# Valuation The Big Picture <br> Aswath Damodaran <br> http://www.damodaran.com 

## Valuing a Firm

- The value of the firm is obtained by discounting expected cashflows to the firm, i.e., the residual cashflows after meeting all operating expenses and taxes, but prior to debt payments, at the weighted average cost of capital, which is the cost of the different components of financing used by the firm, weighted by their market value proportio

$$
\text { Value of Firm }=\sum_{t=1}^{t=n} \frac{C F \text { to Firm }}{(1+\text { WACC })^{t}}
$$

where,
CF to Firm ${ }_{t}=$ Expected Cashflow to Firm in period $t$
WACC $=$ Weighted Average Cost of Capital

## DISCOUNTED CASHFLOW VALUATION





## I. The Cost of Capital

## The Cost of Capital is central to both corporate finance and valuation

- In corporate finance, the cost of capital is important because
- It operates as the hurdle rate when considering new investments
- It is the metric that allows firms to choose their optimal capital structure
- In valuation, it is the discount rate that we use to value the operating assets of the firm.


## I. The Cost of Equity



## A Simple Test

- You are valuing Ambev in U.S. dollars and are attempting to estimate a risk free rate to use in the analysis. The risk free rate that you should use is
- The interest rate on a nominal real denominated Brazilian government bond
- The interest rate on an inflation-indexed Brazilian government bond
- The interest rate on a dollar denominated Brazilian government bond (11.20\%)
- The interest rate on a U.S. treasury bond (4.70\%)


## Everyone uses historical premiums, but..

- The historical premium is the premium that stocks have historically earned over riskless securities.
- Practitioners never seem to agree on the premium; it is sensitive to
- How far back you go in history...
- Whether you use T.bill rates or T.Bond rates
- Whether you use geometric or arithmetic averages.
- For instance, looking at the US:

|  | Arithmetic average <br>  <br> Stocks - Stocks - |  | Geometric Average <br> Stocks |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Historical Period | T.Bills | T.Bonds | T.Bills | T.Bonds |
| $1928-2003$ | $7.92 \%$ | $6.54 \%$ | $5.99 \%$ | $4.82 \%$ |
| $1963-2003$ | $6.09 \%$ | $4.70 \%$ | $4.85 \%$ | $3.82 \%$ |
| $1993-2003$ | $8.43 \%$ | $4.87 \%$ | $6.68 \%$ | $3.57 \%$ |

## Two Ways of Estimating Country Risk Premiums...

- Default spread on Country Bond: In this approach, the country risk premium is based upon the default spread of the bond issued by the country (but only if it is denominated in a currency where a default free entity exists.
- Brazil was rated B2 by Moody's and the default spread on the Brazilian dollar denominated C.Bond at the end of May 2004 was $6.50 \%$. ( $11.2 \%-4.7 \%$ )
- Relative Equity Market approach: The country risk premium is based upon the volatility of the market in question relative to U.S market.
Country risk premium $=$ Risk Premium ${ }_{\text {US }}{ }^{*} \sigma_{\text {Country Equity }} / \sigma_{\text {US Equity }}$
Using a $4.82 \%$ premium for the US, this approach would yield:
Total risk premium for Brazil $=4.82 \%(29.24 \% / 18.59 \%)=7.58 \%$
Country risk premium for Brazil $=7.58 \%-4.82 \%=2.76 \%$
(The standard deviation in weekly returns from 2001 to 2003 for the Bovespa was $29.24 \%$ whereas the standard deviation in the S\&P 500 was $18.59 \%$ )


## And a third approach

- Country ratings measure default risk. While default risk premiums and equity risk premiums are highly correlated, one would expect equity spreads to be higher than debt spreads.
- Another is to multiply the bond default spread by the relative volatility of stock and bond prices in that market. In this approach:
- Country risk premium $=$ Default spread on country bond* $\sigma_{\text {Country Equity }} / \sigma$ Country Bond
- Standard Deviation in Bovespa (Equity) $=29.24 \%$
- Standard Deviation in Brazil C-Bond $=24.15 \%$
- Default spread on C-Bond $=6.50 \%$
- Country Risk Premium for Brazil $=6.50 \%(29.24 \% / 24.15 \%)=7.87 \%$


## From Country Spreads to Corporate Risk premiums

- Approach 1: Assume that every company in the country is equally exposed to country risk. In this case,
$\mathrm{E}($ Return $)=$ Riskfree Rate + Country Spread + Beta (US premium)
Implicitly, this is what you are assuming when you use the local
Government's dollar borrowing rate as your riskfree rate.
■ Approach 2: Assume that a company's exposure to country risk is similar to its exposure to other market risk.
$\mathrm{E}($ Return $)=$ Riskfree Rate + Beta (US premium + Country Spread)
■ Approach 3: Treat country risk as a separate risk factor and allow firms to have different exposures to country risk (perhaps based upon the proportion of their revenues come from non-domestic sales)
E (Return)=Riskfree Rate $+\beta$ (US premium) $+\lambda$ (Country Spread)


## Estimating Company Exposure to Country Risk: Determinants

- Source of revenues: Other things remaining equal, a company should be more exposed to risk in a country if it generates more of its revenues from that country. A Brazilian firm that generates the bulk of its revenues in Brazil should be more exposed to country risk than one that generates a smaller percent of its business within Brazil.
- Manufacturing facilities: Other things remaining equal, a firm that has all of its production facilities in Brazil should be more exposed to country risk than one which has production facilities spread over multiple countries. The problem will be accented for companies that cannot move their production facilities (mining and petroleum companies, for instance).
- Use of risk management products: Companies can use both options/futures markets and insurance to hedge some or a significant portion of country risk.


## Estimating Lambdas: The Revenue Approach

- The easiest and most accessible data is on revenues. Most companies break their revenues down by region. One simplistic solution would be to do the following:
$\lambda=\%$ of revenues domestically ${ }_{\text {firm }} / \%$ of revenues domestically ${ }_{\text {avg firm }}$
- Consider, for instance, Embraer, Embratel and Ambev, all of which are incorporated and traded in Brazil. Embraer gets 3\% of its revenues from Brazil, Embratel gets almost all of its revenues in Brazil and Ambev gets about $92 \%$ of its revenues in Brazil. The average Brazilian company gets about $77 \%$ of its revenues in Brazil:
- Lambda $_{\text {Embraer }}=3 \% / 77 \%=.04$
- Lambda $_{\text {Embratel }}=100 \% / 77 \%=1.30$
- Lambda $_{\text {Ambev }}=92 \% / 77 \%=1.19$
- There are two implications
- A company's risk exposure is determined by where it does business and not by where it is located
- Firms might be able to actively manage their country risk exposures


## Estimating Lambdas: Earnings Approach



## Estimating Lambdas: Stock Returns versus CBond Returns

```
Return \(_{\text {Embraer }}=0.0195+\mathbf{0 . 2 6 8 1}\) Return C Bond \(\quad\) Return \(_{\text {Embratel }}=-0.0308+\mathbf{2 . 0 0 3 0}\) Return \(_{C}\) Bond
Return \(_{\text {Ambev }}=0.0290+\mathbf{0 . 4 1 3 6}\) Return \(_{\mathrm{C} \text { Bond }}\)
Return \(_{\text {Petrobras }}=-0.0308+\mathbf{0 . 6 6 0 0}\) Return \(_{C \text { Bond }}\)
Return \(_{\text {Vale }}=0.02169 \boldsymbol{+ 0 . 3 7 6 0} \cdot\) Return \(_{\mathrm{C} \text { Bond }}\)
```



## Estimating a US Dollar Cost of Equity for Ambev

- Assume that the beta for Ambev is 0.87 , and that the riskfree rate used is $4.70 \%$. Also assume that the risk premium for the US is $4.82 \%$ and the country risk premium for Brazil is $7.87 \%$.
- Approach 1: Assume that every company in the country is equally exposed to country risk. In this case,
$\mathrm{E}($ Return $)=4.70 \%+0.87(4.82 \%)+7.87 \%=16.76 \%$
■ Approach 2: Assume that a company's exposure to country risk is similar to its exposure to other market risk.
$\mathrm{E}($ Return $)=4.70 \%+0.87(4.82 \%+7.87 \%)=15.74 \%$
- Approach 3: Treat country risk as a separate risk factor and allow firms to have different exposures to country risk (perhaps based upon the proportion of their revenues come from non-domestic sales)
$\mathrm{E}($ Return $)=4.70 \%+0.87(4.82 \%)+0.41(7.87 \%)=12.12 \%$


## Implied Equity Premium for the S\&P 500: January 1, 2004

- We can use the information in stock prices to back out how risk averse the market is and how much of a risk premium it is demanding.

In 2003, dividends \& stock buybacks were $2.81 \%$ of the index, generating 31.29 in cashflows

Analysts expect earnings to grow $9.5 \%$ a year for the next 5 years as the economy comes out of a recession.

After year 5, we will assume that earnings on the index will grow at $4.25 \%$, the same rate as the entire economy

January 1, 2004
S\&P 500 is 111 Ifyou pay the current level of the index, you can expect to make a return of $7.94 \%$ on stocks (which is obtained by solving for $r$ in the following equation)

$$
1111.91=\frac{34.26}{(1+r)}+\frac{37.52}{(1+r)^{2}}+\frac{41.08}{(1+r)^{3}}+\frac{44.98}{(1+r)^{4}}+\frac{49.26}{(1+r)^{5}}+\frac{49.26(1.0425)}{(r-.0425)(1+r)^{5}}
$$

■ Implied Equity risk premium $=$ Expected return on stocks - Treasury bond rate $=7.94 \%-4.25 \%=$ 3.69\%

## Implied Premiums in the US

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## Implied Premiums: From Bubble to Bear Market... January 2000 to December 2002




## An Intermediate Solution

- The historical risk premium of $4.82 \%$ for the United States is too high a premium to use in valuation. It is much higher than the actual implied equity risk premium in the market
- The current implied equity risk premium requires us to assume that the market is correctly priced today. (If I were required to be market neutral, this is the premium I would use)
■ The average implied equity risk premium between 1960-2003 in the United States is about 4\%. We will use this as the premium for a mature equity market.


## Implied Premium for Brazil: May 2004

- Level of the Index $=22236$
- Dividends on the Index $=5.17 \%$ of 22236
- Other parameters (all in US dollars)
- Riskfree Rate $=4.70 \%$
- Expected Growth (in dollars)
- Next 5 years $=14 \%$ (Used expected growth rate in Earnings for ADRs)
- After year $5=5.20 \%$ (Slightly higher than riskfree rate because of higher expected real growth)
- Solving for the expected return:
- Expected return on Equity $=12.96 \%$
- Implied Equity premium $=12.96 \%-4.70 \%=8.26 \%$
- Implied Equity premium for US on same day $=3.95 \%$
- Implied country premium for Brazil $=4.31 \%$


## Estimating Beta

- The standard procedure for estimating betas is to regress stock returns $\left(\mathrm{R}_{\mathrm{j}}\right)$ against market returns $\left(\mathrm{R}_{\mathrm{m}}\right)$ -

$$
R_{j}=a+b R_{m}
$$

- where $a$ is the intercept and $b$ is the slope of the regression.
- The slope of the regression corresponds to the beta of the stock, and measures the riskiness of the stock.
- This beta has three problems:
- It has high standard error
- It reflects the firm's business mix over the period of the regression, not the current mix
- It reflects the firm's average financial leverage over the period rather than the current leverage.


## Beta Estimation : The Index Effect



## Determinants of Betas



## The Solution: Bottom-up Betas

- The bottom up beta can be estimated by :
- Taking a weighted (by sales or operating income) average of the unlevered betas of the different businesses a firm is in.

$$
\sum_{j=1}^{j=k} \beta_{j}\left[\frac{\text { Operating Income }_{\mathrm{j}}}{\text { Operating Income }_{\text {Fim }}}\right]
$$

(The unlevered beta of a business can be estimated by looking at other firms in the same business)

- Lever up using the firm's debt/equity ratio

$$
\beta_{\text {levered }}=\beta_{\text {unlevered }}[1+(1-\text { tax rate })(\text { Current Debt/Equity Ratio })]
$$

- The bottom up beta will give you a better estimate of the true beta when
- It has lower standard error $\left(\mathrm{SE}_{\text {average }}=\mathrm{SE}_{\text {firm }} / \sqrt{ } \mathrm{n}(\mathrm{n}=\right.$ number of firms $)$
- It reflects the firm's current business mix and financial leverage
- It can be estimated for divisions and private firms.


## Bottom-up Betas: Embraer, Ambev, Vale and Petrobras

| Company | Business | Unlevered beta | D/E Ratio | Weights | Levered Beta |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Embraer | Aerospace | 0.95 | $18.95 \%$ | $100 \%$ | 1.07 |
| Ambev | Alcoholic beverages | 0.75 | $19.43 \%$ | $80 \%$ | 0.85 |
|  | Soft Drinks | 0.85 | $19.43 \%$ | $20 \%$ | 0.96 |
|  | Company | 0.77 | $19.43 \%$ |  | 0.87 |
| Vale | Mining | 0.98 | $25.66 \%$ | $71 \%$ | 1.15 |
|  | Aluminum | 0.72 | $25.66 \%$ | $9 \%$ | 0.84 |
|  | Steel | 0.63 | $25.66 \%$ | $14 \%$ | 0.74 |
|  | Transportation | 0.73 | $25.66 \%$ | $5 \%$ | 0.85 |
|  | Other | 0.74 | $25.66 \%$ | $1 \%$ | 0.87 |
|  | Company | 0.89 | $25.66 \%$ | $100 \%$ | 1.04 |
| Petrobras | Integrated Oil | 0.56 | $49.58 \%$ | $100 \%$ | 0.74 |

## Gross Debt versus Net Debt Approaches: Embraer in October 2003

- Net Debt Ratio for Embraer $=($ Debt - Cash $) /$ Market value of Equity

$$
=(1953-2320) / 11,042=-3.32 \%
$$

- Levered Beta for Embraer $=0.95(1+(1-.34)(-.0332))=0.93$
- The cost of Equity using net debt levered beta for Embraer will be much lower than with the gross debt approach. The cost of capital for Embraer, though, will even out since the debt ratio used in the cost of capital equation will now be a net debt ratio rather than a gross debt ratio.


## From Cost of Equity to Cost of Capital



## Estimating Synthetic Ratings

- The rating for a firm can be estimated using the financial characteristics of the firm. In its simplest form, the rating can be estimated from the interest coverage ratio

Interest Coverage Ratio = EBIT / Interest Expenses

- For Embraer's interest coverage ratio, we used the interest expenses and EBIT from 2002.

Interest Coverage Ratio $=2166 / 222=9.74$

- For Ambev's interest coverage ratio, we used the interest expenses and EBIT from 2003.

Interest Coverage Ratio $=2213 / 570=3.88$

- For Vale's interest coverage ratio, we used the interest expenses and EBIT from 2003 also

Interest Coverage Ratio $=6,371 / 1989=3.20$

## Interest Coverage Ratios, Ratings and Default Spreads

| If Interest Coverage Ratio is |  |
| :--- | :--- |
| $>8.50$ | $(>12.50)$ |
| $6.50-8.50$ | $(9.5-12.5)$ |
| $5.50-6.50$ | $(7.5-9.5)$ |
| $4.25-5.50$ | $(6-7.5)$ |
| $3.00-4.25$ | $(4.5-6)$ |
| $2.50-3.00$ | $(4-4.5)$ |
| $2.25-2.50$ | $(3.5-4)$ |
| $2.00-2.25$ | $((3-3.5)$ |
| $1.75-2.00$ | $(2.5-3)$ |
| $1.50-1.75$ | $(2-2.5)$ |
| $1.25-1.50$ | $(1.5-2)$ |
| $0.80-1.25$ | $(1.25-1.5)$ |
| $0.65-0.80$ | $(0.8-1.25)$ |
| $0.20-0.65$ | $(0.5-0.8)$ |
| $<0.20$ | $(<0.5)$ |


| Estimated Bond Rating | Default Spread(2003) <br> AAA | Default Spread(2004) <br> $0.75 \%$ |
| :--- | :--- | :--- |
| AA | $1.00 \%$ | $0.35 \%$ |
| A+ | $1.50 \%$ | $0.70 \%$ |
| A | $1.80 \%$ | $0.85 \%$ |
| A- | $2.00 \%$ | $1.00 \%$ |
| BBB | $2.25 \%$ | $1.50 \%$ |
| BB+ | $2.75 \%$ | $2.00 \%$ |
| BB | $3.50 \%$ | $2.50 \%$ |
| B+ | $4.75 \%$ | $3.25 \%$ |
| B | $6.50 \%$ | $4.00 \%$ |
| B - | $8.00 \%$ | $6.00 \%$ |
| CCC | $10.00 \%$ | $8.00 \%$ |
| CC | $11.50 \%$ | $10.00 \%$ |
| C | $12.70 \%$ | $12.00 \%$ |
| D | $15.00 \%$ | $20.00 \%$ |

The first number under interest coverage ratios is for larger market cap companies and the second in brackets is for smaller market cap companies. For Embraer and Ambev, I used the interest coverage ratio table for smaller/riskier firms (the numbers in brackets) which yields a lower rating for the same interest coverage ratio.

## Estimating the cost of debt

| Company | EBIT | Interest | Interest | Rating | Company | Country Cost of |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | Expense | Coverage | Spread | Spread | Debt $(\$)$ |  |
| Embraer (2003) | 2166 | 222 | 9.76 | AA | $1.00 \%$ | $4 \%$ | $9.17 \%$ |
| Ambev | 2213 | 570 | 3.88 | BB+ | $2.00 \%$ | $4 \%$ | $10.70 \%$ |
| Vale | 6371 | 1989 | 3.20 | BB | $2.50 \%$ | $4 \%$ | $11.20 \%$ |
| Petrobras | 14974 | 3195 | 4.69 | A- | $1 \%$ | $4 \%$ | $9.70 \%$ |

Riskfree Rate $=4.17 \%$ for Embraer in 2003, $4.70 \%$ for all other firms
Cost of debt $(\$)=$ Riskfree Rate + Company Spread + Country Spread
(I have assumed that all of these companies will have to bear only a portion of the total country default spread of Brazil which is $4.50 \%$ )

## Weights for the Cost of Capital Computation

- The weights used to compute the cost of capital should be the market value weights for debt and equity.
- There is an element of circularity that is introduced into every valuation by doing this, since the values that we attach to the firm and equity at the end of the analysis are different from the values we gave them at the beginning.
- As a general rule, the debt that you should subtract from firm value to arrive at the value of equity should be the same debt that you used to compute the cost of capital.


## Estimating Cost of Capital: Embraer

- Equity
- Cost of Equity $=4.17 \%+1.07(4 \%)+0.27(7.67 \%)=10.52 \%$
- Market Value of Equity $=11,042$ million BR ( $\$ 3,781$ million $)$
- Debt
- Cost of debt $=4.17 \%+4.00 \%+1.00 \%=9.17 \%$
- Market Value of Debt $=2,093$ million BR ( $\$ 717$ million)
- Cost of Capital

Cost of Capital $=10.52 \%(.84)+9.17 \%(1-.34)(0.16))=9.81 \%$
The book value of equity at Embraer is 3,350 million BR.
The book value of debt at Embraer is 1,953 million BR; Interest expense is 222 mil; Average maturity of debt $=4$ years
Estimated market value of debt $=222$ million (PV of annuity, 4 years, $9.17 \%$ ) + $\$ 1,953$ million $/ 1.0917^{4}=2,093$ million $B R$

## Estimating Cost of Capital: Ambev

- Equity
- Cost of Equity $=4.7 \%+0.87(4 \%)+0.41(7.87 \%)=11.41 \%$
- Market Value of Equity $=29,886$ million BR ( $\$ 9,508$ million)
- Debt
- Cost of debt $=4.7 \%+4.00 \%+2.00 \%=10.70 \%$
- Market Value of Debt $=5,808$ million BR $(\$ 1,848$ million $)$
- Cost of Capital

Cost of Capital $=11.41 \%(.837)+10.7 \%(1-.34)(0.163))=10.70 \%$
The book value of equity at Ambev is 4,209 million BR.
The book value of debt at Ambev is 5,980 million BR; Interest expense is 570 mil; Average maturity of debt $=3$ years
Estimated market value of debt $=570$ million (PV of annuity, 3 years, $10.7 \%$ ) + $\$ 5,980$ million $/ 1.107^{3}=5,808$ million BR

## Estimating Cost of Capital: Vale

- Equity
- Cost of Equity $=4.7 \%+1.04(4 \%)+0.37(7.87 \%)=11.77 \%$
- Market Value of Equity = 56,442 million BR (\$ 17,958 million)
- Debt
- Cost of debt $=4.7 \%+4.00 \%+2.50 \%=11.20 \%$
- Market Value of Debt $=14,484$ million BR ( $\$ 4,612$ million $)$
- Cost of Capital

Cost of Capital $=11.77 \%(.796)+11.2 \%(1-.34)(0.204))=10.88 \%$
The book value of equity at Vale is 15,937 million BR.
The book value of debt at Vale is 13,709 million BR; Interest expense is 1,989
mil; Average maturity of debt $=2$ years
Estimated market value of debt $=1,989$ million ( PV of annuity, 2 years, $11.2 \%$ ) + 13,709 million $/ 1.112^{2}=14,484$ million $B R$

## Estimating Cost of Capital: Petrobras

- Equity
- Cost of Equity $=4.70 \%+0.79(4 \%)+0.66(7.87 \%)=12.58 \%$
- Market Value of Equity $=85,218$ million BR $(\$ 27,114$ million $)$
- Debt
- Cost of debt $=4.7 \%+4.00 \%+1.00 \%=9.70 \%$
- Market Value of Debt $=39,367$ million BR $(\$ 12,537$ million $)$
- Cost of Capital

Cost of Capital $=12.58 \%(.684)+9.7 \%(1-.34)(0.316))=10.63 \%$
The book value of equity at Petrobras is 50.987 million BR.
The book value of debt at Petrobras is 42,248 million BR; Interest expense is
1,989 mil; Average maturity of debt $=4$ years
Estimated market value of debt $=3,195$ million (PV of annuity, 4 years, $9.7 \%$ ) + 42,248 million $/ 1.097^{4}=39,367$ million $B R$

## If you had to do it....Converting a Dollar Cost of Capital to a Nominal Real Cost of Capital Ambev

- Approach 1: Use a BR riskfree rate in all of the calculations above. For instance, if the BR riskfree rate was $12 \%$, the cost of capital would be computed as follows:
- Cost of Equity $=12 \%+0.87(4 \%)+0.41(7.87 \%)=18.71 \%$
- Cost of Debt $=12 \%+2 \%=14 \%$
- (This assumes the riskfree rate has no country risk premium embedded in it.)
- Approach 2: Use the differential inflation rate to estimate the cost of capital. For instance, if the inflation rate in BR is $8 \%$ and the inflation rate in the U.S.

$$
\begin{aligned}
\begin{array}{l}
\text { is } 2 \% \\
\text { st of capital }=
\end{array} & \left(1+\text { Cost of Capital }_{s}\right)\left[\frac{1+\text { Inflation }_{B R}}{1+\text { Inflation }_{S}}\right] \\
& =1.107(1.08 / 1.02)-1=17.21 \%
\end{aligned}
$$

# II. Valuing Control and Synergy Acquisition Valuation 

It is not what you buy but what you pay for it....

## Issues in Acquisition Valuation

- Acquisition valuations are complex, because the valuation often involved issues like synergy and control, which go beyond just valuing a target firm. It is important on the right sequence, including
- When should you consider synergy?
- Where does the method of payment enter the process.
- Can synergy be valued, and if so, how?

■ What is the value of control? How can you estimate the value?

## The Value of Control

- Control has value because you think that you can run a firm better than the incumbent management.
- Value of Control = Value of firm, run optimally - Value of firm, status quo
- The value of control should be inversely proportional to the perceived quality of that management and its capacity to maximize firm value.
- Value of control will be much greater for a poorly managed firm that operates at below optimum capacity than it is for a well managed firm. It should be negligible or firms which are operating at or close to their optimal value


## Price Enhancement versus Value Enhancement




## The Paths to Value Creation

- Using the DCF framework, there are four basic ways in which the value of a firm can be enhanced:
- The cash flows from existing assets to the firm can be increased, by either
- increasing after-tax earnings from assets in place or
- reducing reinvestment needs (net capital expenditures or working capital)
- The expected growth rate in these cash flows can be increased by either
- Increasing the rate of reinvestment in the firm
- Improving the return on capital on those reinvestments
- The length of the high growth period can be extended to allow for more years of high growth.
- The cost of capital can be reduced by
- Reducing the operating risk in investments/assets
- Changing the financial mix
- Changing the financing composition


## I. Ways of Increasing Cash Flows from Assets in Place



## II. Value Enhancement through Growth



## III. Building Competitive Advantages: Increase length of the growth period

Increase length of growth period


## Illustration: Valuing a brand name: Coca Cola

|  | Coca Cola | Generic Cola Company |
| :--- | :--- | :--- |
| AT Operating Margin | $\mathbf{1 8 . 5 6 \%}$ | $\mathbf{7 . 5 0 \%}$ |
| Sales/BV of Capital | 1.67 | 1.67 |
| ROC | $31.02 \%$ | $12.53 \%$ |
| Reinvestment Rate | $65.00 \%(19.35 \%)$ | $65.00 \%(47.90 \%)$ |
| Expected Growth | $20.16 \%$ | $8.15 \%$ |
| Length | 10 years | 10 yea |
| Cost of Equity | $12.33 \%$ | $12.33 \%$ |
| E/(D+E) | $97.65 \%$ | $97.65 \%$ |
| AT Cost of Debt | $4.16 \%$ | $4.16 \%$ |
| D/(D+E) | $2.35 \%$ | $2.35 \%$ |
| Cost of Capital | $12.13 \%$ | $12.13 \%$ |
| Value | $\$ 115$ | $\$ 13$ |

## Gauging Barriers to Entry

- Which of the following barriers to entry are most likely to work for Embraer?
- Brand Name
- Patents and Legal Protection
a Switching Costs
- Cost Advantages

■ What about for Ambev?

- Brand Name
- Patents and Legal Protection
- Switching Costs
- Cost Advantages


## Reducing Cost of Capital



## Embraer : Optimal Capital Structure

| Debt Ratio | Beta | Cost of Equity | Bond Rating | Interest rate on debt | Tax Rate | Cost of Debt (after-tax) | WACC | Firm Value (G) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0 \%$ | 0.95 | $10.05 \%$ | AAA | $8.92 \%$ | $34.00 \%$ | $5.89 \%$ | $10.05 \%$ | $\$ 3,577$ |
| $10 \%$ | 1.02 | $10.32 \%$ | AAA | $8.92 \%$ | $34.00 \%$ | $5.89 \%$ | $9.88 \%$ | $\$ 3,639$ |
| $20 \%$ | 1.11 | $10.67 \%$ | AA | $9.17 \%$ | $34.00 \%$ | $6.05 \%$ | $9.75 \%$ | $\$ 3,690$ |
| $30 \%$ | 1.22 | $11.12 \%$ | A | $9.97 \%$ | $34.00 \%$ | $6.58 \%$ | $9.76 \%$ | $\$ 3,686$ |
| $40 \%$ | 1.37 | $11.72 \%$ | A- | $10.17 \%$ | $34.00 \%$ | $6.71 \%$ | $9.72 \%$ | $\$ 3,703$ |
| $50 \%$ | 1.58 | $12.56 \%$ | B | $14.67 \%$ | $34.00 \%$ | $9.68 \%$ | $11.12 \%$ | $\$ 3,218$ |
| $60 \%$ | 1.89 | $13.81 \%$ | CCC | $18.17 \%$ | $34.00 \%$ | $11.99 \%$ | $12.72 \%$ | $\$ 2,799$ |
| $70 \%$ | 2.42 | $15.90 \%$ | CC | $19.67 \%$ | $34.00 \%$ | $12.98 \%$ | $13.86 \%$ | $\$ 2,562$ |
| $80 \%$ | 3.48 | $20.14 \%$ | CC | $19.67 \%$ | $33.63 \%$ | $13.05 \%$ | $14.47 \%$ | $\$ 2,450$ |
| $90 \%$ | 6.95 | $34.05 \%$ | CC | $19.67 \%$ | $29.90 \%$ | $13.79 \%$ | $15.81 \%$ | $\$ 2,236$ |

## Ambev: Optimal Capital Structure

| Debt Ratio | Beta | Cost of Equity | Bond Rating | Interest rate on debt | Tax Rate | Cost of Debt (after-tax) | WACC | Firm Value (G) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0 \%$ | 0.85 | $11.33 \%$ | AAA | $9.20 \%$ | $34.00 \%$ | $6.07 \%$ | $11.33 \%$ | $\$ 33,840$ |
| $10 \%$ | 0.91 | $11.58 \%$ | AA | $9.20 \%$ | $34.00 \%$ | $6.07 \%$ | $11.03 \%$ | $\$ 35,530$ |
| $20 \%$ | 0.99 | $11.89 \%$ | A- | $9.70 \%$ | $34.00 \%$ | $6.40 \%$ | $10.79 \%$ | $\$ 36,966$ |
| $30 \%$ | 1.09 | $12.29 \%$ | B- | $14.70 \%$ | $34.00 \%$ | $9.70 \%$ | $11.52 \%$ | $\$ 32,873$ |
| $40 \%$ | 1.23 | $12.83 \%$ | CC | $18.70 \%$ | $34.00 \%$ | $12.34 \%$ | $12.63 \%$ | $\$ 28,005$ |
| $50 \%$ | 1.49 | $13.87 \%$ | C | $20.70 \%$ | $25.24 \%$ | $15.48 \%$ | $14.67 \%$ | $\$ 21,930$ |
| $60 \%$ | 1.95 | $15.71 \%$ | D | $28.70 \%$ | $14.22 \%$ | $24.62 \%$ | $21.05 \%$ | $\$ 12,721$ |
| $70 \%$ | 2.59 | $18.30 \%$ | D | $28.70 \%$ | $12.19 \%$ | $25.20 \%$ | $23.13 \%$ | $\$ 1,098$ |
| $80 \%$ | 3.89 | $23.49 \%$ | D | $28.0 \%$ | $10.67 \%$ | $25.64 \%$ | $25.21 \%$ | $\$ 9,804$ |
| $90 \%$ | 7.78 | $39.06 \%$ | D | $28.70 \%$ | $9.48 \%$ | $25.98 \%$ | $27.29 \%$ | $\$ 8,748$ |

## Vale: Optimal Capital Structure

| Debt Ratio | Beta | Cost of Equity | Bond Rating | Interest rate on debt | Tax Rate | Cost of Debt (after-tax) | WACC | Firm Value (G) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0\% | 0.89 | 11.17\% | AAA | 9.20\% | 34.00\% | 6.07\% | 11.17\% | \$67,576 |
| 10\% | 0.95 | 11.43\% | AA | 9.20\% | 34.00\% | 6.07\% | 10.89\% | \$70,723 |
| 20\% | 1.04 | 11.76\% | A+ | 9.40\% | 34.00\% | 6.20\% | 10.65\% | \$73,819 |
| 30\% | 1.14 | 12.18\% | A- | 9.70\% | 34.00\% | 6.40\% | 10.44\% | \$76,537 |
| 40\% | 1.28 | 12.73\% | BB | 11.20\% | 34.00\% | 7.39\% | 10.60\% | \$74,451 |
| 50\% | 1.48 | 13.52\% | B- | 14.70\% | 34.00\% | 9.70\% | 11.61\% | \$63,058 |
| 60\% | 1.77 | 14.69\% | CC | 18.70\% | 34.00\% | 12.34\% | 13.28\% | \$50,122 |
| 70\% | 2.35 | 17.01\% | CC | 18.70\% | 29.68\% | 13.15\% | 14.31\% | \$44,418 |
| 80\% | 3.90 | 23.20\% | D | 28.70\% | 15.46\% | 24.26\% | 24.05\% | \$20,372 |
| 90\% | 7.79 | 38.79\% | D | 28.70\% | 13.74\% | 24.76\% | 26.16\% | \$18,043 |



## Value of stock in a publicly traded firm

- When a firm is badly managed, the market still assesses the probability that it will be run better in the future and attaches a value of control to the stock price today:

Value per share $=\frac{\text { Status Quo Value }+ \text { Probability of control change (Optimal - Status Quo Value) }}{\text { Number of shares outstanding }}$

- With voting shares and non-voting shares, a disproportionate share of the value of control will go to the voting shares. In the extreme scenario where non-voting shares are completely unprotected:

$$
\text { Value per non - voting share }=\frac{\text { Status Quo Value }}{\# \text { Voting Shares }+ \text { \# Non }- \text { voting shares }}
$$

Value per voting share $=$ Value of non - voting share $+\frac{\text { Probability of control change (Optimal }- \text { Status Quo Value) }}{\text { \# Voting Shares }}$

## Valuing Ambev voting and non-voting shares

- Status Quo Value $=\$ 5,304$ million* $3.14=16,655$ million BR
- Optimal Value $=\$ 6,277$ million $* 3.14=19,710$ million BR
- Number of shares
- Voting $=15.735$
- Non-voting $=22.801$
- Total $=38.536$

■ Value/ non-voting share $=16,655 / 38.536=433 \mathrm{BR} /$ share
■ Value/ voting share $=433+(19710-16655) / 15.735=626$ BR/share

## Operating Synergy

- The key to the existence of synergy is that the target firm controls a specialized resource that becomes more valuable if combined with the bidding firm's resources. The specialized resource will vary depending upon the merger:
- In horizontal mergers: economies of scale, which reduce costs, or from increased market power, which increases profit margins and sales. (Examples: Bank of America and Security Pacific, Chase and Chemical)
- In vertical integration: Primary source of synergy here comes from controlling the chain of production much more completely.
- In functional integration: When a firm with strengths in one functional area acquires another firm with strengths in a different functional area, the potential synergy gains arise from exploiting the strengths in these areas.


## Valuing operating synergy

(a) What form is the synergy expected to take? Will it reduce costs as a percentage of sales and increase profit margins (as is the case when there are economies of scale)? Will it increase future growth (as is the case when there is increased market power)? )
(b) When can the synergy be reasonably expected to start affecting cashflows? (Will the gains from synergy show up instantaneously after the takeover? If it will take time, when can the gains be expected to start showing up? )

## A procedure for valuing synergy

(1) the firms involved in the merger are valued independently, by discounting expected cash flows to each firm at the weighted average cost of capital for that firm.
(2) the value of the combined firm, with no synergy, is obtained by adding the values obtained for each firm in the first step.
(3) The effects of synergy are built into expected growth rates and cashflows, and the combined firm is re-valued with synergy.
Value of Synergy = Value of the combined firm, with synergy - Value of the combined firm, without synergy

## Synergy Effects in Valuation Inputs

| If synergy is | Valuation Inputs that will be affected are <br> Cost Savings <br> Operating Margin of combined firm will be greater <br> than the revenue-weighted operating margin of <br> individual firms. |
| :--- | :--- |
| More projects:Higher Reinvestment Rate (Retention) |  |
| Better projects: Higher Return on Capital (ROC) |  |
| Longer Growth Period |  |

## Financial Synergy

- Sources of Financial Synergy
- Diversification: Acquiring another firm as a way of reducing risk cannot create wealth for two publicly traded firms, with diversified stockholders, but it could create wealth for private firms or closely held publicly traded firms.
- Cash Slack: When a firm with significant excess cash acquires a firm, with great projects but insufficient capital, the combination can create value.
- Tax Benefits: The tax paid by two firms combined together may be lower than the taxes paid by them as individual firms. This can arise either because of an increase in depreciation, use of tax-loss carryforwards or (as in Brazil) an increase in tax-deductible interest on capital.
- Debt Capacity: It is possible that a combined firm can have higher debt capacity and/or a lower cost of debt that two firms standing alone. This, in turn, will increase firm value.


## Structure of the Transaction



## J.P. Morgan's estimate of annual operating synergies in Ambev/Labatt Merger



## J.P. Morgan's estimate of total synergies in Labatt/Ambev Merger

## Cost synergles

- Achievement of full synergies by 2008
- Synergies are gradually phased in over four years with $0 \%$, 30\%, 30\%, 40\% and 100\% being realized in 2004 through 2008 Realization of synergies requires cash outlays in the 20052007 period which are reflected in the net present value
- Capex synergies only begin in 2008
- Deoreased tax shield taken into acoount at full statutory tax rate, does not take into acoount potential additional upside as a result of lower effective historioal tax rate
- Synergies realized in Canada discounted at Labatt Canada's WACC (6.5\%) while synergies realized in Brazil disoounted at AmBev's WACC (12.4\%)

Revenue synergles

- Achievement of full synergies by 2008
- Synergies are gradually phased in over four years with 0\%, $30 \%, 30 \%, 40 \%$ and $100 \%$ being realized in 2004 through 2008
- Discounted at $14.4 \%$, refleoting AmBev"s WaCC plus an additional spread to refleot higher risk of realizing such synergies


## Interest on own oapital

- Tax benefits generated through increased interest on AmBev"s oapital payments due to AmBev's inoreaved shareholders" equity after aoquisition of Labatt
- Discounted at AmBev's oost of equity (13.6\%)


## Operational Evidence on Synergy

A stronger test of synergy is to evaluate whether merged firms improve their performance (profitability and growth), relative to their competitors, after takeovers.
o McKinsey and Co. examined 58 acquisition programs between 1972 and 1983 for evidence on two questions -
o Did the return on the amount invested in the acquisitions exceed the cost of capital?
o Did the acquisitions help the parent companies outperform the competition?
o They concluded that $\mathbf{2 8}$ of the $\mathbf{5 8}$ programs failed both tests, and $\mathbf{6}$ failed at least one test.
o KPMG in a more recent study of global acquisitions concludes that most mergers ( $>80 \%$ ) fail - the merged companies do worse than their peer group.
o Large number of acquisitions that are reversed within fairly short time periods. bout $20.2 \%$ of the acquisitions made between 1982 and 1986 were divested by 1988. In studies that have tracked acquisitions for longer time periods (ten years or more) the divestiture rate of acquisitions rises to almost $50 \%$.

## Labatt DCF valuation

- Labatt is the Canadian subsidiary of Interbrew and is a mature firm with sold brand names. It can be valued using a stable growth firm valuation model.
- Base Year inputs
- EBIT (1-t) $=\$ 411$ million
- Expected Growth Rate $=3 \%$
- Return on capital $=9 \%$
- Cost of capital $=7 \%$
- Valuation
- Reinvestment Rate $=\mathrm{g} / \mathrm{ROC}=3 / 9=33.33 \%$
- Value of Labatt $=411(1-.333) /(.07-.03)=\$ 6.85$ billion
- Ambev is paying for Labatt with 23.3 billion shares (valued at about $\$ 5.8$ billion) and is assuming $\$ 1.5$ billion in debt, resulting in a value for the firm of about $\$ 7.3$ billion.


## Who gets the benefits of synergy?



# III. Valuing Equity in Cyclical firms and firms with negative earnings : The Search for Normalcy 

Aswath Damodaran<br>http://www.damodaran.com

## Begin by analyzing why the earnings are not not normal

A Framework for Analyzing Companies with Negative or Abnormally Low Earnings


## 1. If the earnings decline or increase is temporary and will be quickly reversed... Normalize

- You can normalize earnings in three ways:
- Company's history: Averaging earnings or operating margins over time and estimating a normalized earning for the base year
- Industry average: You can apply the average operating margin for the industry to the company's revenues this year to get a normalized earnings.
- Normalized prices: If your company is a commodity company, you can normalize the price of the commodity across a cycle and apply it to the production in the current year.


## Aracruz in 2001: The Effect of Commodity Prices

Aracruz Celulose: Revenues, Profits and the Price of Paper


## Normalizing Earnings

|  | Revenues | Operating Income | Operating Margin | Price of pulp |
| :---: | :---: | :---: | :---: | :---: |
| 1991 | $\$ 131.19$ | $\$ 19.54$ | $14.89 \%$ | 100 |
| 1992 | $\$ 447.84$ | $\$ 125.45$ | $28.01 \%$ | 113.18 |
| 1993 | $\$ 301.93$ | $-\$ 34.86$ | $-11.55 \%$ | 102.89 |
| 1994 | $\$ 614.05$ | $\$ 131.34$ | $21.39 \%$ | 112.54 |
| 1995 | $\$ 703.00$ | $\$ 271.00$ | $38.55 \%$ | 98.71 |
| 1996 | $\$ 493.00$ | $\$ 15.00$ | $3.04 \%$ | 94.86 |
| 1997 | $\$ 536.83$ | $\$ 39.12$ | $7.29 \%$ | 92.93 |
| 1998 | $\$ 535.98$ | $-\$ 4.93$ | $-0.92 \%$ | 99.20 |
| 1999 | $\$ 989.75$ | $\$ 403.35$ | $40.75 \%$ | 102.09 |
| 2000 | $\$ 1,342.35$ | $\$ 665.85$ | $49.60 \%$ | 109.39 |
| Normalized 2000 | $\$ 1,342.35$ | $\$ 324.59$ | $24.18 \%$ |  |
| Normalized 2000 | $\$ 1,258.78$ | $\$ 582.28$ |  | 102.58 |



Aswath Damodaran

## 2. If the earnings are negative because the firm is early in its life cycle...

- When operating income is negative or margins are expected to change over time, we use a three step process to estimate growth:
- Estimate growth rates in revenues over time
- Use historical revenue growth to get estimates of revenue growth in the near future
- Decrease the growth rate as the firm becomes larger
- Keep track of absolute revenues to make sure that the growth is feasible
- Estimate expected operating margins each year
- Set a target margin that the firm will move towards
- Adjust the current margin towards the target margin
- Estimate the capital that needs to be invested to generate revenue growth and expected margins
- Estimate a sales to capital ratio that you will use to generate reinvestment needs each year.


## Discounted Cash Flow Valuation: High Growth with Negative Earnings




## 3. If earnings are negative because the firm has structural/ leverage problems...

- Survival Scenario: The firm survives and solves its structural problem (brings down its financial leverage). In this scenario, margins improve and the debt ratio returns to a sustainable level.
- Failure Scenario: The firm does not solve its structural problems or fails to make debt payments, leading to default and liquidation.



## The Going Concern Assumption

- Traditional valuation techniques are built on the assumption of a going concern, I.e., a firm that has continuing operations and there is no significant threat to these operations.
- In discounted cashflow valuation, this going concern assumption finds its place most prominently in the terminal value calculation, which usually is based upon an infinite life and ever-growing cashflows.
- In relative valuation, this going concern assumption often shows up implicitly because a firm is valued based upon how other firms - most of which are healthy - are priced by the market today.
■ When there is a significant likelihood that a firm will not survive the immediate future (next few years), traditional valuation models may yield an over-optimistic estimate of value.


## DCF Valuation + Distress Value

- A DCF valuation values a firm as a going concern. If there is a significant likelihood of the firm failing before it reaches stable growth and if the assets will then be sold for a value less than the present value of the expected cashflows (a distress sale value), DCF valuations will understate the value of the firm.
■ Value of Equity= DCF value of equity (1-Probability of distress) + Distress sale value of equity (Probability of distress)


## Bond Price to estimate probability of distress

- Global Crossing has a $12 \%$ coupon bond with 8 years to maturity trading at $\$ 653$. To estimate the probability of default (with a treasury bond rate of $5 \%$ used as the riskfree rate):

$$
653=\sum_{t=1}^{t=8} \frac{120\left(1-\pi_{\text {Distress }}\right)^{t}}{(1.05)^{t}}+\frac{1000\left(1-\pi_{\text {Distress }}\right)^{8}}{(1.05)^{\mathrm{N}}}
$$

- Solving for the probability of bankruptcy, we get
- With a 10 -year bond, it is a process of trial and error to estimate this value. The solver function in excel accomplishes the same in far less time.

$$
\pi_{\text {Distress }}=\text { Annual probability of default }=13.53 \%
$$

- To estimate the cumulative probability of distress over 10 years:

■ Cumulative probability of surviving 10 years $=(1-.1353)^{10}=23.37 \%$
■ Cumulative probability of distress over 10 years $=1-.2337=.7663$ or $76.63 \%$

## Valuing Global Crossing with Distress

- Probability of distress
- Cumulative probability of distress $=76.63 \%$
- Distress sale value of equity
- Book value of capital $=\$ 14,531$ million
- Distress sale value $=25 \%$ of book value $=.25 * 14531=\$ 3,633$ million
- Book value of debt $=\$ 7,647$ million
- Distress sale value of equity $=\$ 0$

■ Distress adjusted value of equity

- Value of Global Crossing $=\$ 3.22(1-.7663)+\$ 0.00(.7663)=\$ 0.75$


# Real Options: Fact and Fantasy 

Aswath Damodaran

## Underlying Theme: Searching for an Elusive Premium

- Traditional discounted cashflow models under estimate the value of investments, where there are options embedded in the investments to
- Delay or defer making the investment (delay)
- Adjust or alter production schedules as price changes (flexibility)
- Expand into new markets or products at later stages in the process, based upon observing favorable outcomes at the early stages (expansion)
- Stop production or abandon investments if the outcomes are unfavorable at early stages (abandonment)
- Put another way, real option advocates believe that you should be paying a premium on discounted cashflow value estimates.


## Three Basic Questions

- When is there a real option embedded in a decision or an asset?
- When does that real option have significant economic value?
- Can that value be estimated using an option pricing model?


## When is there an option embedded in an action?

- An option provides the holder with the right to buy or sell a specified quantity of an underlying asset at a fixed price (called a strike price or an exercise price) at or before the expiration date of the option.
- There has to be a clearly defined underlying asset whose value changes over time in unpredictable ways.
- The payoffs on this asset (real option) have to be contingent on an specified event occurring within a finite period.


## Payoff Diagram on a Call



## Example 1: Product Patent as an Option



## Example 2: Undeveloped Oil Reserve as an option



## Example 3: Expansion of existing project as an option



## When does the option have significant economic value?

- For an option to have significant economic value, there has to be a restriction on competition in the event of the contingency. In a perfectly competitive product market, no contingency, no matter how positive, will generate positive net present value.
- At the limit, real options are most valuable when you have exclusivity - you and only you can take advantage of the contingency. They become less valuable as the barriers to competition become less steep.


## Exclusivity: Putting Real Options to the Test

- Product Options: Patent on a drug
- Patents restrict competitors from developing similar products
- Patents do not restrict competitors from developing other products to treat the same disease.
- Natural Resource options: An undeveloped oil reserve or gold mine.
- Natural resource reserves are limited.
- It takes time and resources to develop new reserves
- Growth Options: Expansion into a new product or market
- Barriers may range from strong (exclusive licenses granted by the government - as in telecom businesses) to weaker (brand name, knowledge of the market) to weakest (first mover).


## Determinants of option value

- Variables Relating to Underlying Asset
- Value of Underlying Asset; as this value increases, the right to buy at a fixed price (calls) will become more valuable and the right to sell at a fixed price (puts) will become less valuable.
- Variance in that value; as the variance increases, both calls and puts will become more valuable because all options have limited downside and depend upon price volatility for upside.
- Expected dividends on the asset, which are likely to reduce the price appreciation component of the asset, reducing the value of calls and increasing the value of puts.
- Variables Relating to Option
- Strike Price of Options; the right to buy (sell) at a fixed price becomes more (less) valuable at a lower price.
- Life of the Option; both calls and puts benefit from a longer life.

■ Level of Interest Rates; as rates increase, the right to buy (sell) at a fixed price in the future becomes more (less) valuable.

## When can you use option pricing models to value real options?

- All option pricing models rest on two foundations.
- The first is the notion of a replicating portfolio where you combine the underlying asset and borrowing/lending to create a portfolio that has the same cashflows as the option.
- The second is arbitrage. Since both the option and the replicating portfolio have the same cashflows, they should trade at the same value.
- As a result, option pricing models work best when
- The underlying asset is traded - this yield not only observable prices and volatility as inputs to option pricing models but allows for the possibility of creating replicating portfolios
- An active marketplace exists for the option itself.
- When option pricing models are used to value real assets where neither replication nor arbitrage are usually feasible, we have to accept the fact that
- The value estimates that emerge will be far more imprecise.
- The value can deviate much more dramatically from market price because of the difficulty of arbitrage.


## Illustrating Replication: The Binomial Option Pricing Model



## The Black Scholes Model

## Value of call $=\mathrm{SN}\left(\mathrm{d}_{1}\right)-\mathrm{K}^{-\mathrm{rt}} \mathrm{N}\left(\mathrm{d}_{2}\right)$

where,

$$
\mathrm{d}_{1}=\frac{\ln \left(\frac{\mathrm{S}}{\mathrm{~K}}\right)+\left(\mathrm{r}+\frac{\sigma^{2}}{2}\right) \mathrm{t}}{\sigma \sqrt{\mathrm{t}}}
$$

- $\mathrm{d}_{2}=\mathrm{d}_{1}-\sigma \sqrt{ } \mathrm{t}$

■ The replicating portfolio is embedded in the Black-Scholes model. To replicate this call, you would need to

- Buy $\mathrm{N}(\mathrm{d} 1)$ shares of stock; $\mathrm{N}(\mathrm{d} 1)$ is called the option delta
- Borrow $\mathrm{K} \mathrm{e}^{-\mathrm{rt}} \mathrm{N}\left(\mathrm{d}_{2}\right)$


## The Normal Distribution



| $d$ | $N(d)$ | $d$ | $N(d)$ | $d$ | $N(d)$ |
| :---: | ---: | ---: | ---: | ---: | ---: |
| -3.00 | 0.0013 | -1.00 | 0.1587 | 1.05 | 0.8531 |
| -2.95 | 0.0016 | -0.95 | 0.1711 | 1.10 | 0.8643 |
| -2.90 | 0.0019 | -0.90 | 0.1841 | 1.15 | 0.8749 |
| -2.85 | 0.0022 | -0.85 | 0.1977 | 1.20 | 0.8849 |
| -2.80 | 0.0026 | -0.80 | 0.2119 | 1.25 | 0.8944 |
| -2.75 | 0.0030 | -0.75 | 0.2266 | 1.30 | 0.9032 |
| -2.70 | 0.0035 | -0.70 | 0.2420 | 1.35 | 0.9115 |
| -2.65 | 0.0040 | -0.65 | 0.2578 | 1.40 | 0.9192 |
| -2.60 | 0.0047 | -0.60 | 0.2743 | 1.45 | 0.9265 |
| -2.55 | 0.0054 | -0.55 | 0.2912 | 1.50 | 0.9332 |
| -2.50 | 0.0062 | -0.50 | 0.3085 | 1.55 | 0.9394 |
| -2.45 | 0.0071 | -0.45 | 0.3264 | 1.60 | 0.9452 |
| -2.40 | 0.0082 | -0.40 | 0.3446 | 1.65 | 0.9505 |
| -2.35 | 0.0094 | -0.35 | 0.3632 | 1.70 | 0.9554 |
| -2.30 | 0.0107 | -0.30 | 0.3821 | 1.75 | 0.9599 |
| -2.25 | 0.0122 | -0.25 | 0.4013 | 1.80 | 0.9641 |
| -2.20 | 0.0139 | -0.20 | 0.4207 | 1.85 | 0.9678 |
| -2.15 | 0.0158 | -0.15 | 0.4404 | 1.90 | 0.9713 |
| -2.10 | 0.0179 | -0.10 | 0.4602 | 1.95 | 0.9744 |
| -2.05 | 0.0202 | -0.05 | 0.4801 | 2.00 | 0.9772 |
| -2.00 | 0.0228 | 0.00 | 0.5000 | 2.05 | 0.9798 |
| -1.95 | 0.0256 | 0.05 | 0.5199 | 2.10 | 0.9821 |
| -1.90 | 0.0287 | 0.10 | 0.5398 | 2.15 | 0.9842 |
| -1.85 | 0.0322 | 0.15 | 0.5596 | 2.20 | 0.9861 |
| -1.80 | 0.0359 | 0.20 | 0.5793 | 2.25 | 0.9878 |
| -1.75 | 0.0401 | 0.25 | 0.5987 | 2.30 | 0.9893 |
| -1.70 | 0.0446 | 0.30 | 0.6179 | 2.35 | 0.9906 |
| -1.65 | 0.0495 | 0.35 | 0.6368 | 2.40 | 0.9918 |
| -1.60 | 0.0548 | 0.40 | 0.6554 | 2.45 | 0.9929 |
| -1.55 | 0.0606 | 0.45 | 0.6736 | 2.50 | 0.9938 |
| -1.50 | 0.0668 | 0.50 | 0.6915 | 2.55 | 0.9946 |
| -1.45 | 0.0735 | 0.55 | 0.7088 | 2.60 | 0.9953 |
| -1.40 | 0.0808 | 0.60 | 0.7257 | 2.65 | 0.9960 |
| -1.35 | 0.0885 | 0.65 | 0.7422 | 2.70 | 0.9965 |
| -1.30 | 0.0968 | 0.70 | 0.7580 | 2.75 | 0.9970 |
| -1.25 | 0.1056 | 0.75 | 0.7734 | 2.80 | 0.9974 |
| -1.20 | 0.1151 | 0.80 | 0.7881 | 2.85 | 0.9978 |
| -1.15 | 0.1251 | 0.85 | 0.8023 | 2.90 | 0.9981 |
| -1.10 | 0.1357 | 0.90 | 0.8159 | 2.95 | 0.9984 |
| -1.05 | 0.1469 | 0.95 | 0.8289 | 3.00 | 0.9987 |
| -1.00 | 0.1587 | 1.00 | 0.8413 |  |  |
|  |  |  |  |  |  |

## 1. Obtaining Inputs for Patent Valuation

| Input | Estimation Process |
| :--- | :--- |
| 1. Value of the Underlying Asset | - Present Value of Cash Inflows from taking project <br> now <br> - This will be noisy, but that adds value. |
| 2. Variance in value of underlying asset | - Variance in cash flows of similar assets or firms <br> - Variance in present value from capital budgeting <br> simulation. |
| 3. Exercise Price on Option | - Option is exercised when investment is made. <br> - Cost of making investment on the project ; assumed <br> to be constant in present value dollars. |
| 4. Expiration of the Option | - Life of the patent |
| 5. Dividend Yield | - Cost of delay <br> - Each year of delay translates into one less year of <br> value-creating cashflows <br> Annual cost of delay $=\frac{1}{n}$ |

## Valuing a Product Patent as an option: Avonex

- Biogen, a bio-technology firm, has a patent on Avonex, a drug to treat multiple sclerosis, for the next 17 years, and it plans to produce and sell the drug by itself. The key inputs on the drug are as follows:
PV of Cash Flows from Introducing the Drug Now $=\mathrm{S}=\$ 3.422$ billion PV of Cost of Developing Drug for Commercial Use $=\mathrm{K}=\$ 2.875$ billion Patent Life $=t=17$ years Riskless Rate $=r=6.7 \%$ (17-year T.Bond rate) Variance in Expected Present Values $=\mathbf{s}^{2}=0.224$ (Industry average firm variance for bio-tech firms)
Expected Cost of Delay $=\mathrm{y}=1 / 17=5.89 \%$

$$
\mathrm{d} 1=1.1362 \quad \mathrm{~N}(\mathrm{~d} 1)=0.8720
$$

$$
\mathrm{d} 2=-0.8512 \quad \mathrm{~N}(\mathrm{~d} 2)=0.2076
$$

Call Value $=3,422 \exp ^{(-0.0589)(17)}(0.8720)-2,875\left(\exp ^{(-0.067)(17)}(0.2076)=\$\right.$ 907 million

## 2. Valuing an Oil Reserve

- Consider an offshore oil property with an estimated oil reserve of 50 million barrels of oil, where the present value of the development cost is $\$ 12$ per barrel and the development lag is two years.
- The firm has the rights to exploit this reserve for the next twenty years and the marginal value per barrel of oil is $\$ 12$ per barrel currently (Price per barrel-marginal cost per barrel).
- Once developed, the net production revenue each year will be $5 \%$ of the value of the reserves.
- The riskless rate is $8 \%$ and the variance in $\ln$ (oil prices) is 0.03 .


## Valuing an oil reserve as a real option

- Current Value of the asset $=\mathrm{S}=$ Value of the developed reserve discounted back the length of the development lag at the dividend yield $=\$ 12 * 50 /(1.05)^{2}=\$ 544.22$
- (If development is started today, the oil will not be available for sale until two years from now. The estimated opportunity cost of this delay is the lost production revenue over the delay period. Hence, the discounting of the reserve back at the dividend yield)
■ Exercise Price $=$ Present Value of development cost $=\$ 12 * 50=$ $\$ 600$ million
- Time to expiration on the option $=20$ years

■ Variance in the value of the underlying asset $=0.03$

- Riskless rate $=8 \%$

■ Dividend Yield $=$ Net production revenue $/$ Value of reserve $=5 \%$

## Extending the option pricing approach to value natural resource firms

Input to model
Value of underlying asset

Exercise Price

Corresponding input for valuing firm
Value of cumulated estimated reserves of the resource owned by the firm, discounted back at the dividend yield for the development lag.
Estimated cumulated cost of developing estimated reserves
Time to expiration on option Average relinquishment period across all reserves owned by firm (if known) or estimate of when reserves will be exhausted, given current production rates.
Riskless rate corresponding to life of the option
Variance in the price of the natural resource
Estimated annual net production revenue as percentage of value of the reserve.

## Valuing Gulf Oil

- Gulf Oil was the target of a takeover in early 1984 at $\$ 70$ per share (It had 165.30 million shares outstanding, and total debt of $\$ 9.9$ billion).
- It had estimated reserves of 3038 million barrels of oil and the average cost of developing these reserves was estimated to be $\$ 10$ a barrel in present value dollars (The development lag is approximately two years).
- The average relinquishment life of the reserves is 12 years.
- The price of oil was $\$ 22.38$ per barrel, and the production cost, taxes and royalties were estimated at $\$ 7$ per barrel.
- The bond rate at the time of the analysis was $9.00 \%$.
- Gulf was expected to have net production revenues each year of approximately $5 \%$ of the value of the developed reserves. The variance in oil prices is 0.03 .


## Valuing Undeveloped Reserves

- Inputs for valuing undeveloped reserves
- Value of underlying asset $=$ Value of estimated reserves discounted back for period of development lag= $3038 *(\$ 22.38-\$ 7) / 1.05^{2}=\$ 42,380.44$
- Exercise price $=$ Estimated development cost of reserves $=3038 * \$ 10=\mathbf{\$ 3 0 , 3 8 0}$ million
- Time to expiration $=$ Average length of relinquishment option $=\mathbf{1 2}$ years
- Variance in value of asset $=$ Variance in oil prices $=\mathbf{0 . 0 3}$
- Riskless interest rate $=\mathbf{9 \%}$
- Dividend yield $=$ Net production revenue/ Value of developed reserves $=\mathbf{5 \%}$
- Based upon these inputs, the Black-Scholes model provides the following value for the call:

$$
\begin{array}{ll}
\mathrm{d} 1=1.6548 & \mathrm{~N}(\mathrm{~d} 1)=0.9510 \\
\mathrm{~d} 2=1.0548 & \mathrm{~N}(\mathrm{~d} 2)=0.8542
\end{array}
$$

■ Call Value $=42,380.44 \exp ^{(-0.05)(12)}(0.9510)-30,380\left(\exp ^{(-0.09)(12)}(0.8542)=\$\right.$ 13,306 million

## Valuing Gulf Oil

- In addition, Gulf Oil had free cashflows to the firm from its oil and gas production of $\$ 915$ million from already developed reserves and these cashflows are likely to continue for ten years (the remaining lifetime of developed reserves).
- The present value of these developed reserves, discounted at the weighted average cost of capital of $12.5 \%$, yields:
- Value of already developed reserves $=915\left(1-1.125^{-10}\right) / .125=\$ 5065.83$
- Adding the value of the developed and undeveloped reserves

Value of undeveloped reserves<br>Value of production in place<br>Total value of firm<br>Less Outstanding Debt<br>Value of Equity<br>Value per share

$=\$ 13,306$ million

$$
\begin{aligned}
& =\$ 5,066 \text { million } \\
& =\$ 18,372 \text { million } \\
& =\$ 9,900 \text { million } \\
& =\$ 8,472 \text { million }
\end{aligned}
$$

$$
=\$ 8,472 / 165.3=\$ 51.25
$$

## 3. An Example of an Expansion Option

- Ambev is considering introducing a soft drink to the U.S. market. The drink will initially be introduced only in the metropolitan areas of the U.S. and the cost of this "limited introduction" is \$ 500 million.
- A financial analysis of the cash flows from this investment suggests that the present value of the cash flows from this investment to Ambev will be only $\$ 400$ million. Thus, by itself, the new investment has a negative NPV of $\mathbf{\$ 1 0 0}$ million.
■ If the initial introduction works out well, Ambev could go ahead with a full-scale introduction to the entire market with an additional investment of \$1 billion any time over the next 5 years. While the current expectation is that the cash flows from having this investment is only $\$ 750$ million, there is considerable uncertainty about both the potential for the drink, leading to significant variance in this estimate.


## Valuing the Expansion Option

- Value of the Underlying Asset ( S ) = PV of Cash Flows from Expansion to entire U.S. market, if done now $=\$ 750$ Million
- Strike Price $(K)=$ Cost of Expansion into entire U.S market $=\$ 1000$ Million
- We estimate the standard deviation in the estimate of the project value by using the annualized standard deviation in firm value of publicly traded firms in the beverage markets, which is approximately $34.25 \%$.
- Standard Deviation in Underlying Asset's Value = 34.25\%
- Time to expiration $=$ Period for which expansion option applies $=5$ years

Call Value= \$234 Million

## Opportunities and not Options...



## Key Tests for Real Options

- Is there an option embedded in this asset/ decision?
- Can you identify the underlying asset?
- Can you specify the contigency under which you will get payoff?
- Is there exclusivity?
- If yes, there is option value.
- If no, there is none.
- If in between, you have to scale value.
- Can you use an option pricing model to value the real option?
- Is the underlying asset traded?
- Can the option be bought and sold?
- Is the cost of exercising the option known and clear?

