Note on the BIRD ACS Reference Frames

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O. Montenbruck, E. Gill, Th. Terzibaschian

Abstract

This note describes the relation between various coordinate frames employed in the joint analysis of attitude measurements from the BIRD star camera (SC) and position data from the BIRD on-board navigation system (ONS). It is suggested to adopt the true-of-date (TOD) system as the nominal inertial reference system for use in the attitude control system (ACS). Differences between the nominal system and the actual realizations by the star camera and the ONS software are discussed.

List of Symbols

$\boldsymbol{N}$	Nutation matrix
$\boldsymbol{P}$	Precession matrix
$\boldsymbol{R}_x$	Elementary rotation matrix describing the transformation of a vector into a system that arises from a right handed rotation around the *x*-axis
$\boldsymbol{R}_y$	Elementary rotation matrix describing the transformation of a vector into a system that arises from a right handed rotation around the *y*-axis
$\boldsymbol{R}_z$	Elementary rotation matrix describing the transformation of a vector into a system that arises from a right handed rotation around the *z*-axis
t	Time
Θ	Earth rotation matrix
$\bar{\Theta}$	True Sidereal Time
Θ	Mean Sidereal Time
Π	Polar motion matrix
$w_\oplus$	Earth angular velocity

For clarity, the explicit representations of the elementary rotation matrices are given below:

$$\boldsymbol{R}_x(f) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & +\cos f & +\sin f \\ 0 & -\sin f & +\cos f \end{pmatrix} \quad \boldsymbol{R}_y(f) = \begin{pmatrix} +\cos f & 0 & -\sin f \\ 0 & 1 & 0 \\ +\sin f & 0 & +\cos f \end{pmatrix} \quad \boldsymbol{R}_z(f) = \begin{pmatrix} +\cos f & +\sin f & 0 \\ -\sin f & +\cos f & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

1 Definition of Reference Frames

The reference frames employed in the discussion of the BIRD attitude control system are specified in the sequel. In addition to well established systems like ICRF, MOD, TOD, and WGS84, various other systems are introduced and designated below that arise from intermediate or approximate transformations. An overview is given in Fig. 1.

1.1 International Celestial Reference Frame (ICRF) and Earth Mean Equator and Equainox of Date (EME2000)

The International Celestial Reference Frame (ICRF) provides a realization of the International Celestial Reference System (ICRS) using a catalog of distant VLBI radio sources [1]. In the optical regime, the realization is provided by the Hipparcos catalog, which is also used as basis of the ASTRO-5 star camera [2]. The ICRF matches the EME2000 system, which has previously been employed in planetary ephemerides and star catalogs, on a level of some tens of milli-arcseconds. The EME2000 system is aligned with the *mean* equator and equinox at the reference epoch J2000 (1.5 Jan. 2000) [3]. Its *z*-axis is parallel to the mean rotation axis of the Earth and the *x*-axis points into the direction of the mean vernal equinox, i.e. the ascending node of the Earth's mean orbital plane on the mean equator, at

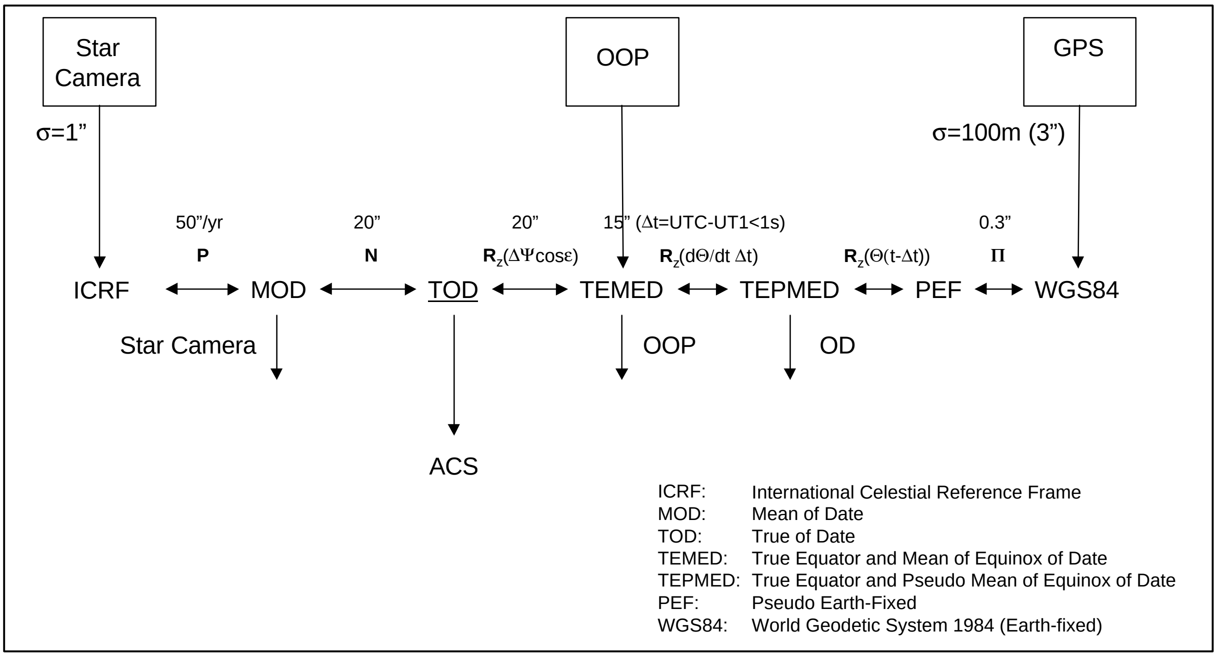

Fig. 1 Summary of reference systems

the fixed epoch J2000. Here, the term "mean" indicates that oscillations of the equator and ecliptic with periods of 2 weeks to 18.6 years (i.e. nutation) have been averaged.

1.2 Mean Equator and Equinox of Date (Mean of Date, MOD)

The MOD system is aligned with the *mean* equator and equinox at the time of consideration (i.e. the time of a measurement or the epoch of a state vector). It's *z*-axis is parallel to the mean rotation axis of the Earth and the *x*-axis points into the direction of the mean vernal equinox at the concerned time. Again, the term "mean" indicates that only secular (or long-periodic) changes in the orientation of the Earth's rotation axis (or its equator) and the ecliptic are considered. Coordinates in the MOD and ICRF system are related by the transformation

$$\boldsymbol{x}_{\mathrm{MOD}} = \boldsymbol{P}(t)\boldsymbol{x}_{\mathrm{ICRF}} = \boldsymbol{R}_z(-z)\boldsymbol{R}_y(J)\boldsymbol{R}_z(-z)\boldsymbol{x}_{\mathrm{ICRF}} \quad , \tag{1}$$

where the angles z, J, and z are expressed as third order polynomials in time in the adopted IAU 1976 theory of precession [1]. While the obliquity of the ecliptic changes by as little as 47" per century, the luni-solar precession results in a regression of the vernal equinox by 5029" per century, or about 50" per year.

1.3 True Equator and Equinox of Date (True of Date, TOD)

The TOD system is aligned with the *true* equator and equinox at the time of consideration. It's *z*-axis is parallel to the instantaneous rotation axis of the Earth and the *x*-axis points into the direction of the true vernal equinox at the concerned time. Coordinates in the true of date system are related to the mean of date coordinates by the expression

$$\boldsymbol{x}_{\mathrm{TOD}} = \boldsymbol{N}(t)\boldsymbol{x}_{\mathrm{MOD}} = \boldsymbol{R}_x(-e-\Delta e)\boldsymbol{R}_z(-\Delta y)\boldsymbol{R}_x(+e)\boldsymbol{x}_{\mathrm{MOD}} \quad , \tag{2}$$

where e denotes the mean obliquity of the ecliptic. The angles Δy (nutation in longitude) and Δe (nutation in obliquity) are expressed as harmonic series with a total of 106 terms down to 0.0001". The leading terms are caused by the precession of the Moon's orbital plane and exhibit a period of 18.6 years. Their amplitude amounts to 17" (longitude) and 9" (obliquity).

1.4 Pseudo Earth Fixed (PEF)

The pseudo Earth-fixed system is aligned with the instantaneous Earth rotation axis and the Greenwich meridian. It is related to the TOD system by a single *z*-axis rotation about an angle that is equal to the true sidereal time *Q*:

$$\boldsymbol{x}_{\text{PEF}} = \Theta(t)\boldsymbol{x}_{\text{TOD}} = \boldsymbol{R}_z(\Theta(t))\boldsymbol{x}_{\text{TOD}} \quad . \tag{3}$$

The true sidereal time

$$\Theta(t) = \overline{\Theta}(t) + \Delta\Psi \cos e \quad . \tag{4}$$

is expressed as the sum of the mean sidereal time $\overline{\Theta}$ and the "equation of the equinoxes", which describes the angle between the directions to the mean and true equinox. The mean sidereal time is related to the UT1 time scale by a conventional third order polynomial. It is noted that UT1 differs from the internationally adopted Coordinated Universal Time (UTC) by up to 0.9s. The difference of both time scales is monitored and published by the International Earth Rotation Service (IERS).

1.5 World Geodetic System 1984 (WGS84)

The WGS84 system is a commonly adopted realization of an Earth-fixed reference system from a set of well established ground station coordinates. Its axes are aligned with the adopted International Reference Pole and Meridian that are fixed with respect to the surface of the Earth. The difference between a truly Earth-fixed system and the pseudo Earth-fixed system results from the motion of the Earth's rotation axis with respect to the Earth's surface. This is known as polar motion and expressed by the transformation

$$\boldsymbol{x}_{\text{WGS84}} = \Pi(t)\boldsymbol{x}_{\text{PEF}} = \boldsymbol{R}_y(-x_p)\boldsymbol{R}_z(-y_p)\boldsymbol{x}_{\text{PEF}} \quad , \tag{5}$$

where x_p and y_p denote the coordinates of the rotation axis with respect to the adopted reference pole. The difference between Earth-fixed (WGS84) coordinates and pseudo Earth-fixed coordinates amounts to roughly 0.3" or about 10m near the surface of the Earth.

1.6 True Equator and Mean Equinox of Date (TEMED)

The TEMED system is a less common reference system that is aligned with the *true* equator and the *mean* equinox at the time of consideration. It's *z*-axis is parallel to the instantaneous rotation axis of the Earth but the *x*-axis points into the direction of the *mean* vernal equinox at the concerned time. The TEMED system arises implicitly in an orbit determination program, when true sidereal time is substituted by mean sidereal time in the transformation of measurements from the Earth-fixed frame to inertial frame. Accordingly, the TOD and TEMED systems are related to each other by the transformation

$$\boldsymbol{x}_{\text{TOD}} = \boldsymbol{R}_z(\Delta y \cos e)\boldsymbol{x}_{\text{TEMED}} \quad . \tag{6}$$

The difference between mean and true sidereal time (or likewise the difference between the mean and true equinox) amounts to roughly 15"=1s at maximum and varies with the longitude of the Moon's ascending node on the ecliptic. The TEMED system is e.g. applied in the generation of two-line elements by NORAD [3], whereas a rigorous true of date system is applied in GSOC's flight dynamics system.

1.7 True Equator and Pseudo Mean Equinox of Date (TEPMED)

The TEPMED system is introduced here to denote a system which is aligned with the true Earth equator but exhibits an offset from the mean equinox which corresponds to the time difference between UT1 and UTC. The TEMED system arises implicitly in an orbit determination program, when true sidereal time is substituted by mean sidereal time and at the same time the UT1-UTC time difference is neglected. Accordingly, it is related to the TEMED system by the transformation

$$\boldsymbol{x}_{\text{TEMED}} = \boldsymbol{R}_z(w_\oplus(\text{UT1} - \text{UTC}))\boldsymbol{x}_{\text{TEPMED}} \quad , \tag{7}$$

where $w_\oplus$ is the Earth rotation rate.

2. Reference Systems Employed in ACS Subsystems

2.1 Star Camera

The BIRD star camera attitude determination is based on an input catalogue referred to the Earth mean equator and equinox of J2000. A linear approximation of the precession angles with an accuracy of better than 1" is used to convert the resulting attitude measurements into the mean of date system [4].

2.2 On-board Navigation System

The on-board navigation system processes GPS measurements that are obtained in the Earth-fixed WGS84 reference frame. For the sake of simplicity, an inertial system aligned with the true equator and the pseudo mean equinox of date (TEPMED) is chosen for the numerical trajectory integration. This saves the need for on-board knowledge of the UT1-UTC difference as well as the need to apply a nutation correction. Since the geocoding of the BIRD image data involves a back-transformation to the Earth-fixed system, the adjusted and predicted WGS84 positions are unaffected by the particular choice of the inertial reference system. For attitude control purposes, on the other hand, consistency with the star camera data must be ensured within the specified accuracy limits.

In back-up cases, the ONS processes two-line elements referred to either the TEMED (NORAD element sets) or TOD system (GSOC element sets). In view of the moderate accuracy of these data, no distinction between both systems need to be applied.

3. Recommendation

Taking into account the present implementation of the star camera and ACS software, it is suggested to adopt the true of date system (TOD) as the nominal inertial reference system for ACS computations. It is a well defined and commonly adopted inertial system, which very well matches the needs of the BIRD ACS sysytem. The proper delivery of TOD related data is left to the respective subsystems (SC and ONS). Within the overall error budget of about 0.5' (30"), the currently applied reference systems of the star camera (MOD) and the on-board navigation system (TEPMED) are already consistent with this convention.

References

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