

CUT-FOLD-STAPLE

Rotate a flat shape into the third dimension.

[45 minutes]

Materials:

- Eight pieces of paper—four sheets of 8½" x 11" paper cut in half, widthwise, so each piece is 8½" x 5½". (For a prettier result, you can use colored paper or old wrapping paper.)
- 3" x 5" index card
- Stapler
- Scissors
- Flat surface
- Pencil
- Ruler

Try This:

- Step 1 Draw a shape on your index card that has at least one straight edge. Then cut out your shape.
- Step 2 Take two pieces of paper, one on top of the other, and fold them in half, like a little book. (It should be 5½ x 4¼ inches.)
- Step 3 Place your index card shape on the folded paper stack with one straight edge against the fold line. Trace the shape.
- Step 4 Keeping the stack folded, cut out your traced shape from both folded pieces of paper at the same time. What do you think these shapes will look like when you unfold them?



Step 5 Carefully unfold your shapes. Are you surprised by how they look? The shape on one side of the fold line looks like a mirror image of the other side. A shape whose two halves are similar in this way has *bilateral* or *line symmetry*.



- Step 6 Repeat steps 2 through 5 three more times, so that you have eight identical unfolded shapes. Shapes that are identical to one another are called *congruent* shapes. Flip over one of the shapes. Does it look the same as before? Even if you turn (*rotate*) or flip over (*reflect*) one of your shapes, it is still considered to be congruent to the others.
- Step 7 Collect all the unfolded shapes in one stack. Staple them together in two or three places along the fold line. Make sure your staples are in line with the fold.
- Step 8 Take the stapled stack of papers in both hands, slide a thumb under the first layer of paper and fold completely over the stapled center.
- Step 9 Continue to fold over the layers until the "pages" fan out apart from one another.



Voilà! You have just transformed a collection of 2D stapled shapes into a 3D object! How is this shape similar or different from the one you drew? Does your shape remind you of anything?

What's Going On?

When you flip over a shape, you see the reflection of the original shape. In a similar way, the two halves of the unfolded shape are reflections—or mirror images—of each other. In other words, they have bilateral symmetry. Many animals and plants exhibit bilateral symmetry, including you. The left and right halves of your own body are almost mirror images of each other.

In your final creation, you have many congruent, or identical, shapes attached to each other. Together they form a single, three-dimensional (3D) shape. If you took your index card shape, and simply rotated it around the flat side, you would trace out the same 3D space as the shape you've made. In this case, the flat side of the starting shape would be called the *axis of rotation*. From the National Council of Teachers of Mathematics (NCTM)

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Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships:

• Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes.

Apply transformations and use symmetry to analyze mathematical situations:

- Predict and describe the results of sliding, flipping, and turning two-dimensional shapes;
- Describe a motion or a series of motions that will show that two shapes are congruent;
- Identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs.

Understand measurable attributes of objects and the units, systems, and processes of measurement:

• Understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute.

Apply appropriate techniques, tools, and formulas to determine measurements:

• Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles.