Laboratory 6 - The Orbit of the Moon

Materials Used: Vernier caliper, moon photographs, Excel spreadsheet with Polar Plotter add-in.

Objective: To measure the eccentricity of the Moon's orbit.

Discussion: The Moon has a very low orbital eccentricity. Knowledge of the Moon's slight variance from a circular orbit extends all the way back to the Classical age of astronomy. The Greeks were aware of small periodic changes in the Moon's size. We can compute the eccentricity for the Moon's orbit by making the same observation as the Greeks, i.e., by noting the apparent change in the Moon's diameter throughout a lunar cycle.

Consider Figure 1 below. *D* is the apparent diameter of the Moon as viewed from Earth, θ the angle subtended by the Moon as viewed from Earth, and *d* the distance to the Moon. In this procedure we will measure *D* in a series of photographs and use this value to compute θ and *d* for each image and ultimately the eccentricity of the Moon's orbit.

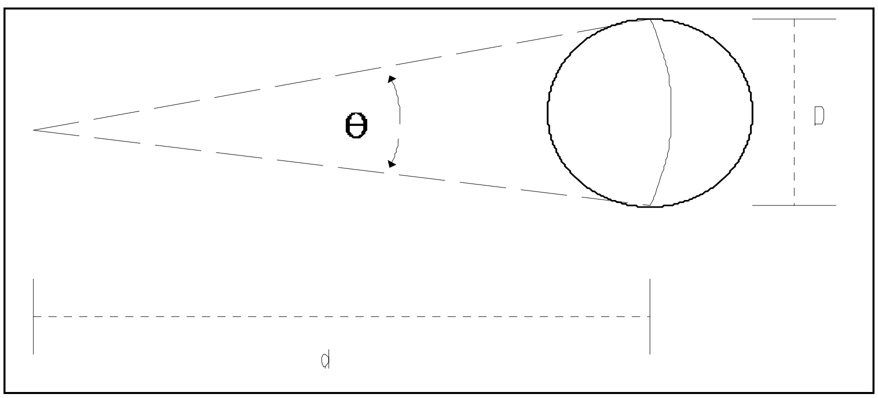


Figure 1. The geometry of the arc length relationship.

The relationship: $d = \dfrac{D}{\theta}$ will be used to compute the radius (*d*) of the Moon's orbit about the Earth for each image. We will use the small variances in *d* to compute the eccentricity of the Moon's orbit about the Earth.

Vernier Calipers

Vernier scales are used to make precise measurements. Vernier devices employ dual scales to allow measurements with one digit greater accuracy than that afforded by ruled scales. A vernier caliper is a caliper with a vernier scale that measures the distance between the caliper arms.

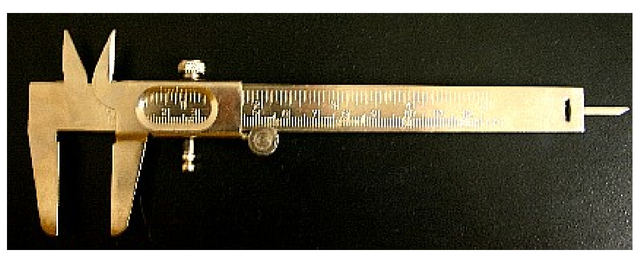
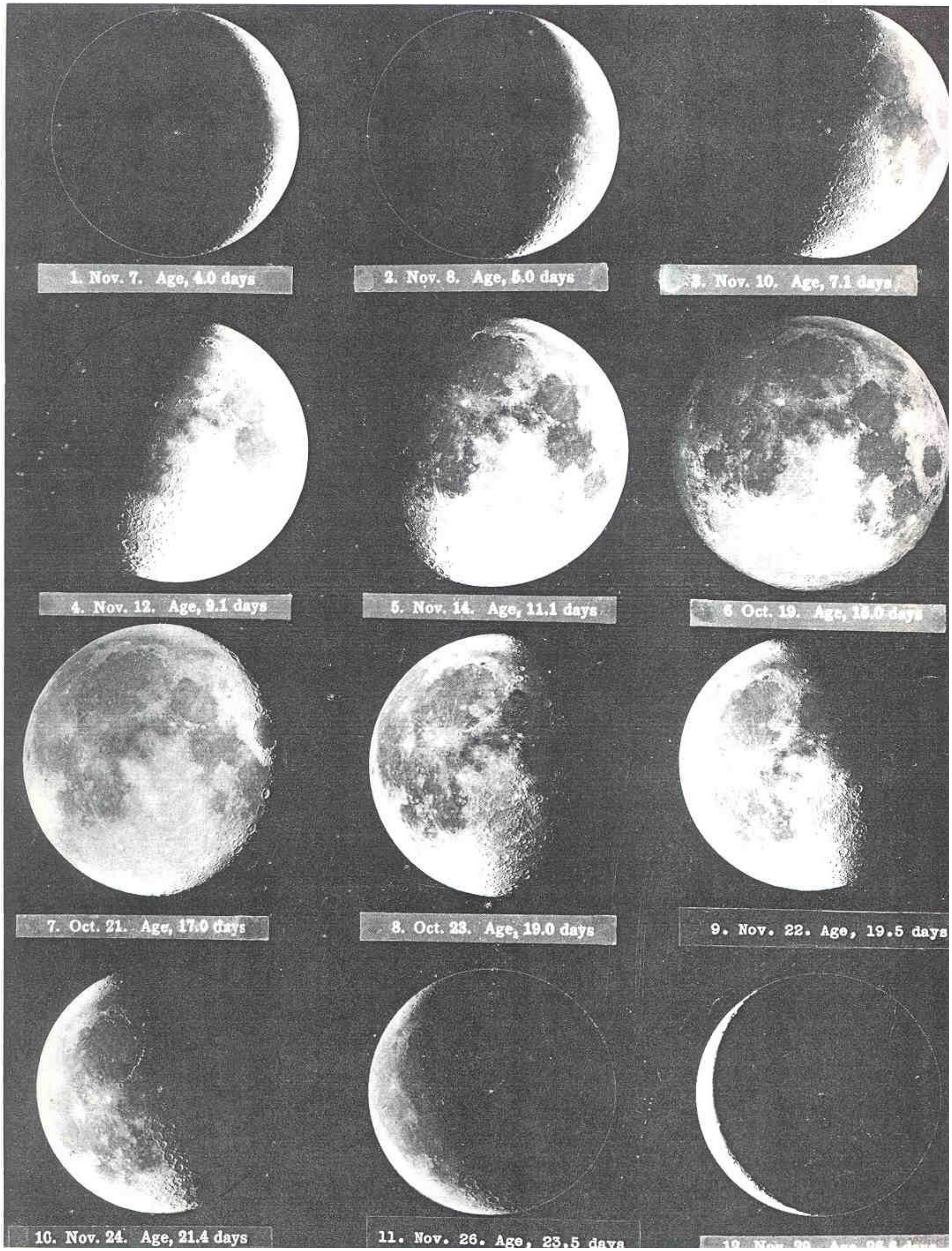


Figure 2. Vernier Calipers.

Figure 3.

Calibration Bar (.01 radians)



To use a set of vernier calipers the arms are spread to accommodate the item being measured and then closed until they just grasp the item along the desired dimension (the thumb dial in the center of Fig. 2 works well for this). The first digits of the measurement are read from the ruled scale while the last is read from the lower vernier scale.

Consider the vernier device in Figure 4. Note that the zero mark on the lower vernier scale lies between 3.2 and 3.3 centimeters on the upper scale. The first two digits are read from the upper scale and will be 3.2. A third digit would indicate where the measurement lies between 3.2 and 3.3 cm. On a ruled device we'd just make an estimate. On a vernier device we employ the vernier scale to determine the extra digit.

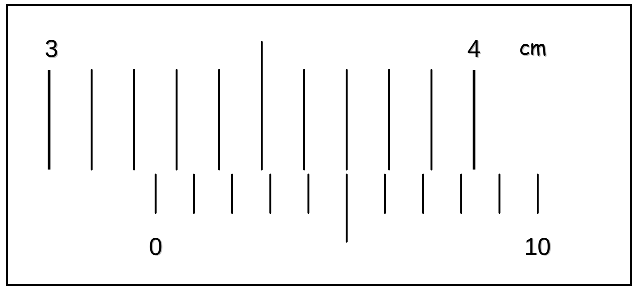


Figure 4. A vernier scale.

Note that the 5th mark on the lower vernier scale is almost perfectly aligned with a mark (which one doesn't matter) on the upper scale. We are looking for the closest alignment of a mark on the lower scale with the upper scale. This alignment is the measure of the distance between 3.2 and 3.3 cm that we are looking for. Since the mark on the lower scale that is most closely aligned with one on the upper scale is the 5th the measurement is 3.25 cm.

Now have a look at Figure 5. What is the reading on this vernier scale?

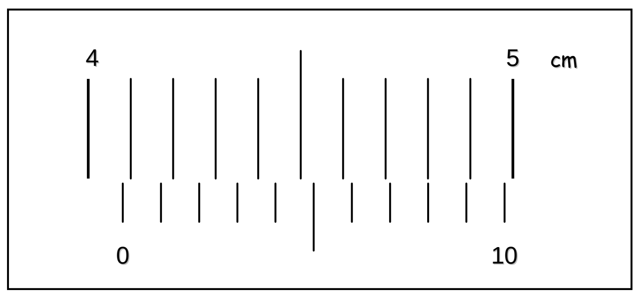


Figure 5.

Now use your vernier calipers to measure the length of bar below. You'll have to exercise some care and judgment in adjusting the calipers so that the arms just straddle the ends of the bar.

Computing the Eccentricity of the Moon's Orbit

A spreadsheet (astmoon.xls) has been prepared to assist you in computing the moon's eccentricity and plotting its orbit. Your lab instructor will show you how to access and use this spreadsheet.

A sequence of 12 Moon photographs (similar to Figure 3) has been provided for you. The images are numbered 1 - 12 in rows from left to right. Please do not mark on these sheets. You will use vernier calipers to measure at least two Moon diameters from each photograph and determine the average value (D_{ave}) of your measurements.

A calibration bar is displayed at the top of your set of Moon photos. Begin by measuring the length of the calibration bar with vernier calipers and entering the value you obtain into the appropriate cell in the spreadsheet.

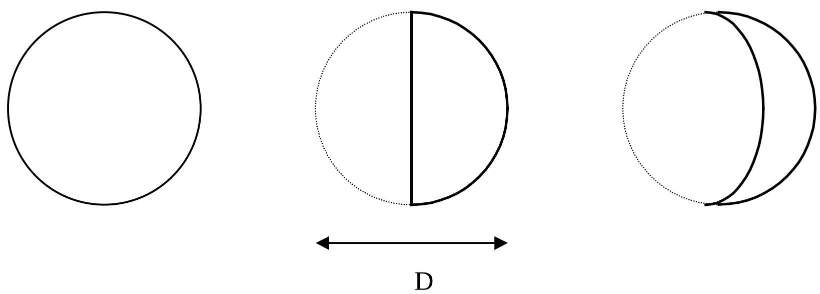


Figure 6. Measuring the diameter of the moon with vernier calipers.

Next use your calipers to obtain two measurements of the moon's diameter in each of the 12 photographs. Enter the values into the spreadsheet in columns D_1 and D_2. The spreadsheet will compute the average value of your measurements for you.

Since the calibration bar represents an arc length of 0.01 radians, dividing .01 by the length of the calibration bar gives us a scaling factor for each photo (rad/cm). By multiplying the average diameter of each photograph by this scaling factor we obtain the angle subtended by the moon in each photograph (the viewing angle), in radians. The spreadsheet will do this calculation for you and the results are displayed in column "θ". The spreadsheet then converts these viewing angles into Earth-Moon distances. From these the spreadsheet will estimate the eccentricity of the moon's orbit.

The eccentricity of an ellipse is between zero and one, i.e., $0 < \varepsilon < 1$. You should arrive at a number between 0 and 1 if you have done everything right. The actual value of *e* is quite small (<< 1). Why do you suppose that the eccentricity of the Moon's orbit is so small?

The point at which the Earth and Moon are closest to each other during a lunar cycle is known as *perigee*, and the point at which they are farthest apart is known as *apogee*. With this in mind, what do the radii of the inner and outer diameters represent in your Earth-Moon plot?

The spreadsheet will compute your perigee (r_{min}) and apogee (r_{max}) distances and the eccentricity of the moon's orbit from these values. The eccentricity of the orbit is the difference of these radii divided by their sum, or:

$$e = \frac{r_o - r_i}{r_o + r_i}$$

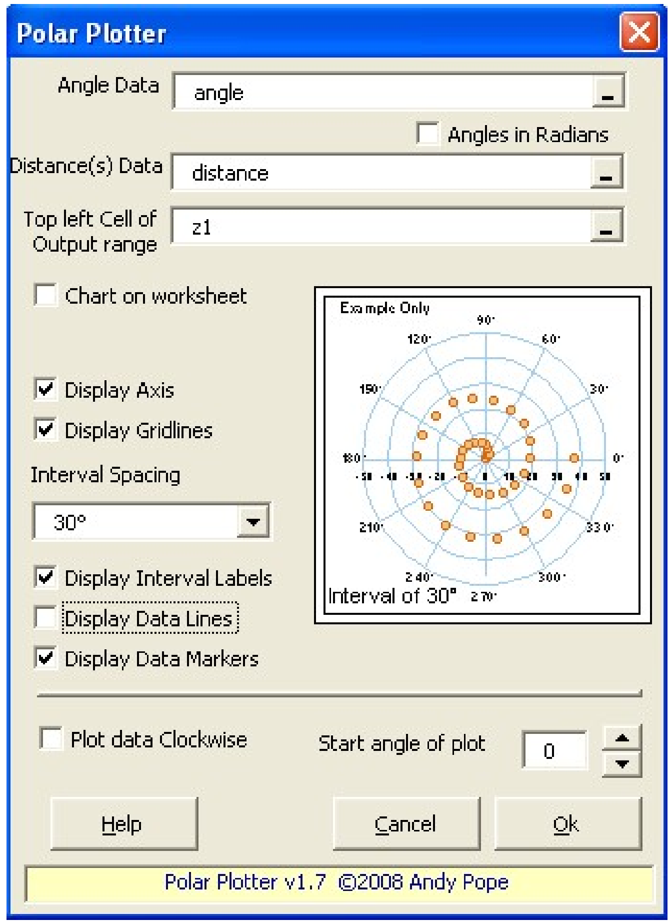
Table 1. An Example data set. These values are not actual values.

Length of Calibration bar (*C*) 6cm

Image #	Diameters	D_{ave}	Subtended Angle (θ = D/*C* x .01)	Distance (km) (d = (1/θ)*3430)	Longitude φ
1					270^0
2					283^0
3					309^0
4					336^0
5					5^0
6					27^0
7					57^0
8					87^0
9					127^0
10					151^0
11	4.71, 4.75cm	4.73cm	4.73/6 x .01 = .0079	434,177	176^0
12					212^0

Finally you will use the spreadsheet data to create a polar plot of the moon's orbit. There is a very useful Excel Add-In (created by Andy Pope) that plots Excel data in polar (angle and distance) format.

Click the Tools button on the menu bar near the top of the screen to activate the Excel Tools menu. Near the bottom of the drop down box that appears you'll see a "Polar Plot" button. Clicking this button activates the menu shown at the right. Enter data into the cells as shown and be sure to click or unclick all of the boxes exactly as shown. When your menu looks exactly like the one at the right you have finished click OK and a plot of the moon's orbit based on your data will appear.



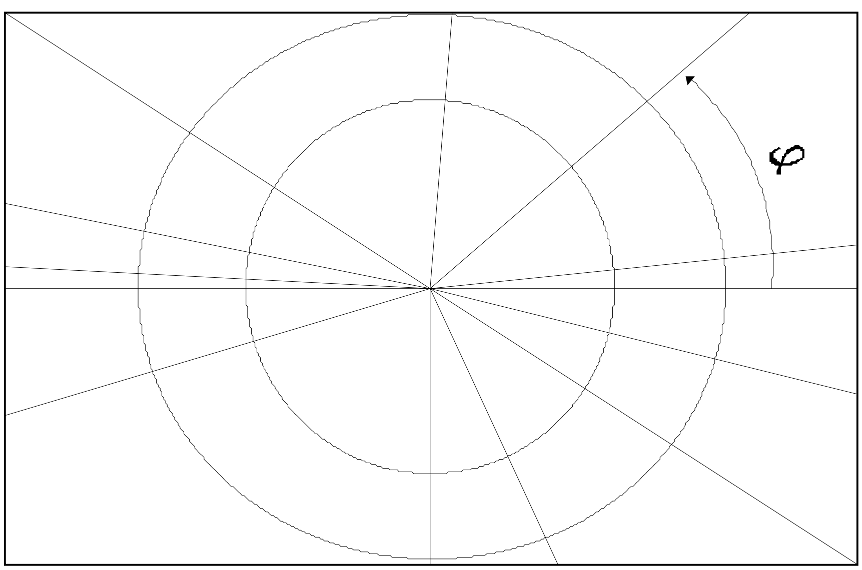


Figure 7. Example plot of the Earth-Moon System. Azimuthal angles φ are indicated by radial lines. The inner and outer radii are indicated with circles. Data points are not shown.

Exercises

1. To what is the apparent difference in the size of the moon during a lunar cycle attributable?

2. How do vernier scales differ from rules scales?

3. A satellite of a planet has a perigee distance of 350,000 km and an apogee distance of 375,000 km. What is the eccentricity of its orbit?

4. What, approximately, are the moon's perigee and apogee distances?

5. Aside from size, what appears different about the moon from photo to photo and why?

6. What is a polar plot?