says (I, 4) that of his predecessors only Hipparchus had given the latitudes of a few cities. Longitudinal differences could be determined from simultaneous observations of an eclipse at two places, but again almost no such observations existed. Ptolemy seems to have known only one, the lunar eclipse of 20 September 331 B.C., observed at Arbela in Assyria and at Carthage. Unfortunately an error in the observation at Arbela led Ptolemy to assume a time difference of about three hours between the two places instead of about two; this error was probably a major factor in the most notorious distortion in his map, the excessive length of the Mediterranean. In the absence of anything resembling a modern survey, Ptolemy, like his predecessors, had to rely on "itineraries" derived from milestones along the main roads and the reports of merchants and soldiers. In view of the inaccuracies in lengths, and especially directions, inevitable in such works, the *Geography* is a remarkable factual as well as scientific achievement.

The *Optics*, in five books, is lost in Greek. An Arabic translation was made from a manuscript lacking book I and the end of book V; from this translation, which is also lost, Eugenius of Sicily produced the extant Latin translation in the twelfth century. Despite the incompleteness and frequent obscurity of the text, the outlines of Ptolemy's optical theory are clear enough. The lost book I dealt with the general theory of vision. Like most ancient theoreticians Ptolemy believed that vision takes place by means of a "visual flux" emanating from the eye in the form of a cone-shaped bundle of "visual rays," the apex of which lies within the eyeball; this flux produces sensations in the observer when it strikes colored objects. References back in the surviving books show that besides enunciating the above theory, Ptolemy demonstrated that vision is propagated in a straight line and determined the size of the visual field (both probably by experiments).

In book II he deals with the role of light and color in vision (he believes that the presence of light is a "necessary condition" and that color is an inherent quality of objects); with the perception of the position, size, shape, and movement of objects; and with various types of optical illusion.

Books III and IV treat the theory of reflection ("catoptrics," to use the ancient term). First, three laws are enunciated: (1) the image appears at some point along the (infinite) line joining the eye to the point of reflection on the mirror. (2) The image appears on the perpendicular from the object to the surface of the mirror. (3) Visual rays are reflected at equal angles. The laws are then demonstrated experimentally. There follows a remarkable discussion on the propriety of assimilating binocular to monocular vision in geometric proofs. Ptolemy incidentally determines the relationships between the images seen by the left and right eyes and the composite image seen by both, using an ingenious experimental apparatus with lines of different colors. He then develops from the three laws a series of theorems on the location, size, and appearance of images, first for plane mirrors, then for spherical convex mirrors, then (in book IV) for spherical concave mirrors and various types of "composite" (such as cylindrical) mirrors.







Book V deals with refraction. The phenomenon is demonstrated by the experiment (at least as old as Archimedes) of the coin in the vessel that appears when water is poured in. There follows a very interesting experiment to determine the magnitude of refraction from air to glass, from water to glass, and from air to water. Ptolemy does this experiment by means of a disk with a graduated circumference. For each pair of media he tabulates the angles of refraction corresponding to angles of incidence of 10°, 20°, · · ·, 80°. The results cannot be raw observations, since the second differences are constant in all cases; but they are remarkably close to what one can derive from Snell's law $\frac{\sin i}{\sin \rho} = \text{const.}$, with a suitable index of refraction. The rest of the surviving part of the book is devoted to discussion of astronomical refraction, the relationship between the amount of refraction and the density of the media, and the appearance of refracted images.

It is difficult to evaluate Ptolemy's achievement in the *Optics* because so little remains of his predecessors' work. In "pure optics" we have only the work of Euclid (*ca.* 300 B.C.), consisting of some elementary geometrical theorems derived from a few postulates that are a crude simplification of the facts of vision. In catoptrics we have a corrupted Latin version of the work of Hero (*ca.* A.D. 60) and a treatise compiled in late antiquity from authors of various dates, falsely attributed to Euclid. From these we get only an occasional glimpse of Archimedes' catoptrics, which was probably highly original, particularly in its use of experiment. The establishment of theory by experiment, frequently by constructing special apparatus, is the most striking feature of Ptolemy's *Optics*. Whether the subject matter is largely derived or original, the *Optics* is an impressive example of the development of a mathematical science with due regard to the physical data, and is worthy of the author of the *Almagest*.

A work on music theory (Ἁρμονικά), in three books, deals with the mathematical intervals (on a stretched string) between notes and their classification according to various traditional Greek systems. It seeks a middle ground between the two schools of the Pythagoreans and the followers of Aristoxenus, of whom the former, according to Ptolemy (I, 2), stressed mathematical theory at the expense of the ear's evidence, while the latter did the reverse. Again we see Ptolemy's anxiety to erect a theory that is mathematically satisfactory but also takes due account of the phenomena. According to the commentary of Porphyry (late third century), the *Harmonica* is mostly derivative, especially from the work of one Didymus (first century).[30] We have no means of checking this statement.

Ptolemy's philosophical standpoint is Aristotelian, as is immediately clear from the preface to the *Almagest*. But it is clear from his astronomy alone that he does not regard Aristotle as holy writ; and influences from later philosophy, notably Stoicism, have been detected. An insignificant philosophical work entitled *Περὶ κριτηρίου καὶ ἡγεμονικοῦ* ("On the Faculties of Judgment and Command") goes under his name. There is nothing in its contents conflicting with Ptolemy's general philosophical position, but the style bears little resemblance to his other works; and the ascription, while generally accepted, seems dubious. Ptolemy's high reputation in later times caused a number of spurious works, mostly astrological, to be foisted on him. An example is the *Καρπός* (*Centiloquium*, in its Latin translation), a collection of 100 astrological aphorisms.

Ptolemy was active in almost every mathematical science practiced in antiquity, but several of his works are known to us only from references in ancient authors. These are a work on mechanics (*Περὶ ῥοπῶν*), in three books; a work in which he attempted to prove Euclid's parallel postulate; *On Dimension* (*Περὶ διαστάσεως*), in one book, in which he "proved" that there can be no more than three dimensions; and *On the Elements* (*Περὶ τῶν στοιχείων*).

In estimating Ptolemy's stature and achievement as a scientist, it is unfortunately still necessary to react against the general tendency of nineteenth-century scholarship to denigrate him as a mere compiler of the scientific work of his predecessors. This extreme view, exemplified in the writings of Delambre,[31] is no longer held by anyone competent but still persists in handbooks. It need not be refuted in detail. With a candor unusual in ancient authors Ptolemy freely acknowledges what parts of his theory he owes to Hipparchus. To say that he is lying when he claims other parts as his own work is a gratuitous slander, which, when it can be tested by independent evidence (which is rare), has proved false. It is certain that a great part of the theory in the *Almagest* is his personal contribution, and it is unlikely that the situation was radically different in all his other scientific work. On the other hand, his was not an original genius: his method was to take existing theory and to modify and extend it so as to get good agreement with observed facts. In this method, however, Ptolemy was no different from the vast majority of scientists of all periods; and he in no way deserves the reputation of a hack. His work is remarkable for its blend of knowledge, ingenuity, judgment, and clarity. The authority that it achieved in several fields is not surprising.

The *Almagest* was the dominant influence in theoretical astronomy until the end of the sixteenth century. In antiquity it became the standard textbook almost immediately. Commentaries to it were composed by Pappus (*fl.* 320) and Theon of Alexandria, but neither they nor any other Greek advanced the science beyond it. It was translated into Arabic about 800; and improved translations were made during the ninth century, notably in connection with the astronomical activity patronized by Caliph al-Ma'mūn. The Islamic astronomers soon recognized its superiority to what they had derived from Persian and Hindu sources; but since they practiced observation, they also recognized the deficiencies in its solar theory. An example of the influence of Ptolemy on Islamic astronomy and its improvements on his theory is the *Zīj* of al-Battānī (*ca.* 880). The first part of this work is closely modeled on the *Almagest*, the second on the *Handy Tables* (also translated earlier). Al-Battānī greatly improves on Ptolemy's values for the obliquity of the ecliptic, the solar mean motion (and hence precession), the eccentricity of the sun, and the longitude of its apogee. He substitutes the sine function (derived from India) for the chord function. Otherwise his work is mostly a restatement of Ptolemy's. This is typical of Islamic astronomy: the solar theory was refined (so that even the proper motion of the apogee of the sun was enunciated by al-Zarqāl *ca.* 1080), but Ptolemy's lunar and planetary theories were accepted as they stood. Such attempts as were made to revise them were based not on observation but on philosophical objections, principally to the equant. Alternative systems, preserving uniform circular rotation, were devised by Naṣīr al-Dīn al-Ṭūsī (*fl.* 1250) and his followers at Maragha, and also by Ibn al-Shāṭir (*fl.* 1350). The latter's lunar model is in one sense a real improvement over Ptolemy's, since it avoids the exaggerated variation in the lunar distance. But the influence of these reformers was very small (except for a hypothetical transmission from Ibn ash-Shāṭir to Copernicus, who adopted almost identical models).

The *Almagest* became known in western Europe through Gerard of Cremona's Latin translation from the Arabic in 1175 (a version made in Sicily from the Greek *ca.* 1160 seems to have been little known). The arrival from Islamic sources of this and other works based on Ptolemy led to a rise in the level of Western astronomy in the thirteenth century, but until the late fifteenth serious attempts to make independent progress were sporadic and insignificant. The first major blow at the Ptolemaic system was Copernicus' *De revolutionibus* (1543). Yet even this work betrays in its form and in much of its content the overwhelming influence of the *Almagest*. However great the cosmological implication of the change to a heliocentric system, Copernican astronomy is cast in a firm Ptolemaic mold. This influence was not broken until Tycho Brahe realized that the "reform of astronomy" must be based on systematic accurate observation. So he compiled the first star catalog since Ptolemy based entirely on independent observation (earlier star catalogs, such as the famous one of al-Ṣūfī, epoch 964, had been mostly repetitions of Ptolemy's with the addition of a constant of precession; star coordinates independent of Ptolemy, such as those in the list of the *Zīj al-Mumtaḥan* compiled at the order of al-Ma'mūn, are very few in number); Brahe was also able to make the first real improvements in lunar theory since Ptolemy. Most important, he provided Kepler with the essential material for his treatise on Mars, justifiably entitled *Astronomia nova.* Kepler's work finally made the *Almagest* obsolete except as a source of ancient observations.

Ptolemy's system for establishing the exact distances and sizes of the heavenly bodies was also enthusiastically adopted in Islamic astronomy. One of the numerous adaptations of it found its way to the West in the twelfth-century Latin translation of the epitome of the *Almagest* by al-Farghānī (*fl.* 850). In this version it became part of the standard world picture of the later Middle Ages (for instance, in Dante's *Divina commedia*) and persisted into the sixteenth century, although its basis was removed by Copernicus.

The *Tetrabiblos* went through a process of transmission to the Islamic world and then to western Europe similar to that of the *Almagest* and also enjoyed an immense reputation in both, although never so unrivaled. The *Geography* was also translated into Arabic early in the ninth century. Its inadequacy for much of the territory subject to Islam was immediately obvious, and so it was soon replaced by revisions more accurate for those parts. The earliest is the *Kitāb Ṣūrat al-Arḍ* of al-Khwārizmī (*ca.* 820). The original Arabic translation of the *Geography* is lost, but its direct and indirect influence on Islamic geographical works is considerable. The *Geography* did not reach western Europe until the fifteenth century (in a Latin translation from the Greek by Jacobus Angelus, *ca.* 1406), when exploration was already making it obsolete. Nevertheless it became very popular, since it was still the best guide to much of the known world; most cartographic publications of the fifteenth and sixteenth centuries are based on it. Ptolemy's two map projections were an important stimulus to the development of cartography in the sixteenth century. The first is reflected in Mercator's map of Europe (1554), the second in various maps





beginning with one by Bernardus Sylvanus in his 1511 edition of the *Geography*, which uses an equivalent of the Bonne projection.[32]

The *Optics* had little direct influence on western Europe. Eugenius' version was known to Roger Bacon and probably to Witelo in the later thirteenth century, but they are exceptions. It did however, inspire the great optical work of Ibn al-Haytham (*d.* 1039). The latter was an original scientist who made some notable advances (of which the most important was his correct explanation of the role of light in vision), but the form and much of the content of his *Kitāb al-Manāẓir* are taken from Ptolemy's work; and the inspiration for his remarkable experiments with light must surely also be attributed to Ptolemy. His treatise was translated into Latin and is the basis of the *Perspectiva* of Witelo (*ca.* 1270), which became the standard optical treatise of the later Middle Ages. The indirect influence of the *Optics* persisted until the early seventeenth century.

Ptolemy's treatise on musical theory never attained the authority of his other major works, since rival theories continued to flourish. But it was extensively used by Boethius, whose work was the main source of knowledge of the subject in the Latin West; and hence some of Ptolemy's musical doctrine was always known. When it became available to western Europe in Greek, it was no more than a historical curiosity. But it had a strange appeal for one great scientist. The last three chapters of book III of the *Harmonica* are missing; they dealt with the relationships between the planetary spheres and musical intervals. Kepler intended to publish a translation of book III, with a "restoration" of the last chapters and a comparison with his own kindred speculations, as an appendix to his *Harmonice mundi.* The appendix never appeared, but the whole work is a tribute to his predecessor.

*NOTES*

1. *Almagest* XI, 5; IX, 7 (Manitius, II, 228, 131).
2. Boll, "Studien," p. 53.
3. *Opera astronomica minora*, p. 155. An alternative MS reading is "the fifteenth year" (152/153).
4. Olympiodorus, . . . *In Platonis Phaedonem* . . ., Norwin ed., p. 59, l. 9.
5. Boll, *op. cit.*, pp. 54–55.
6. Buttmann, "Ueber den Ptolemäus in der Anthologie . . .," pp. 483 ff.
7. II, 13 (Manitius, I, 129).
8. I, 3 (Manitius, I, 9).
9. III, 59; Lejeune ed., p. 116.
10. V, 23–30; Lejeune ed., pp. 237–242.
11. E.g., X, 1 (Manitius, II, 156).
12. See Pappus, *Collectio*, VI, intro.; Hultsch ed., p. 474, with Hultsch's note *ad. loc.*
13. Aristotle, *Metaphysica*, *Λ*, 1073b17 ff.; Simplicius, *In Aristotelis De caelo*, Heiberg ed., pp. 491 ff.
14. *Almagest* XII, 1 (Manitius, II, 267 ff.).
15. *Almagest* IX, 2 (Manitius, II, 96–97).
16. Theon, *Commentary on the Almagest*, I, 10; Rome ed., p. 451. See Toomer, "The Chord Table . . .," pp. 6–16, 19–20.
17. Kugler, *Die Babylonische Mondrechnung*, pp. 4 ff.
18. See Toomer, "The Size of the Lunar Epicycle . . .," pp. 145 ff.
19. The first to compute and draw the path of an eclipse seems to have been Cassini I in 1664; see Lalande, *Astronomie*, II, 358.
20. Shown by A. Ricius, *De motu octavae sphaerae*, f. 39 (following Levi ben Gerson); see also Laplace, *Exposition*, p. 383.
21. Delambre, *Histoire de l'astronomie ancienne*, II, 250 ff.
22. By Vogt in "Versuch einer Wiederherstellung von Hipparchs Fixsternverzeichnis."
23. E.g., Peters and Knobel, *Ptolemy's Catalogue of Stars, passim.*
24. See *Almagest* VII, 4 (Manitius, II, 30).
25. See Caspar's intro. to his trans. of Kepler's *Neue Astronomie*, pp. 60*–61*.
26. IX, 1 (Manitius, II, 93–94).
27. Vitruvius, *De architectura* IX, 7.
28. Suidas, "*Πτολεμαῖος ὁ Κλαύδιος*"; Adler ed., IV, 254.
29. Synesius, *Opuscula*, Terzaghi ed., p. 138.
30. Porphyry, *Commentary on the Harmonica*; Düring ed., p. 5.
31. E.g., *Histoire de l'astronomie ancienne*, II, *passim.*
32. See Hopfner in Mžik, *Des Klaudios Ptolemaios Einführung*, p. 105.

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**PUISEUX, VICTOR** (*b.* Argenteuil, Val-d'Oise, France, 16 April 1820; *d.* Frontenay, Jura, France, 9 September 1883), *mathematics, mechanics, celestial mechanics.*

Puiseux spent his youth in Lorraine, where his father, a tax collector, was posted in 1823. He was educated at the Collège de Pont-à-Mousson and, from 1834, at the Collège Rollin in Paris, where he attended C. Sturm's course in special mathematics. After winning the grand prize in physics (1836) and mathematics (1837) in the *concours général*, he was admitted in 1837 to the École Normale Supérieure. There he became friends with his future colleagues Briot and Bouquet. In 1840 Puiseux placed first in the *agrégation* in mathematics and then spent an additional year at the École Normale Supérieure as *chargé de conférences*, completing his training and preparing a dissertation in astronomy and mechanics, which he defended 21 August 1841.

Puiseux was professor of mathematics at the royal college of Rennes (1841–1844) and at the Faculty of Sciences of Besançon (1844–1849). During this period he published about ten articles on infinitesimal geometry and mechanics in Liouville's *Journal de mathématiques pures et appliquées*. In 1849 he was called to Paris as *maître de conférences* of mathematics at the École Normale Supérieure, a post he held until 1855 and again from 1862 to 1868.

In addition to his teaching duties, for several years Puiseux attended Cauchy's courses and became one of his closest followers. Under this fruitful influence, Puiseux wrote several important memoirs on the theory of functions of a complex variable before turning to celestial mechanics. In 1857, having substituted for various professors, including the astronomer Jacques Binet at the Collège de France and Sturm and Le Verrier at the Faculty of Sciences, Puiseux succeeded Cauchy in the chair of mathematical astronomy at the latter institution. He retained this post until 1882, publishing several important memoirs. Brief tenures as director of the Bureau de Calculs at the Paris observatory (1855–1859) and at the Bureau des Longitudes (1868–1872) permitted him to display his mastery of the techniques of astronomical computation. In 1871 he became a member of the mathematics section of the Académie des Sciences, succeeding Lamé.

In 1849 Puiseux married Laure Jeannet; of their six children only Pierre and André survived childhood; both became astronomers. An austere teacher and tireless worker, Puiseux devoted himself to the education of his children, was active in various catholic organizations, and took a passionate interest in botany and alpinism. He was, in fact, a pioneer in the latter sport and in 1848 was the first to scale one of the peaks (now bearing his name) of Mount Pelvoux.

Puiseux's scientific work encompassed infinitesimal geometry, mechanics, mathematical analysis, celestial mechanics, and observational astronomy. His first publication (1841), his doctoral dissertation, dealt with the invariability of the major axes of the planetary orbits and with the integration of the equations of motion of a system of material points. Although well-executed, the work lacked great originality. Similarly, his papers on infinitesimal geometry, most of which were published at the beginning of his career, attested his analytic virtuosity but constituted a rather limited contribution to the subject—notwithstanding his discovery of new properties of evolutes and involutes. The most interesting among these papers pertain to questions related to mechanics: the motion of the conical pendulum, tautochrones, a generalization of the top problem, and the study of the apparent movements of the surface of the earth.

In 1850 and 1851, however, Puiseux accomplished much more original work, developing, correcting, and completing major aspects of the theory of functions of a complex variable that had been elaborated by Cauchy. Examining functions of a complex variable $z$ defined by an algebraic equation of the form $f(u,z) = 0$, Puiseux succeeded in separating the various branches and in formulating the expansions in corresponding series. He clearly distinguished, for the first time, the different types of singular points (poles, essential points, and branch points); determined the integrals of algebraic differentials over the paths of integration; specified the "mode of existence of non-uniform functions" (C. Hermite); and pointed out the applications of series containing fractional powers of the variable. Despite its intrinsic interest, Puiseux's theory was surpassed in 1857 when Riemann, in his *Theorie der Abelschen Funktionen*, approached the topic from a topological point of view and introduced the famous "Riemann surfaces." Puiseux subsequently turned to the study of celestial mechanics and astronomy and virtually never returned to his theory.

Following Cauchy, Puiseux sought to apply the most recent mathematical methods to the fundamental problems of celestial mechanics. His papers on the series expansions of the perturbation function, on long-term inequalities in planetary motions, and on related questions constitute an elaboration and refinement of earlier work by Cauchy. After presenting the lucid exposition "Sur les principales inégalités du mouvement de la lune" (*Annales scientifiques de l'École Normale Supérieure*, **1** [1864], 39–80), Puiseux took up the difficult problem of the acceleration of the mean motion of the moon. Although Laplace (1787) thought he could explain this phenomenon by the secular decrease in the eccentricity of the orbit of the earth, J. Adams showed in 1853 that Laplace's theory accounted for only half of the observed effect. After extensive calculations, Puiseux established (*Journal de mathématiques pures et appliquées*, 2nd ser., **15** [1870], 9–116) that the secular displacement of the ecliptic had no significant influence on the acceleration. Although a purely negative conclusion, Puiseux's finding led to a better delimitation of the problem, which was investigated by G. Hill in 1877.

Puiseux was also concerned with improving the computational methods employed in basic astronomy. At the Bureau de Calculs, he directed the reduction of both the lunar observations made at Paris from 1801 to 1829 and the meridional observations of 1837–1838. After comparing the different methods available for deducing the solar parallax from the observation of the transits of Venus, Puiseux participated in the preparations carried out for the observation of the 1874 and 1882 transits; he also worked on the observations made in 1874 by French astronomers. During his brief tenure at the Bureau des Longitudes, he served as principal editor of the *Connaissance des temps ou des mouvements célestes.*

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**PULFRICH, CARL** (*b.* Strässchen [near Burscheid], Solingen, Germany, 24 September 1858; *d.* Baltic Sea, near Timmendorferstrand, Germany, 12 August 1927), *physics.*