Accelerated Mathematics 2

This is the second in a sequence of mathematics courses designed to prepare students to take AB or BC Advanced Placement Calculus. It includes right triangle trigonometry; exponential, logarithmic, and higher degree polynomial functions; matrices; linear programming; vertex-edge graphs; conic sections; planes and spheres; population means, standard deviations, and normal distributions. (*Prerequisite: Accelerated Math 1 or Mathematics 2.*)

Instruction and assessment should include the appropriate use of manipulatives and technology. Topics should be represented in multiple ways, such as concrete/pictorial, verbal/written, numeric/data-based, graphical, and symbolic methods. Concepts should be introduced and used, where appropriate, in the context of realistic phenomena.

ALGEBRA

Students will investigate exponential, logarithmic, and polynomial functions of higher degree; understand matrices and use them to solve problems; and solve linear programming problems in two variables.

MA2A1. Students will explore exponential functions.

a. Extend properties of exponents to include all integer exponents.
b. Investigate and explain characteristics of exponential functions, including domain and range, asymptotes, zeros, intercepts, intervals of increase and decrease, rates of change, and end behavior.
c. Graph functions as transformations of $f(x) = a^x$.
d. Solve simple exponential equations and inequalities analytically, graphically, and by using appropriate technology.
e. Understand and use basic exponential functions as models of real phenomena.
f. Understand and recognize geometric sequences as exponential functions with domains that are sets of whole numbers.
g. Interpret the constant ratio in a geometric sequence as the base of the associated exponential function.

MA2A2. Students will explore inverses of functions.

a. Discuss the characteristics of functions and their inverses, including one-to-oneness, domain, and range.
b. Determine inverses of linear, quadratic, and power functions and functions of the form $f(x) = a/x$, including the use of restricted domains.
c. Explore the graphs of functions and their inverses.
d. Use composition to verify that functions are inverses of each other.

MA2A3. Students will analyze graphs of polynomial functions of higher degree.

a. Graph simple polynomial functions as translations of the function $f(x) = ax^n$.
b. Understand the effects of the following on the graph of a polynomial function: degree, lead coefficient, and multiplicity of real zeros.
c. Determine whether a polynomial function has symmetry and whether it is even, odd, or neither.

d. Investigate and explain characteristics of polynomial functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior.

MA2A4. Students will explore logarithmic functions as inverses of exponential functions.

a. Define and understand the properties of n^{th} roots.
b. Extend properties of exponents to include rational exponents.
c. Define logarithmic functions as inverses of exponential functions.
d. Understand and use properties of logarithms by extending laws of exponents.
e. Investigate and explain characteristics of exponential and logarithmic functions including domain and range, asymptotes, zeros, intercepts, intervals of increase and decrease, and rate of change.
f. Graph functions as transformations of $f(x) = a^x$, $f(x) = \log_a x$, $f(x) = e^x$, $f(x) = \ln x$.
g. Explore real phenomena related to exponential and logarithmic functions including half-life and doubling time.

MA2A5. Students will solve a variety of equations and inequalities.

a. Find real and complex roots of higher degree polynomial equations using the factor theorem, remainder theorem, rational root theorem, and fundamental theorem of algebra, incorporating complex and radical conjugates.
b. Solve polynomial, exponential, and logarithmic equations analytically, graphically, and using appropriate technology.
c. Solve polynomial, exponential, and logarithmic inequalities analytically, graphically, and using appropriate technology. Represent solution sets of inequalities using interval notation.
d. Solve a variety of types of equations by appropriate means choosing among mental calculation, pencil and paper, or appropriate technology.

MA2A6. Students will perform basic operations with matrices.

a. Add, subtract, multiply, and invert matrices, when possible, choosing appropriate methods, including technology.
b. Find the inverses of two-by-two matrices using pencil and paper, and find inverses of larger matrices using technology.
c. Examine the properties of matrices, contrasting them with properties of real numbers.

MA2A7. Students will use matrices to formulate and solve problems.

a. Represent a system of linear equations as a matrix equation.
b. Solve matrix equations using inverse matrices.
c. Represent and solve realistic problems using systems of linear equations.

MA2A8. Students will solve linear programming problems in two variables.

a. Solve systems of inequalities in two variables, showing the solutions graphically.
b. Represent and solve realistic problems using linear programming.

MA2A9. Students will understand and apply matrix representations of vertex-edge graphs.

a. Use graphs to represent realistic situations.
b. Use matrices to represent graphs, and solve problems that can be represented by graphs.

GEOMETRY

Students will explore right triangles and right triangular trigonometry. They will understand and apply properties of conic sections, planes, and spheres.

MA2G1. Students will identify and use special right triangles.

a. Determine the lengths of sides of 30°-60°-90° triangles.
b. Determine the lengths of sides of 45°-45°-90° triangles.

MA2G2. Students will define and apply sine, cosine, and tangent ratios to right triangles.

a. Discover the relationship of the trigonometric ratios for similar triangles.
b. Explain the relationship between the trigonometric ratios of complementary angles.
c. Solve application problems using the trigonometric ratios.

MA2G3. Students will investigate the relationships between lines and circles.

a. Find equations of circles.
b. Graph a circle given an equation in general form.
c. Find the equation of a tangent line to a circle at a given point.
d. Solve a system of equations involving a circle and a line.
e. Solve a system of equations involving two circles.

MA2G4. Students will recognize, analyze, and graph the equations of the conic sections (parabolas, circles, ellipses, and hyperbolas).

a. Convert equations of conics by completing the square.
b. Graph conic sections, identifying fundamental characteristics.
c. Write equations of conic sections given appropriate information.

MA2G5. Students will investigate planes and spheres.

a. Plot the point (x, y, z) and understand it as a vertex of a rectangular prism.
b. Apply the distance formula in 3-space.
c. Recognize and understand equations of planes and spheres.

DATA ANALYSIS AND PROBABILITY

Students will make informal inferences about means and standard deviations. Students will use a normal distribution to calculate probabilities. Students will organize, represent, investigate, interpret, and make inferences from both observational studies and experiments.

MA2D1. Using sample data, students will make informal inferences about population means and standard deviations.

a. Pose a question and collect sample data from at least two different populations.
b. Understand and calculate the means and standard deviations of sets of data.
c. Use means and standard deviations to compare data sets.
d. Compare the means and standard deviations of random samples with the corresponding population parameters. Observe that the different sample means vary from one sample to the next. Observe that the distribution of the sample means has less variability than the population distribution.

MA2D2. Students will create probability histograms of discrete random variables, using both experimental and theoretical probabilities.

MA2D3. Students will solve problems involving probabilities by interpreting a normal distribution as a probability histogram for a continuous random variable (*z*-scores are used for a general normal distribution).

a. Determine intervals about the mean that include a given percent of data.
b. Determine the probability that a given value falls within a specified interval.
c. Estimate how many items in a population fall within a specified interval.

MA2D4. Students will understand the differences between experimental and observational studies by posing questions and collecting, analyzing, and interpreting data.

Terms/Symbols: exponential function, logarithmic function, $\log_a x$, n^{th} root, $\sqrt[n]{\ }$, asymptote, half-life, asymptote, geometric sequence, inverse of a function, one-to-one function, composition of functions, f^{-1}, degree of polynomial, multiplicity of roots, matrix, inverse matrix, A^{-1}, vertex-edge graph, sine, cosine, tangent, trigonometric ratio, complementary angles, trigonometry, conic sections, parabola, ellipse, hyperbola, 3-space, inference, population mean, standard deviation, discrete random variable, normal distribution, discrete random variable, continuous random variable, z-score, experimental and observational studies

Process Standards

The following process standards are essential to mastering each of the mathematics content standards. They emphasize critical dimensions of the mathematical proficiency that all students need.

MA2P1. Students will solve problems (using appropriate technology).

a. Build new mathematical knowledge through problem solving.
b. Solve problems that arise in mathematics and in other contexts.
c. Apply and adapt a variety of appropriate strategies to solve problems.
d. Monitor and reflect on the process of mathematical problem solving.

MA2P2. Students will reason and evaluate mathematical arguments.

a. Recognize reasoning and proof as fundamental aspects of mathematics.
b. Make and investigate mathematical conjectures.
c. Develop and evaluate mathematical arguments and proofs.
d. Select and use various types of reasoning and methods of proof.

MA2P3. Students will communicate mathematically.

a. Organize and consolidate their mathematical thinking through communication.
b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
c. Analyze and evaluate the mathematical thinking and strategies of others.
d. Use the language of mathematics to express mathematical ideas precisely.

MA2P4. Students will make connections among mathematical ideas and to other disciplines.

a. Recognize and use connections among mathematical ideas.
b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
c. Recognize and apply mathematics in contexts outside of mathematics.

MA2P5. Students will represent mathematics in multiple ways.

a. Create and use representations to organize, record, and communicate mathematical ideas.
b. Select, apply, and translate among mathematical representations to solve problems.
c. Use representations to model and interpret physical, social, and mathematical phenomena.

Reading Standard Comment

After the elementary years, students are seriously engaged in reading for learning. This process sweeps across all disciplinary domains, extending even to the area of personal learning. Students encounter a variety of informational as well as fictional texts, and they experience text in all genres and modes of discourse. In the study of various disciplines of learning (language arts, mathematics, science, social studies), students must learn through reading the communities of discourse of each of those disciplines. Each subject has its own specific vocabulary, and for students to excel in all subjects, they must learn the specific vocabulary of those subject areas *in context.*

Beginning with the middle grades years, students begin to self-select reading materials based on personal interests established through classroom learning. Students become curious about science, mathematics, history, and literature as they form contexts for those subjects related to their personal and classroom experiences. As students explore academic areas through reading, they develop favorite subjects and become confident in their verbal discourse about those subjects.

Reading across curriculum content develops both academic and personal interests in students. As students read, they develop both content and contextual vocabulary. They also build good habits for reading, researching, and learning. The Reading Across the Curriculum standard focuses on the academic and personal skills students acquire as they read in all areas of learning.

MRC. Students will enhance reading in all curriculum areas by:

a. Reading in all curriculum areas
  - Read a minimum of 25 grade-level appropriate books per year from a variety of subject disciplines and participate in discussions related to curricular learning in all areas
  - Read both informational and fictional texts in a variety of genres and modes of discourse
  - Read technical texts related to various subject areas

b. Discussing books
  - Discuss messages and themes from books in all subject areas.
  - Respond to a variety of texts in multiple modes of discourse.
  - Relate messages and themes from one subject area to messages and themes in another area.
  - Evaluate the merit of texts in every subject discipline.
  - Examine author's purpose in writing.
  - Recognize the features of disciplinary texts.

c. Building vocabulary knowledge
  - Demonstrate an understanding of contextual vocabulary in various subjects.
  - Use content vocabulary in writing and speaking.
  - Explore understanding of new words found in subject area texts.

d. Establishing context

- Explore life experiences related to subject area content.
- Discuss in both writing and speaking how certain words are subject area related.
- Determine strategies for finding content and contextual meaning for unknown words.