Theory of Computation

Harald Röck

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1 Sipser Problem 1.42

If A is any language, let $A_{\frac{1}{2}-}$ be the set of all first halves of strings in A so that

$$A_{\frac{1}{2}-} = \{x| \text{ for some } y, |x| = |y| \text{ and } xy \in A\}$$

Show that, if A is regular, then so is $A_{\frac{1}{2}-}$.

Solution: Definition 1.7: A language is called a regular language if some finite automaton recognizes it.

If A is regular there is a DFA $m = (Q, \Sigma, \delta, q_0, F)$ which we transform to a NFA $m' = (Q', \Sigma, \delta', q'_0, F')$ which recognizes $A_{\frac{1}{2}-}$. The set of states $Q' = Q \times Q \times Q$ where each state is a triple $< p, q, r >$ of the original states. The alphabet is the same as in m, hence Σ. The transition function $\delta' : Q' \times \Sigma \longrightarrow Q'$ is defined as:

$$\delta'(< p, q, r >, a) \;=\; < \delta(p, a),\; q,\; \delta(r, b)|b \in \Sigma >$$

where b is non-det. choosen. The start state $q'_0 =< q_0, p_m, p_m >$ where p_m is the state of m when it would be in the middle of the input string(non-det guess). The set of accept states is $F' = \{< p_m, p_m, q > |q \in F\}$.

Idea of construction: We place a marker $\triangle$ at the middle of the input, because we don't know it yet we make a non-deterministic guess. We keep that marker at this position(see the q in δ'). Each time we advance at the beginning of the input (the p) we move also from the middle of the input (the r). If we reach the middle marker with p and an accept state of m with r we accept.

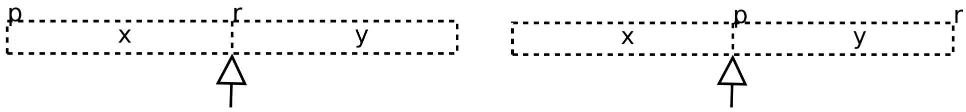

Figure 1: The start state and the accept state of m'

2 Sipser Problem 2.24

Let language A be:

$$A = \left\{ a^i b^j c^k | i, j, k \leq 0 \text{ and either } i = j \text{ or } j = k \right\}$$

Show that A is inherently ambiguous.

Solution: See [1]

3 Sipser Problem 2.25

Let CFG G be

$$S \rightarrow aSb \mid bY \mid Ya \quad (1)$$
$$Y \rightarrow bY \mid aY \mid \epsilon \quad (2)$$

Give a simple description of $L(G)$ in English. Use that description to give a CFG for $\overline{L(G)}$, the complement of $L(G)$.

Solution: $L(G)$ is the set of strings with a least one character. If all as are before the bs then there is a different number of as than bs, or else an a comes after a b.

$\overline{L(G)} = \{a^n b^n | n \leq 0\}$ is the set of strings where all as are before the bs and the number of as is the same as the number of bs.

4 Sipser Problem 2.26

Let $C = \{x\#y|x, y \in \{0,1\}*$ and $x \neq y\}$. Show that C is a context-free language.

Solution: Use Threorem 2.12: A language is context-free if and only if some pushdown automaton recognizes it.

Two strings $x = x_1x_2\dots x_n; x_i \in \{0,1\}$ and $y = y_1y_2\dots y_m; y_i \in \{0,1\}$ differ if $n \neq m$ or $n = m$ and $\exists i : i \leq n$ and $x_i \neq y_i$.

We construct a non-deterministic PDA which recognizes C by (1) guessing if $|x| \neq |y|$ and (2) guessing the index i where y_i differs from x_i. See figure 2. Notation in the figure: a is for any input, e for ϵ transitions.

References

[1] Herman A. Maurer. A direct proof of the inherent ambiguity of a simple context-free language. *J. ACM*, 16(2):256–260, 1969.

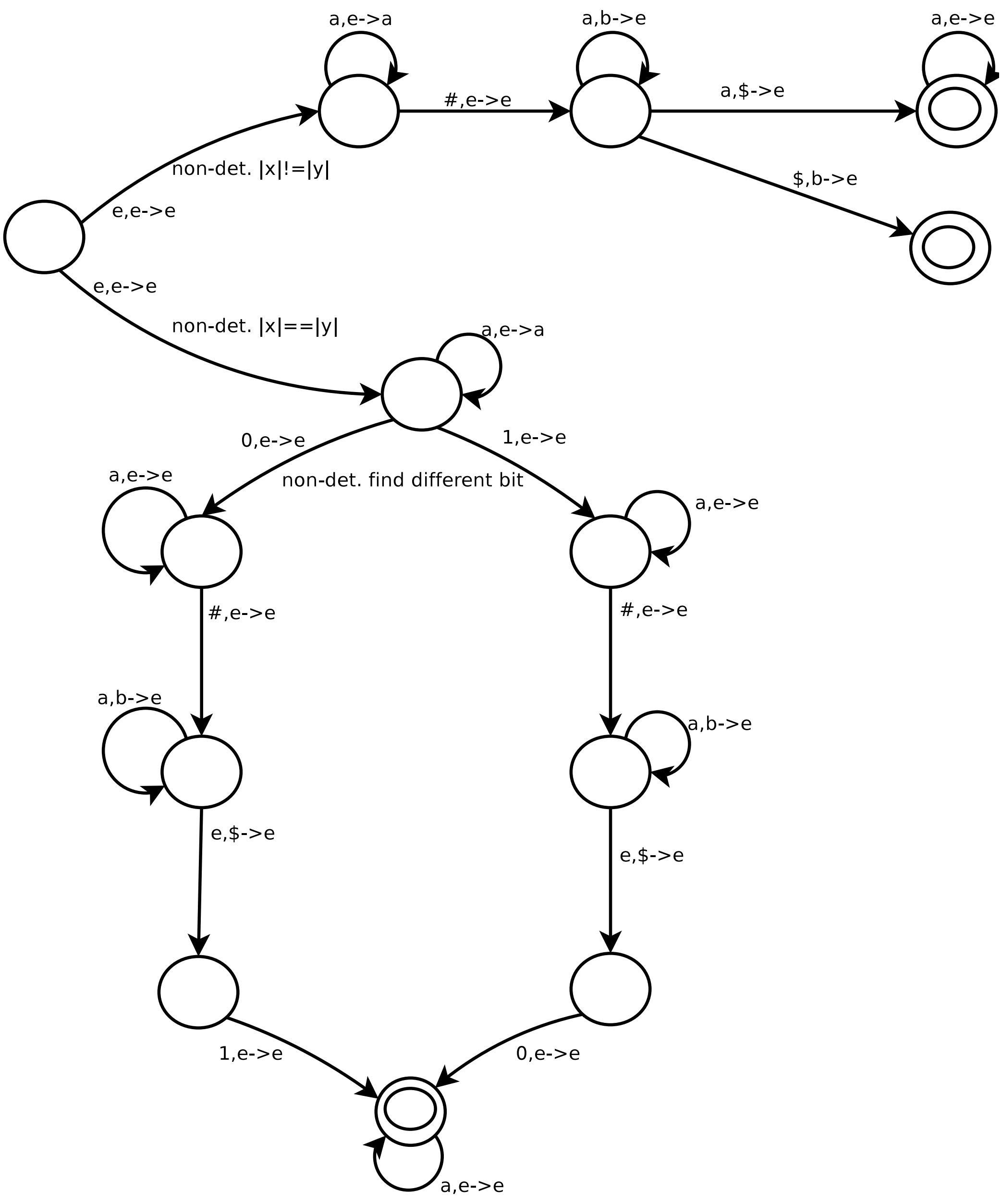

Figure 2: A Non-Deterministic Pushdown Automata which recognizes C