Algebraic Combinatorics

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It is usually said that the essence of mathematics is dealing with the infinite and that it is far more difficult than dealing with the finite. Is this belief really true? As it is well known, the classification of simple Lie groups, which are continuous infinite groups, was solved a long time ago by using Dynkin diagrams. On the other hand, the classification of finite simple groups which was completed in the beginning of the 1980's, is far more difficult and complicated compared with the Lie group case. The simple group called Monster has

$$2^{46}\cdot 3^{20}\cdot 5^{9}\cdot 7^{6}\cdot 11^{2}\cdot 13^{3}\cdot 17\cdot 19\cdot 23\cdot 29\cdot 31\cdot 41\cdot 47\cdot 59\cdot 71\ (\simeq 8\times 10^{53})$$

elements. Such an explicit enormous finite number is so complex that it can almost be said to be larger than infinity. We firmly stand on the viewpoint that the finite and discrete mathematics are very important. We believe that to study finite objects is more essential and reflects the real world, and that infinite objects are important (only) for the first approximation of the finite objects. Some people might think that problems on finite objects can all be solved easily by using computers. However, the fact is not so simple. A slightly large finite size easily exceeds the capacity of the computer.

Algebraic combinatorics has been given its mathematical depth based on the thoughts and philosophy of other branches of mathematics, such as group theory. The name algebraic combinatorics was first used by Bannai in the late 1970's, and it seems that the name became popular and was then accepted by the mathematical community through the publication of the book "Algebraic Combinatorics I" by Bannai and Ito in 1984. The essence of algebraic combinatorics may be described as either "representation theoretical combinatorics" or "group theory without groups".

Association schemes play an important role in algebraic combinatorics. They are essentially edge-colored complete graphs which enjoy certain regularity conditions. Association schemes are a purely combinatorial concept which was extracted from the concept of transitive permutation groups. Since there are too many examples of association schemes, their complete classification seems to be out of reach. However, there has been much research on the existence and the classification problems of those with good extra properties. Bannai has led this research direction, and alongside many researchers, has worked on distance-regular graphs, in particular on P- and Q-polynomial association schemes, as well as their connections with the theory of orthogonal polynomials. The classification problem of primitive and commutative association schemes is an important (and a bit distant) goal of future research. We can regard the classification problem of finite simple groups as a part of that of primitive and commutative case, and we are hoping to look at the classification of finite simple groups through the view point of association schemes and algebraic combinatorics. This whole revisionism project might still be premature at the present stage, but we have so far been successful in the study of character tables of commutative association schemes. Another important feature of association schemes is that they are very useful as a framework to study codes and designs in a unified way. The purpose of coding theory is to find the objects in which the elements are most separated (i.e., the minimum distance of two distinct elements become maximum), and its usefulness in practical applications such as telecommunications is well recognized. The purpose of design theory is to find good subsets which well approximate the original space. The concept of designs originated in the theory of designs of experiments in statistics, and it is also closely connected to some branches of analysis such as approximation theory. Bannai has been successful in the study of the existence and the classification problems of very good structures, such as perfect codes and tight designs.

Theory of codes and designs can be developed in various spaces such as spheres. The book "Algebraic Combinatorics on Spheres" (by Eiichi Bannai and Etsuko Bannai Springer Tokyo, 1999) written in Japanese deals with these topics. Recent works of Bannai include: Classification of certain tight spherical designs (Bannai-Munemasa-Venkov), "Codes and designs on Grassmannian spaces"(Bachoc-Bannai-Coulangeon),and "Tight Euclidean designs (Bannai-Bannai). Bannai is also interested in the kissing number problem (i.e., how many unit spheres can one put around a given unit sphere in the n-dimensional Euclidean space so that they do not overlap), and what the best (i.e., the densest) sphere packings in the n-dimensional Euclidean spaces are. In passing, the kissing number in 24 dimensional space is known to be 196560 (Odlyzko Sloane, 1977), and the uniqueness of such kissing configuration was obtained by Bannai-Sloane (1981).

profile

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