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Nonexistence of Humphreys' "Volume Cooling" for Terrestrial Heat Disposal by Cosmic Expansion

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The young-earth RATE project posits accelerated nuclear decay during the Flood. To dispose of heat, Humphreys appeals to cosmic expansion and Einstein's general theory of relativity. However, cosmic expansion is irrelevant to terrestrial physics. The static gravitational field on Earth conserves terrestrial energy and so is not a heat sink. One can understand why the relevant gravitational field is static from either a matching problem or an averaging problem. In the averaging problem, one averages Einstein's equations over cosmic distances to get effective field equations for cosmic parameters; these equations tend to differ from Einstein's equations due to nonlinearity. The conserved terrestrial energy is derived rigorously using the divergence theorem and tensor calculus. Difficulties with gravitational energy localization might be due to an unjustified assumption of uniqueness, as Peter Bergmann hinted long ago. It is recalled that unwillingness to posit miracles in Noah's Flood was largely a later seventeenth-century rationalist Protestant innovation, making the Flood story empirically vulnerable and contributing to its ultimate rejection.

Einstein's general theory of relativity and the cosmic expansion provide no assistance in disposing of excess heat because terrestrial energy is conserved.

Recent work by some prominent young-earth creationists involved in the RATE (Radioisotopes and the Age of The Earth) project has posited accelerated nuclear decay during Noah's Flood to explain the presence of isotopes that are construed by mainstream geologists as indicating decay during much longer periods of terrestrial history. Accelerated nuclear decay, it is conceded by the RATE group, would produce a prodigious quantity of heat in the earth in a short period of time.

Some time ago D. Russell Humphreys proposed a "white hole cosmology" that is supposed to serve the purposes of young-earth creation science by allowing for distant stars to be seen quickly.¹ In fact, the result of positing a bounded

matter distribution (with the matter-filled region being localized and surrounded by a vast emptiness) is simply a modest variant of Big Bang cosmology that provides no help for young-earth creationists' light transit time problem,² though it may be of interest for other reasons,³ including the fact that it suggests a new difficulty for arguing from Big Bang cosmology to theism.⁴

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Recently Humphreys has appealed to cosmic expansion and Einstein's general theory of relativity (GTR) as a mechanism for getting rid of some heat caused by accelerated nuclear decay.⁵ However, the phenomenon of cosmic expansion is irrelevant to terrestrial physics, so no heat disposal mechanism is to be found there, as this paper will show.

This paper is not the first criticism of Humphreys' heat sink proposal to appear, but it seems to be the most detailed. Some criticisms of Humphreys' proposal have been published by Glenn Morton and George Murphy.⁶ Randy Isaac has observed briefly that expansion-induced cooling does not apply to bound systems (in which the parts do not have enough energy to separate significantly).⁷ An exchange between the RATE group and Isaac has occurred recently in these pages.⁸ However, a detailed explanation of why the relevant space-time metric in the vicinity of the earth is static (that is, unchanging over time and unchanged by reversing the direction of increasing time coordinate) and how static character entails terrestrial energy conservation, though available in the GTR literature, remains to be applied explicitly in this context. GTR is conceptually and technically intricate⁹ in comparison to other classical field theories in physics, such as Maxwell's electromagnetism, so the discussion here might have pedagogical as well as polemical value.

There are two ways of understanding why the relevant space-time metric is static, one from a matching problem, the other from an averaging problem. This averaging problem, which is a matter of current research interest in GTR and cosmology, is relevant to contemporary debates about dark energy as well.

Cosmic Expansion and the Terrestrial Space-Time Metric

Contemporary cosmology, based on the Robertson-Walker space-time metric, is often glossed as involving the "expansion of the universe." Presumably the "universe" includes the whole physical world, including galaxies and planets and tables, so it is tempting to infer that all physical objects are expanding and are doing so in the same fashion. The question then arises how the expansion can be noticed. For example, measuring the height of an expanding man with a correspondingly expanding ruler would yield a constant height.

In principle, the correct way to treat this problem is to give an exact (microscopic and quantum-mechanical) description of matter with an exact (presumably quantum mechanical) theory of gravity. A demonstrably satisfactory quantum theory of gravity being not yet available, and likely being well approximated by GTR anyway, one naturally relies on GTR, which has proven highly satisfactory both empirically and explanatorily. GTR treats matter as a classical continuum or field, not a quantum field, so some approximation is involved in giving a classical rather than quantum treatment of matter. Furthermore, the variations in density on atomic scales are routinely neglected in successful applications of GTR to macroscopic bodies, such as the planets and sun in our solar system. Thus some sort of implicit averaging must be occurring already in taking the classical limit of quantum physics.¹⁰

To consider the relevance (if any) of cosmic expansion on a bound system such as the solar system or a planet, there are two plausible approaches, matching and averaging. The matching approach treats the bound system as a localized region with some noncosmological metric (perhaps the Schwarzschild metric outside gravitating bodies) matched to a surrounding homogeneous (the same at all points) Robertson-Walker cosmological metric, like a universe of cheese with one hole. (Hence the term "Swiss cheese" is applied; here the cheese corresponds to the exterior homogeneous Robertson-Walker metric and the hole to the interior inhomogeneous region, which might not be empty of matter itself.)

On this approach, many studies have concluded that the cosmic expansion has either no effect or a negligible effect for small systems. Previously I reviewed a number of these studies.¹¹ If the space-time metric (which includes the gravitational potential in GTR) in the hole is independent of time ("stationary"), then the energy of the matter with gravitational interaction is conserved. (If the space-time metric is not stationary, then one must also include gravitational energy to get a conserved quantity, and further complications arise.¹²) Conservation of energy implies that there is no mechanism for disposing of heat to be found, *pace* Humphreys. If the earth is taken as the localized body, matched to a surrounding expanding universe, then the energy of the earth is conserved because the time dependence of the terrestrial gravitational field is

negligible. The energy of the earth is also conserved in the more realistic case where the hole in the cheese is larger than the earth.

In calling the effect of cosmic expansion nonexistent or, at most, negligible, I am assuming the expansion rates implied by standard Big Bang cosmology. If Humphreys wishes to appeal to some novel super-fast expansion during terrestrial history, the effects might not be negligible. However, Humphreys then owes us an argument that the effect would be mere cooling of the earth, as opposed, for example, to its disruption.¹³

Of course, the matter outside the solar system, or the earth, or whatever bound system of interest, is not really homogeneous, because there are stars, clusters of stars, galaxies, etc. Thus a fully satisfactory approach would not involve the fiction of replacing the actual matter distribution outside the hole with a homogeneous distribution. Rather, one would take the true inhomogeneous matter distribution and apply a systematic averaging procedure over some distance scales and perhaps time scales to obtain from Einstein's equations a set of equations for the evolution of the *averaged* matter distribution and gravitational field. This project, which saw rather little work until the 1980s, and which has only become popular recently, is called the "averaging problem" in relativistic cosmology.¹⁴ Thus one needs to average Einstein's equations over cosmic distances in order to find equations for the cosmic parameters.

The analogous procedure for electromagnetism in a medium is well-known¹⁵ and comparatively simple due to the linearity of Maxwell's equations and the flat space-time geometry. The cosmic expansion, therefore, is an emergent effect that presumably arises from large-scale averaging of Einstein's equations, much as macroscopic electromagnetism, with constitutive relations between the electric fields \mathbf{D} and \mathbf{E} and between the magnetic fields \mathbf{B} and \mathbf{H} , arises from averaging microscopic electromagnetism spatially. For Maxwell's equations, the dynamics of the average bears a simple relation to the average of the dynamics, largely due to the linearity of Maxwell's equations; linearity makes the mathematical whole just the sum of the parts. For Einstein's equations, the dynamics of the average bears a complicated relation to the average of the dynamics, largely due to the nonlinearity of

Einstein's equations. Thus the averaged variables will presumably satisfy equations quite different from Einstein's equations in general, whereas the averaged Maxwell electromagnetic equations differ only a bit from the original equations. The curved space-time geometry in GTR makes the addition of vectors at different points path-dependent as well.

While the averaging problem for Einstein's equations is technically involved, the main point needed here, as with electromagnetism, is that *different* gravitational fields are ascribed to the *same* spatio-temporal locations, depending on the distance (and perhaps time) scales employed. When one recognizes that the appropriate space-time metric in the region of the earth depends on the distance scale over which one averages (cosmic vs. stellar vs. terrestrial or the like), one ceases to be tempted to apply the cosmological Robertson-Walker metric at planetary scales as Humphreys does to dispose of terrestrial heat. Given averaging over terrestrial distance scales, again the appropriate metric is independent of time (or very nearly so—the rotation of the earth and the like being insignificant by relativistic standards). Whether or not this calculation has yet been performed using a modern systematic approach to the averaging problem, there is little reason to doubt the outcome. Some authors claim that a careful treatment of the averaging problem can reduce or eliminate the need to posit "dark energy;" this question is a matter of contemporary research.¹⁶

Conservation of Energy from Static Metric

Whether one uses the matching approach or the averaging approach, the relevant metric for the earth is independent of time ("stationary"). Because the earth is not changing (at least on scales relevant to this problem), neither is its energy. It is a familiar point from mechanics that time-independent systems conserve energy.¹⁷ In GTR, the notion of being independent of time depends strongly on which four-vector field is used to represent time translation. Time translation involves identifying (largely by stipulation) space-time points at different moments as being in the same place or in different places, among other things. For a stationary space-time metric, there exists a vector field called a time-like Killing vector field. That means that the space-time

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metric tensor $g_{\mu\nu}$ can be written in a fashion that is independent of the space-time coordinate picked out by the Killing vector field, and that this vector field is time-like with respect to the metric.¹⁸ Stationary character then means independence of time as picked out by this particular vector field. This vector field need not be unique. In the case at hand, the relevant field is proportional to the earth's 4-velocity in the vicinity of the earth. In a coordinate system in which the earth is at rest, this vector field takes a very simple form.

A stronger condition holds in the present context. Whereas a uniform flux in one direction is consistent with stationary character, the relevant terrestrial gravitational field, in fact, does not distinguish (or not much) between the future and past directions. The gravitational field is (to a suitable approximation) time-reversal invariant as well as stationary, and so is called "static." A static metric clearly does not lead to cosmological red-shifting or other energy loss for material at rest in the earth. To a good approximation (by relativistic standards), the earth is a motionless nonrotating solid ball with increasing density toward the middle, static, and, hence, not expanding. With the metric thus unchanging, there is no red-shifting or other energy loss to use up radiogenic heat.

Given these simplifications, one can demonstrate rigorously the conservation of energy for the earth in the following manner. The matter stress-energy tensor $T^{\mu\nu}$ is covariantly "conserved" (using the matter or gravitational field equations) in the sense of having zero four-dimensional covariant divergence: $\sum_{\mu} \nabla_{\mu} T^{\mu\nu} = 0$. (All Greek indices run from 0 to 3, where the 0th coordinate x^0 is time and the remaining three coordinates x^i are spatial, whether approximately Cartesian, spherical, or whatever.) For the time-like Killing vector field ξ^{μ} , one has $\nabla_{\mu} \xi_{\nu} + \nabla_{\nu} \xi_{\mu} = 0$, where $\xi_{\nu} = \sum_{\mu} g_{\mu\nu} \xi^{\mu}$. This equation, which is equivalent to the vanishing of the Lie derivative of the metric $g_{\mu\nu}$ with respect to ξ^{α} , shows that the space-time metric $g_{\mu\nu}$ does not depend essentially on the time coordinate adapted to the Killing vector field ξ^{μ} .¹⁹ Because the covariant derivative of the metric is zero ($\nabla_{\alpha} g_{\mu\nu} = 0$), the covariant derivative of the metric's determinant g , or of $\sqrt{-g}$, also is zero.²⁰ Thus

$$\sum_{\mu} \nabla_{\mu} \left(\sum_{\nu} T^{\mu\nu} \sqrt{-g} \xi_{\nu} \right) = \sum_{\nu} \left(\sum_{\mu} \nabla_{\mu} T^{\mu\nu} \right) \sqrt{-g} \xi_{\nu} + \sum_{\mu} \sum_{\nu} T^{\mu\nu} \left(\nabla_{\mu} \sqrt{-g} \right) \xi_{\nu} +$$

$$\sum_{\mu} \sum_{\nu} T^{\mu\nu} \sqrt{-g} \nabla_{\mu} \xi_{\nu} = 0 + 0 + \sum_{\mu} \sum_{\nu} \frac{1}{2} T^{\mu\nu} \sqrt{-g} \left(\nabla_{\mu} \xi_{\nu} + \nabla_{\nu} \xi_{\mu} \right) = 0, \quad (1)$$

where the Leibniz product rule has been used twice in the second line and the symmetry of $T^{\mu\nu}$ and the Killing vector character of ξ^{μ} have been employed in the last line. Precisely because $\sum_{\nu} T^{\mu\nu} \sqrt{-g} \xi_{\nu}$ is a weight 1 contravariant vector density, its covariant divergence (with the symbol ∇_{μ}) equals its coordinate divergence (with the symbol $\frac{\partial}{\partial x^{\mu}}$) and is a scalar

density of weight 1.²¹ That result is just what one needs for the divergence theorem in a form suitable for generalized coordinates and hence slightly generalized from that in vector calculus, in order to get results independent of the merely conventional choice of coordinates.²² (In basic vector calculus, the use of Cartesian coordinates obliterates the distinction between covariant and ordinary differentiation, but in GTR, Cartesian coordinates generally do not exist.) To express the divergence theorem conveniently, two useful bits of notation are $d^4x = dx^0 dx^1 dx^2 dx^3$ for the element of coordinate 4-volume and $dS_{\mu} = d^4x / dx^{\mu} = (dx^1 dx^2 dx^3, dx^0 dx^2 dx^3, dx^0 dx^1 dx^3, dx^0 dx^1 dx^2)$ for the element of 3-area of the hypersurface enclosing the 4-volume. Integrating the divergence 4-dimensionally over the relevant part of the earth's history between two moments (and throwing in a minus sign) gives

$$\begin{aligned} 0 &= \int 0 d^4x = - \int d^4x \sum_{\mu} \nabla_{\mu} \left(\sum_{\nu} T^{\mu\nu} \sqrt{-g} \xi_{\nu} \right) = \\ &= - \int d^4x \sum_{\mu} \frac{\partial}{\partial x^{\mu}} \left(\sum_{\nu} T^{\mu\nu} \sqrt{-g} \xi_{\nu} \right) = \\ &= - \sum_{\mu} \int dS_{\mu} \sum_{\nu} T^{\mu\nu} \sqrt{-g} \xi_{\nu} = \\ &= - \sum_{\mu} \int dS_{\mu} \sum_{\nu} T^{\mu\nu} \sqrt{-g} \sum_{\alpha} g_{\nu\alpha} \xi^{\alpha} = \\ &= - \sum_{\mu} \sum_{\nu} \sum_{\alpha} \int dS_{\mu} T^{\mu\nu} \sqrt{-g} g_{\nu\alpha} \xi^{\alpha}. \end{aligned} \quad (2)$$

Because ξ^{α} is the time-translation vector field for the coordinates employed, it has components $\delta_0^{\alpha} = (1, 0, 0, 0)$, where δ_{β}^{α} is the Kronecker symbol that is 0 if the values of the indices α and β disagree and 1 if they match. The static character of the metric implies that $g_{\nu 0} = g_{00} \delta_{\nu}^0$. Because the matter in question has vanishing energy flux density, some of the stress-energy tensor components vanish: $T^{\mu 0} = T^{00} \delta_{\mu}^0$. Thus

$$0 = - \sum_{\mu} \sum_{\nu} \sum_{\alpha} \int dS_{\mu} T^{\mu\nu} \sqrt{-g} g_{\nu\alpha} \xi^{\alpha} =$$

$$\begin{aligned}
 & -\sum_{\mu} \sum_{\nu} \sum_{\alpha} \int dS_{\mu} T^{\mu\nu} \sqrt{-g} g_{\nu\alpha} \delta_0^{\alpha} = \\
 & -\sum_{\mu} \sum_{\nu} \int dS_{\mu} T^{\mu\nu} \sqrt{-g} g_{\nu 0} = \\
 & -\sum_{\mu} \sum_{\nu} \int dS_{\mu} T^{\mu\nu} \sqrt{-g} g_{00} \delta_{\nu}^0 = \\
 & -\sum_{\mu} \int dS_{\mu} T^{\mu 0} \sqrt{-g} g_{00} = \sum_{\mu} \int dS_{\mu} T^{00} \delta_0^{\mu} \sqrt{-g} (-g_{00}) = \\
 & \int_{final} dS_0 T^{00} \sqrt{-g} (-g_{00}) - \int_{initial} dS_0 T^{00} \sqrt{-g} (-g_{00}), \quad (3)
 \end{aligned}$$

which is the spatial integral of the matter’s energy density at final time minus the spatial integral of matter’s energy at initial time. There being no dependence on time in the problem, the two integrals are equal. Thus the energy of matter is conserved; it follows that there is no volume cooling. One can be a bit more explicit when the matter in question is an elastic solid.²³ The relevant component of the stress-energy tensor is $T^{00} = \rho U^0 U^0$ with $\sum_{\mu} \sum_{\nu} U^{\mu} g_{\mu\nu} U^{\nu} = -1$ for the 4-velocity U^{μ} due to the $-+++$ signature of the metric tensor. Because the earth is at rest in the coordinates employed, $U^{\mu} = U^0 \delta_0^{\mu}$. Then $(U^0)^2 = -\frac{1}{g_{00}}$. Thus the integrals

appearing in the last line of equation (3) have the form

$$\begin{aligned}
 \int dS_0 T^{00} \sqrt{-g} (-g_{00}) &= \int dS_0 \left(\frac{\rho}{-g_{00}} \right) \sqrt{-g} (-g_{00}) = \\
 \int dx^1 dx^2 dx^3 \rho \sqrt{-g}
 \end{aligned}$$

evaluated at the final and initial moments, thereby canceling to 0. The factor $\sqrt{-g}$ provides the analog of the familiar Jacobian factor $r^2 \sin \theta$ for volume integrals in spherical coordinates, for example.

Another way to discuss the problem of energy conservation is to consider gravitational energy-momentum explicitly rather than (as in the previous paragraph) implicitly. One approach is the method of a gravitational energy-momentum pseudotensor(s), such as one finds in older GTR texts.²⁴ A fairly nontechnical outline of pseudotensors appeared here previously.²⁵ The relevant idea described there is that in GTR there is conservation of the combined energy-momentum of gravity and matter together, though typically not of either one separately. When the gravitational energy is unchanging, as in the static earth, the energy of matter is conserved. Whereas Robert Gentry claimed that Big Bang cosmology violated energy conservation and so was absurd,²⁶ Humphreys claims that Big Bang cosmology provides a sink for terrestrial heat and so is

a resource for young-earth creation science. A fully satisfactory treatment of gravitational energy has not yet appeared, or at least has not been recognized, due to the conceptual intricacies involving individuation of space-time points in GTR. However, there are reasonable notions of energy conservation that apply, *pace* Gentry and perhaps Humphreys. A good recent review is that by Szabados.²⁷ In some respects, the problem is a conceptual excess of different conserved energies, with equally good claims on being “the” energy (if there is just one true energy), rather than the lack of any conserved energy.²⁸ It is often assumed that there should be just one energy rather than many energies in GTR, as in other theories,²⁹ but this assumption is wrong.³⁰

Conclusion

Some historical perspective on Flood geology in relation to natural laws might be useful. Unwillingness to posit miracles somewhere or other in Noah’s Flood was largely a later seventeenth-century rationalist Protestant innovation.³¹ This move made the Flood story empirically vulnerable and contributed to its ultimate rejection. Perhaps there is a lesson here for contemporary defenders of a global Flood. In any case, GTR and the cosmic expansion provide no assistance in disposing of excess heat because terrestrial energy is conserved. ✂

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