The Canadian National Energy Program and Its Aftermath: A Game-theoretic Analysis*

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L'analyse à l'aide de la théorie des jeux des politiques publiques a été jusqu'à maintenant plus de nature prescriptive que descriptive, soulevant ainsi des questions quant à son utilité pratique. Une façon de rectifier ceci est de rendre opérationnels les concepts de solution de la théorie des jeux de façon à permettre la comparaison avec des choix sociaux observés. Ce texte se penche sur une période intéressante en soi de la politique énergétique, soit la politique d'énergie nationale du Canada et ses conséquences durant l'année qui suit son introduction. Il se divise en cinq parties. Premièrement, nous décrivons la structure du jeu qui porte sur la confrontation intergouvernementale au sujet de cette politique d'octobre 1980 à septembre 1981. Deuxièmement, les solutions théoriques pertinentes à ce jeu de négociation sont identifiées. Troisièmement, des valeurs mesurables sont dérivées de ces concepts. Quatrièmement, les valeurs obtenues des solutions de la théorie des jeux sont comparées à la distribution des paiements correspondants à l'entente Canada–Alberta de septembre 1981. Finalement, les conséquences de ces résultats et les voies de recherche futures sont examinées.

Game-theoretic analysis of public policy has followed a prescriptive rather than descriptive path, thereby raising questions about its practical relevance. One way to rectify this divergence is to operationalize solution concepts from game theory, in order to permit comparison with observed instances of social choice. The purpose of the present study is to examine an intrinsically interesting phase of energy policy, the Canadian National Energy Program (NEP) and its year-long aftermath, in those terms. There will be five stages to the investigation. First, the game-theoretic setting will be described, referring to the phase of intergovernmental confrontation in Canada over the NEP from October 1980 to September 1981. Second, relevant solution concepts for this bargaining game will be identified. Measurements are to be derived for these concepts in the third stage. In the fourth phase, the values generated by the game-theoretic solutions will be compared to the pay-off distribution corresponding to the Canada-Alberta Agreement of September 1981. Fifth, and finally, implications of the findings will be explored, along with possibilities for further research.

> I think a categorical disavowal of *descriptive* content is implicit in the entire game-theoretical approach. Game theory is definitely normative in spirit and method. Its goal is a *prescription* of how a rational player should behave in a given game situation when the preferences of this player and of all the other players are given in utility units (Rapoport, 1960: 226–7).

Over the last three decades, game theorists appear to have followed Rapoport's advice. Yet the separation of description from prescription has entailed certain drawbacks, especially in the context of policy analysis. On the one hand, contemporary solution concepts in game theory are presented in increasingly rigorous, formal expositions. On the other, these prescriptions about rational behaviour have become so far removed from the practical aspects of social choice that, at least in some instances, it is reasonable to question their normative relevance.[1] Prescriptive viability would seem to entail that an ostensible solution have either some empirical basis or intuitive plausibility. But these aspects tend to be more descriptive in orientation and therefore have received relatively little attention from game theorists. By contrast, other scholars usually have studied negotiation through the 'descriptive account' of a given interaction (Zartman, 1983:6). Thus it is tenable to argue that game theory as policy analysis might benefit from efforts toward practical application, while understanding of policy-oriented negotiations might be enhanced by a rigorous approach.

Given these concerns, one way to proceed is to operationalize solution concepts from game theory, in order to permit comparison with observed instances of social choice.[2] The purpose of the present study is to examine an intrinsically interesting phase of energy policy, referring to the Canadian National Energy Program (NEP) and its year-long aftermath, in those terms. There will be five stages to the investigation. First, the game-theoretic setting will be identified, referring to the phase of intergovernmental confrontation in Canada from October 1980 to September 1981. Second, relevant solution concepts for the game will be identified. Measurements are to be derived for these concepts in the third stage. In the fourth phase, the values generated by the game-theoretic solutions will be compared to the pay-off distribution corresponding to the Canada-Alberta Agreement of September 1981. Fifth, and finally, implications of the findings will be explored, along with possibilities for further research.

The Game-theoretic Setting

Several components must be identified in order to proceed effectively with a game-theoretic framework of analysis: the number and identity of the players, whether the play results in constant-sum pay-offs, and co-operative versus non-co-operative conditions.

Most fundamental among the questions to answer about the energy game are the number and identity of the players. James (1989) found compelling evidence that the crisis phase in energy politics at the outset of the current decade involved two players, the Canadian federal government and the Government of the Province of Alberta. The normal configuration of interest groups receded during that period, with bargaining restricted to governments that engaged in a struggle over substantial and presumably escalating economic rents from energy resources. The game of confrontation politics between the central government and the principal energy-producing region, Alberta, lasted from the announcement of the NEP in October 1980 to the signing of the Canada-Alberta Agreement in September 1981. Given the potentially vast revenues from taxation of oil and natural gas at the start of the 1980s, it is not surprising that energy politics crystallized into an intense competition between relatively autonomous levels of government.[3]

Some description of the NEP's formulation and implementation, along with the politics it inspired, may be useful in demonstrating that, although societal actors did not disappear from the scene, the usual levels of consultation with interest groups did not hold true during the phase of confrontation. To begin, the NEP consisted of a set of decisions prepared by a small circle of policy-makers. These individuals designed its provisions within a broad direction of policy set forth by Energy Minister Marc Lalonde. Thus public officials responsible for energy policy, not societal demands, provided the impetus for the NEP. Of course, as Doern and Phidd (1983:476) observed, 'the NEP was much more a marriage of interests between ministers and bureaucrats, spawned by intense partisan conflict,' as opposed to a 'bureaucratic imposition'.

For society-centred models, such as pluralism, groups like the energy industry would be expected to have input into the policy-making process. The energy program of the Liberal government, however, took virtually no notice of the demands of energy producing corporations. The NEP emerged almost entirely from within the government and bureaucracy. This result is not surprising, given the pre-NEP situation. It will become apparent – based on a later review of Alberta's July 1980 Proposals – that the Liberal government perceived the need to respond decisively. The Proposals put the Liberal government at a great disadvantage in terms of economic rent shares and the energy industry would have been very unlikely to respond positively to a revision favouring Ottawa. The government therefore acted under two constraints: time and boundaries on the content of its reply, which had to redistribute rent shares.

With regard to the year that followed announcement of the NEP, the evidence clearly favours a two-player game. Collective action by the oil and gas industries, consumers and other groups does not explain the subsequent process of bargaining. Instead, acting alone, Alberta retaliated against the federal government almost immediately, creating a deadlock that lasted nearly a year. With its program of 'province-building' in disarray, the Lougheed government eventually approached its federal rival and entered into bilateral negotiations. As indicated previously, these prominent events entailed action by governments, as opposed to logrolling among interest groups.

None of these observations is intended to dismiss the impact of interest groups on the formulation and implementation of public policy under normal circumstances. Prominent examples would include the Canadian Banking Association's role in shaping evolution of the *Bank Act* and the business lobby's ability to weaken or eliminate successive Competition Policy bills. Furthermore, professional associations influence policy in other fields, for example, the legal and medical associations with regard to justice and health care, respectively (Doern and Phidd, 1983:80–81). While the oil and gas industries, along with Ontario utilities and consumers, looked on with great concern, these groups did not participate directly in the phase of intergovernmental bargaining over the NEP.[4] Both before and after the confrontation period, however, such interest groups contributed to the evolution of energy policy through voting, lobbying and other mechanisms. In sum, the two-player designation is intended for a specific interval, not the long-term process of making energy policy in Canada.

Whether or not the game is constant-sum is also fundamental to its analysis. A constant-sum game is one of pure conflict; one adversary's gain is the other's loss (Davis, 1983:75). The alternative to that is the variable-sum game, which offers some prospect of mutual gain through agreement.[5]

By consensus, the bargaining initiated by the NEP over economic rents is regarded as a variable-sum game. Simeon (1980:182) observed that energy revenues had contributed over $5 billion to the Alberta Heritage Trust Fund, while Courchene and Melvin (1980:192) drew attention to Alberta's 'rapidly rising energy revenues'. In later years, Norrie (1984) and Ruitenbeek (1985) also noted the expectation of higher economic rents that had existed during the era of the Program; further evidence regarding increasing revenues from oil and natural gas at the outset of the decade is plentiful. Since the game is deemed to be variable-sum, subsequent analysis focusses on interdependent choice, as opposed to security levels and maximizing minimal pay-offs in a constant-sum game (Brams, 1975:4–5).

Of course, there is a sense in which the game could be considered constant-sum: when the oil firms are included in the bargaining, a fixed amount of recoverable oil and gas is assumed and regulation occurs at a single designated point in time. But the *price* of energy is a variable and the amount of *economically* recoverable oil is a positive function of price. At any arbitrarily chosen moment the world prices of oil and natural gas are fixed, but it is still possible to describe the game as variable-sum, because the aggregate level of economic rents can change from one scenario to the next.

Another aspect of the game structure that must be settled is whether play is co-operative or non-co-operative. 'The fundamental distinction between co-operative and non-co-operative games,' according to Friedman (1986:148), 'is that cooperative games allow binding agreements while noncooperative games do not'. This description is of more practical value when treated as a continuum rather than a dichotomy. It is difficult to imagine any agreement in political life that is completely enforceable; examples to the contrary are easy to cite. Support from interest groups may be transferred from one political party to another; coalitions at nominating conventions change allegiance despite solemn statements to the contrary; and alliances between nation-states may be discarded if one partner sees an opportunity to benefit from doing so.

However, there are constraints upon such actions and, in a political system such as that of Canada, blatant disregard by governments for negotiated settlements will be prohibitively expensive. A reputation for abrogating agreements – especially those of a prominent nature, such as the 1981 Canada-Alberta Agreement – could seriously impede subsequent efforts to co-operate with other governments. A record of capricious dealings also would be unsettling for the economy, because important interest groups (like labour and business) do not look favourably upon such behaviour from either provincial or federal governments.

Given these constraints, it is reasonable to expect that intergovernmental bargaining within Canada, such as the case at hand, will lean toward the co-operative end of the gaming spectrum. The costs associated with unilateral abrogation serve as a deterrent, especially with regard to breaking prominent agreements. Although societal actors and other governments within confederation did not participate in the Canada-Alberta bargaining game over energy, each of the adversaries had to be aware of a keenly interested audience composed of future negotiating partners over other issues. With this factor in mind, it is appropriate to consider the phase of federal-provincial confrontation over energy revenues in the context of co-operative game theory.[6]

Appropriate solution concepts for the game of federal-provincial bargaining can be identified, given one further piece of information. The rivalry over energy revenues belongs to the class of fixed threat bargaining games. The latter are 'two-person situations in which the players each obtain fixed utility levels if they fail to make an agreement' (Friedman, 1986:151). These games focus on the distribution of pay-offs between two players, as in the case of the conflict over resource-based economic rents between Alberta and the Government of Canada.

Solution Concepts

One of the principal shortcomings of game theory has been the lack of a single, compelling solution concept (Brander, 1985:62).[7] This ongoing problem, which can be traced to the relatively abstract nature of conventional game theory, is as true for fixed threat bargaining games as for any other kind. Solution concepts abound but share an arbitrary character (Young, 1989): Why should one be considered better than another? Since the present focus is more explanatory than normative, a natural means of judgment is whether a given outcome is in line with the result predicted by a particular solution concept.

For example, suppose that a set of solution concepts is derived from various theo-

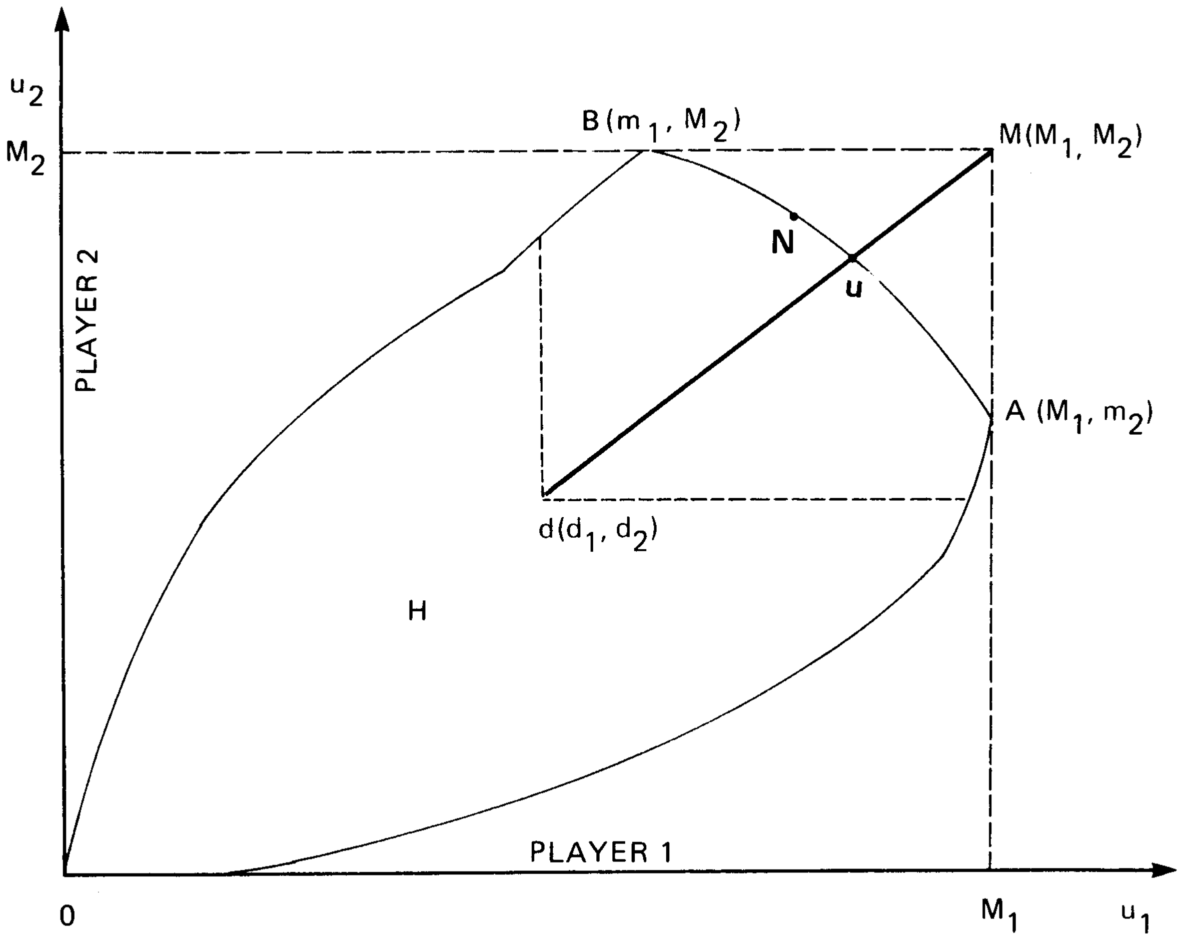

Figure 1
The Nash and Raiffa-Kalai-Smorodinsky solutions

ries of bargaining. Through a series of tests, the respective underlying theories could be compared with regard to predictive accuracy; outcomes in specific issue-areas or concerning fixed groups of players might tend to support some of the concepts more than others. Along those lines, the current investigation is intended as a modest beginning in one realm: intergovernmental bargaining in Canada over economic rents from natural resources.

Friedman (1986:160) has summarized the nature of the class of solution concepts that is relevant to fixed threat bargaining games:

> The basic intuitive notion is that each player naturally aspires to the largest payoff available in the game that is consistent with individual rationality. These two individual payoffs are, in general, not attainable simultaneously and the proposed solution is to settle at the largest attainable payoff point that is proportional to them.

There are five solution concepts to consider.[8] These concepts will be explained in two ways. The basic presentation will be diagrammatic and non-technical. A mathematical exposition of each solution concept appears in the Appendix.

Two of the solution concepts appear in diagrammatic form in Figure 1 as points **N** and **u**. (From this point onward, solution points will appear in bold print.) These points correspond to the Nash and Raiffa-Kalai-Smorodinsky (RKS) solutions, respectively. In the abstract case that will be used to bring out the properties of the Nash and RKS solutions, Players 1 and 2 exist in a spatial game represented by Figure 1, with fixed pay-offs in the absence of an agreement, and a range of other pay-off combinations if one is attained.[9]

Turning to more specific aspects of Figure 1, the horizontal and vertical axes represent the utilities derived by Players 1 and 2, u_1 and u_2, respectively. The set H refers to all feasible pay-off pairs within this game. Its boundaries are $OABO$, with O as the origin and A and B being two other prominent limitations. At point A, Player 1 obtains pay-off M_1, the highest that is possible among all points that are individually rational (i.e. those which are at least as good for Player 1 as what could be obtained without an agreement), while the pay-off for Player 2 at point A is designated as m_2. The point B plays the same role for Player 2. Thus, at points A and B, the pay-offs for Players 1 and 2 are represented by (M_1, m_2) and (m_1, M_2). The curve AB represents the set of efficient outcomes, with a trade-off in value for Players 1 and 2. This curve also is referred to as the possibility frontier or Pareto set.[10] It is presented in a mathematical form in the Appendix.

Within the diagram, $d = (d_1, d_2)$ is the threat point. It designates the pay-offs for Players 1 and 2 in the absence of an agreement. The point d is inside H, rather than along the possibility frontier. In Figure 1, d is located between the origin and the curve composed of efficient outcomes connecting A and B. The threat point therefore is sub-optimal (as in all cases), because an agreement could produce an outcome preferred by both players.

Most familiar among the five solutions is the Nash bargaining point. The point d determines the Nash solution in the following manner: The area within H that is above and to the right of d represents the attainable and individually rational points within the spatial game. These points are individually rational because each is at least as good for both players as what could be obtained at d, in the absence of an agreement. The Nash solution is the point that maximizes the *product* of gains for the players from agreement. The gains, respectively, correspond to $(u_1 - d_1)$ and $(u_2 - d_2)$.[11]

Similar to Nash, the RKS solution ($\mathbf{u}$) depends on the position of the threat point d. However, it also is affected by the location of point M (M_1, M_2). More specifically, for Player 1, M_1 plays the role of an 'aspiration level'; the greater its magnitude, the more that, a priori, should be be expected from an eventual outcome. The pay-off M_2 obtained at point B is analogous for Player 2. In other words, a comparison of ideal states (i.e. M_1 versus M_2) effectively serves as a measurement of intensity of expectations regarding the outcome. Thus the respective utilities are rendered comparable for purposes of the game, which eventually will be played for monetary pay-offs.

Given the constraint imposed by the production possibility frontier, the intersection point $M = (M_1, M_2)$ cannot be achieved. This is not surprising. The pay-offs M_1 and M_2 are not feasible simultaneously because each entails a maximal distribution of the marginal pay-offs to just one player. In other words, no efficient allocation of resources could be ideal in a distributional sense for both Players 1 and 2. Thus the *ideal point*, M, is intended for reference only and does not fall within H.[12] The solution $\mathbf{u}$ is the point on the possibility frontier that falls on the line connecting d to M.

There is some value in explaining each solution concept, beginning with Nash and RKS, in more practical terms. The standard trade-off to consider in a problem of revenue sharing is distributive justice versus efficiency. Since all of the solution points to be considered are on the possibility frontier, efficiency can be recast in terms of power or advantage; in other words, which has more impact in deriving a given solution: the balance of positional power within the game or notions of distributive justice based on other criteria? Various solution concepts will tend to balance these concerns somewhat differently.

With regard to distributive justice and relative advantage within the game, Davis (1983:122) observes that 'the Nash outcome often appears to be unfair, it tends to make the poor poorer and the rich richer. This is

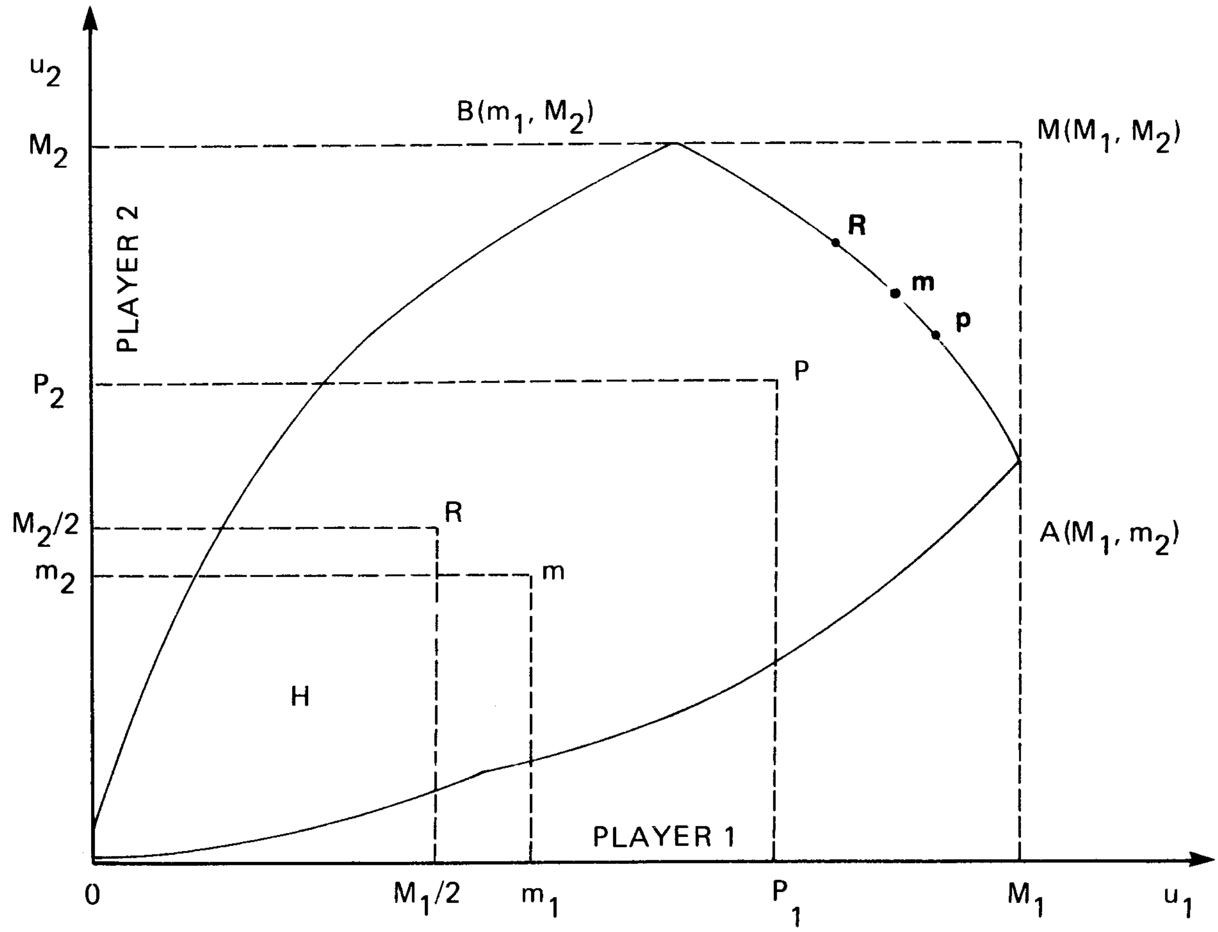

Figure 2
The minimal expectations, minimal compromise and middle of the rectangle solutions

to be expected, however. A rich player is often in a stronger position than a poor one.' For example, if d_1 is substantially greater than d_2, then u_1 must be that much greater relative to u_2 in order to maximize the product of gains. The magnitude of a product depends upon expansion of each component – $(u_1 - d_1)$ and $(u_2 - d_2)$ – meaning that neither relative need nor equality in distribution affects the assignment of revenue shares.

The RKS solution emphasizes relative power and positional advantage over other criteria for division of revenue. Varying the location of point d in Figure 1, with M fixed, will distribute greater pay-offs to a given player as d approaches that player's axis. Thus the player with the favoured position in the game is rewarded, constituting recognition of greater power. For example, in the abstract case displayed by Figure 1, d is closer to the u_1 axis, meaning that the line from d to M will intersect the curve AB at a point closer to A, the preferred outcome for Player 1. With d held constant, the player with the higher relative expectations receives a greater revenue share. Thus a greater magnitude for M_1 relative to M_2 would move the outcome closer to A, while the reverse would produce an outcome closer to B. The RKS solution focusses on each player's advantages and expectations within the game, rather than relative need or notions of equal sharing. But, in comparative terms, the Nash solution corresponds even more directly than RKS to the balance of power within the game, since aspiration levels are not incorporated.

Three other solutions related to that of Nash will also be derived.[13] These appear in Figure 2. Each requires derivation of a reference point, which then can be used to designate a specific solution point on the possibility frontier. As in the case of **N**, the respective solutions maximize the product of gains to the players from bargaining, but the point of reference is not d.

The reference point of minimal expectations, m, is based on what each player can expect to receive when the adversary obtains the ideal pay-off. Thus, Player 1 gets m_1 at B, the ideal outcome for Player 2, while the latter obtains m_2 at A, the best point for Player 1. The solution is the point on the possibility frontier that maximizes the product of gains for each player ($(u_1 - m_1)$ and $(u_2 - m_2)$, respectively) relative to m. The resulting point **m** resembles the other two that will be derived because each 'depends on the shape of H, but has no dependence on the threat point d' (Friedman, 1986:168).[14] In other words, members of the class of solutions linked to Nash are based on various properties of the set of feasible results, as opposed to the nature of the outcome without an agreement.

Under the regime of minimal expectations, each player is in a position to gain relative to a 'minimax' outcome. Thus the solution **m** is not power-based in the sense of Nash (or even RKS) because the non-agreement point (d) does not influence its division of pay-offs. Instead, a distribution is derived by comparing levels of minimal aspiration under conditions of agreement.

Two other Nash-oriented solutions remain to be identified. The reference point of minimal compromise is an average of what the player expects in the ideal and worst-case scenarios along the curve AB. (For example, Player 1 would be expecting M_1 and m_1 at each respective location. This results in $P_1 = (M_1 + m_1)/2$.) The values generated for the reference point, $P = (P_1, P_2)$, reflect the initial boundaries for H created by A and B. Thus the solution will depend directly on properties of H. The solution **P** is the point on the possibility frontier that maximizes the product of the players' gains relative to P.

Minimal compromise is a more comprehensive solution concept than minimal expectations. Its reference point incorporates the worst- *and* best-case scenarios for each player under conditions of agreement. In a more general sense, like **m**, **P** is a function of aspirations within the game, as opposed to relative power, because a player's fate under non-agreement is not considered in dividing the pay-offs.

Third and last among the Nash-oriented solutions is the middle of the rectangle. This refers to the center of the smallest rectangle containing H as a reference point for the solution. With zero as the origin, and A and B determining the dimensions of the rectangle, the relevant point is $R = (M_1/2, M_2/2)$. The solution **R** is the point on the possibility frontier that maximizes the product of the players' gains relative to R.

Middle of the rectangle is the solution that most directly measures relative aspirations within the game. Although RKS and minimal compromise also depend upon each player's pay-off at its ideal outcome, the middle of the rectangle is influenced *only* by the respective best points. As in the cases of minimal compromise and minimal expectations, gains relative to the reference point are independent of the balance of power between Players 1 and 2. Of course, like these solutions, relative need is not a matter of concern for **R**, either.

This completes the presentation of solution concepts. In a comparative sense, the five points fall along a continuum with respect to the trade-off between power within the game and other distributive criteria. The Nash solution depends strictly on positional advantage at the threat point, while the three Nash-oriented solutions – minimal expectations, minimal compromise and middle of the rectangle – emphasize criteria derived from the players' aspirations within the game. Finally, the RKS solution rests somewhere in the middle, depending on both ideal and threat points.

In order to obtain values for the five solutions in the context of the energy game, at this stage it is necessary to operationalize the respective reference points.

Table 1
Expected economic rent shares and aggregate values from non-frontier oil and gas for 1982*

	Provincial governments		Federal government		Oil and gas producers and consumers	
Scenario	Share of rents	Aggregate rent value	Share of rents	Aggregate rent value	Share of rents	Aggregate rent value
Alberta's July 1980 proposals	0.37	10.804	0.08	2.336	0.55	16.060
Unrevised NEP, 28 October 1980	0.30	8.670	0.21	6.069	0.49	14.161
Stalemate, with Alberta's retaliation	0.32	8.448	0.22	5.808	0.46	12.144
Canada-Alberta Agreement	0.34	9.996	0.19	5.586	0.47	13.818

*The aggregate rent values are expressed in billions of dollars.
SOURCE: Adapted from Helliwell and McRae (1982:17).

Measuring Points of Reference and Deriving Solutions

Several points in Figure 1 require operationalization: the origin, d, A and B. Once values exist for these points, it will be possible to generate the five solutions.

Given the nature of the game, the origin in Figure 1 is identified as the conventional point (O,O). Pay-offs lower than zero are impossible for both players. The utilities are economic rents for two governments from oil and gas production; these monetary pay-offs obviously reach a minimum of zero for each player when there is no production.

Table 1 displays data on expected economic rents from oil and gas for the relevant levels of government and other recipients in 1982.[15] All of the aggregate rent values are expressed in billions of dollars. The table has been constructed on the basis of data from Helliwell and McRae (1982). Using a macroeconomic model of the non-frontier oil and gas industry, they estimated the provincial and federal shares of energy revenues under various conditions. Four of these scenarios, listed in Table 1, are directly relevant to the task at hand.

Before describing the scenarios, it is appropriate to discuss the data from Helliwell and McRae in the context of validity and reliability. To begin, the data generated by the macroeconomic model have 'face' validity. The scenarios are based on assumptions built into the actual policies put forward by the governments. Thus the revenues listed in Table 1 reflect the expectations of the players during the game. For example, the four scenarios build in a 2 per cent annual increase in the world oil price, as anticipated by all parties in 1980–1981 (Helliwell and McRae, 1982:14).[16]

Another question related to validity concerns the use of data on revenues projected for 1982 (which appear in Table 1), as opposed to 1986, which Helliwell and McRae also estimated. Given the usual time horizons associated with elected governments, it is reasonable to assume that short-term considerations carried more weight. Use of the data for 1986, the final year of the Canada-Alberta Agreement, also would require measurement of discount parameters. Estimating the immediate value of future revenues would be, at best, a problematic task.

Considerations of validity appear to favour the 1982-oriented data from Helliwell and McRae; reliability points in the same direction. The modelling procedures described in great detail by Helliwell and McRae (1981) do not appear to have provoked criticism. Thus the scenarios listed in Table 1 deserve to be treated as reproducible evidence about expected economic rents, pending reasons to the contrary.

Another aspect of reliability concerns the comparability of statistics. The crucial points within the spatial game, such M and d, along with pay-offs for the Agreement, are generated by the same model. Other sources, such as Gorbet (1980) or Copithorne et al. (1985), do not provide data on the complete set of reference points. Taken together, considerations of validity and reliability are positive for the data presented by Helliwell and McRae (1982).[17]

Turning to the scenarios listed in Table 1, the first is based on proposals made by Alberta's government in July 1980. The proposals covered conventional oil and gas pricing, oil sands development, revenue sharing, interprovincial loans and taxation.[18] Premier Peter Lougheed proposed to raise conventional oil prices over the course of three years to 75 per cent of the Chicago price with no increase in provincial royalty rates for either conventional oil or gas. Development of the oil sands would be accelerated, including a $7 billion investment directly in the sands and provision of an infrastructure for the workforce by the Province of Alberta. The federal share of royalties from the oil sands would increase also. With respect to natural gas, the price would be fixed at 85 per cent of the oil price, with producers paying costs of transportation of new gas to eastern Canadian markets. New mineral leases would be subject to federal taxation and Alberta would provide funding for various interprovincial projects and loans. In return, Lougheed asked that Ottawa refrain from imposing a wellhead tax on either oil or natural gas and also not tax natural gas exports.

Since these proposals came directly from the Government of Alberta, the economic rents under this scenario are taken to approximate those of point A in Figure 1, with Alberta as Player 1. For Alberta, the resulting distribution of revenues would correspond to the perceived ideal outcome. The province would retain control over resources and advance its interests within confederation through the Heritage Fund. Thus $M_1 = 10.804$ and $m_2 = 2.336$, based on Table 1.

With regard to the unrevised NEP of 28 October 1980, it is easy to see a federal orientation. Helliwell and McRae (1981:15) outline the components of the Program that are relevant to the macroeconomic model:

> (i) the federally-set wellhead prices for crude oil and natural gas ...;
> (ii) the petroleum compensation charge levied on all users of oil products;
> (iii) the transfer of 50% of the revenues of the oil export tax to the producing provinces;
> (iv) the new natural gas and liquids tax, starting at $.30/mcf and rising to $.75/mcf by 1983;
> (v) the 8% petroleum and gas revenue tax; and
> (vi) the phasing out of depletion allowances and the introduction of incentive grants.

These policies, collectively speaking, aggrandized the power and influence of Ottawa at the expense of the Government of Alberta. The Federal Government used its powers of taxation to generate revenues for the central treasury and thereby counteract the trend toward a decentralized confederation. The pay-off for Ottawa under this scenario therefore is considered to be that of its perceived ideal point, B, so $m_1 = 8.670$ and $M_2 = 6.069$.

Lougheed retaliated against the NEP on 30 October 1980, with the principal action being a proposed three-stage reduction of conventional oil shipments to eastern Canada. Over the course of a nine-month period, the cutbacks eventually would reach 15 per cent of the prior production level of 1.2 million barrels per day (*Globe and Mail*, 1 November 1980:14). Alberta also issued a court challenge to the NEP and suspended high-profile megaprojects. In pursuing these responses, Lougheed had obvious motivations: the courting of interest groups to promote electoral success and protection of constitutional rights under Section 109.

Combined with the NEP, Alberta's

Table 2
Points of reference for the provincial and federal governments in the spatial game

Solution or Scenario	Pay-off for player 1, provincial government*		Pay-off for player 2, federal government*	
	Segment A	Segment B	Segment A	Segment B
Nash (**N**)	9.194	8.976**	8.810	5.999**
RKS(**u**)	9.900	9.253**	5.971	5.899**
Minimal expectation (**m**)	9.737	13.910	6.627	4.222
Minimal compromise (**P**)	10.039**	11.850	5.413**	4.964
Middle of rectangle (**R**)	8.016	11.306	13.546	5.160
Canada-Alberta agreement	9.996		5.586	

*Each payoff is expressed in billions of dollars.
**These payoffs fall within the acceptable range of values.

Table 3
Squared differences for solutions relative to the Canada-Alberta agreement

Solution	Squared difference for player 1*		Squared difference for player 2*		Sum of squared differences	
	Segment A	Segment B	Segment A	Segment B	Segment A	Segment B
Nash	0.643	1.040**	10.394	0.171**	11.037	1.211**
RKS	0.009	0.552**	0.148	0.098**	0.157	0.650**
Minimal expectations	0.067	15.319	1.084	1.860	1.161	17.179
Minimal compromise	0.002**	3.437	0.030**	0.387	0.032**	3.824
Middle of rectangle	3.897	1.716	63.362	0.181	67.362	1.897

*Each difference is expressed in billions of dollars squared.
**These squared differences are based on solution points which fall within the feasible set.

riposte initiated a phase of confrontation that lasted almost a year. This era therefore can be treated as one of non-agreement. In the context of the spatial game, the revenues generated for Edmonton and Ottawa by the macroeconomic model would approximate those in the absence of an agreement, therefore $d_1 = 8.448$ and $d_2 = 5.808$.

Finally, there is the Canada-Alberta Agreement to consider. The pay-offs here do not correspond to those at any previously specified point in the spatial game. These pay-offs, according to Helliwell and McRae (1982:16) are higher (in aggregate) than those 'in any of the other cases'. Thus the Agreement, allowing for economic and political constraints, should be regarded as a point approximately on the possibility frontier. At a later stage, this point will be compared to the five solutions which, given measurements for the origin, d and M (based on A and B), can now be identified.

In sum, the points A, B and the Agreement all are considered to be on the frontier. The first two points are perceived to be ideal for Alberta and the Government of Canada, respectively, in distributional terms, while the Agreement is the scenario identified by Helliwell and McRae as the highest in overall revenues among those simulated. The mathematical derivation of the connections between these points (e.g., points A and B and the Agreement) appears in the Appendix.

Comparing Solutions

Table 2 displays the pay-offs derived for every solution, with the aggregate values for the Canada-Alberta Agreement included to facilitate comparison. (The relevant calculations are performed in the Appendix.) Each solution has been calculated twice, relative to (1) the segment of the frontier connecting point A to the Agreement (Segment A) and (2) the segment of the frontier connecting the Agreement to point B (Segment B). The Nash solution, for example, is (9.194, 8.810) along Segment A and (8.976, 5.999) along Segment B. However, all but three of the potential solution points are inadmissable because they lie *beyond* the boundaries set by the connecting segments. For example, to obtain a maximum for the Nash reference point relative to Segment A, it is necessary to move the players to (9.194, 8.810), a point that lies beyond the upper boundary of Segment A. The three feasible solution points are Nash and RKS relative to Segment B and minimal compromise relative to Segment A.

Table 3 compares the solutions to the Canada-Alberta Agreement in terms of overall descriptive accuracy. For each solution, the difference in pay-offs for the two players is calculated relative to the Agreement and then squared. These squared values then are added together, in order to estimate how similar each solution point is to the actual agreement achieved by Alberta and the federal government.[19] Squaring the differences has the advantage of emphasizing large discrepancies; a solution concept that makes a relatively small error in estimating each pay-off is considered more accurate in an overall sense than one which is more exact for one player but far off target for the other. The differences will be estimated for all of the points, although it is understood that the margin of error for those outside the boundaries of the frontier would be difficult to interpret. As it turns out, one of the feasible solution points has the smallest margin of error: The solution generated by minimal compromise, **P**, is quite close to the point corresponding to the Agreement. Returning to Table 2, the percentage differences for the provincial and federal governments are 0.43 and 3.10, respectively.

Conclusion

One of the standard complaints against game theory is that it lacks practical relevance. While game theory has a self-pronounced focus on normative issues, that orientation becomes problematic when viewed in the context of policy analysis. The present study has attempted to use game theory in a more descriptive and, potentially, socially relevant manner. The phase of confrontation initiated by the NEP has been modeled as a two-party, variable-sum, co-operative game. Statistics on governmental revenues from energy have been used to estimate pay-offs for game-theoretic solution concepts, with the latter being compared to the values from the Canada-Alberta Agreement. Several observations can be made about this process of evaluation.

First, it is interesting that one of the Nash-oriented solutions, minimal compromise, corresponds very closely to the Canada-Alberta Agreement. This solution's point of reference depends on each player's expectations based on its ideal and worst cases. In the federal-provincial energy game, rational choice therefore seemed to focus on 'averaging' ideal and minimal pay-offs. Perhaps that is true of other bargaining games within confederation. If so, that would identify a 'form of rationality' (Laver, 1986:33) specific to the Canadian political process.

Second, there is a normative point to consider. Costs of transaction from bargaining could be reduced by using minimal compromise as a principle. Assuming that governmental players think in those terms, they might tend to see the explicit comparison and integration of ideal and minimal expectations as fair, or at least tolerable. More productive intergovernmental bargaining, along with relatively stable outcomes, could result from the use of minimal compromise to stimulate negotiations in good faith.

A third point focusses more specifically on the federal government's outlook on the energy game. Since the Canada-Alberta Agreement gave Ottawa less than the stalemate (point d) in strictly monetary terms (see Table 1), it could be that Ottawa placed a positive value on resolving the conflict with Alberta, at least temporarily. This observation is consistent with an interpretation of federal priorities which emphasizes Ottawa's desire to build support for centralization through constitutional measures.

Fourth, and finally, the present study represents only an initial effort toward application of game theory to Canadian public policy. Among many potential avenues for research, it is possible to mention only a few. Further sources of quantitative data could be consulted, in order to assess the reliability of the measurements derived from Helliwell and McRae (1982).[20] In addition, interviews with participants in the bargaining over energy revenues could produce useful supplementary information regarding intensity of preferences over the outcomes.

Another approach would be to model the confrontation phase in energy politics as an iterated (i.e. multiple stage) game, in order to confront these and other questions: What are the roles played by incomplete information and perceptions of power? Does risk propensity affect the strategies pursued in rent-seeking? More specifically, could the Canada-Alberta Agreement have been achieved without prior institution of a policy like the NEP?

More generally, there are other games within confederation to consider. Further application of game theory to Canadian politics could reveal patterns that otherwise might not be discerned.

Notes

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1 For example, all but a very few articles in the *International Journal of Game Theory* are inaccessible to those without advanced training in mathematics. Furthermore, a survey of that journal over the last decade revealed only three articles (Saaty, 1979; Straffin and Heaney, 1981; Grotte and Brooks, 1983) that linked game theory to public policy.

2 For a highly readable introduction to game theory, consult Davis (1983); a more general treatment of rational choice as a mode of analysis appears in McLean (1987). An introduction to rational choice as applied to natural resources is available in Sproule-Jones (1982). For an excellent application to US and Canadian energy politics, see Uslaner (1989).

3 Doern and Toner (1985) and James and Michelin (1989) provide a wide range of evidence that favours state autonomy as a model of formulation and implementation of energy policy during the period in question, leading to a conclusion in favour of a two-party, intergovernmental bargaining game. Both the NEP and its subsequent revision reflected competition between governments, as opposed to a class struggle over the means of production or logrolling among interest groups. These latter two scenarios correspond to Marxism and pluralism, respectively, the usual competition for state autonomy in the explanation of the policy process.

4 For example, Jenkins (1986:157–8) described the provincial government and multinational corporations as allies but also implied that the coalition had a clear hierarchy during the phase of confrontation: 'Alberta provided considerable bargaining leverage to the MNC's opposition to the NEP simply because it was a political actor whose views could not be ignored by a federal government trying desperately to fight western alienation within the confederation.'

5 For a comparative analysis of strategic choice in constant-sum and variable-sum games, see Davis (1983). In the literature of game theory, the respective terms constant- and variable-sum often are used interchangeably with the designations zero- and non-zero-sum.

6 It could be argued that a co-operative game would never include a year of stalemate, such as that seen in the case of the Federal-Alberta game over energy revenues. However, describing a game as co-operative implies that binding agreements may occur; it does not *require* that a deal must be made at any given time. Thus the year of deadlock following Alberta's retaliation against the NEP does not fall outside the boundaries of a co-operative game or rational choice. For a treatment of the phase of confrontation as an iterated game, consult James (1989).

7 A less draconian version of this critique would focus on the lack of a relatively small *subset* of convincing solution concepts. Each member of such a subset would predict well in specific settings.

8 The following description of solution concepts is based primarily upon Friedman (1986:160–2,167–70).

9 The point identified as *d* in Figure 1 sets the boundaries for individual rationality in this game. It designates the pay-off that each player would receive in the absence of an agreement. Individual rationality requires that neither player accept a pay-off less than that obtained at *d*, assuming that monetary pay-offs are all that matter to the player.

10 A pay-off allocation satisfies Pareto optimality if there does not exist another distribution at which all players are at least as well off and one player is better off than before. In Figure 1, each point along *AB* is Pareto optimal because none of the superior points can be attained.

11 The Nash bargaining solution satisfies a set of rigorously specified conditions from Friedman (1986:155–6), including individual rationality and invariance under positive utility transformations.

12 The points along the lines AM_1 and BM_2 cannot be attained in this game. Thus, aside from the curve *AB*, the possibility frontier consists of curves from the origin to *A* and *B*, respectively. The exact shapes of the latter two curves (*OA* and *OB*) cannot be estimated from available data.

13 Perles and Maschler (1981) have analysed a 2-person, Nash bargaining game in a dynamic context, deriving a solution concept that builds in potential changes in the feasible set. However, the present study is an exercise in comparative statics and assumes that *H* is fixed over the course of bargaining.

14 Friedman (1986:161–2,167–8) refers to a proof that many solutions can be generated by combining basic derivations from Nash, such as those considered here, with *d*. However, these solutions lack an intuitive basis and are interesting more for their mathematical properties than for practical application.

15 The pay-offs for the provincial governments are taken to approximate those of Alberta, which held the overwhelming proportion of oil and gas reserves among the provinces.

16 It is easy to argue that more recent data would give more accurate estimates of revenues that the players could have expected to obtain under each scenario. However, the *perceived*, as opposed to actual, expectations from the game are more relevant to measurement of the ideal and threat points.

17 Another aspect of measurement, which lies beyond the scope of this analysis, is the internal validity of the macroeconomic model as viewed by economists. As Jump (1981:35) has noted, in reviewing a different model, simulation results are 'specific to the model being used, and while the author may be familiar with its inner structure, the audience generally is not'. Thus the statistics conveyed by Table 1 should be regarded with caution, despite the advantages already noted.

18 The following description of the Alberta proposals is based on Scarfe (1981:11).

19 For example, in the case of the RKS solution, relative to Segment A, the pay-off difference for Alberta – compared to the Agreement – is 9.900 – 9.996 = -0.096. The squared difference is 0.009. For the federal government, the difference is 5.971 – 5.586 = 0.385 and the squared value is 0.148. The sum of squares therefore is 0.009 + 0.148 = 0.157.

20 In that regard, the recently published study by Helliwell et al. (1989) would be one possibility. Among other subjects, it covers regulation of crude oil and natural gas, taxation and the impact of alternative policies on revenues and rents.

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Appendix

With respect to estimating the possibility frontier, Figure 1 shows a curve connecting A to B which is concave to the origin. In other words, a continuous trade-off is assumed for u_1 and u_2. However, lacking further data on the precise shape of the curve, a straight line will be used as an approximation. Since the ultimate concern is with proximity of points along the line segment AB to each other, this approximation does not create any practical difficulties. Equation 1 expresses u_2 as a linear function of u_1 and two constants, intended to represent the line segment AB:

(1) $u_2 = k_1 u_1 + k_2$

where

k_1 = slope of line from A to B.

k_2 = intercept of line along u_2 axis.

Substitution of $A(M_1, m_2)$ and $B(m_1, M_2)$ in Equation 1 yields the following expression:

(1') $u_2 = \frac{(M_2 - m_2)u_1}{(m_1 - M_1)} + M_2 - \frac{(M_2 - m_2)m_1}{(m_1 - M_1)}$

Having derived an expression for the possibility frontier, each of the five solution points can be expressed in mathematical terms. For the Nash solution, the function f represents the product of the utility gains for Players 1 and 2:

(2) $f = (u_1 - d_1)(u_2 - d_2)$

The point at which f reaches its maximum corresponds to the Nash solution, **N**. (After substitution for either u_1 or u_2, the second derivative of the function f has a negative value. It therefore is certain that the point identified as **N** represents the maximum of the function $f(u_1, u_2)$. This also is true of the other functions: g, k and q.)

With regard to the RKS solution, the line from d to M is conveyed by Equation 3:

(3) $u_2 = au_1 + b$

where

a = slope of line from d to M.

b = intercept along u_2 axis.

The point **u** must satisfy both Equations 3 and 1'; the former connects d to M while the latter guarantees Pareto optimality. Equation 3 means that the RKS solution links the expectations of the players to both the non-co-operative and ideal outcomes.

For the point of minimal expectations, the solution **m** corresponds to the maximum of the function g:

(4) $g = (u_1 - m_1)(u_2 - m_2)$

The reference point for minimal compromise is generated by the following equations:

(5) $P_1 = \frac{M_1 + m_1}{2}$

(6) $P_2 = \frac{M_2 + m_2}{2}$

where

P_1 = pay-off to Player 1.

P_2 = pay-off to Player 2.

For minimal compromise, the solution **P** maximizes the function k:

(7) $k = (u_1 - P_1)(u_2 - P_2)$

Finally, the solution point **R** for the middle of the rectangle is the point on the possibility frontier that maximizes the function q:

(8) $q = (u_1 - R_1)(u_2 - R_2)$

where

$R_1 = M_1/2$

$R_2 = M_2/2$

Using the data from Table 1, it is possible to estimate the possibility frontier, although Equation 1' must be modified in order to achieve that goal. Alberta's July Proposals (point A), the Canada/Alberta Agreement, and the Unrevised NEP (point B) all occupy positions on the frontier. Thus there are *two* line segments to be estimated. One will connect point A to the Agreement, with the latter also being connected to point B. Equations 1* and 1** represent these respective line segments:

(1*) $u_2 = -4.02u_1 + 45.77$

(1**) $u_2 = -0.36u_1 + 9.23$

Nash's bargaining solution maximizes Equation 2, subject to the constraints imposed by membership in H. Substitution of values for d and u_2 in Equation 2 yields the following expression:

(2') $f = (u_1 - 8.448)((-4.02u_1 + 45.77) - 5.808)$

The substitution for u_2 is taken from Equation 1*. The maximum of f also can be calculated relative to the other line segment, from the point of Agreement to point B. Each result will be reported for every solution, although only the calculations based on Equation 1* will be shown. To continue, the maximum of f is derived by taking its first derivative and setting that expression to zero. The value obtained for u_1 is 9.194. Substitution for u_1 in Equation 1* yields **N** = (9.194, 8.810) as the Nash solution. Based on Equation 1**, the maximum is (8.976, 5.999). The first of these results is out of the range set by the line segments, while the second is acceptable. As will become apparent, most of the solution points fall beyond the boundaries set by the connected line segments.

To derive the RKS point (**u**), the first step is to identify the straight line connecting d = (8.448, 5.808) and M = (10.804, 6.069). Substitution of the M and d values in Equation 3 yields the following result:

(3') $u_2 = 0.111u_1 + 4.872$

The line given by Equation 3' intersects with the possibility frontier to produce **u**. Equation 1* then can be used to substitute for u_2 in Equation 3'.

(3") $-4.02u_1 + 45.77 = 0.111u_1 + 4.872$

Thus $u_1 = 9.900$; substitution in Equation 3' establishes $u_2 = 5.971$. The RKS solution turns out to be **u** = (9.900, 5.971). Based on Equation 1**, the solution is (9.253, 5.899). Only the latter falls within the acceptable range.

With respect to the point of minimal expectations, the solution is derived by combining Alberta's pay-off under the unrevised NEP with the federal pay-off derived from Alberta's July proposals and maximizing the product of gains relative to this point. The reference point is m = (8.670, 2.336); substitution in Equation 4 yields the following expression:

(4') $g = (u_1 - 8.670)(u_2 - 2.336)$

Substitution for u_2 in Equation 4', using Equation 1*, generates the following expression:

(4") $g = (u_1 - 8.670)((-4.02u_1 + 45.77) - 2.336)$
This equation reaches its maximum when $u_1 = 9.737$. Substitution in Equation 1* yields **m** = (9.737, 6.627). Based on Equation 1**, the solution is (13.910, 4.222). Each solution falls outside of the feasible range.

Substitution of values for M and m in Equations 5 and 6 yields the reference point for minimal compromise:

(5') $P_1 = \dfrac{10.804 + 8.670}{2}$

(6') $P_2 = \dfrac{6.069 + 2.336}{2}$

The result is $P = (9.737, 4.203)$. Substitution in Equation 7 yields the following expression:
(7') $k = (u_1 - 9.737)(u_2 - 4.203)$
Once again, substitution for u_2 is appropriate:
(7") $k = (u_1 - 9.737)((-4.02u_1 + 45.77)-4.203)$
The maximum of k occurs when $u_1 = 10.039$. Substitution in Equation 1* yields **P** = (10.039, 5.413) as the solution. Based on Equation 1**, the solution is (11.850, 4.964). The latter of the two solution points falls outside of the feasible range.

As for the last solution, the middle of the rectangle, that is based on adjusted values for M_1 and M_2. The reference point is $R = (5.402, 3.035)$. Substitution in Equation 8 yields
(8') $q = (u_1 - 5.402)(u_2 - 3.035)$
Substituting for u_2 gives
(8") $q = (u_1 - 5.402)((-4.02u_1 + 45.77)-3.035)$
The maximum of q occurs when $u_1 = 8.017$. Substitution in Equation 1* gives **R** = (8.016, 13.546) as the solution. Based on Equation 1**, the solution is (11.306, 5.160). Each solution is out of the feasible set.

Having performed all of the relevant calculations, only three of the solution points fall within an acceptable range: Nash and RKS, relative to the line segment connecting point B to the Agreement, and minimal compromise, relative to the segment linking the Agreement to A.