CS 70 Spring 2005 — Discrete Mathematics for CS — Clancy/Wagner — HW 1

# Due Tuesday, January 25

**Note concerning due dates:** Homework submissions after this one will be due on Thursdays at 3:30, so that solutions may be distributed in class. Late homework will not be accepted.

**Coverage:** This assignment involves topics from the January 18 and 20 lectures and from chapter 1 of Rosen.

**Administrative details of preparing and submitting homework submissions:** In CS 70 this semester, you will be submitting all homework solutions online. (You will probably recognize the submission system from CS 61ABC.) To get ready for online submission, do the following.

1. Get an EECS instructional computer account. We will distribute account forms at class on Tuesday, January 18 and Thursday, January 20; thereafter, you may pick one up at 385 Soda.
2. Register with the grading system. See the CS 70 home page for instructions on how to do this.

We will accept only unformatted text files or PDF files for homework submission. In particular, we will *not* accept files produced by Microsoft Word unless they are converted to PDF, and the submission program will be configured to reject files whose names end in ".doc". (We will also not accept paper homework submissions.)

We encourage you to learn the TeX document processing language to produce your homework submissions. The command

```
latex hw1.tex
dvipdf hw1.dvi hw1.pdf
```

given on the instructional computing systems, will produce a PDF file from TeX source. Make sure you check carefully that the PDF comes out correctly before submitting, and correct any errors. We suggest using

```
acroread hw1.pdf
```

Each homework assignment will be accompanied with a TeX template for you to fill in your answers. The templates will be accessible from the CS 70 home page.

Whatever the format of your submission, it should start with the following information:

Your full name
Your login name

The name of the homework assignment (e.g. hw3)
Your section number
Your list of partners

To submit your answers to this homework assignment, create a directory named `hw1`, copy your answer file to that directory, `cd` to that directory, and then give the command

```
submit hw1
```

**Homework exercises:**

**1. (6 pts.) Getting started**

(a) Read the course web page. Write on your homework, immediately after your name, the following sentence: "I understand and will comply with the academic integrity policy."

(b) What is David Wagner's favorite number? The answer is found on the course newsgroup, `ucb.class.cs70`. Look for the post from David Wagner titled "The answer to question 1(b)," and write down the answer you find there. Instructions on how to access the newsgroup may be found on the course web page.

(Why are we having you do this? The class newsgroup is your best source for recent announcements, clarifications on homeworks, and related matters, and we want you to be familiar with how to read the newsgroup.)

**2. (12 pts.) Inference rules**

For each of the following, define proposition symbols for each simple proposition in the argument (for example, $P$ = "I will ace this homework"). Then write out the logical form of the argument. If the argument form corresponds to a known inference rule, say which it is. If not, show that the proof is correct using truth tables.

(a) I will ace this homework and I will have fun doing it. Therefore, I will ace this homework.

(b) It is hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees today. Therefore, the pollution is dangerous.

(c) Tina will join a startup next year. Therefore, Tina will join a startup next year or she will be unemployed.

(d) If I work all night on this homework, I will answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, I will understand the material.

**3. (12 pts.) Quantifier crazy**

Recall that $\mathbf{N} = \{0, 1, \ldots\}$ denotes the set of natural numbers, and $\mathbf{Z} = \{\ldots, -1, 0, 1, \ldots\}$ denotes the set of integers.

(a) Define $P(n)$ by

$$P(n) = \forall m \in \mathbf{N}.\ m < n \implies \neg(\exists k \in \mathbf{N}.\ n = mk \wedge k < n).$$

Concisely, for which numbers $n \in \mathbf{N}$ is $P(n)$ true?

(b) Re-write the following in a way that removes all negations ("$\neg$, $\neq$") but remains equivalent.

$$\forall i.\ \neg\forall j.\ \neg\exists k.\ (\neg\exists\ell.\ f(i,j) \neq g(k,\ell)).$$

(c) Prove or disprove: $\forall m \in \mathbf{Z}.\ \exists n \in \mathbf{Z}.\ m \geq n$.

(d) Prove or disprove: $\exists m \in \mathbf{Z}.\ \forall n \in \mathbf{Z}.\ m \geq n$.

**4. (6 pts.) Chess**

Alice and Bob are playing a game of chess, with Alice to move first. If $x_1, \ldots, x_n$ represents a sequence of possible moves (i.e., first Alice will make move $x_1$, then Bob will make move $x_2$, and so on), we let $W(x_1, \ldots, x_n)$ denote the proposition that, after this sequence of moves is completed, Bob is checkmated.

(a) State using quantifier notation the proposition that Alice can force a checkmate on her second move, no matter how Bob plays.

(b) Alice has many possibilities to choose from on her first move, and wants to find one that lets her force a checkmate on her second move. State using quantifier notation the proposition that $x_1$ is *not* such a move.

**5. (8 pts.) Knights and Knaves**

Joan is either a knight or a knave. Knights always tell the truth, and only the truth; knaves always tell falsehoods, and only falsehoods. Someone asks Joan, "Are you a knight?" She replies, "If I am a knight then I'll eat my hat."

(a) Must Joan eat her hat?

(b) Let's set this up as problem in propositional logic. Introduce the following propositions:

$$P = \text{"Joan is a knight"}$$
$$Q = \text{"Joan will eat her hat".}$$

Translate what we're given into propositional logic, i.e., re-write the premises in terms of these propositions.

(c) Using proof by enumeration, prove that your answer from part (1) follows from the premises you wrote in part (2). (No inference rules allowed.)

**6. (6 pts.) Is it true?**

For each claim below, prove or disprove the claim.

(a) Every positive integer can be expressed as the sum of two perfect squares. (A perfect square is the square of an integer. 0 may be used in the sum.)

(b) For all rational numbers $a$ and $b$, $a^b$ is also rational.